QCD Critical Point : The Race is on

Rajiv V. Gavai
T. I. F. R., Mumbai, India

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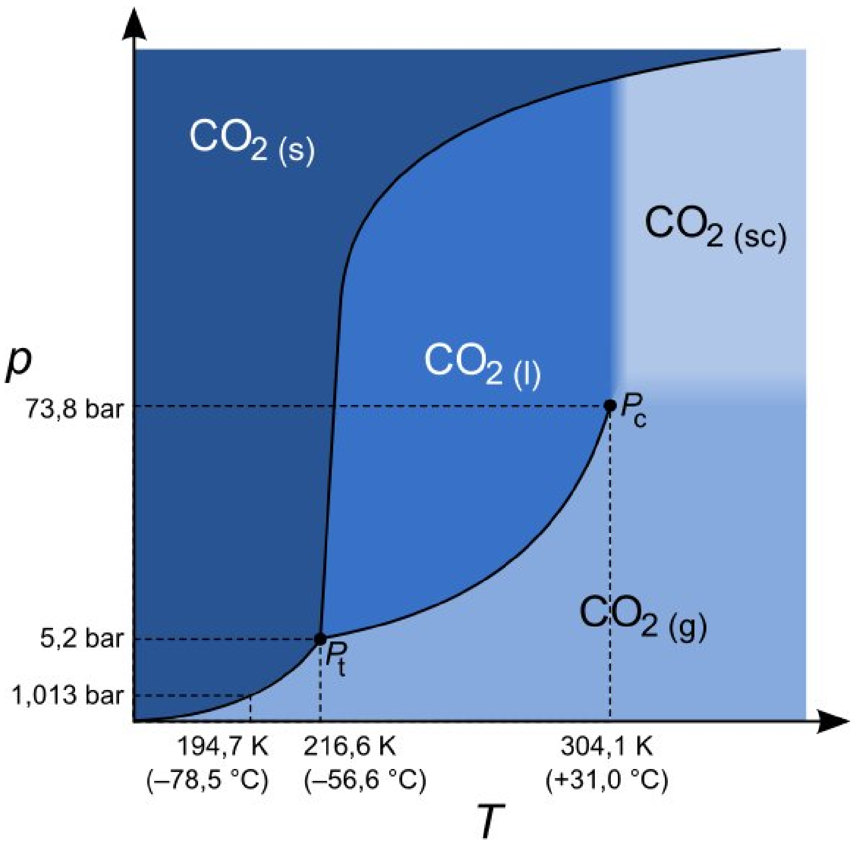

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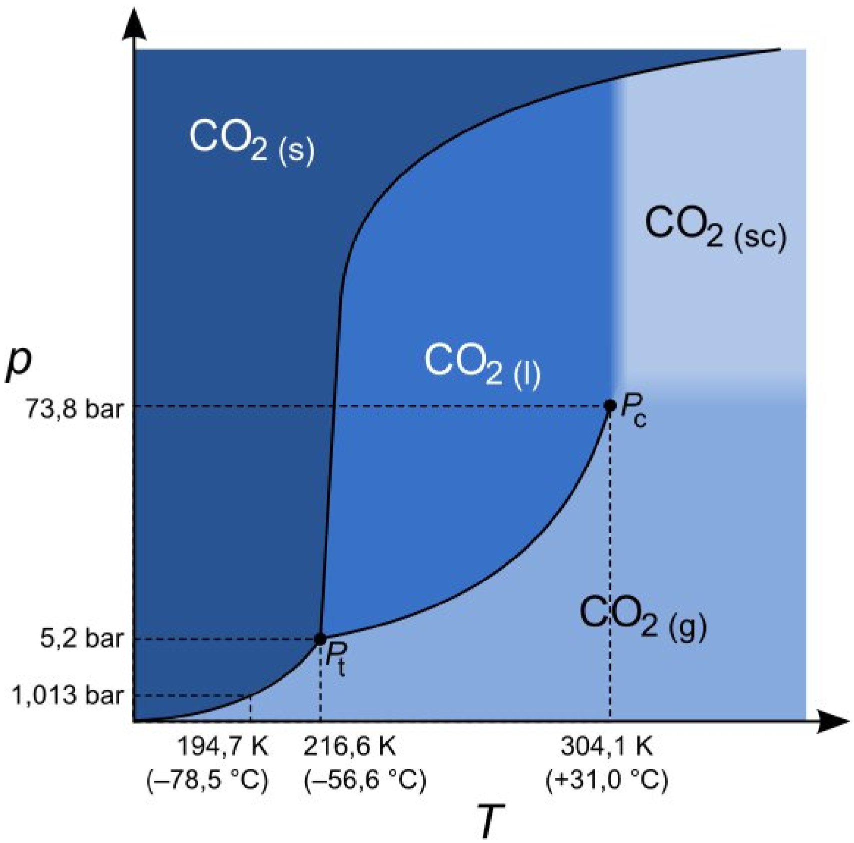

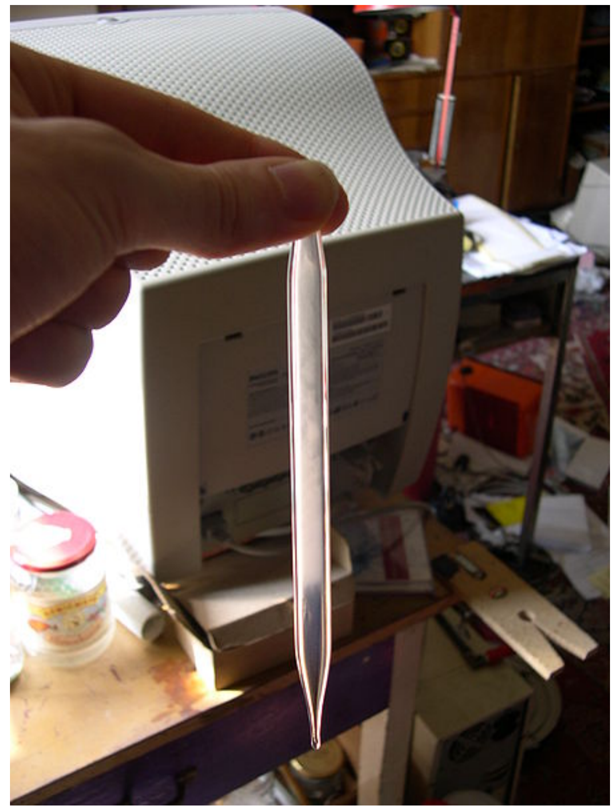

From Wikipedia

Introduction

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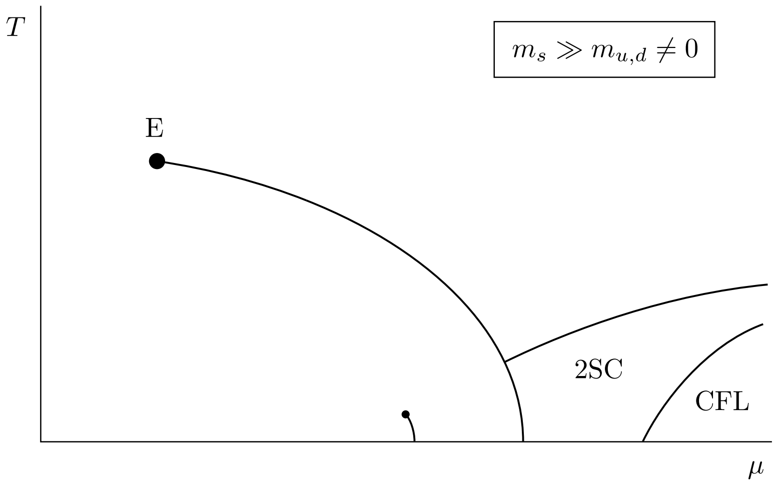

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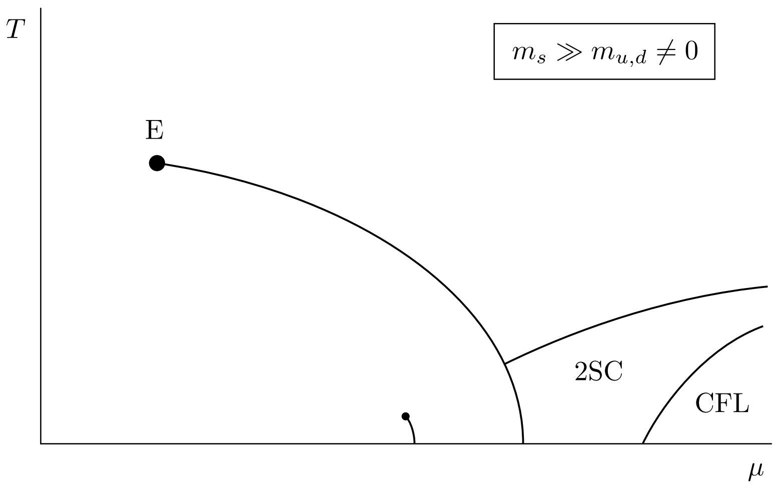

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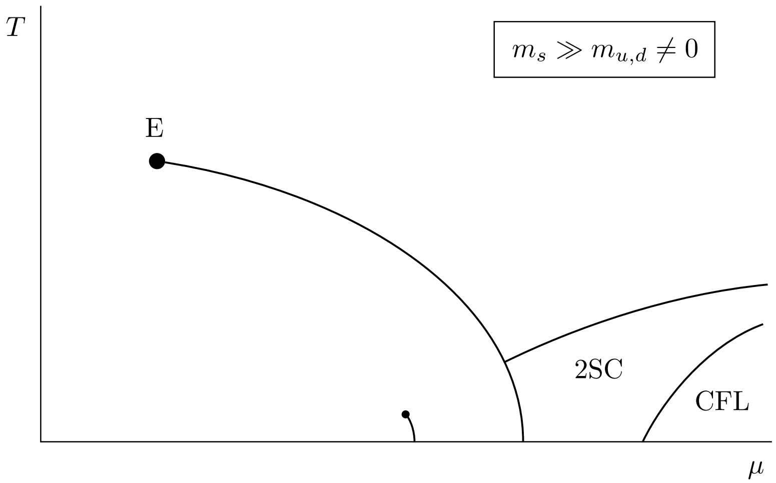

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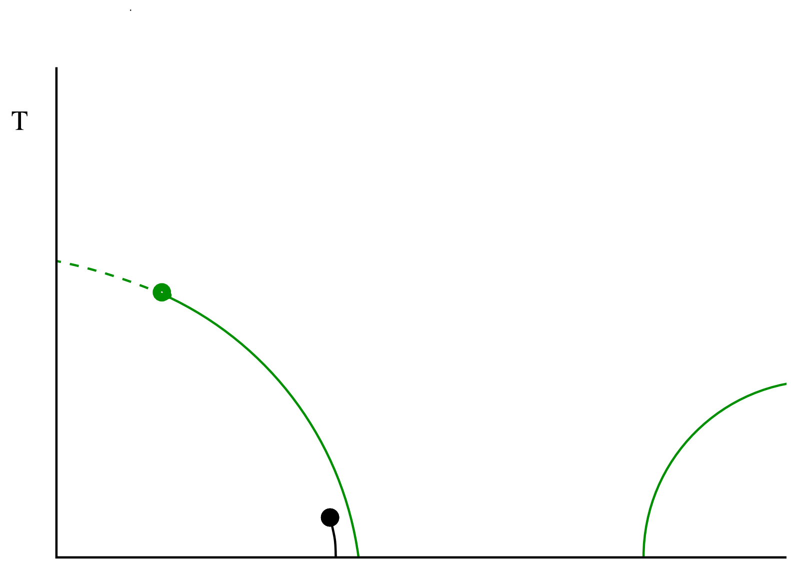

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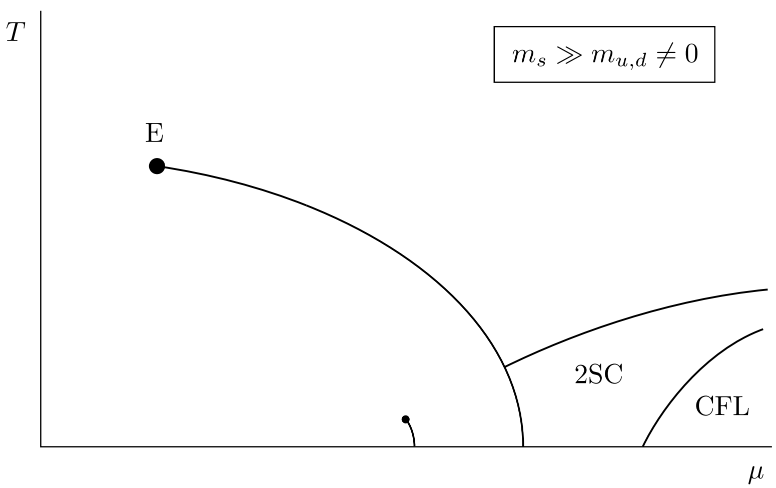

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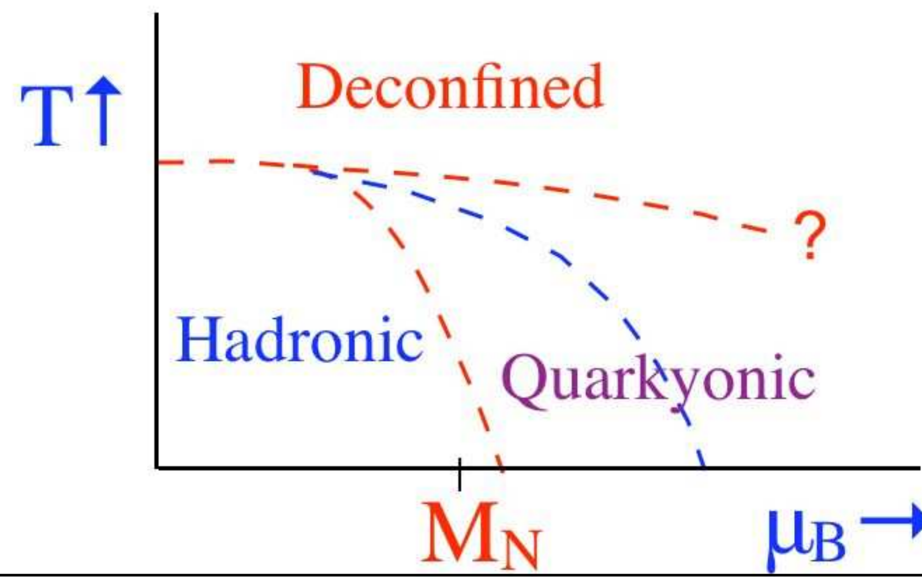

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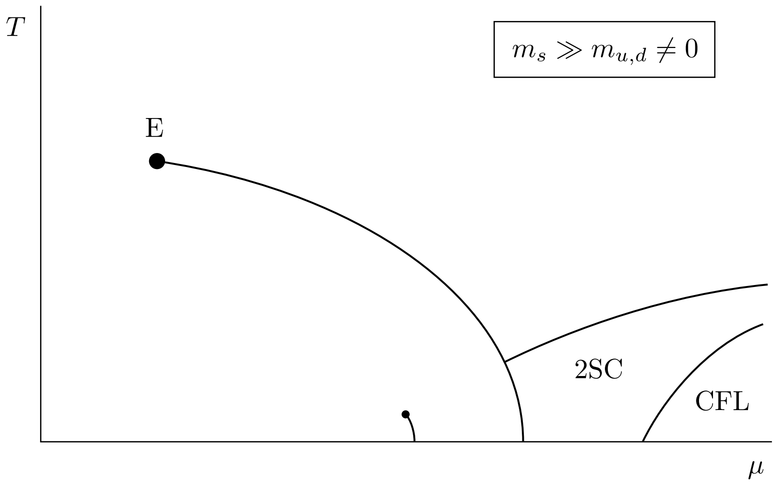

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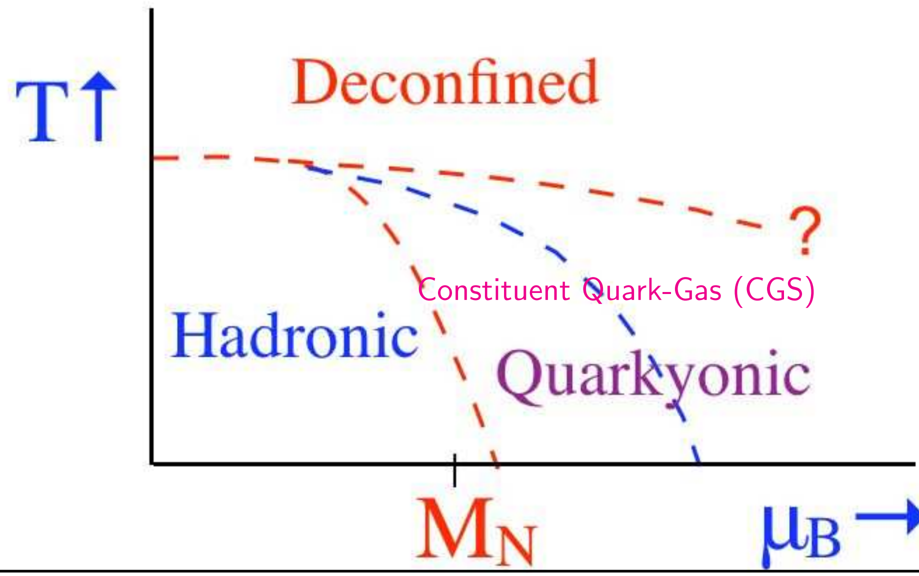

The Race is ON

- Between the theorists, to claim a patch on the QCD phase diagram,
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♡ In the true "consumer" spirit, careful evaluation of each of the claims is necessary.

Lattice QCD Results

- QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics.
- Completely parameter-free : Λ_{QCD} and quark masses from hadron spectrum.
- The Transition Temperature T_c, the Equation of State, Flavour Correlations (C_{BS}) and the Wróblewski Parameter λ_s are some examples for Heavy Ion Physics. (Gavai-Gupta, PRD 2006 & PRD 2002)

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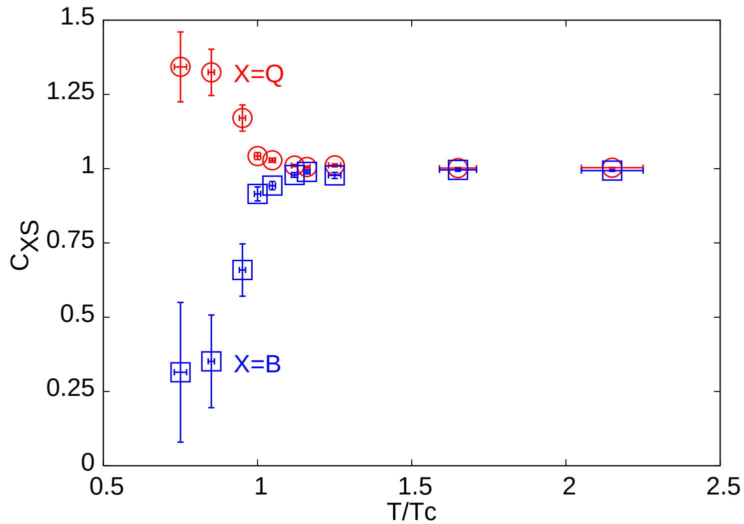

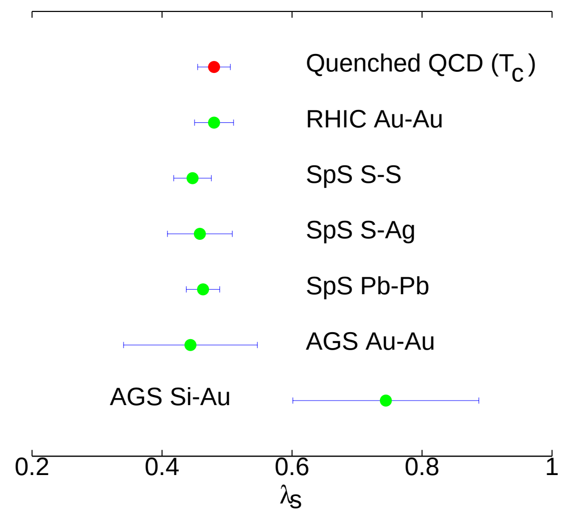

The $\mu \neq 0$ problem : Quark Type

- Mostly staggered quarks used in these simulations. Broken flavour and spin symmetry on lattice $\Longrightarrow N_f = 2$ simulations may be fine in $a \to 0$ limit but 3 or 2 $+1$ problematic.

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- Unfortunately breaks chiral symmetry ! (Banerjee, Gavai & Sharma PRD 2008; PoS (Lattice 2008); PRD 2009)

- Desperately needed : Formalism with Continuum-like (flavour & spin) symmetries for quarks at nonzero μ and T.

The $\mu \neq 0$ problem : The Measure

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_f \mathrm{Det} \ M(m_f, \mu_f) \quad ,$$

and the thermal expectation value of an observable $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle = \frac{\int DU \exp(-S_G) \ \mathcal{O} \prod_f \mathrm{Det} \ M(m_f, \mu_f)}{\mathcal{Z}}.$$

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However, det M is a complex number for any $\mu \neq 0$: The Phase/sign problem

Lattice Approaches

Several Approaches proposed in the past two decades : None as satisfactory as the usual $T \neq 0$ simulations. Still scope for a good/great idea !

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- Canonical Ensemble (K. -F. Liu, IJMP B16 (2002) 2017, S. Kratochvila and P. de Forcrand, Pos LAT2005 (2006) 167.)
- Complex Langevin (G. Aarts and I. O. Stamatescu, arXiv:0809.5227 and its references for earlier work).

How Do We Do This Expansion?

Canonical definitions yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, $\lambda_s \ldots$)

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!} \left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!} \left(\frac{\mu_d}{T}\right)^{n_d} \tag{1}$$

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- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence obtained from these using $\sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}T^2}}$ or $\left(n!\frac{\chi_B^{(2)}}{\chi_B^{(n+2)}T^2}\right)^{1/n}$. We use both and terms up to 8th order in μ.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (Gavai-Gupta '05, '09) use up to 8^{th} order (B-RBC so far has up to 6^{th} order). Need 20 inversions of $(D+m)$ on $\sim$ 500 vectors for a single measurement.

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CRAY X1 of I L G T I , T I F R, Mumbai

Our Simulations & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.

- $m_\rho/T_c = 5.4 \pm 0.2$ and $m_\pi/m_\rho = 0.31 \pm 0.01$ (MILC)

- Earlier Lattice : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005)

- Lattice used : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, PRD 2009). Needed to determine β_c. Our result ($\beta_c = 5.425(5)$) well bracketed by MILC for $m/T_c = 0.075$ and 0.15.

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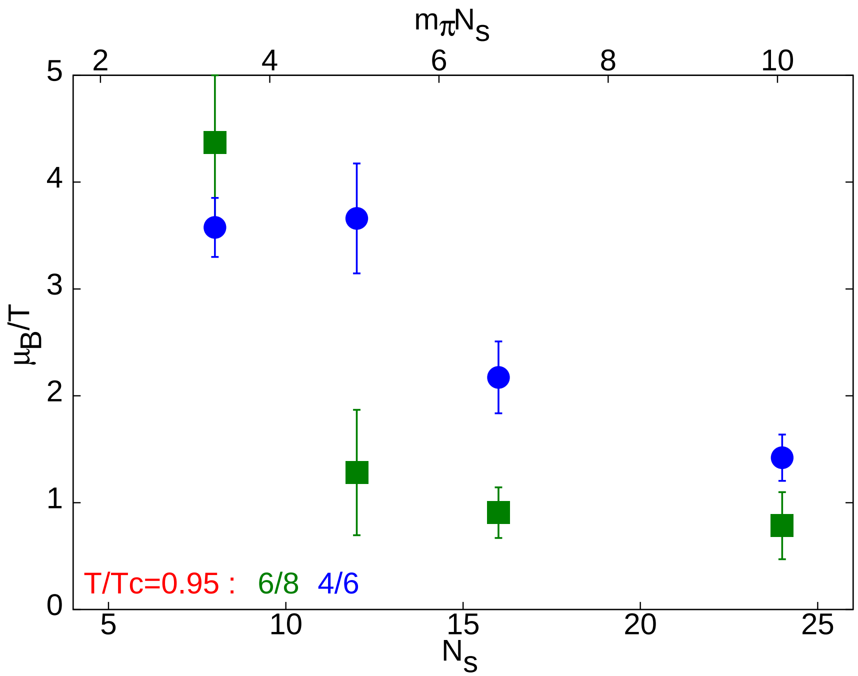
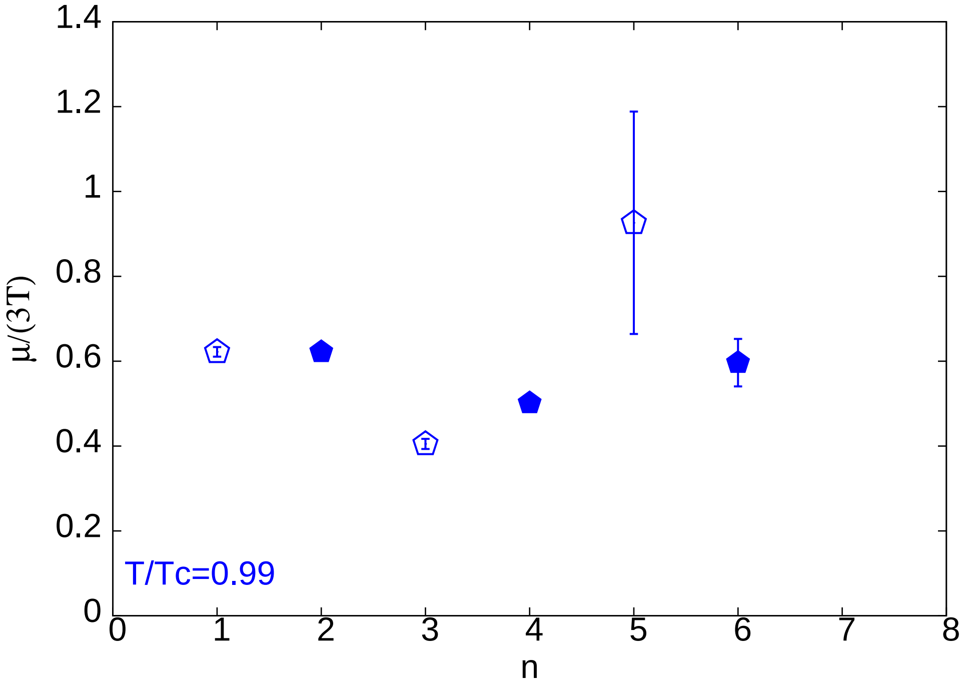
- New Simulations made at T/T_c = 0.89(1), 0.92(1), 0.94(1), 0.97(1), 0.99 (1) 1.00(1), 1.21(1), 1.33(1), 1.48(3) and 1.92(5)
- Typical stat. 50-200 in max autocorrelation units.

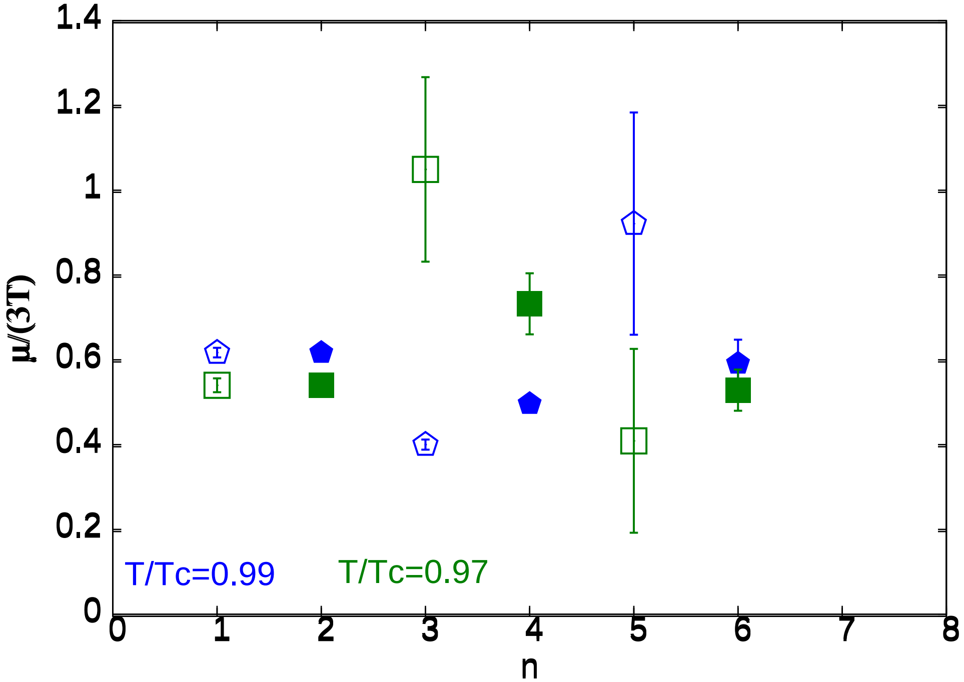
Our $N_\tau = 4$ Results



- Our estimate consistent with Fodor & Katz (2002) [$m_\pi/m_\rho = 0.31$ and $n_s m_\pi \sim$ 3-4].

- Strong finite size effects for small N_s. A strong change around $N_s m_\pi \sim 6$.


1.4
1.2
1
0.8
0.6
0.4
0.2
0
μ/(3T)
T/Tc=0.99
0 1 2 3 4 5 6 7 8
n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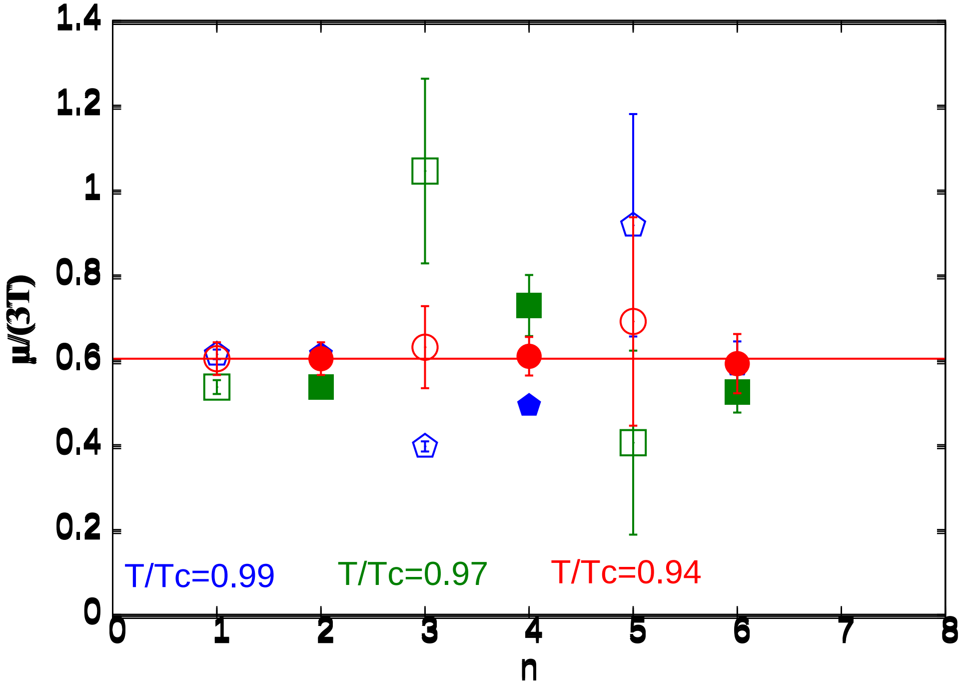
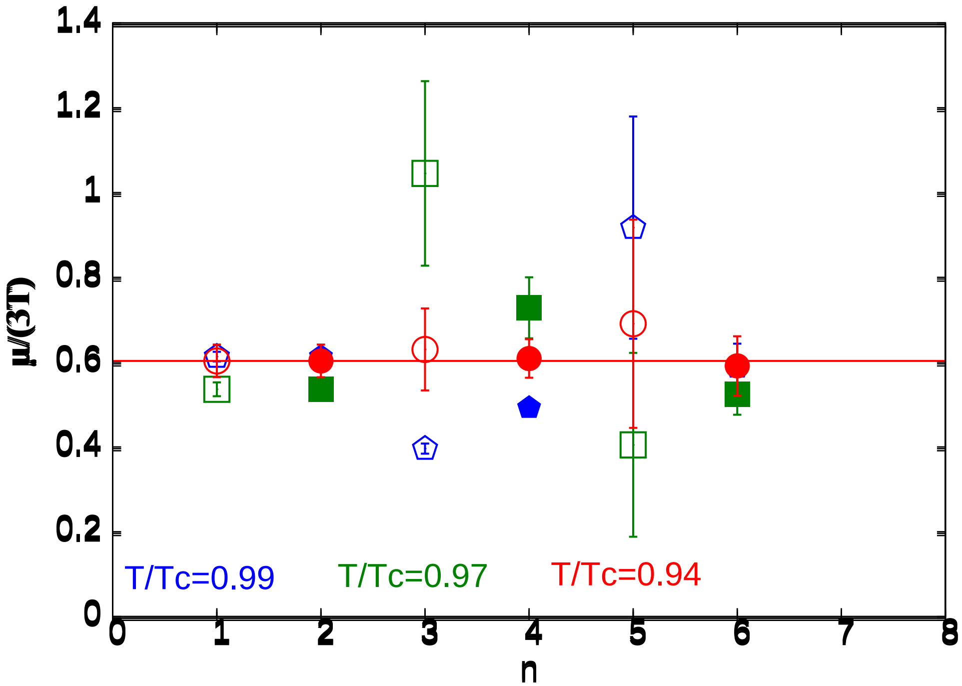


T/Tc=0.99
T/Tc=0.97
T/Tc=0.94
μ/(3T)
n

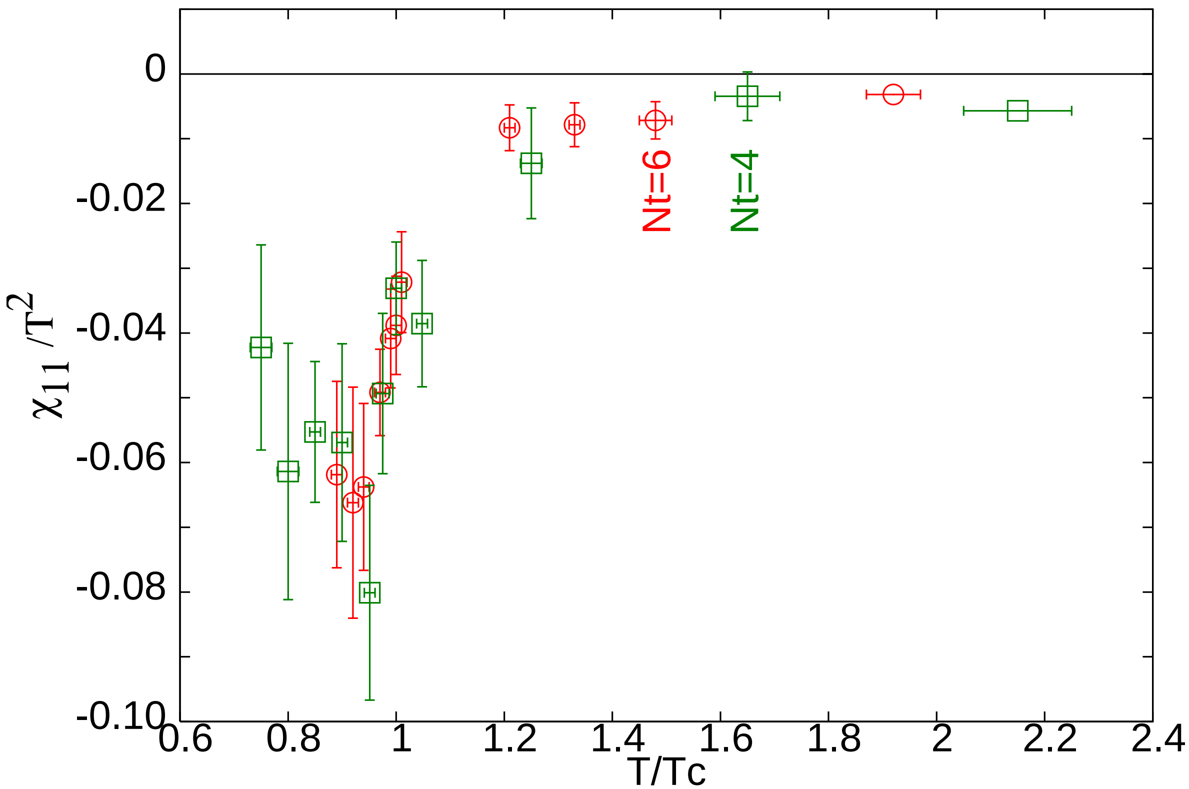


- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\mu_B^E/T^E = 1.3 \pm 0.3$. Infinite volume result: $\downarrow$ to 1.1(1)

- Critical point at smaller $\mu_B/T \sim 1 - 2$.

More Details

Measure of the seriousness of sign problem : χ_{11}; $N_t = 4$ & 6 agree.



Cross Check on μ^E/T^E

♠ Use Padé approximants for the series to estimate the radius of convergence.

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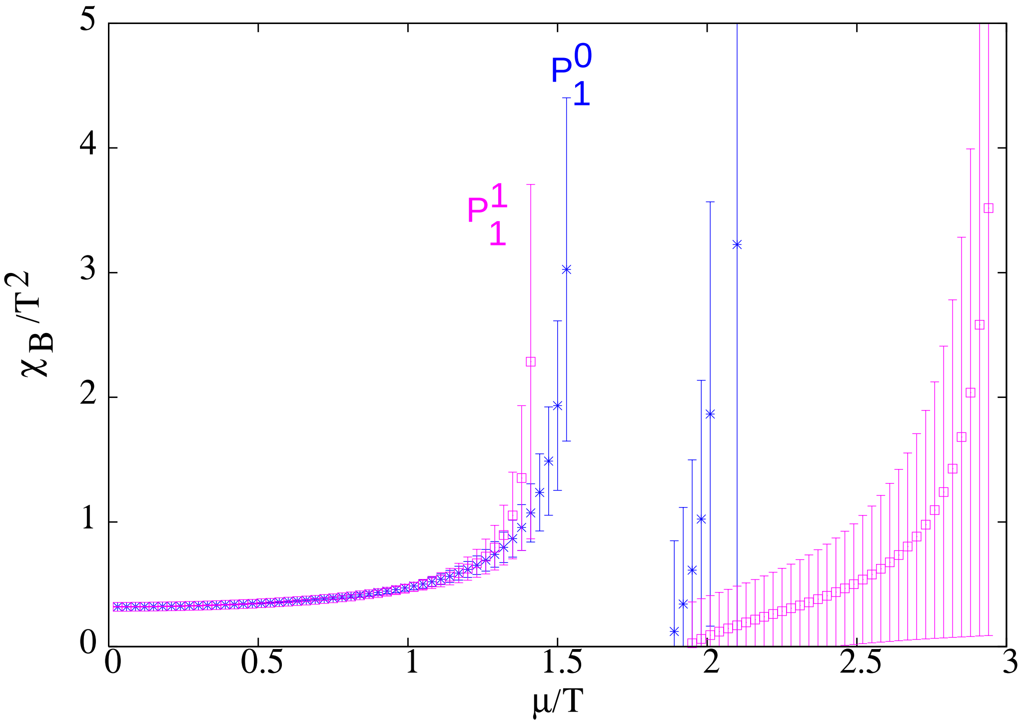
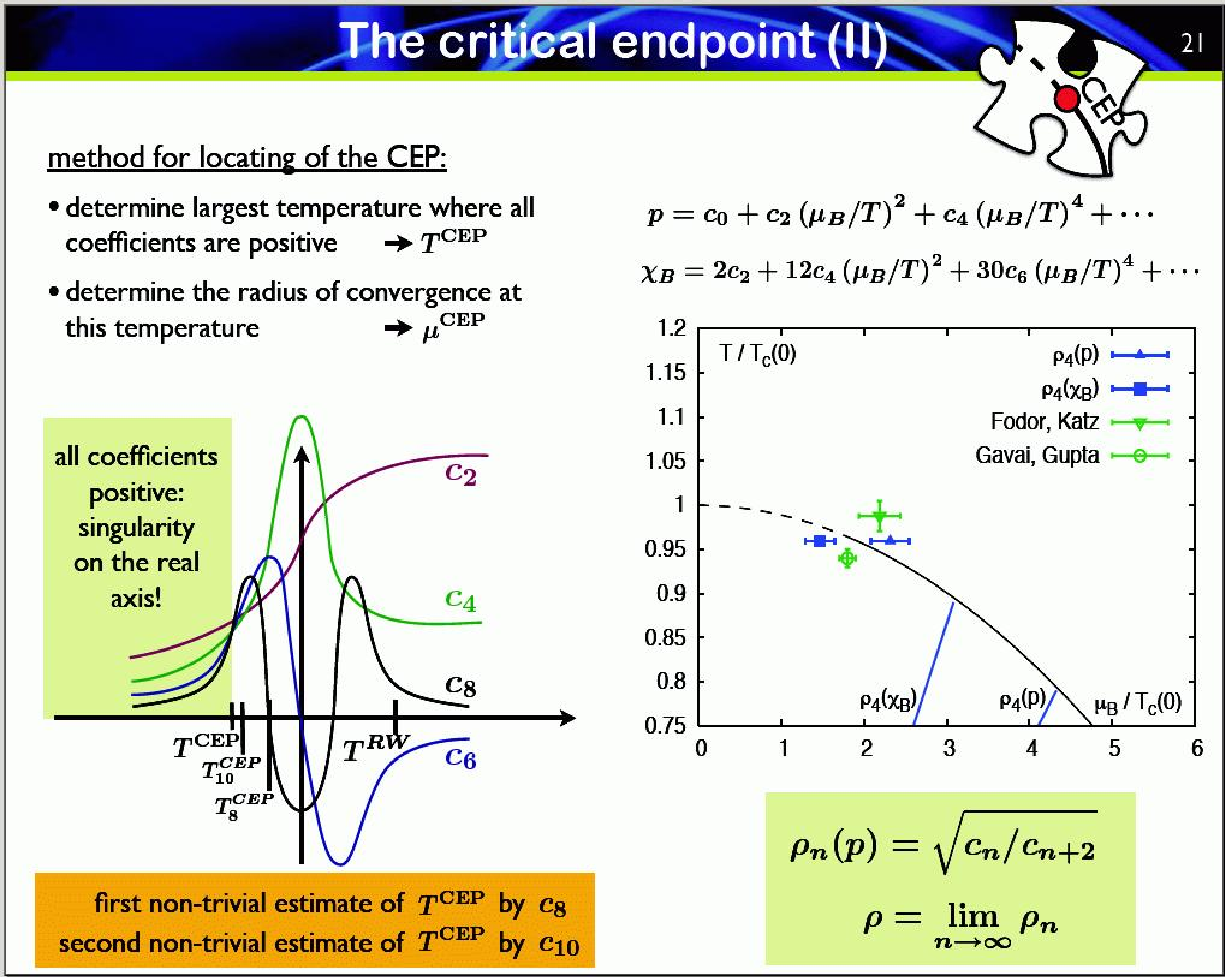


♡ Consistent Window with our other estimates.



(Ch. Schmidt FAIR Lattice QCD Days, Nov 23-24, 2009.)

Searching Experimentally

- Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases μ_B (Rajagopal, Shuryak & Stephanov PRD 1999)

- Look for nonmontonic dependence of the event-by-event fluctuations with colliding energy.

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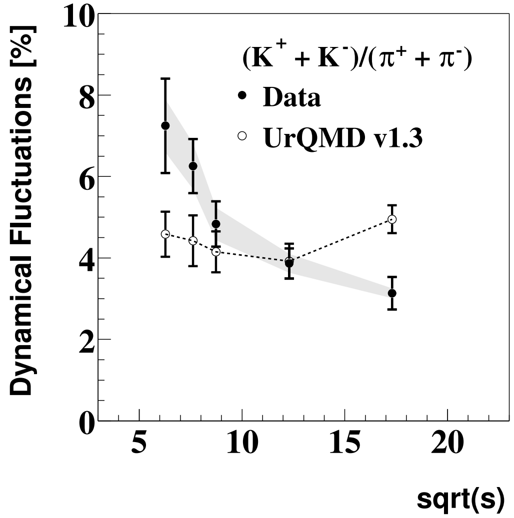

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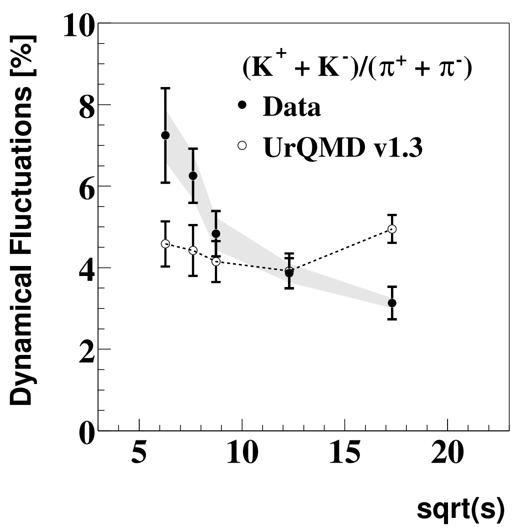

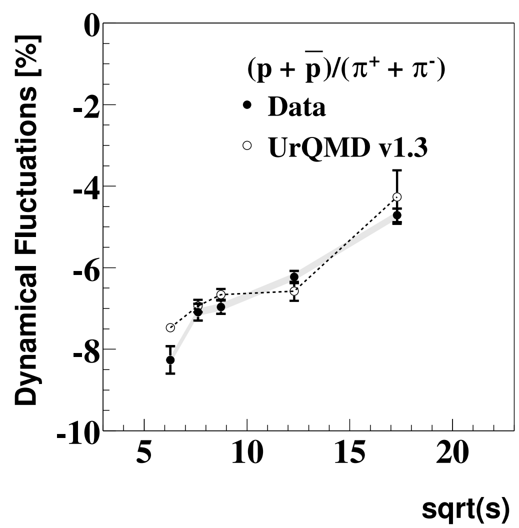

Lattice predictions along the freezeout curve

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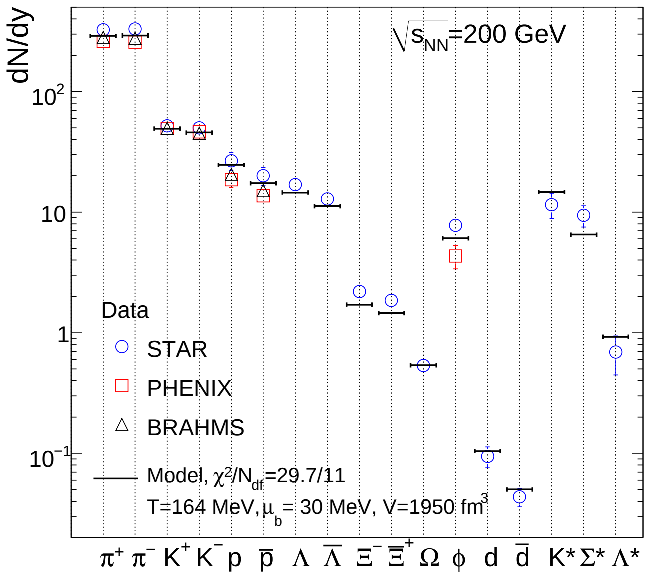

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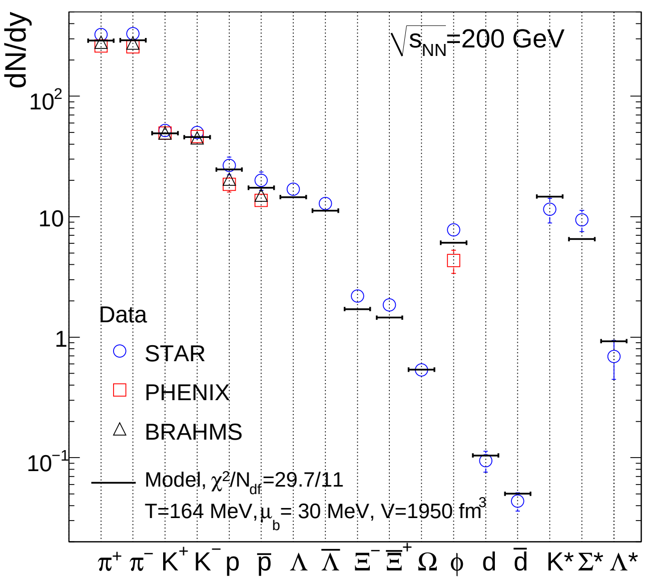

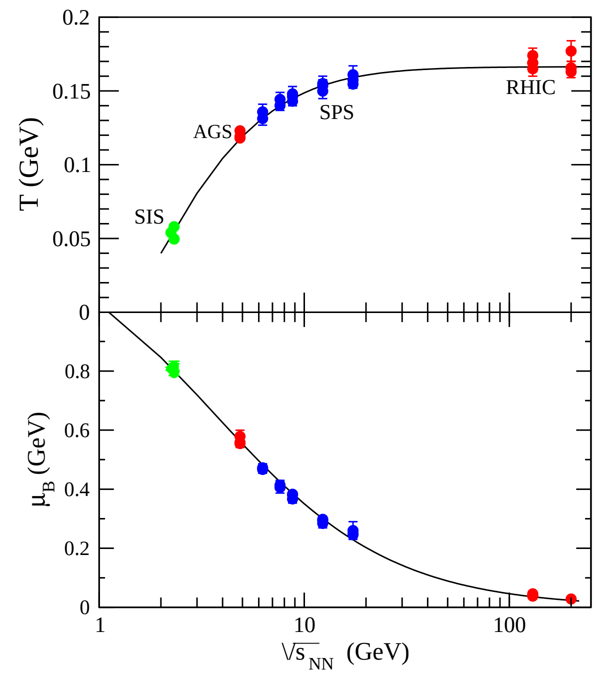

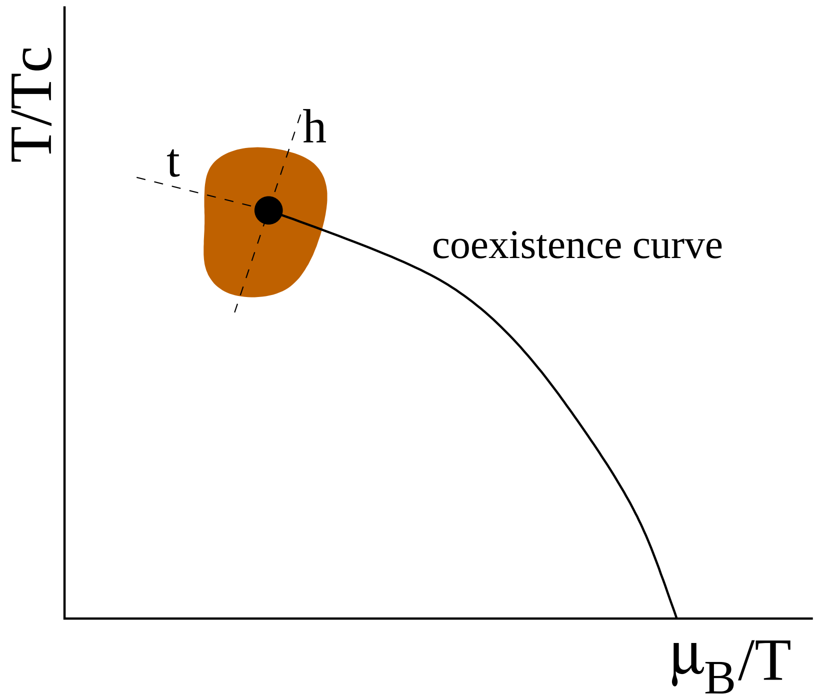
T/Tc
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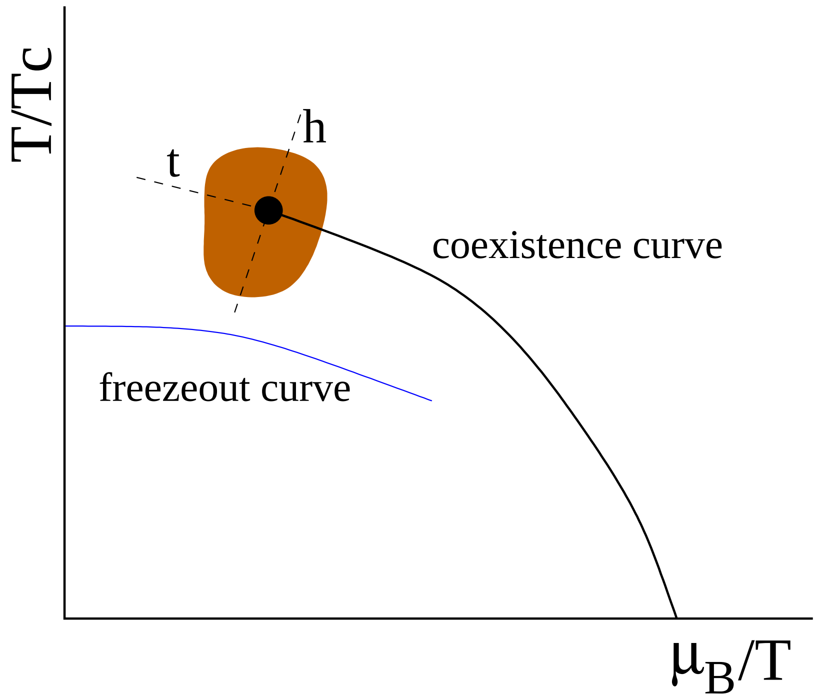
coexistence curve
μB/T


T/Tc
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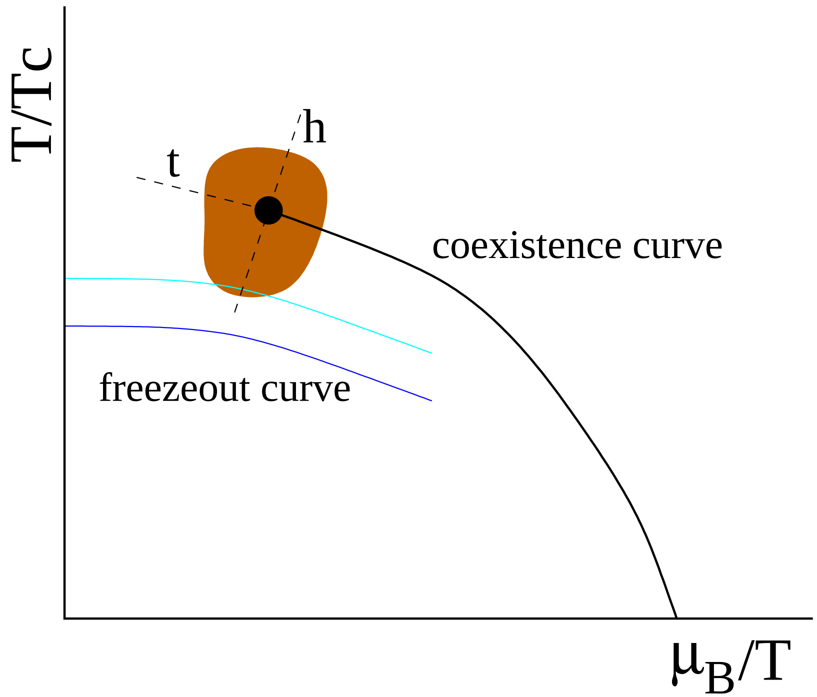
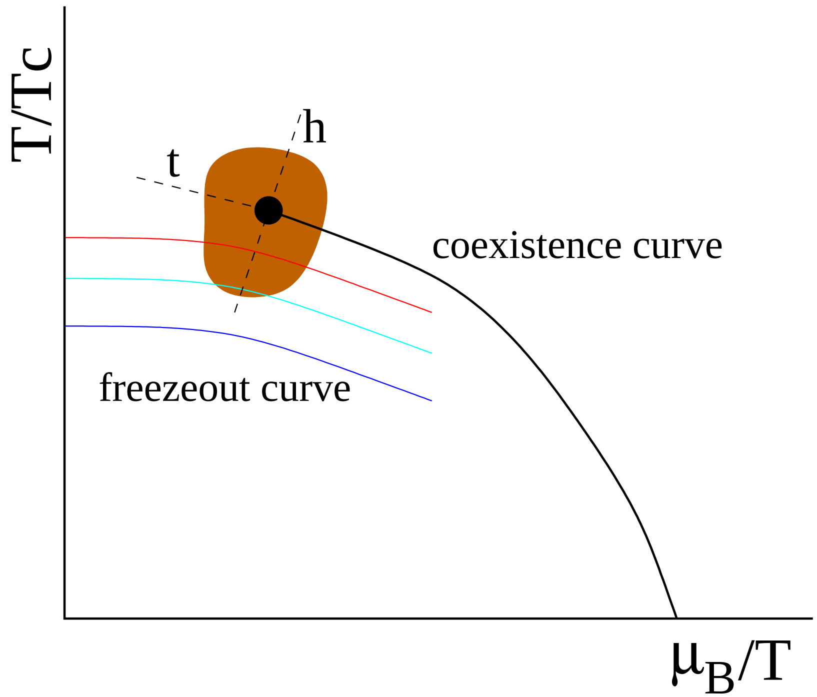
freezeout curve
μ_B/T


T/Tc
h
t
coexistence curve
freezeout curve
μB/T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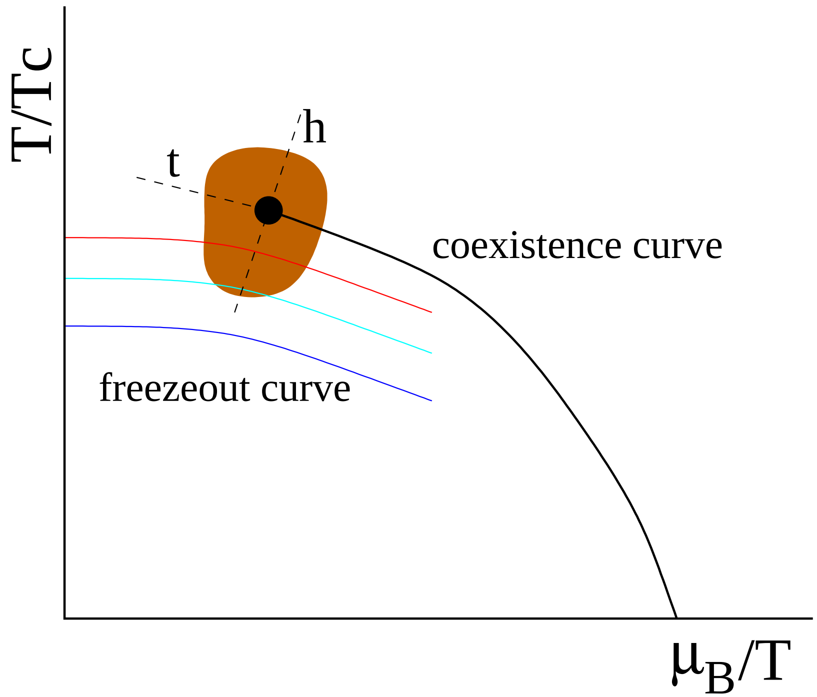
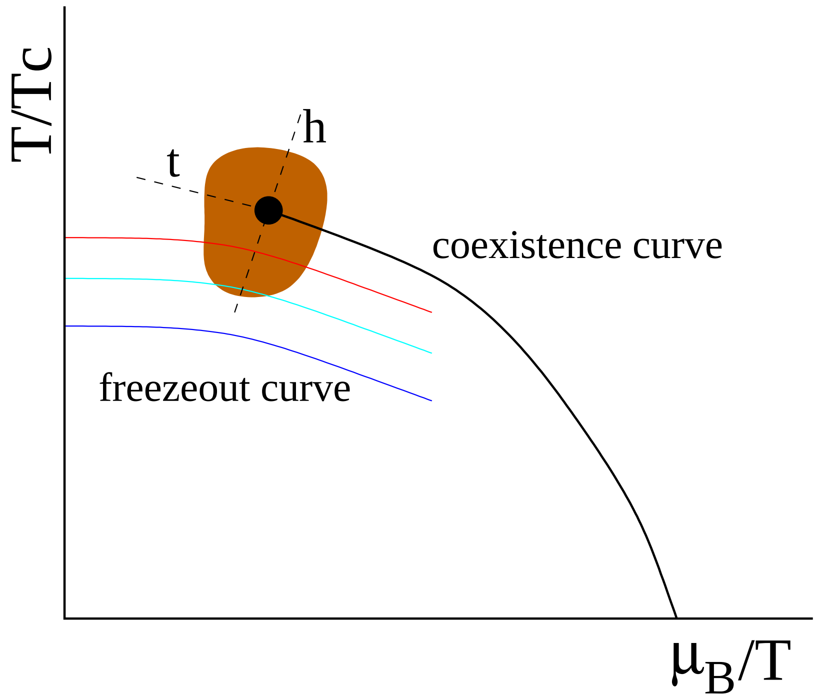


- Use the freezeout curve computed from hadron abundances to relate $(T, \mu_B)$to $\sqrt{s}$ and employ lattice QCD predictions along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)



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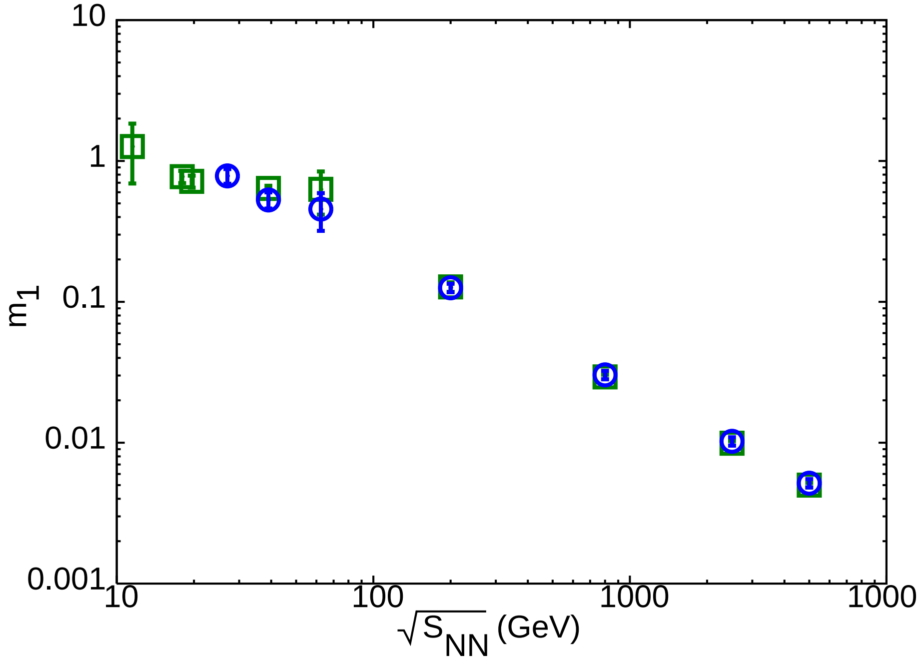
- Define $m_1 = \frac{T\chi^{(3)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)}$, $m_3 = \frac{T\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)}$, and $m_2 = m_1 m_3$ and use the Padè method to construct them.

- Use $T_c(\mu = 0) = 175$ MeV ; for $N_t = 4$ (boxes) and 6 (circles).



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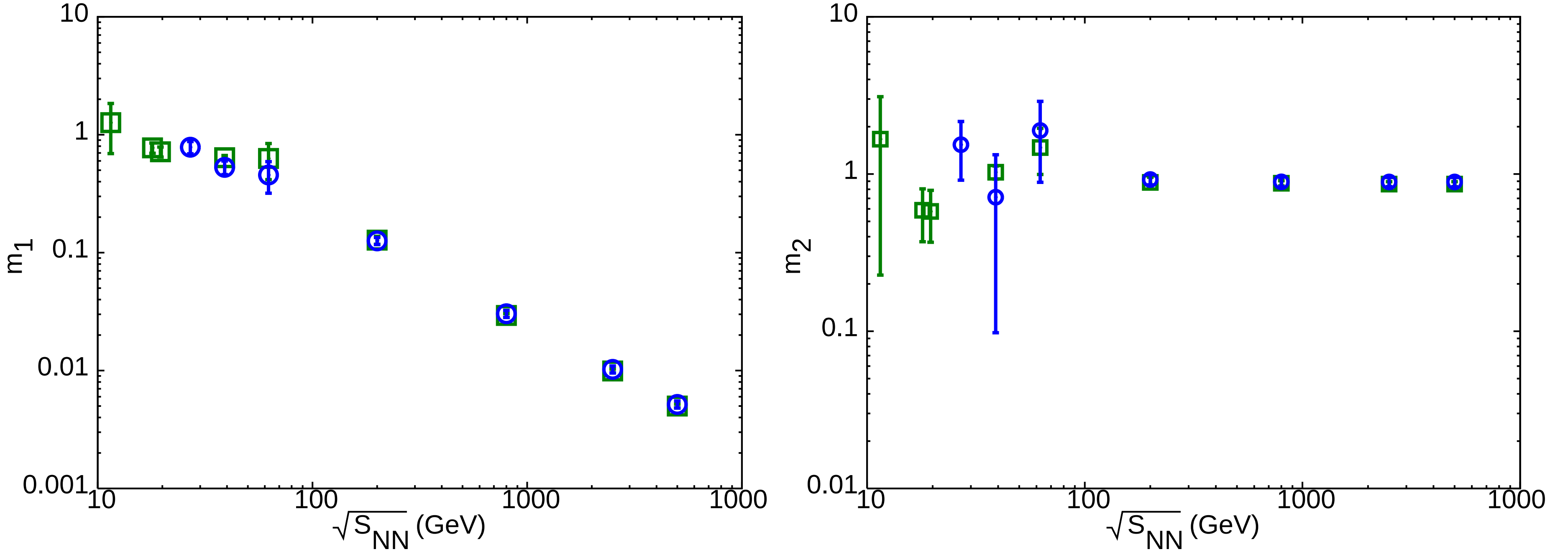

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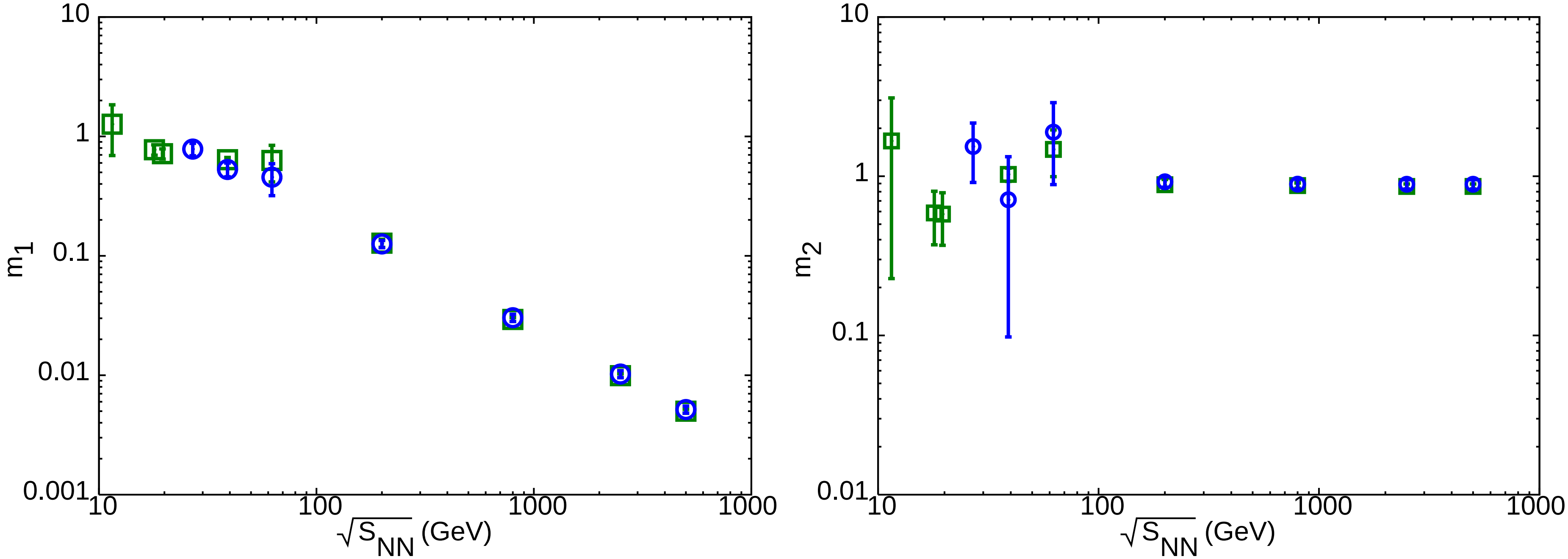

- Our estimated critical point suggests a peak-like structure in all m_i which would be accessible to the low energy scan of RHIC BNL !!

- Dull, smooth & monotonic behaviour without the critical point.

- Proton number fluctuations (Hatta-Stephenov, PRL 2003)

- Neat idea : directly linked to the baryonic susceptibility which ought to diverge at the critical point. Since diverging ξ is linked to σ mode, which cannot mix with any isospin modes, expect χ_I to be regular.

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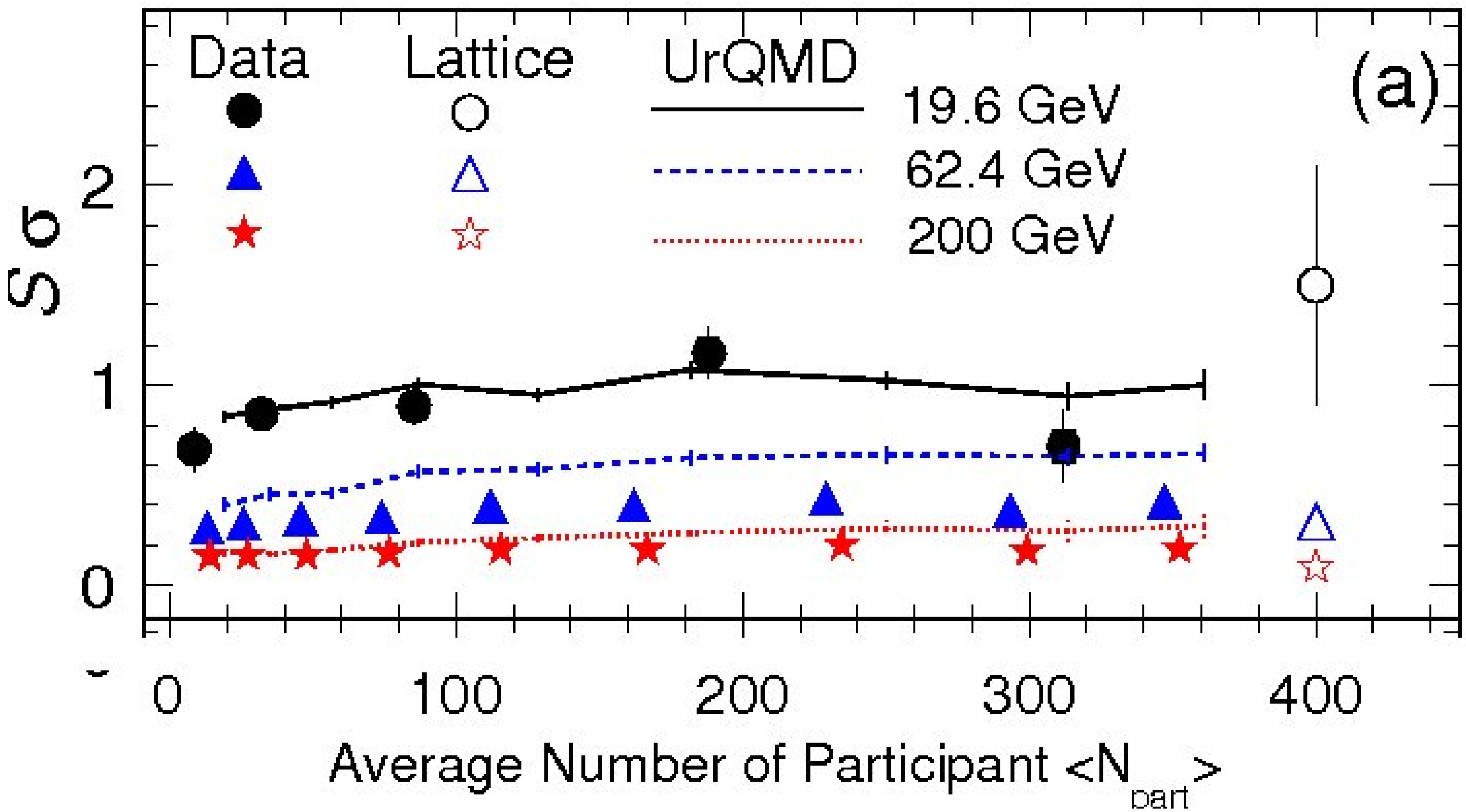
- Leads to a ratio χ_Q:χ_I:χ_B = 1:0:4

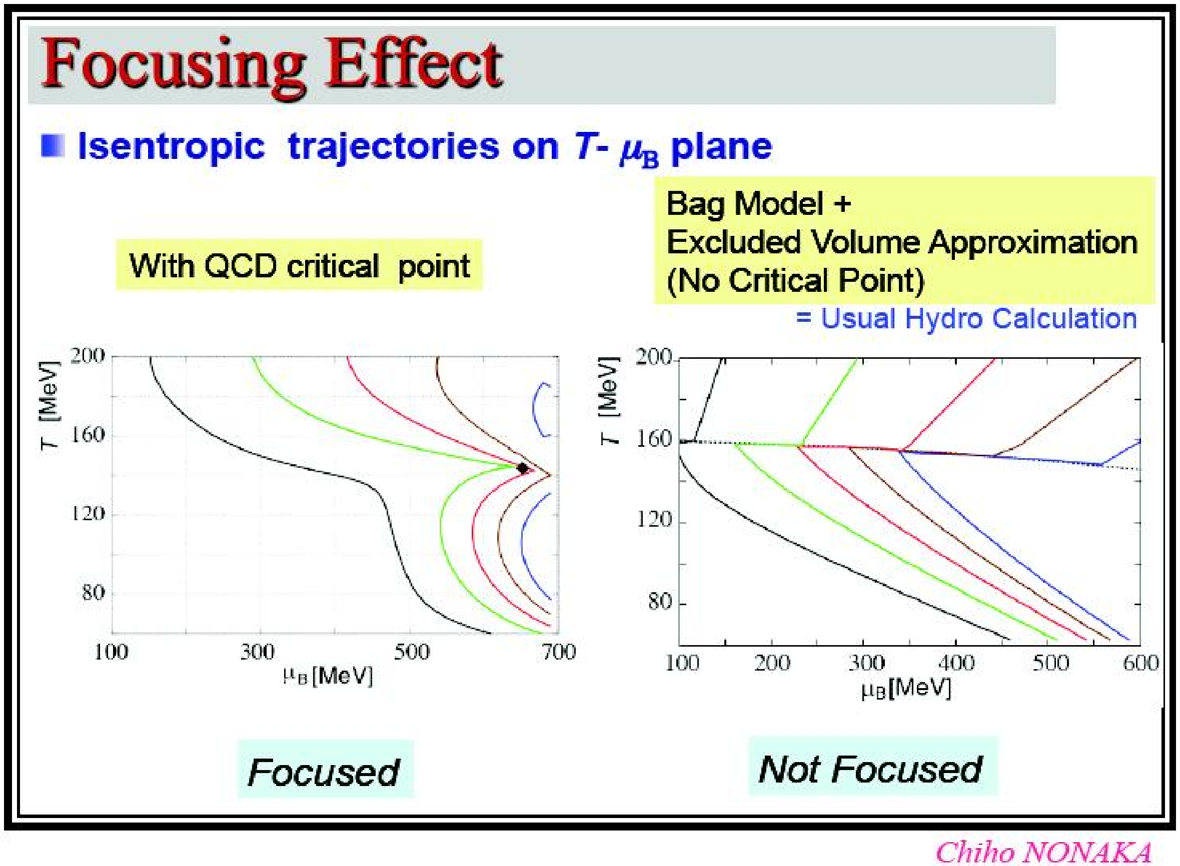
- Assuming protons, neutrons, pions to dominate, both χ_Q and χ_B can be shown to be proton number fluctuations only.

- STAR has recently used this idea and constructed the ratio m_2 we have.



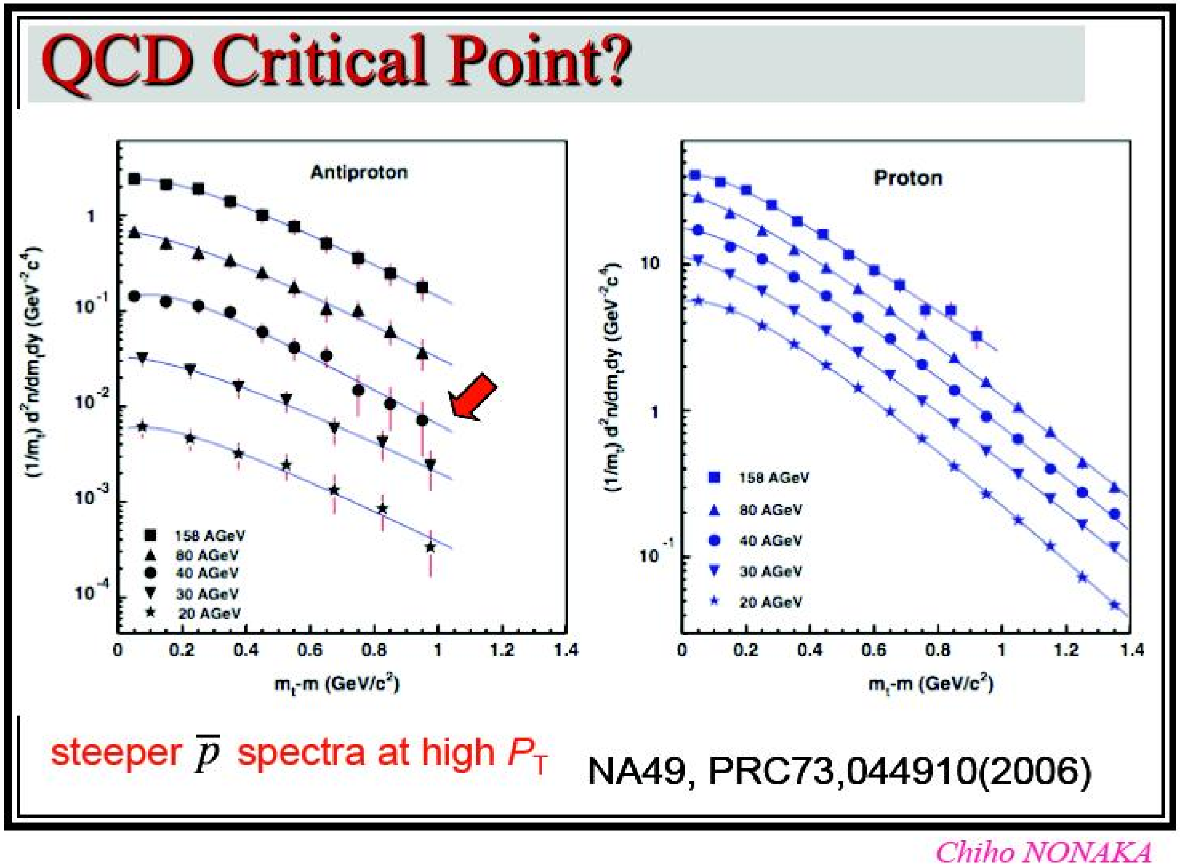
- Reasonable agreement with our lattice results. Where is the critical point ?

- Isentropic trajectories focus at the critical point (Asakawa-Nonaka, PRC 2005).



- This leads to the emission of high p_T particles at earlier times.

(Asakawa-Bass-Nonaka-Müller, INT 2008 workshop).



- Note this is NOT a fluctuations signal but model (EoS) dependent ?

Summary

- Phase diagram in $T - \mu$ has begun to emerge: Different methods, $\rightsquigarrow$ similar qualitative picture. Critical Point at $\mu_B/T \sim 1 - 2$.

- Our results for $N_t = 6$ first to begin the crawling towards continuum limit.

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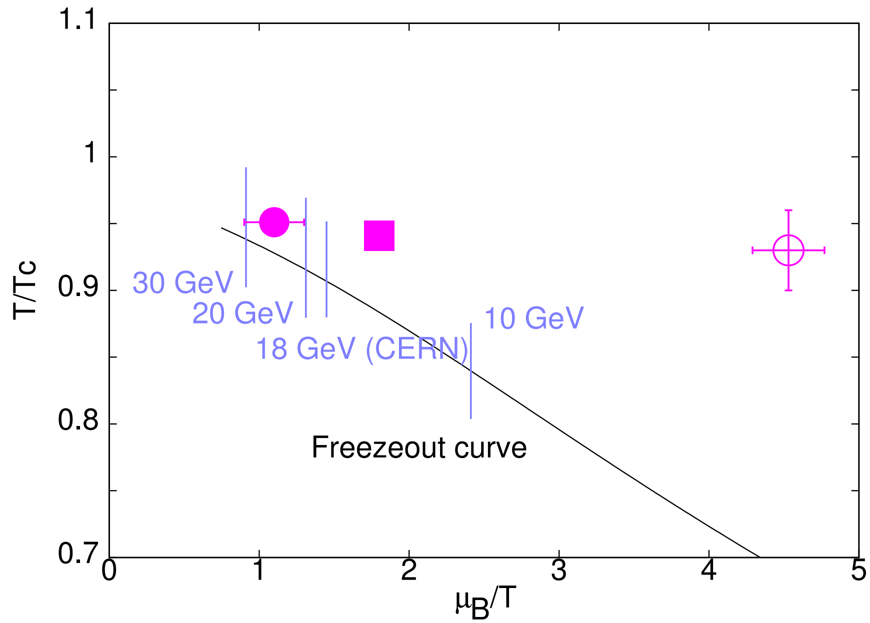

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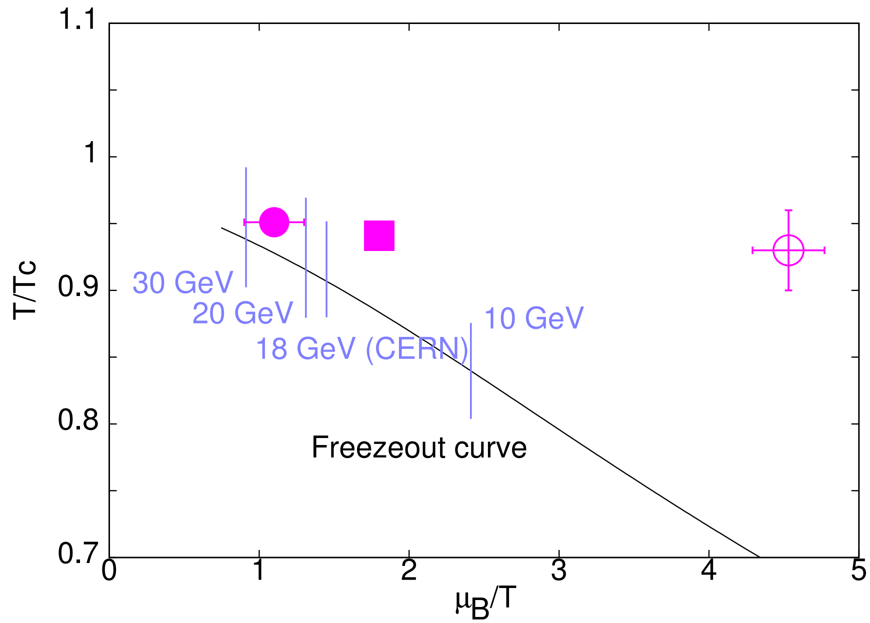

So far no signs of a critical point in the experimental results at CERN.
Will RHIC deliver it for us ? and/or Will it be FAIR ?

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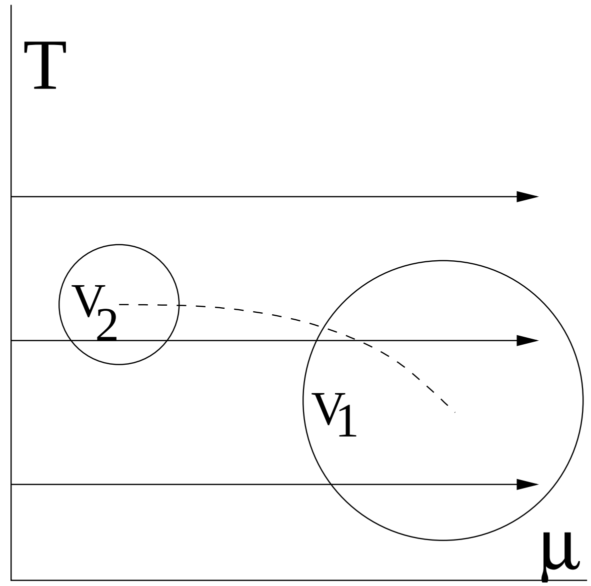
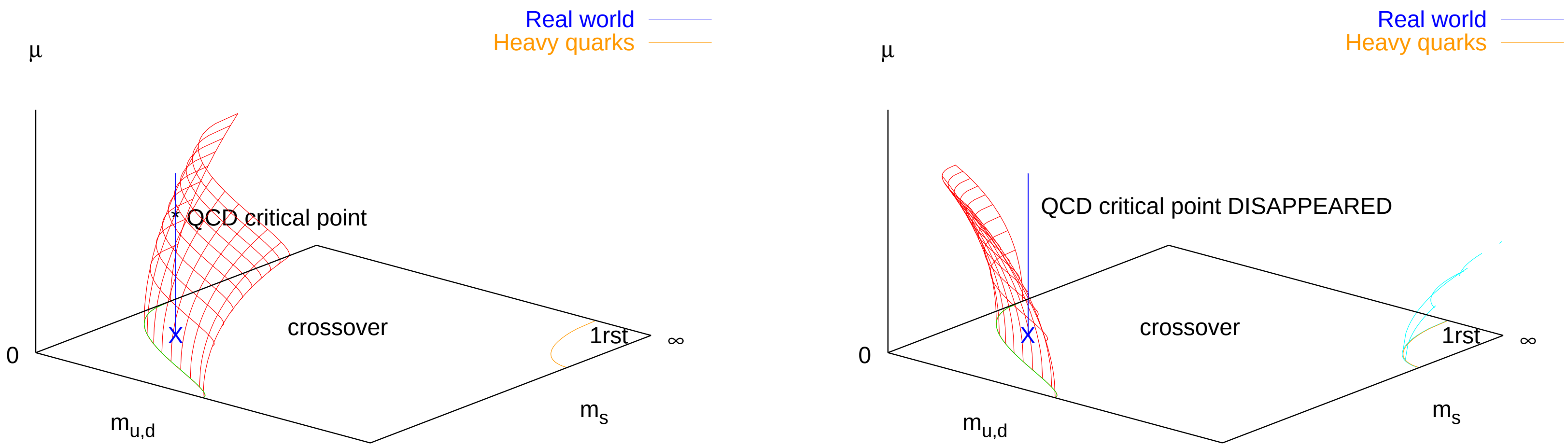


We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

Imaginary Chemical Potential

deForcrand-Philpsen JHEP 0811



For $N_f = 3$, they find $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T_c}\right)^2 - 47(20)\left(\frac{\mu}{\pi T_c}\right)^4$, i.e., m_c shrinks with μ.

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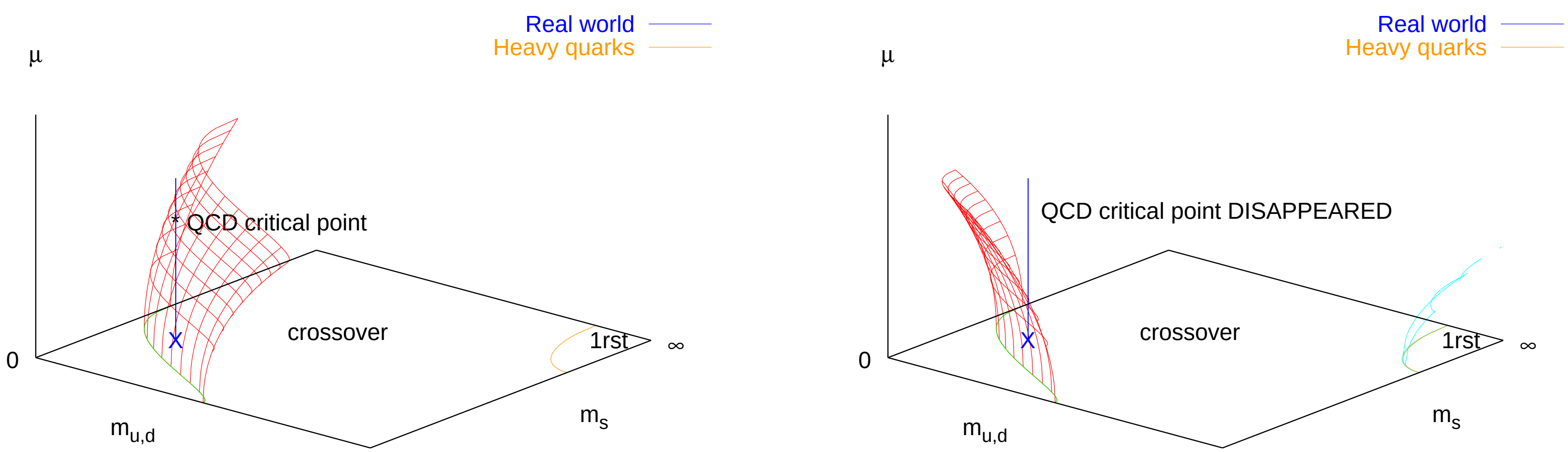

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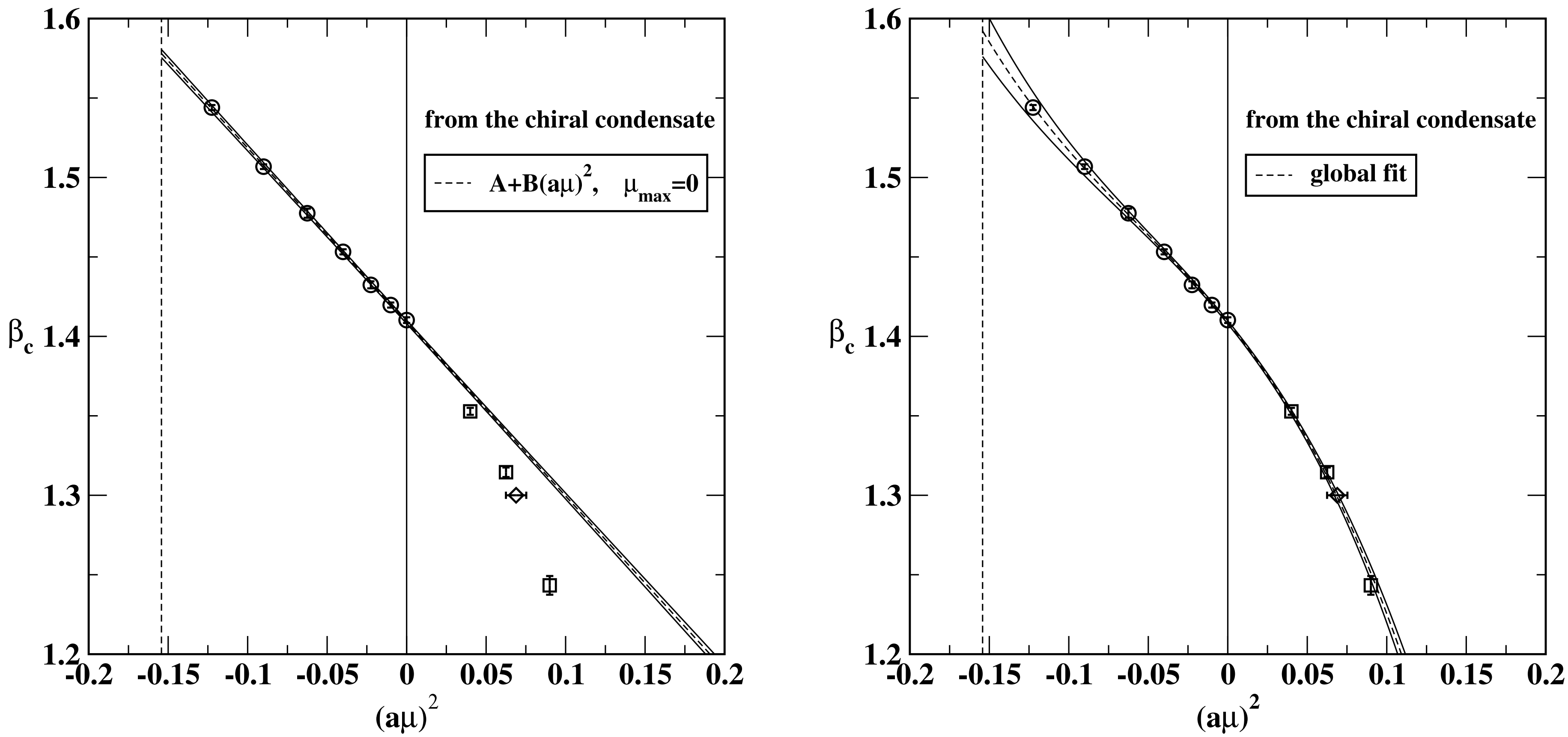
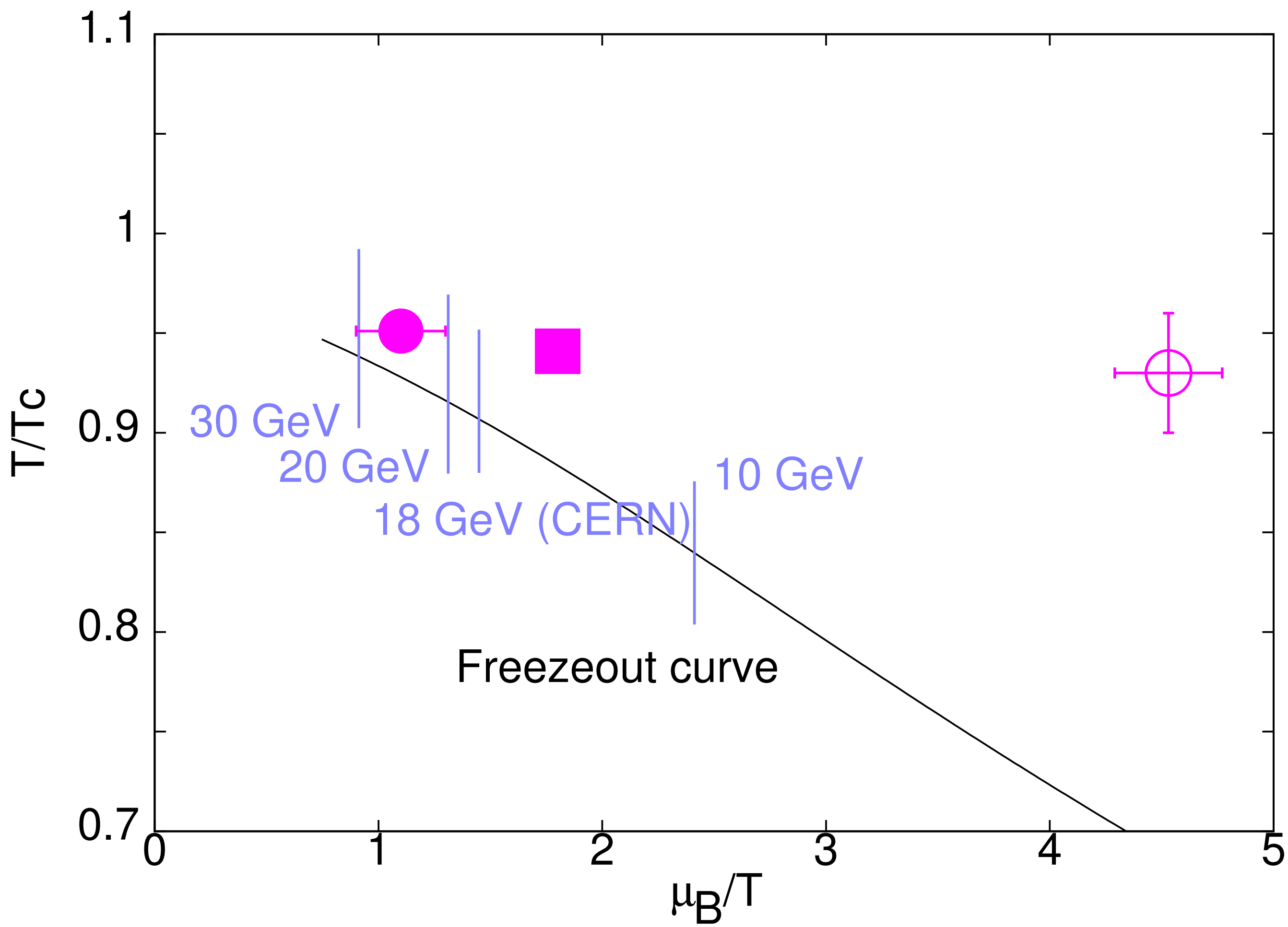
Problems : i) Positive coefficient for finer lattice (Philipsen, CPOD 2009), ii) Known examples where shapes are different in real/imaginary μ, “The Critical line from imaginary to real baryonic chemical potentials in two-color QCD”, P. Cea, L. Cosmai, M. D’Elia, A. Papa, PR D77, 2008




1.1
1
0.9
0.8
0.7
0
1
2
3
4
5
T/Tc
μ_B/T
30 GeV
20 GeV
18 GeV (CERN)
10 GeV
Freezeout curve