Towards QCD Thermodynamics using Exact Chiral Symmetry on Lattice

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T. I. F. R., Mumbai

* *arXiv : 0803.3925, to appear in Phys. Rev. D, & in preparation.*

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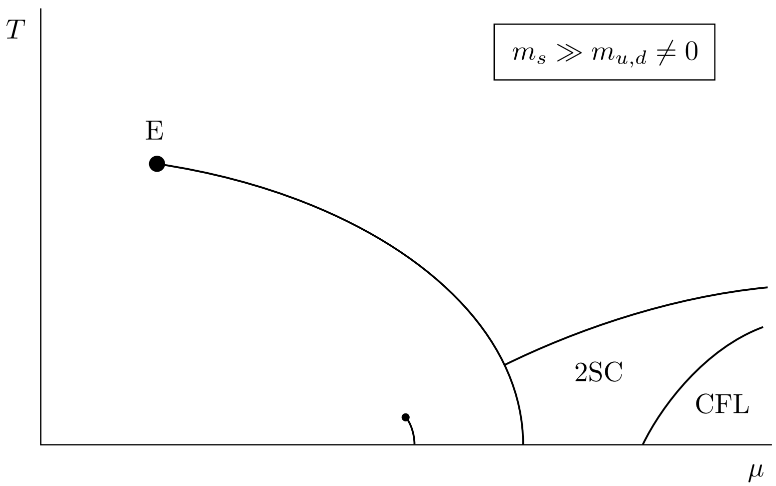

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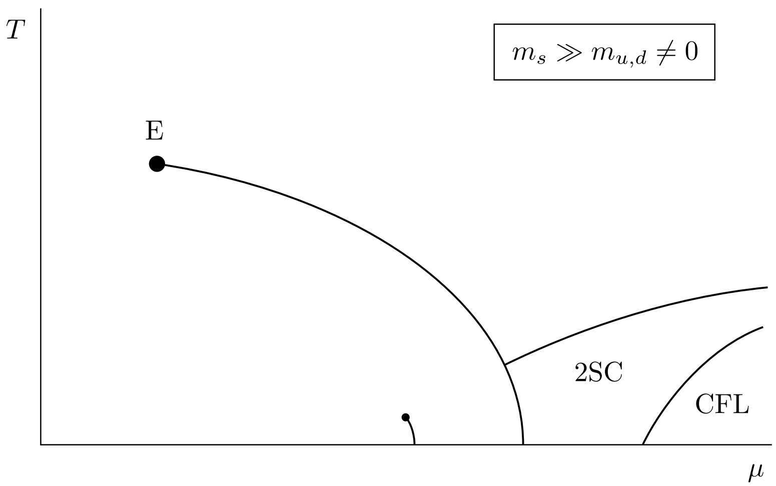

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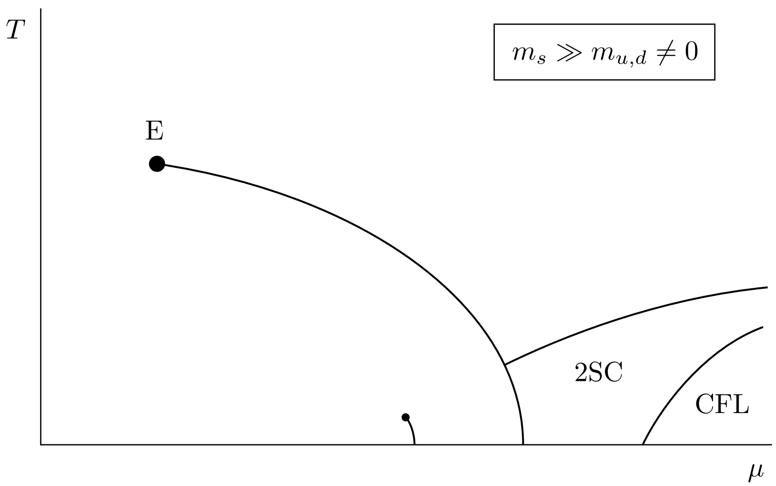

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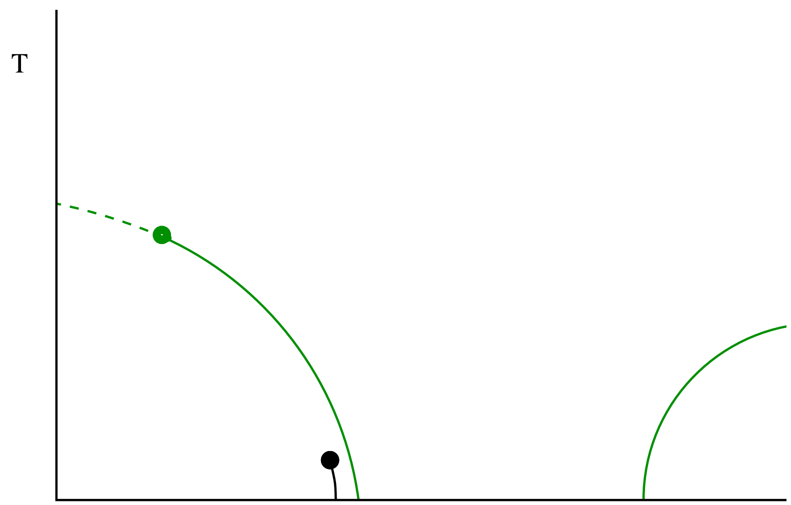

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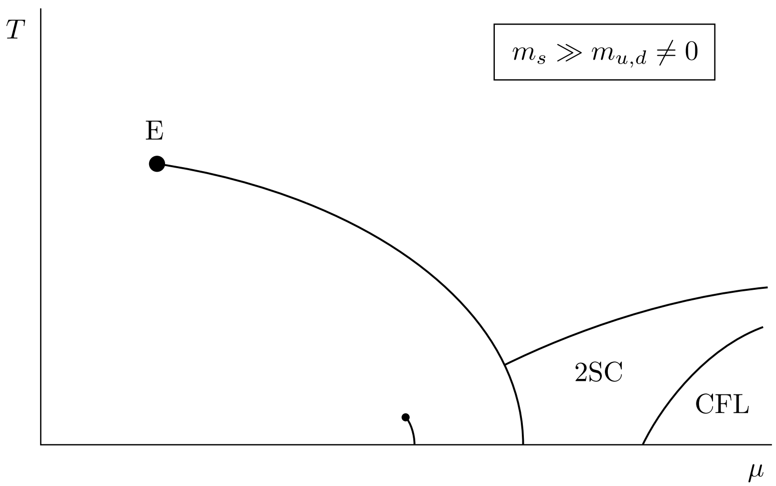

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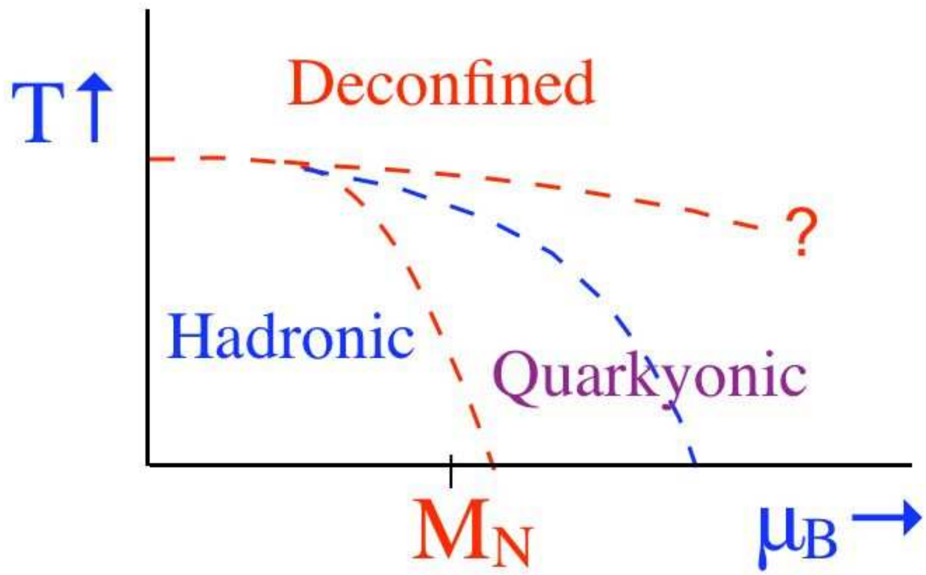

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$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \Big[\gamma_5 D + D\gamma_5 - \frac{a}{2} D\gamma_5 D - \frac{a}{2} D\gamma_5 D\Big]_{xy} \psi_y = 0 \qquad (1)$$

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♠ Overlap fermions, and Domain Wall fermions in the limit of large fifth dimension satisfy this relation.

Domain Wall Fermions

♠ Proposed by Kaplan (PLB 1992), a convenient form for Domain Wall fermion action (Shamir, NPB, 1993) is:

$$S_F = \sum_{s,s'=1}^{N_5} \sum_{x,x'} \bar{\psi}(x,s) D_{dw}(x,s;x',s') \psi(x',s') \ , \qquad (2)$$

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$$D_{dw}(x,s;x',s') = [a_5 D_w + 1]\delta_{s,s'} - [P_- \delta_{s,s'-1} + P_+ \delta_{s,s+1'}] \ , \tag{3}$$

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with boundary conditions $P_+\psi(x,0) = -am \ P_+\psi(x,N_5)$ and $P_-\psi(x,N_5+1) = -am \ P_-\psi(x,1)$.

♠ Only light modes attached to the wall(s) are physical. Divide out heavy modes by having the $D_{dw}(am)/D_{dw}(am=1)$ as the effective Domain Wall operator in $\mathcal{Z}$.

♡ As outlined in Edwards & Heller (PRD 63, 2001), one can integrate out the fermionic fields in the fifth direction to rewrite the above ratio as

$$[(1+am) - (1-am)\gamma_5 \tanh(\frac{N_5}{2}\ln|T|)] \ , \tag{4}$$

with $T = (1 + a_5\gamma_5 D_w P_+)^{-1}(1 - a_5\gamma_5 D_w P_-)$.

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♡ Taking the limit $N_5 \to \infty$ for $a_5 = 1$, one obtains sign function of log $|T|$, proving that the DWF satisfy the Ginsparg-Wilson relation in this limit.

♡ Taking the limit $a_5 \to 0$ such that $L_5 = a_5 N_5 =$ constant, one can show $N_5 \ln T \to L_5 \gamma_5 D_{dw}$. Further, for $L_5 \to \infty$, DWF reduce to the overlap fermions.

♡ We use this form in our numerical work.

Introducing Chemical Potential

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- Note $\gamma_5 D_w(a\mu)$ is no longer Hermitian, requiring an extension of the sign function. B & W proposal : For complex $\lambda = (x + iy)$, sign(λ) $=$ sign (x).
- Gattringer-Liptak, PRD 2007, showed for $M = 1$ numerically that no μ^2 divergences exist for the free case (U $=$1).

- We show this to be true analytically and for all M as well. Furthermore, this holds for all functions such that $K(a\mu) \cdot L(a\mu) = 1$ for Overlap (Banerjee, Gavai, Sharma, PRD 2008) and Domain Wall Fermions (Gavai, Sharma 2008).

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- We claim that chiral invariance is lost for nonzero μ. Note that

$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \Big[\gamma_5 D(a\mu) + D(a\mu)\gamma_5 - \frac{a}{2} D(0)\gamma_5 D(a\mu) - \frac{a}{2} D(a\mu)\gamma_5 D(0) \Big]_{xy} \psi_y \ , \tag{5}$$

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which is not sufficient to make $\delta S = 0$. True for both Overlap and Domain Wall fermions and any K,L.

Our Results

- We investigated thermodynamics of free overlap and domain wall fermions with an aim to examine the continuum limit analytically and numerically.

- Analytically, we prove the absence of μ^2-divergences for general K and L. Our numerical results were for tuning the irrelevant parameter M to obtain small deviations from continuum limit on coarse lattices.

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- Energy density and pressure can be obtained from $\ln Z = \ln\ \det\ D_{ov}$ by taking T and V, or equivalently a_4 and a, partial derivatives.

- Dirac operator is diagonal in momentum space. Use its eigenvalues to compute $\mathcal{Z}$:
  $\lambda_\pm = 1 - [sgn\left(\sqrt{h^2 + h_5^2}\right) h_5 \pm i\sqrt{h^2}]/\sqrt{h^2 + h_5^2}$, with
  $h_i = -\sin ap_i$, i =1, 2 and 3, $h_4 = -a\ \sin(a_4 p_4)/a_4$ and
  $h_5 = M - \sum_{i=1}^{3}[1 - cos(ap_i)] - a[1 - cos(a_4 p_4)]/a_4$.

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- Hiding p_i-dependence in terms of known functions g, d and f, the energy density on an $N^3 \times N_T$ lattice is found to be

$$\epsilon a^4 = \frac{2}{N^3 N_T} \sum_{p_i,n} F(\omega_n) = \frac{2}{N^3 N_T} \sum_{p_i,n} \left[(g + \cos\omega_n) + \sqrt{d + 2g\cos\omega_n} \right] \times \left[\frac{(1-\cos\omega_n)}{d + 2g\cos\omega_n} + \frac{\sin^2\omega_n (g+\cos\omega_n)}{(d+2g\cos\omega_n)(f+\sin^2\omega_n)} \right] \quad (7)$$

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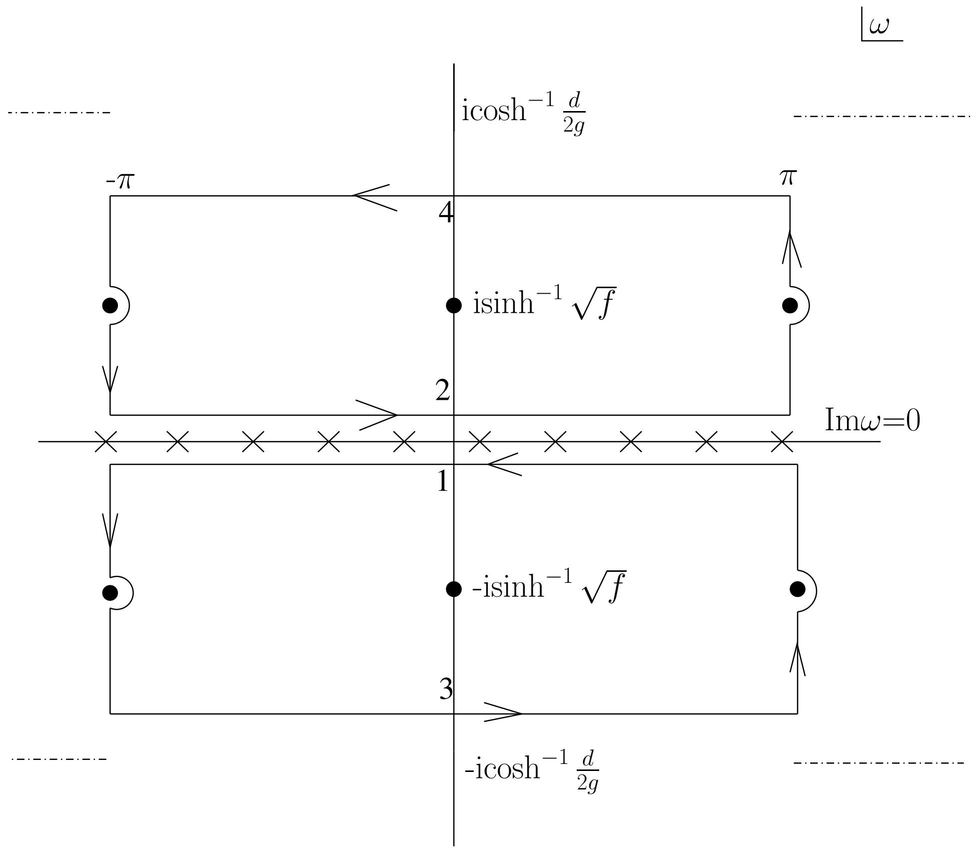
- Can be evaluated using the standard contour technique or numerically.

Analytic Evaluation : $\mu = 0$.



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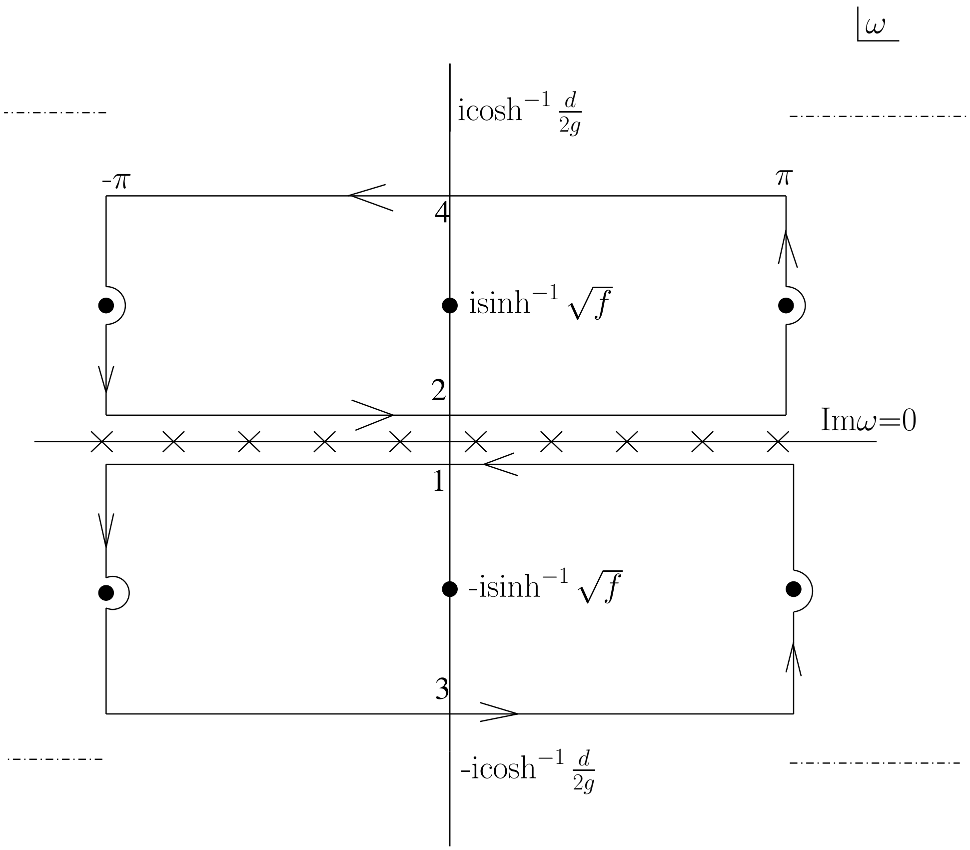

- Poles at $\omega = \pm i \sinh^{-1}\sqrt{f}$ and Poles (branch points) at $\pm i \cosh^{-1}\frac{d}{2g}$.

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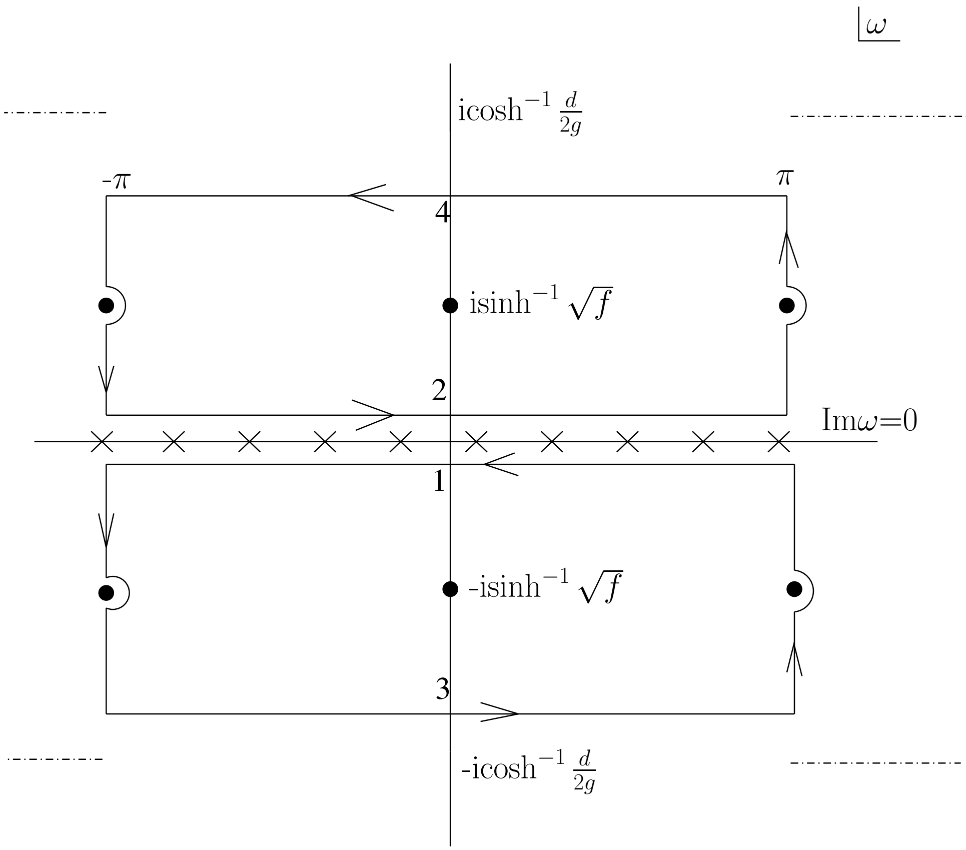

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- Evaluating integrals, $\epsilon a^4 = 4N^{-3}\sum_{p_j}\left[\sqrt{f/1+f}\right]$ $[\exp(N_T\sinh^{-1}\sqrt{f})+1]^{-1}$ $+\epsilon_3 + \epsilon_4$, where $f = \sum_i \sin^2(ap_i)$.

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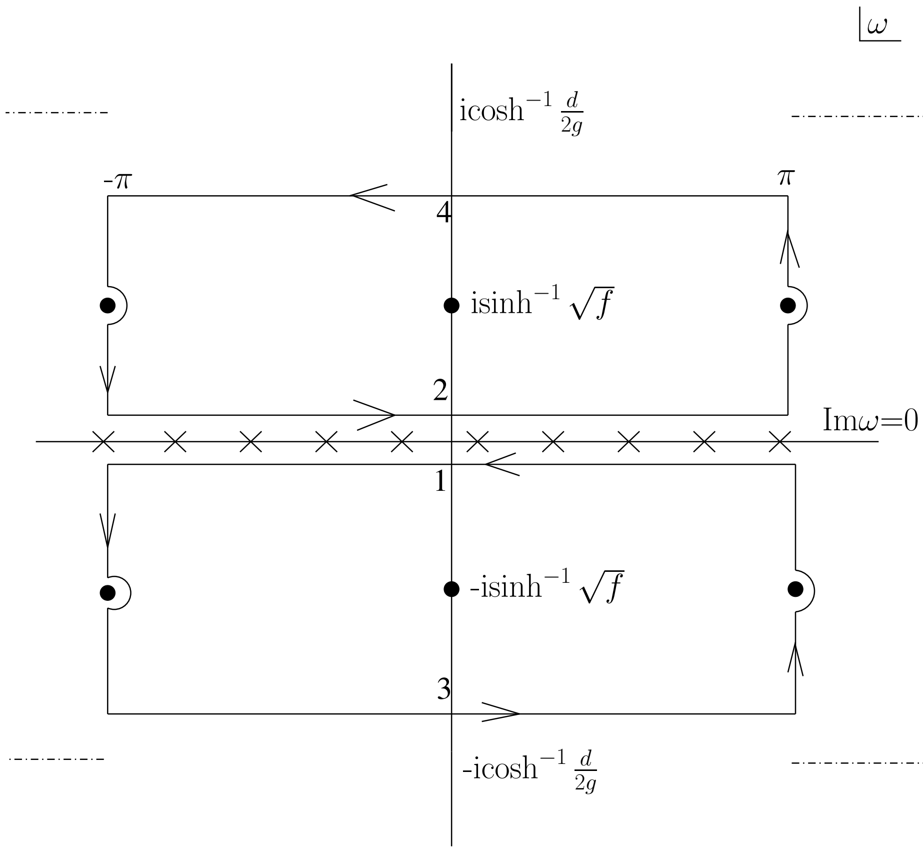

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- Can be seen to go to ϵ_{SB} as $a \to 0$ for all M.

More Details : $T = 0$, $\mu \neq 0$

- Defining $K(\mu) + L(\mu) = 2R \cosh\theta$ and $K(\mu) - L(\mu) = 2R \sinh\theta$, the same treatment as above goes through by substituting $\sin\omega_n \rightarrow R\sin(\omega_n - i\theta)$ and $\cos\omega_n \rightarrow R\cos(\omega_n - i\theta)$.

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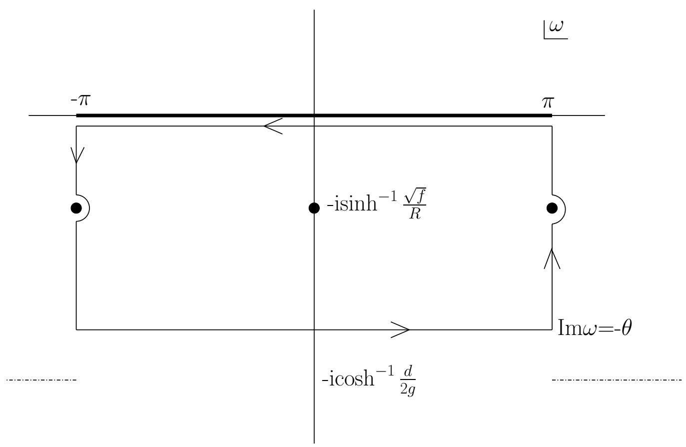
- Energy density is also functionally the same with $F(1, \omega_n) \rightarrow F(R, \omega_n - i\theta)$.

- Additional observable, number density : Has the same pole structure so similar computation.



Divergence Cancellation at $T = 0$, $\mu \neq 0$

- Doing the contour integral, the energy density turns out to be :

$$\epsilon a^4 = (\pi N^3)^{-1} \sum_{p_j} \Big[2\pi \mathrm{Res}\ F(R,\omega) \Theta \left(K(a\mu) - L(a\mu) - 2\sqrt{f} \right) + \int_{-\pi}^{\pi} F(R,\omega) d\omega - \int_{-\pi}^{\pi} F(1,\omega) d\omega \Big].$$

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- $R = K(a\mu) \cdot L(a\mu) = 1$ ensures cancellation of the last two terms and the canonical result in the continuum limit $a \to 0$.

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- K and L should be such that $K(a\mu) - L(a\mu) = 2a\ \mu + \mathcal{O}(a^3)$ with $K(0) = 1 = L(0)$.

- Generalization to $T \neq 0$ and $\mu \neq 0$ case straightforward. One merely needs two different contours depending on pole locations and value of θ.

Numerical Evaluation

♣ Zero temperature contribution : as $N_T \to \infty$, ω sum becomes integral which we estimated numerically.
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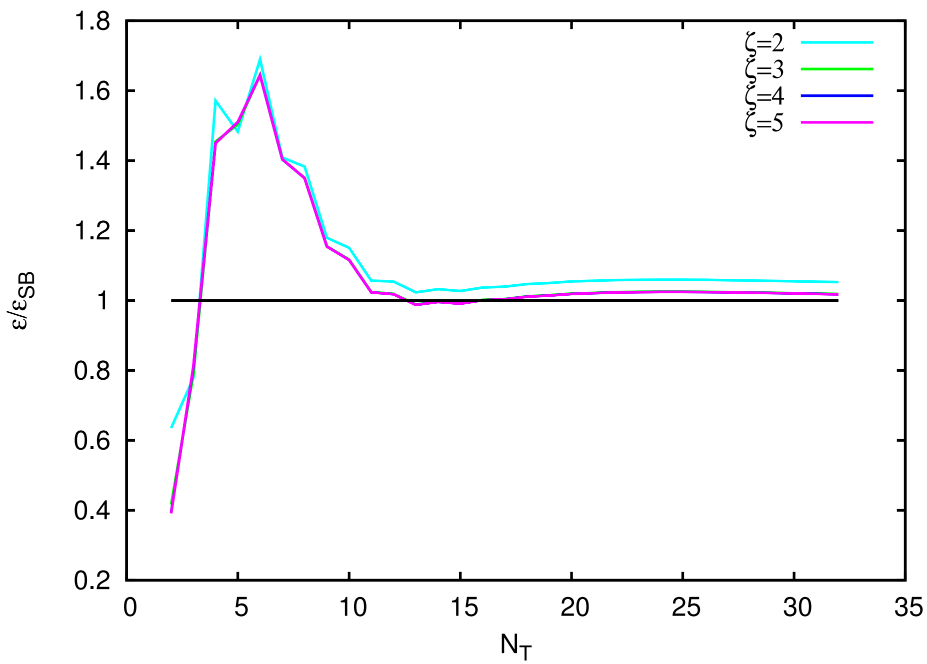

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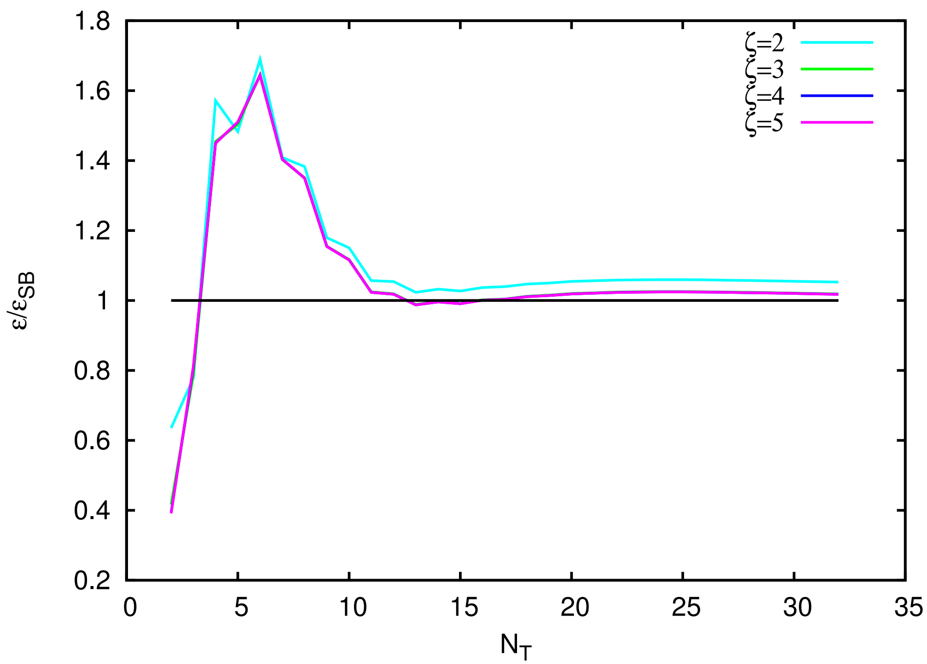

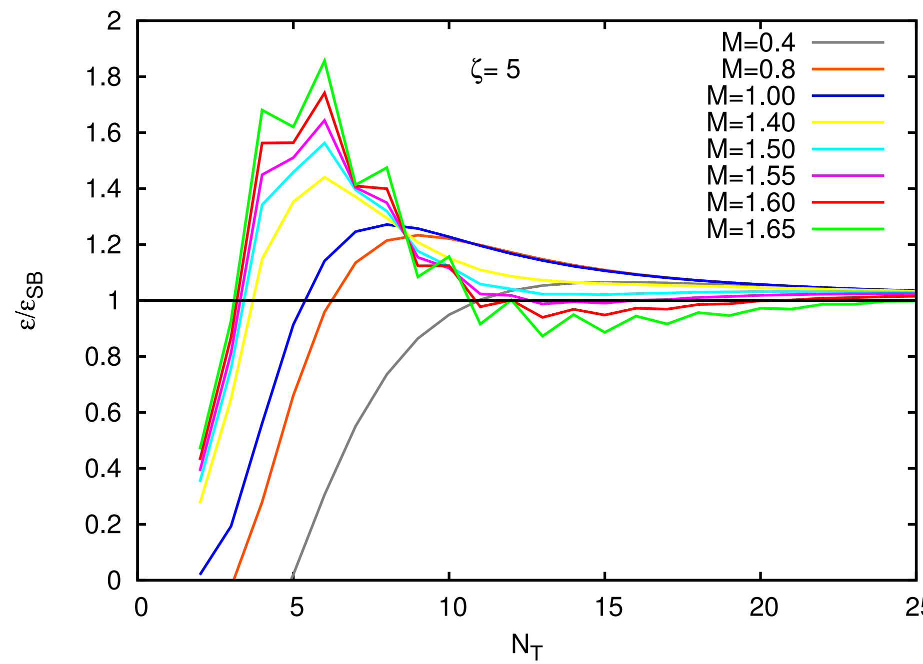

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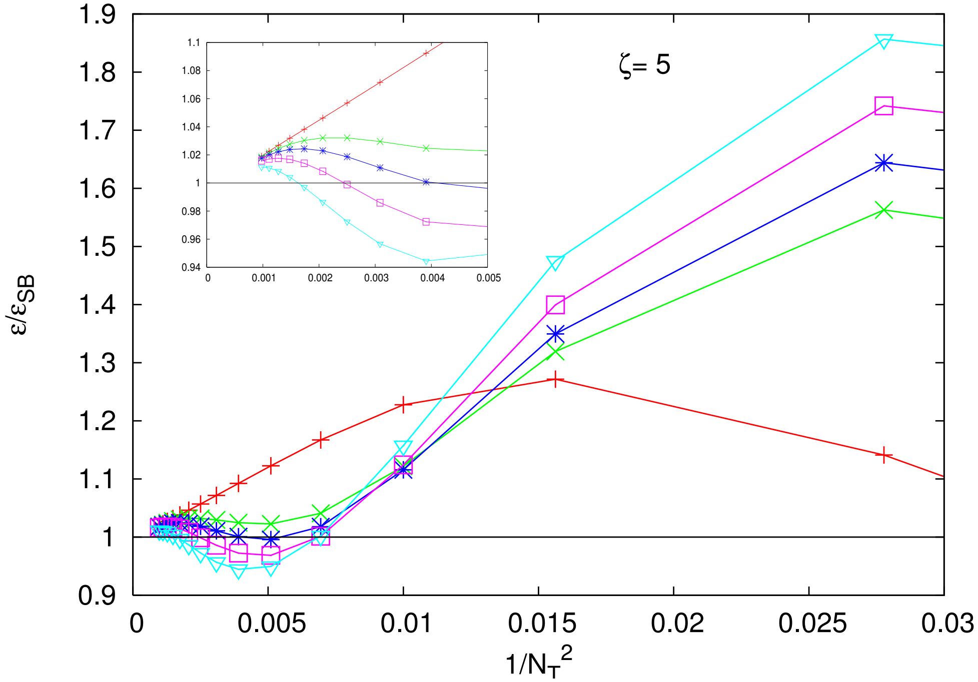

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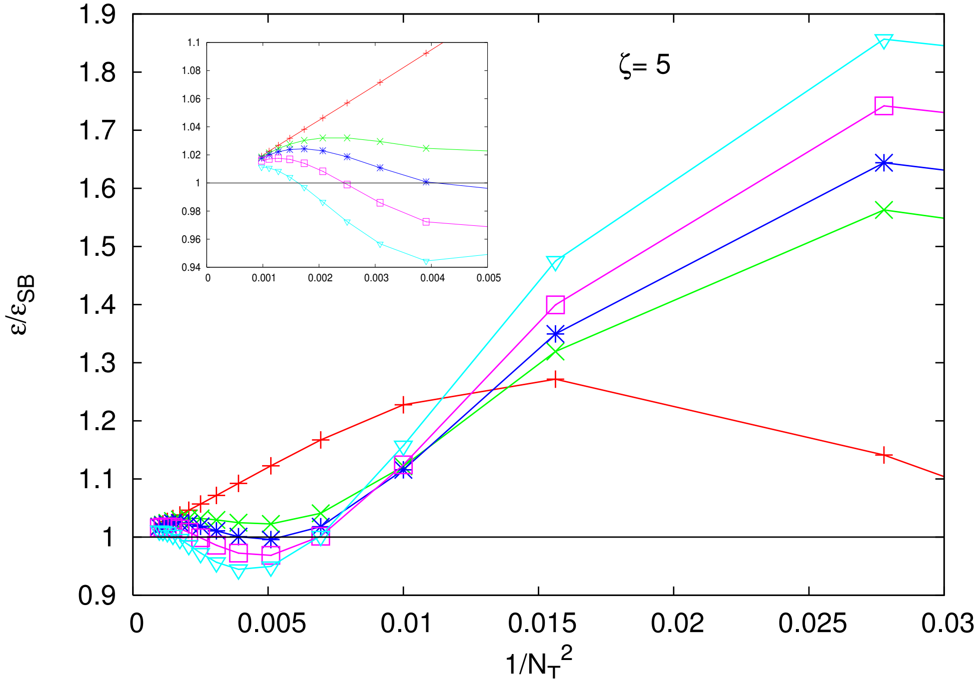
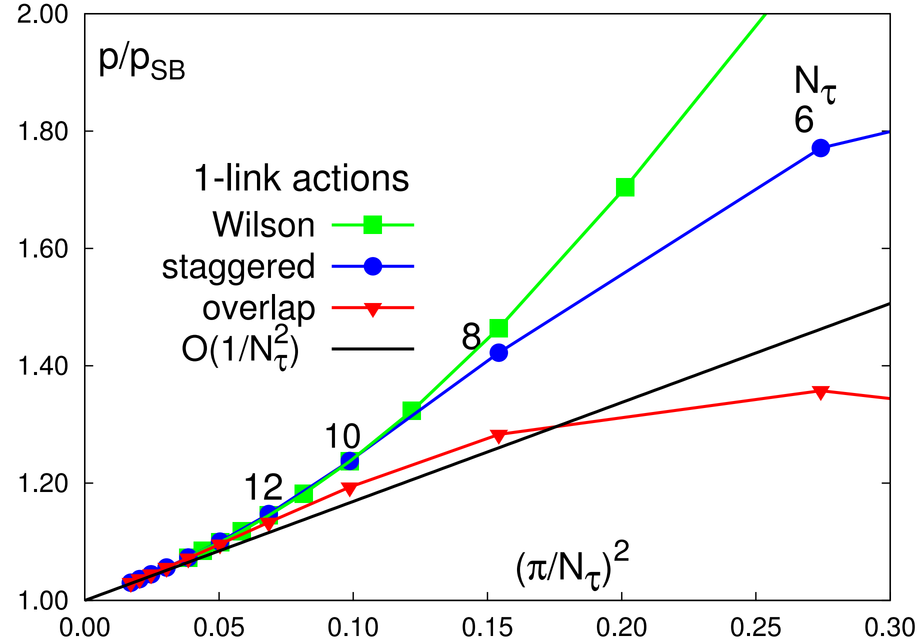


Banerjee, Gavai & Sharma , arXiv:0803.3925



Hegde, Karsch, Laermann & and Shcheredin, arXiv:0801.4883

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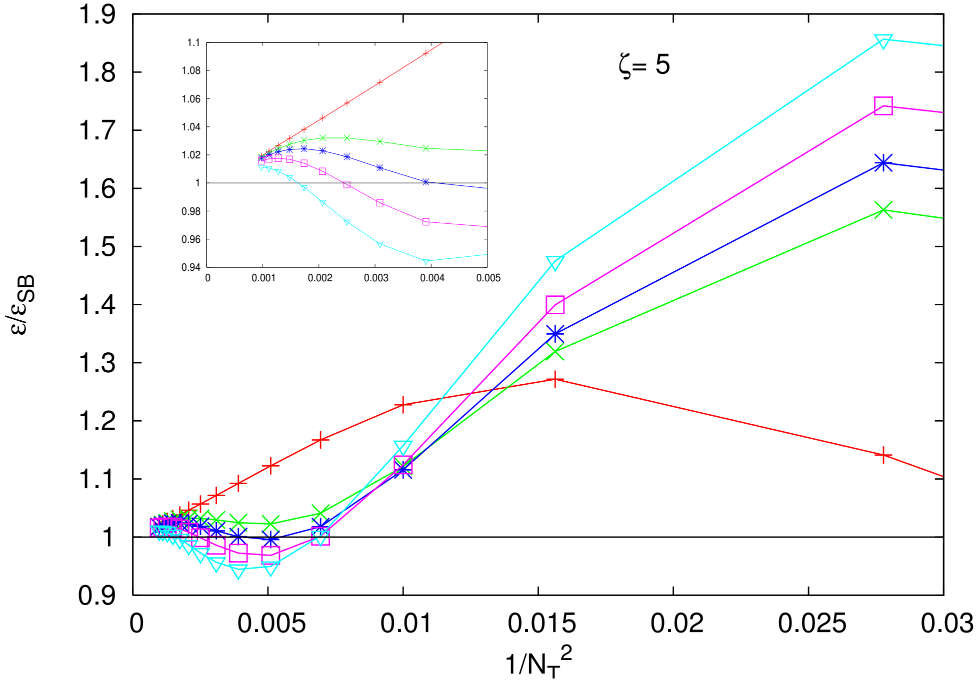
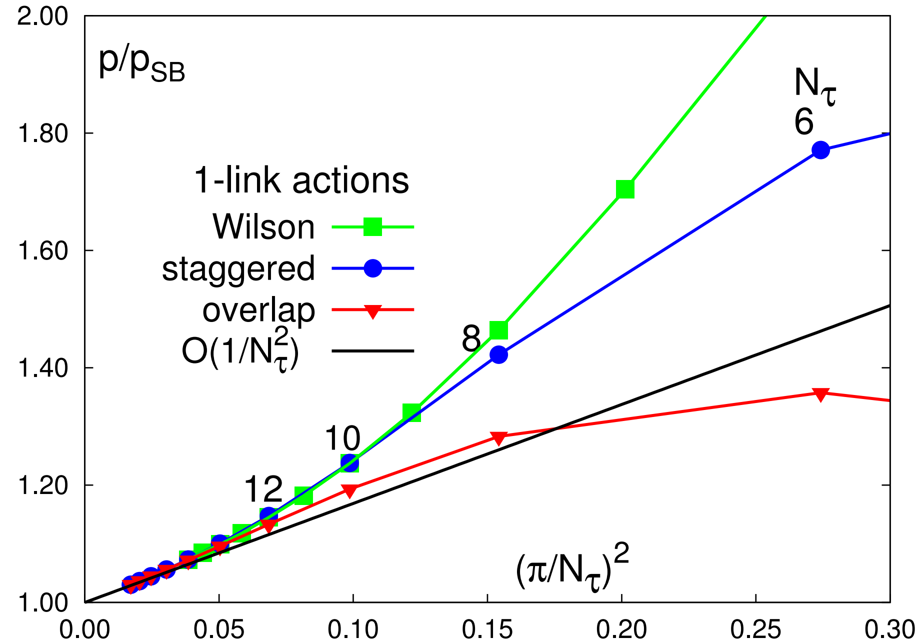


Banerjee, Gavai & Sharma , arXiv:0803.3925



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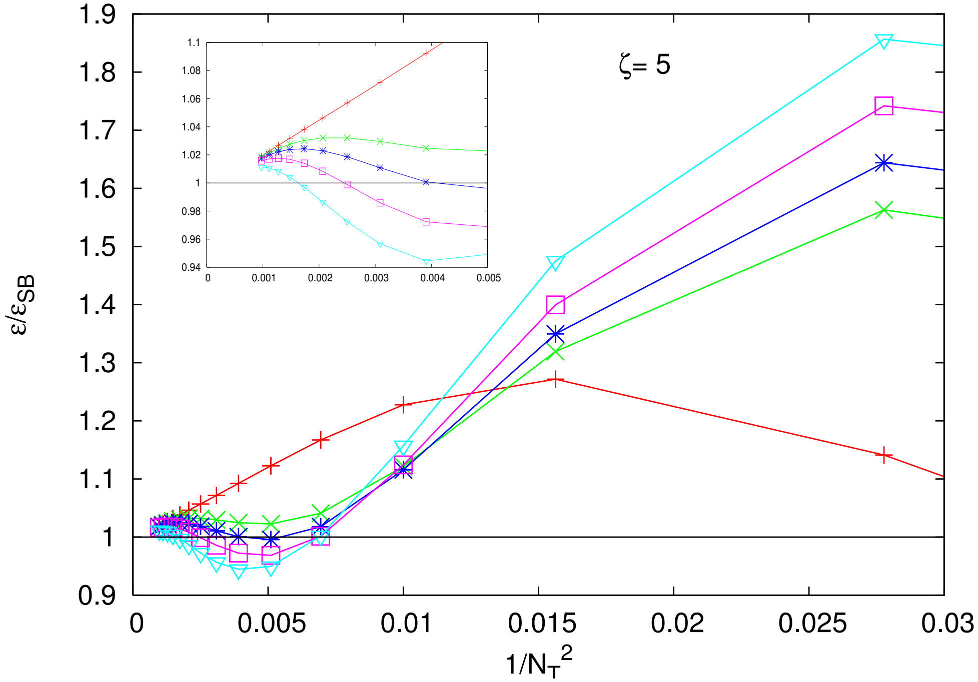

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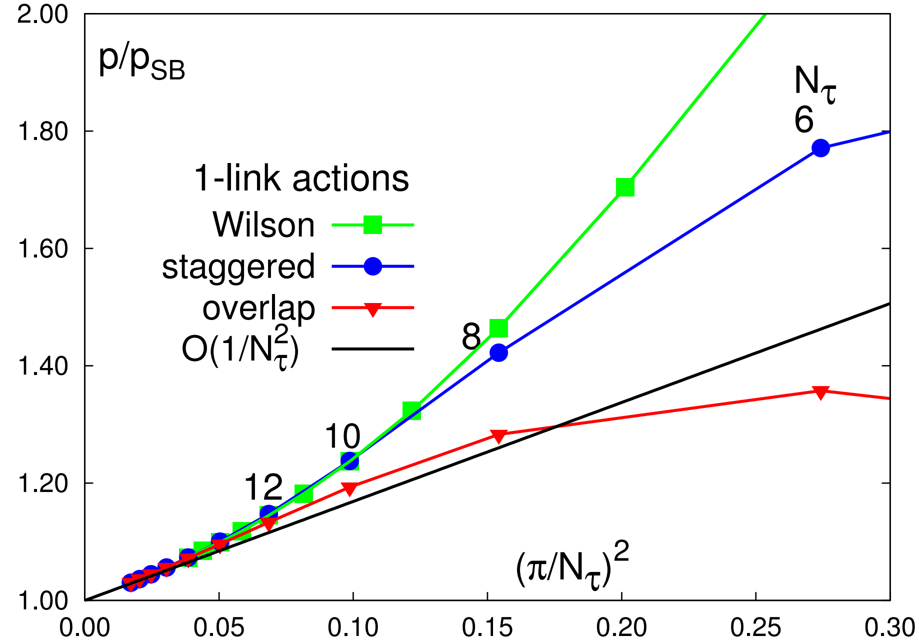

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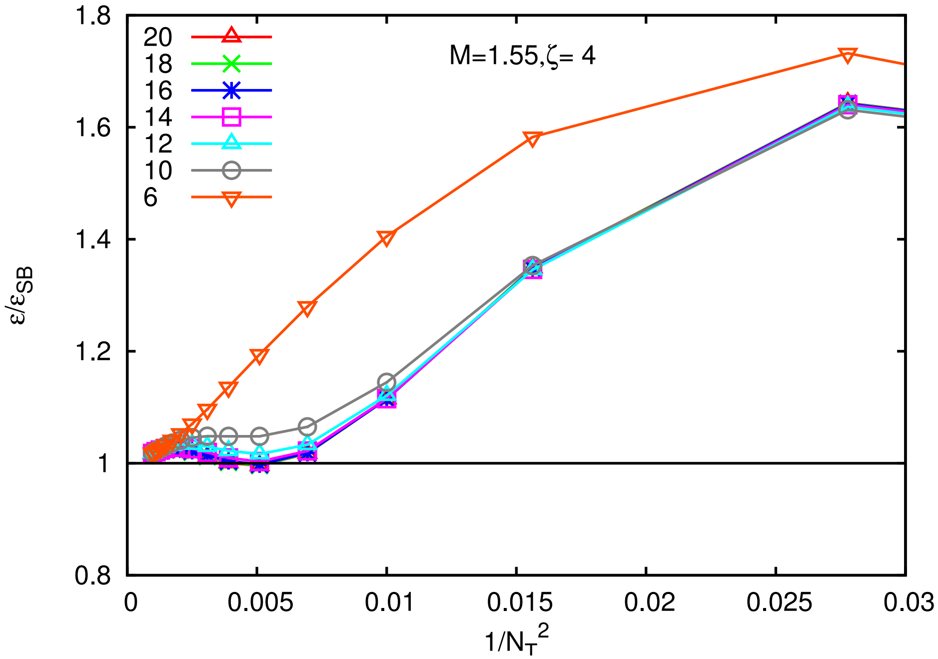
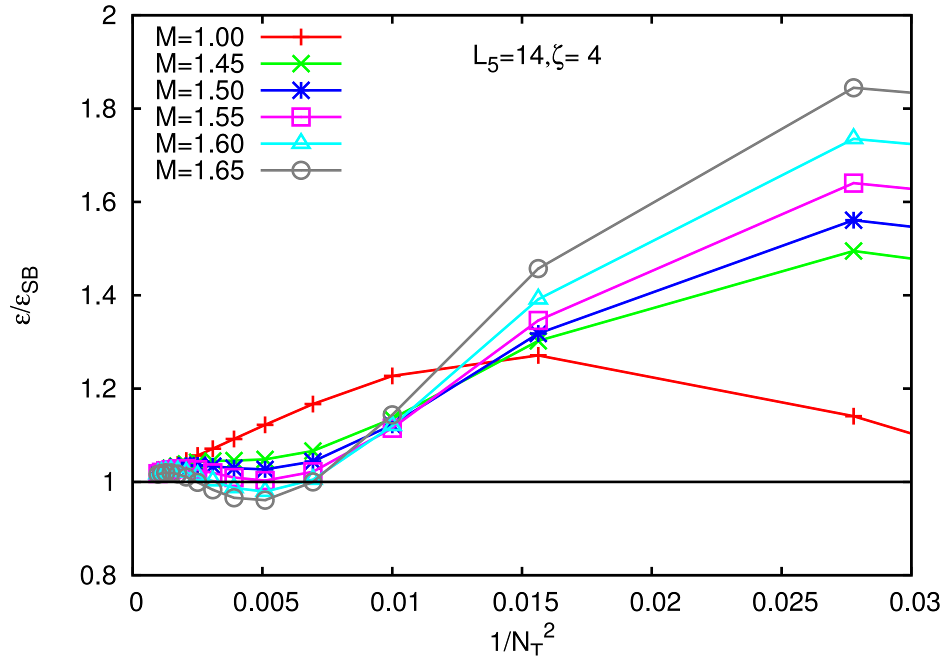
Domain Wall Fermions ($a_5 \to 0$)





Rajiv V. Gavai and Sayantan Sharma, in preparation.

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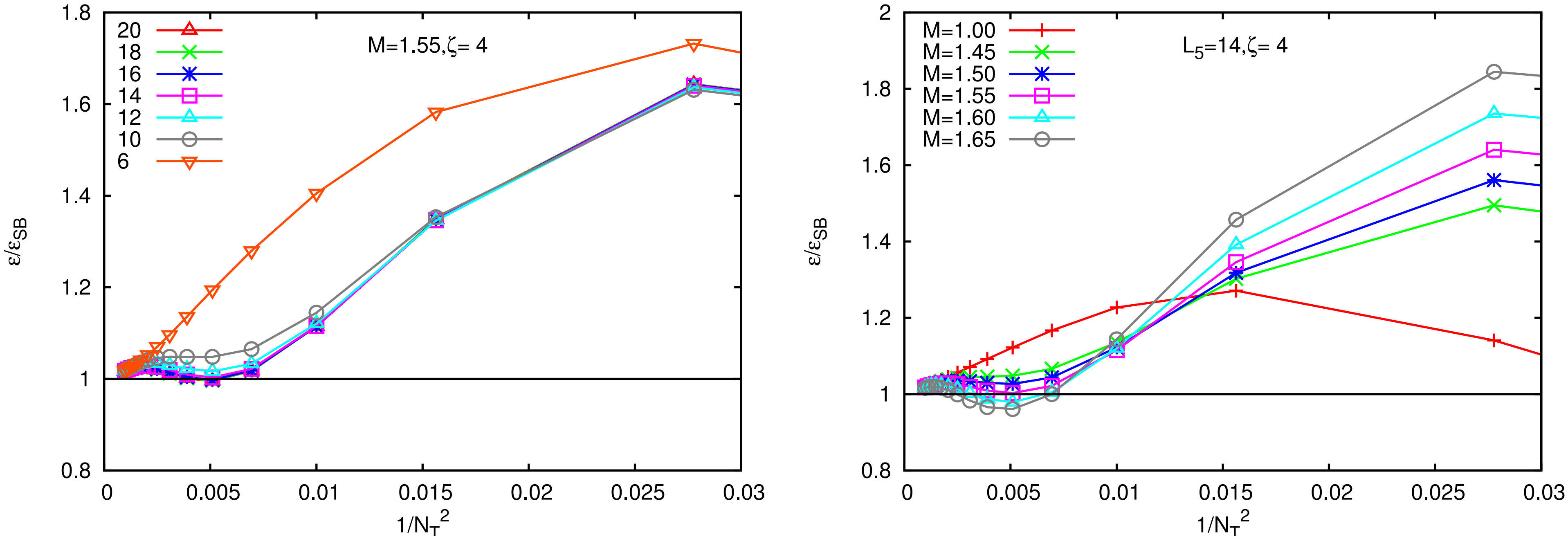

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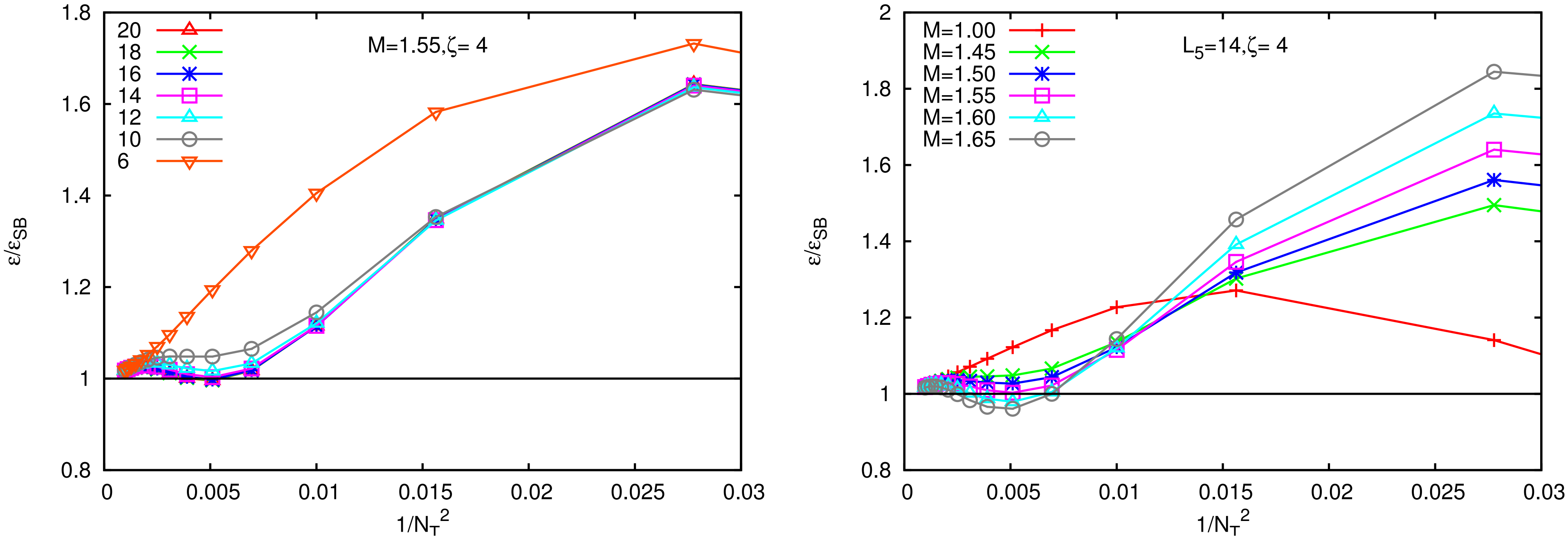

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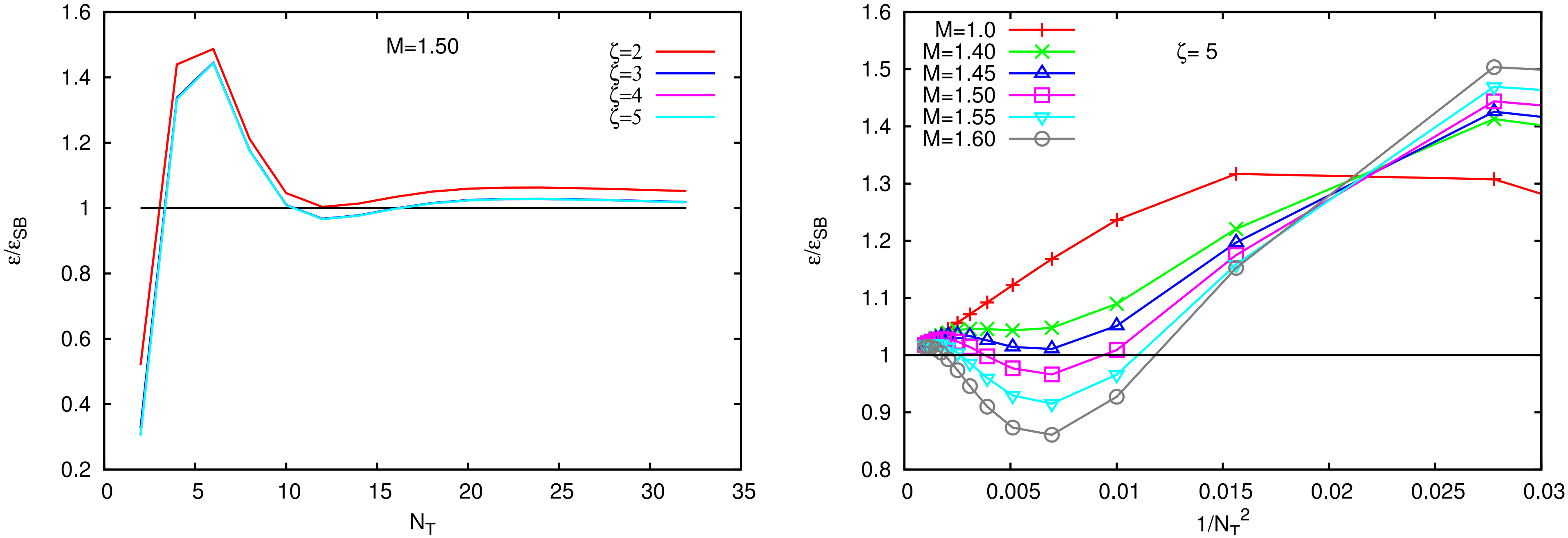
Domain Wall Fermions ($a_5 = 1$)



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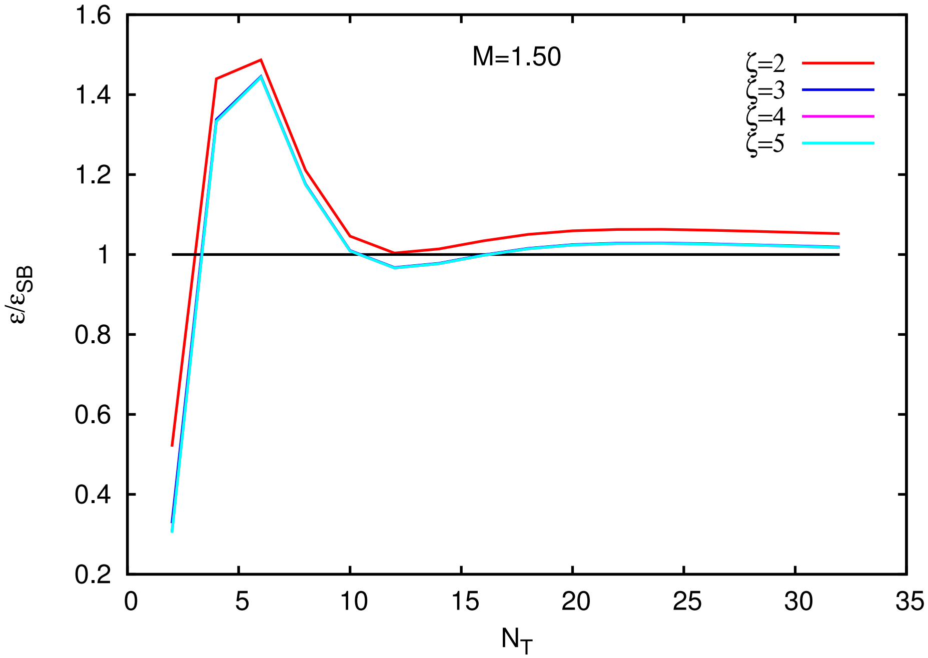

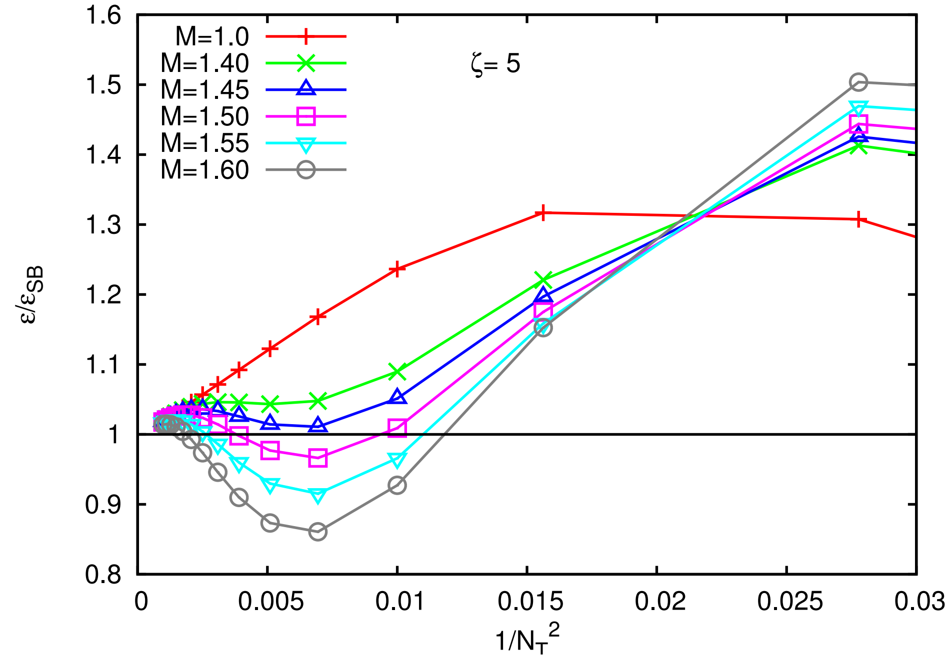

Rajiv V. Gavai and Sayantan Sharma, in preparation.

◊ $\zeta \geq 4$ seems to be large enough to get thermodynamic limit.
◊ Optimal range now seems to be $1.40 \leq M \leq 1.50$; $M = 1.9$ used by Chen et al. (PRD 2001) in their study of order parameters of FTQCD.

Numerical Evaluation

♢ Two Observables : $\Delta\epsilon(\mu, T) = \epsilon(\mu, T) - \epsilon(0, T)$ and Susceptibility, $\sim \partial^2 \ln \mathcal{Z}/\partial\mu^2$.

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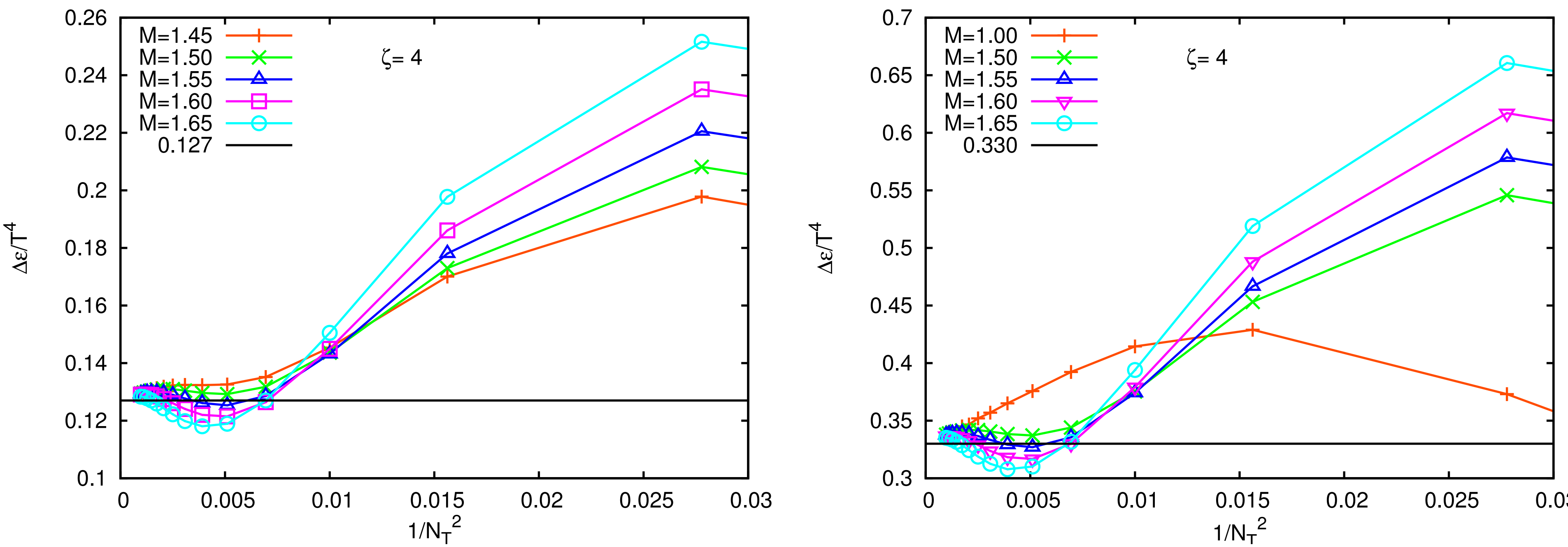
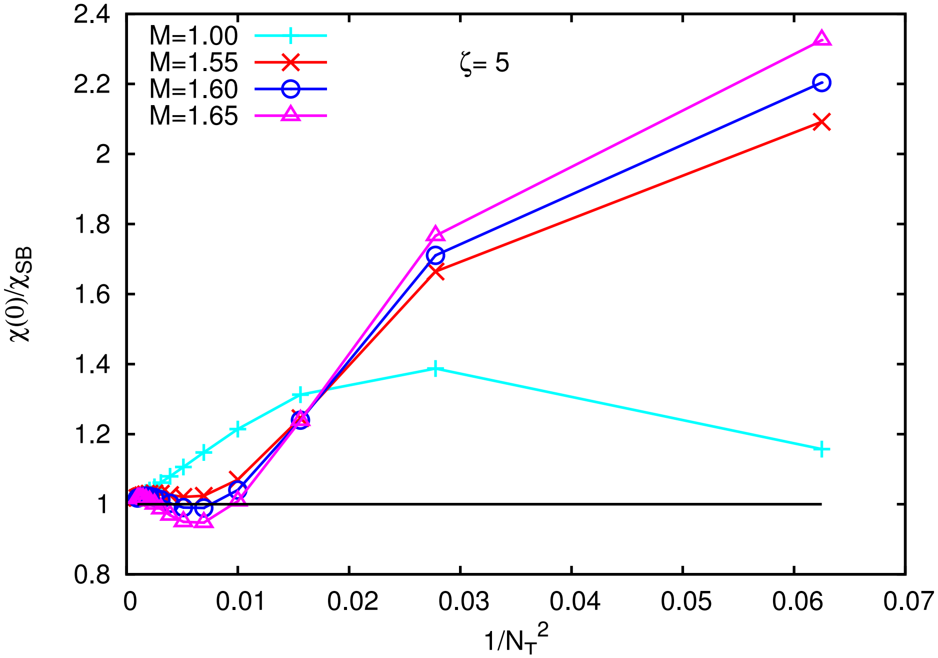
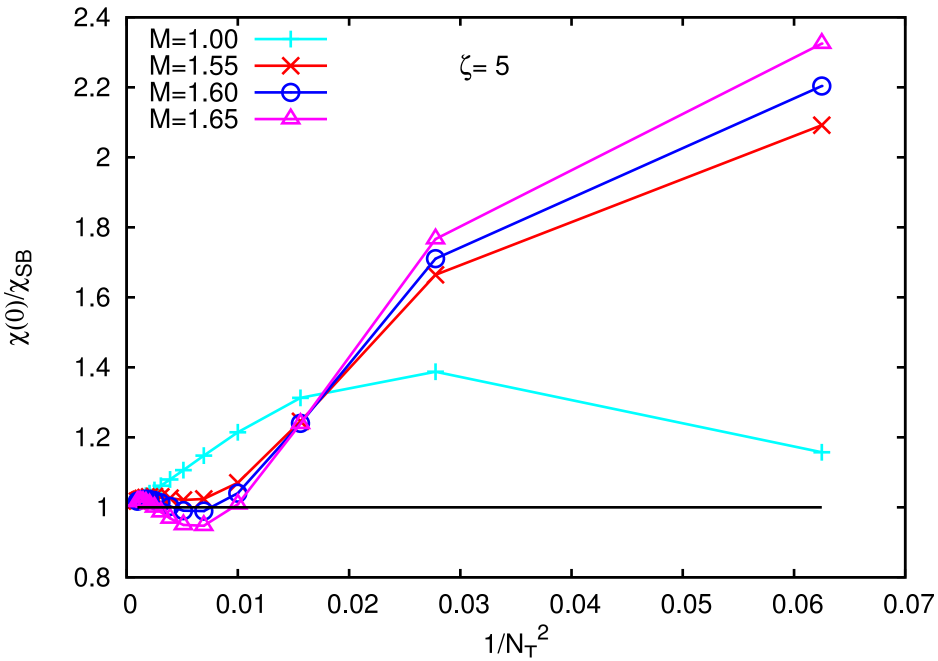
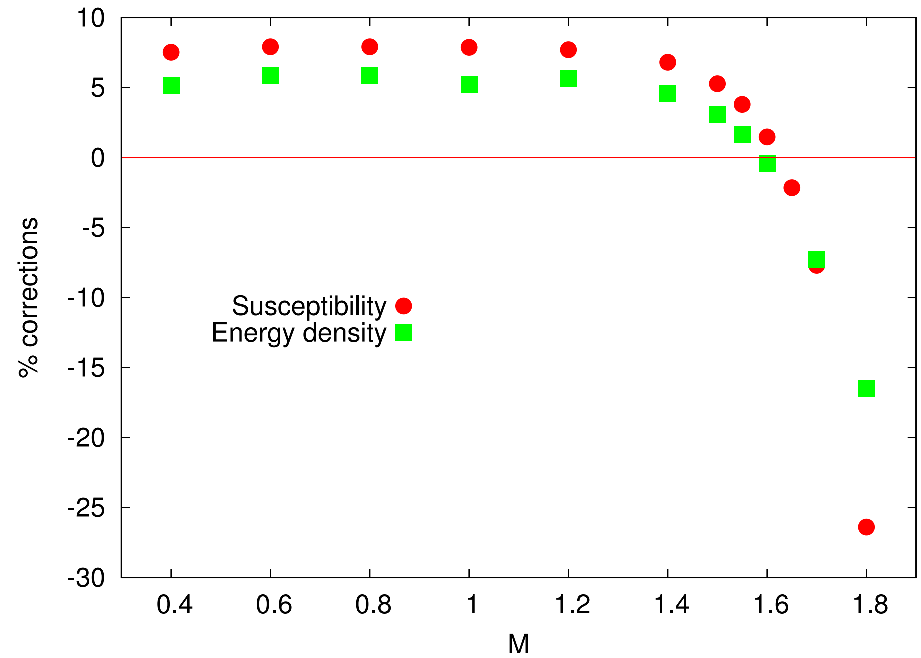


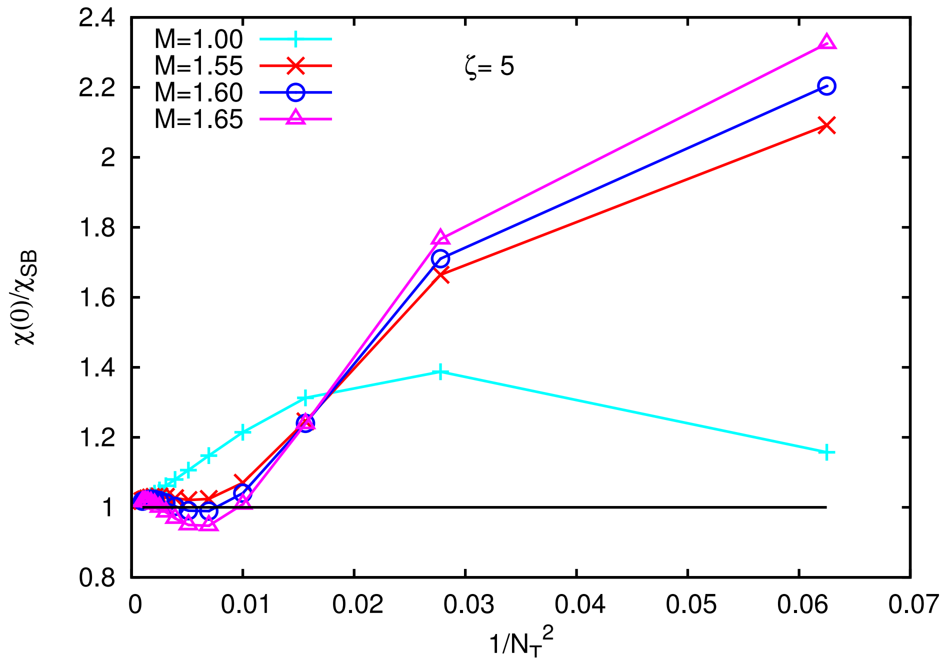


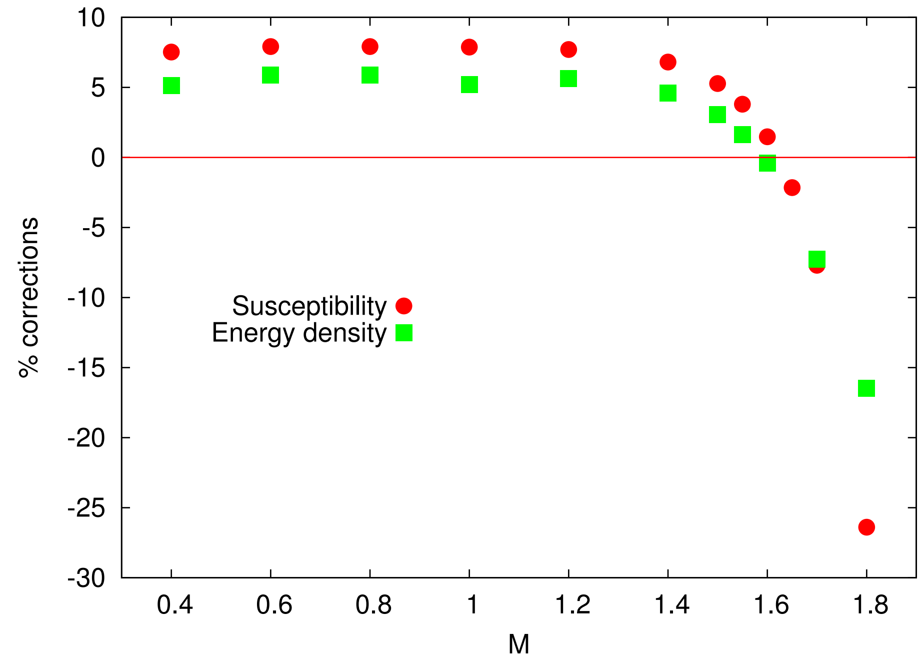
M=1.00
M=1.55
M=1.60
M=1.65
ζ= 5
χ(0)/χ_SB
1/N_T^2
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07





♡ Susceptibility too behaves the same way as the energy density.





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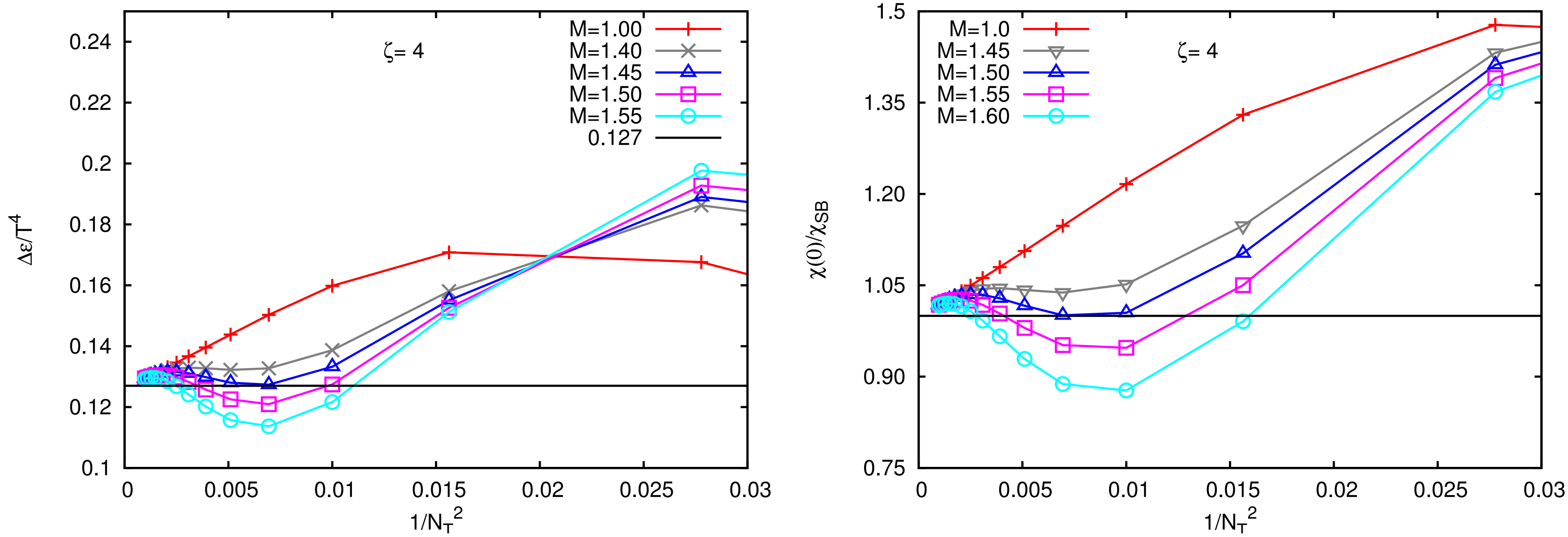

Domain Wall Fermions ($a_5 = 1$)

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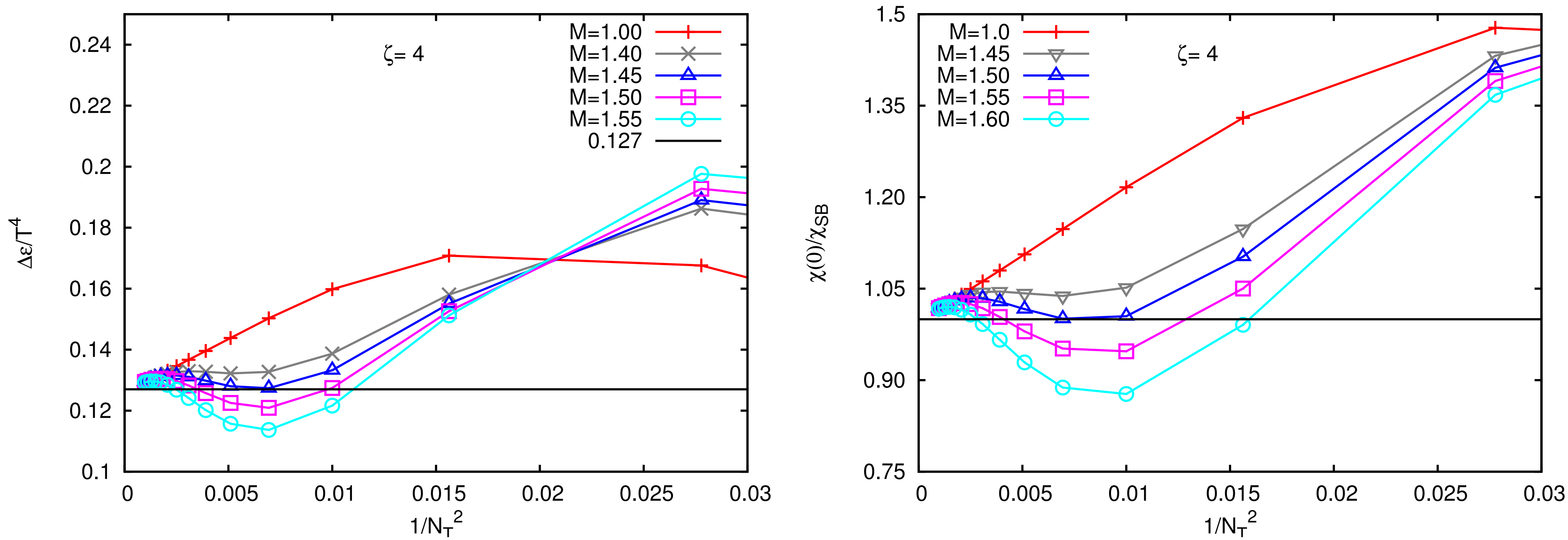
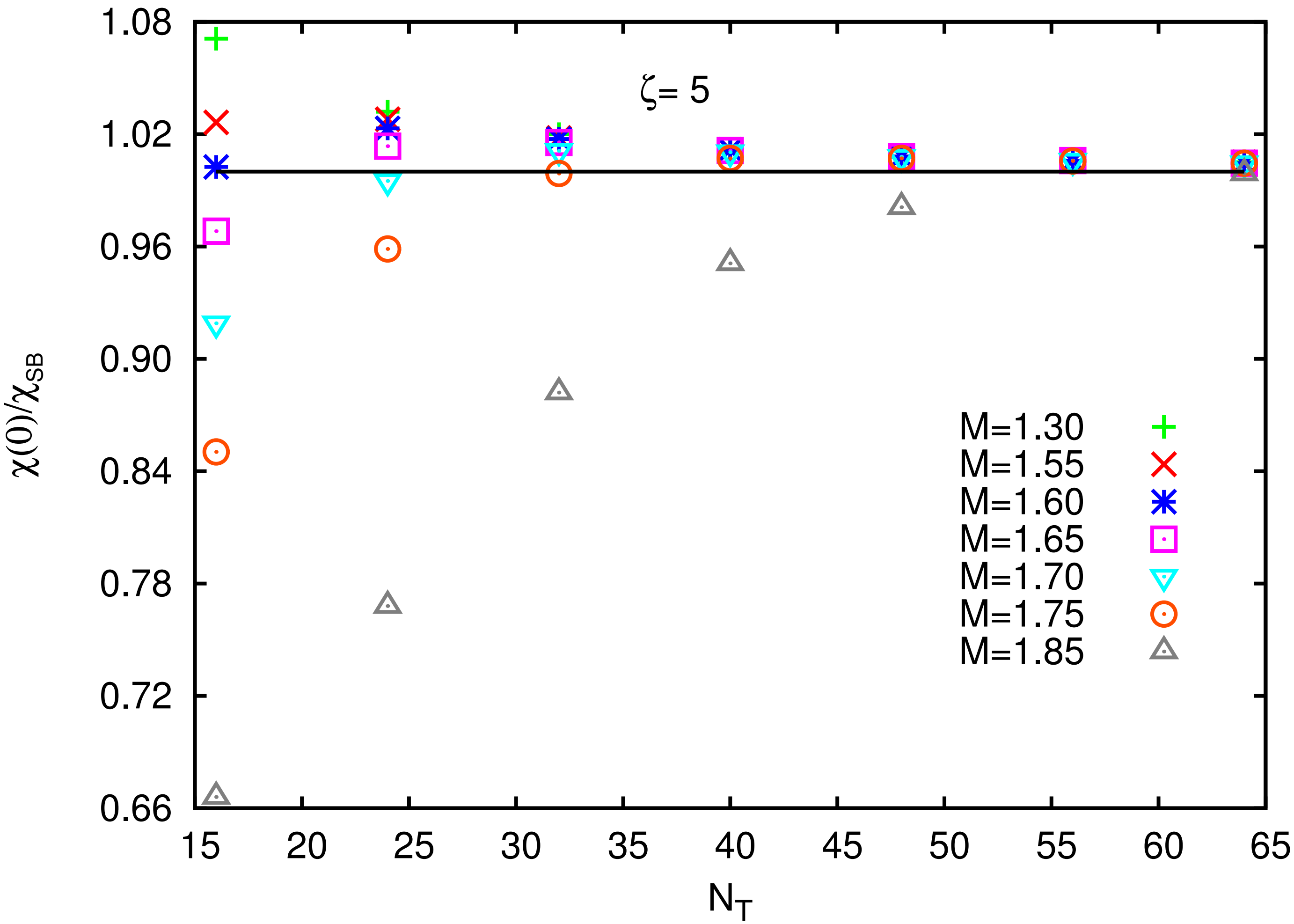


Domain Wall Fermions ($a_5 = 1$)

♡ Again Susceptibility behaves the same way as the energy density.

♡ Again $1.40 \leq M \leq 1.50$ seems optimal, with small deviations already $N_T = 12$.


ζ= 5
χ(0)/χ_SB
N_T
1.08
1.02
0.96
0.90
0.84
0.78
0.72
0.66
15
20
25
30
35
40
45
50
55
60
65
M=1.30
M=1.55
M=1.60
M=1.65
M=1.70
M=1.75
M=1.85

Summary

- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

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- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

- However, any μ^2-divergence in the continuum limit is avoided for it and an associated general class of functions $K(\mu)$ and $L(\mu)$ with $K(\mu) \cdot L(\mu) = 1$.

- For the choice of $1.5 \leq M \leq 1.6$ ($1.4 \leq M \leq 1.5$), both the energy density and the quark number susceptibility at $\mu = 0$ exhibited the smallest deviations from the ideal gas limit for $N_T \geq 12$ for Overlap (Domain Wall) Fermions.

Consequences

- Exact Chiral Symmetry on lattice lost for any $\mu \neq 0$: Real or Imaginary! Note $D_w(a\mu)$ is Hermitian for the latter case.

- μ-dependent mass for even massless quarks.

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- μ-dependent mass for even massless quarks.
- Only smooth chiral condensates : No (clear) chiral transition for any (large) μ possible. How small a, or large N_T may suffice ?
- All coefficients of a Taylor expansion in μ do have the chiral invariance but the series will be smooth and should always converge.

What if . . .

♠ the chiral transformations were $\delta\psi = \alpha\gamma_5(1 - \frac{a}{2}D(a\mu))\psi$ and $\delta\bar{\psi} = \alpha\bar{\psi}(1 - \frac{a}{2}D(a\mu))\gamma_5$?

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- Moreover, symmetry groups *different* at each μ. Recall we wish to investigate $\langle\bar{\psi}\psi\rangle(a\mu)$ to explore if chiral symmetry is restored.

- The symmetry group remains *same* at each T with $\mu = 0$ $\Longrightarrow \langle\bar{\psi}\psi\rangle(am = 0, T)$ is an order parameter for the chiral transition.