for QCD Critical Point

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Importance of Being Critical
Lattice QCD Results
Searching Experimentally
Summary
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Phase Diagram of Water
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- One, possibly two, critical points
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- Extreme density fluctuations \(\implies\) Opalescent turbidity
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- Dielectric constant & Viscosity $\downarrow$. 
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Phase Diagram of Water

- One, possibly two, critical points
- Extreme density fluctuations $\Rightarrow$ Opalescent turbidity
- Dielectric constant & Viscosity $\downarrow$.
- Many liquid fueled engines exploit such supercritical transitions.
• Discontinuous $\epsilon$ – Nonzero Latent Heat – & finite $C_v$ → First order PT.
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• Continuous $\epsilon$, & diverging $C_v$ → Second order PT.

• In(Finite) Correlation Length at 2nd (1st) Order transition.
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• Continuous $\epsilon$, & diverging $C_v$ → Second order PT.

• In(Finite) Correlation Length at 2nd (1st) Order transition.

• “Cross-over” – mere rapid change in $\epsilon$, with maybe a sharp peaked $C_v$.
QCD Phase diagram

♣ A fundamental aspect – Critical Point in $T-\mu_B$ plane;
QCD Phase diagram

♠ A fundamental aspect – Critical Point in $T$-$\mu_B$ plane; Based on symmetries and models,
Expected QCD Phase Diagram

From Rajagopal-Wilczek Review
QCD Phase diagram

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... but could, however, be ... McLerran-Pisarski 2007

From Rajagopal-Wilczek Review
From M. Stephanov, Lattice 2007 Plenary.
Lattice QCD Results

• QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics.
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- The Transition Temperature $T_c$, the Equation of State, Flavour Correlations ($C_{BS}$) and the Wróblewski Parameter $\lambda_s$ are some examples for Heavy Ion Physics.
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- Domain Wall or Overlap Fermions better. BUT Computationally expensive and introduction of $\mu$ unfortunately breaks chiral symmetry! (Banerjee, Gavai & Sharma PRD 2008; arXiv:0809.4535 & arXiv:0811.3026)
The $\mu \neq 0$ problem

Assuming $N_f$ flavours of quarks, and denoting by $\mu_f$ the corresponding chemical potentials, the QCD partition function is

$$Z = \int DU \exp(-S_G) \prod_f \text{Det} M(m_f, \mu_f),$$

and the thermal expectation value of an observable $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle = \frac{\int DU \exp(-S_G) \mathcal{O} \prod_f \text{Det} M(m_f, \mu_f)}{Z}. $$
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$$\langle O \rangle = \frac{\int DU \exp(-S_G) O \prod_f \text{Det} M(m_f,\mu_f)}{Z}.$$

Simulations can be done IF $\text{Det} M > 0$ for any set of $\{U\}$ as probabilistic methods are used to evaluate $\langle O \rangle$.

However, $\text{det} M$ is a complex number for any $\mu \neq 0$ : The Phase/sign problem
Lattice Approaches

Several Approaches proposed in the past two decades: None as satisfactory as the usual $T \neq 0$ simulations. Still scope for a good/great idea!
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- Better control of systematic errors.

We study volume dependence at several $T$ to i) bracket the critical region and then to ii) track its change as a function of volume.
How Do We Do This Expansion?

Canonical definitions yield various number densities and susceptibilities:

\[ n_i = \frac{T}{V} \frac{\partial \ln Z}{\partial \mu_i} \quad \text{and} \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_i \partial \mu_j}. \]

These are also useful by themselves both theoretically and for Heavy Ion Physics (Flavour correlations, \( \lambda_s \ldots \))

Denoting higher order susceptibilities by \( \chi_{n_u,n_d} \), the pressure \( P \) has the expansion in \( \mu \):

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\frac{\Delta P}{T^4} \equiv \frac{P(\mu, T)}{T^4} - \frac{P(0, T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!} \left( \frac{\mu_u}{T} \right)^{n_u} \frac{1}{n_d!} \left( \frac{\mu_d}{T} \right)^{n_d} \quad (1)
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\]
● From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

● Successive estimates for the radius of convergence can be obtained from these using
\[ \sqrt{\frac{n(n+1)\chi_B^{(n+1)}}{\chi_B^{(n+3)}}} \text{ or } \left( n! \frac{\chi_B^{(2)}}{\chi_B^{(n+2)}} \right)^{1/n} \]. We use both and terms up to 8th order in \( \mu \).

● All coefficients of the series must be POSITIVE for the critical point to be at real \( \mu \), and thus physical.

● Coefficients for the off-diagonal susceptibility, \( \chi_{11} \), can be constructed similarly.

● The ratio \( \chi_{11}/\chi_{20} \) can be shown to yield the ratio of widths of the measure in the imaginary and real directions at \( \mu = 0 \).
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CRAY X1 of I L G T I , T I F R, Mumbai
Our Simulations & Results

• Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.

• $m_\rho/T_c = 5.4 \pm 0.2$ and $m_\pi/m_\rho = 0.31 \pm 0.01$ (MILC)

• Earlier Lattice : $4 \times N_s^3$, $N_s = 8, 10, 12, 16, 24$ (Gavai-Gupta, PRD 2005)

• Lattice used : $6 \times N_s^3$, $N_s = 12, 18, 24$ (Gavai-Gupta, arXiv:0806.2233, PRD in press). Needed to determine $\beta_c$. Our result ($\beta_c = 5.425(5)$) well bracketed by MILC for $m/T_c = 0.075$ and 0.15.
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- New Simulations made at \( T/T_c = 0.89(1), 0.92(1), 0.94(1), 0.97(1), 0.99(1) \), 1.00(1), 1.21(1), 1.33(1), 1.48(3) and 1.92(5)

- Typical stat. 50-200 in max autocorrelation units.
$m_πN_S$

$\mu_B/T$

$T/T_c=0.95 : 6/8 4/6$

$T/T_c=0.99$

$\mu/(3T)$

QGP Meet 2008, VECC, Kolkata, November 25, 2008
• We (RVG & S. Gupta, PRD 2005 and arXiv:0806.2233) use terms up to 8th order in $\mu$.

• Our estimate consistent with Fodor & Katz (2002) [$m_\pi/m_\rho = 0.31$ and $N_s m_\pi \sim 3-4$].
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• Strong finite size effects for small $N_s$. A strong change around $N_s m_\pi \sim 6$. (Compatible with arguments of Smilga & Leutwyler and also seen for hadron masses by Gupta & Ray)
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• $T^E_c = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$ for finer lattice: Our earlier coarser lattice result was $\frac{\mu_B^E}{T^E} = 1.3 \pm 0.3$. Infinite volume result: $\downarrow$ to 1.1(1)
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• Critical point shifted to smaller $\mu_B/T \sim 1 - 2$. 

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Measure of the seriousness of sign problem: $\chi_{11}$; $N_t = 4$ & 6 agree.
Cross Check on $\mu^E/T^E$

♠ Use Padé approximants for the series to estimate the radius of convergence.
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♠ Use Padé approximants for the series to estimate the radius of convergence.

♥ Consistent Window with our other estimates.
Estimating $T_c(\mu_c)$ and $\mu_c/T$

Status of the RBC-BI project

- calculations for $N_\tau = 4$ and 6; $N_\sigma = 4N_\tau$
- uses an $O(a^2)$ improved staggered action (p4fat3)
- estimator for $\mu_c$:
  $$\left( \frac{\mu_c(T)}{T_c(0)} \right)_n \equiv \rho_n = \frac{T}{T_c(0)} \sqrt{\frac{c_n}{c_{n+2}}}$$

- slight quark mass dependence
- weak cut-off dependence
- $O(\mu^8)$ requires more statistics
Imaginary Chemical Potential

deForcrand-Philpsen JHEP 0811

QGP Meet 2008, VECC, Kolkata, November 25, 2008

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Top
For $N_f = 3$, they find $\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3) \left( \frac{\mu}{\pi T_c} \right)^2 - 47(20) \left( \frac{\mu}{\pi T_c} \right)^4$, i.e., $m_c$ shrinks with $\mu$. 
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Problems: i) $N_f = 3 \rightarrow$ Anomaly and Staggered quarks? ii) Known examples where shapes are different in real/imaginary $\mu$,
Searching Experimentally

- Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases $\mu_B$ (Rajagopal, Shuryak & Stephanov PRD 1999)

- Look for nonmonotonic dependence of the event-by-event fluctuations with colliding energy.
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• Fluctuations in mean $p_T$ of low $p_T$ pions.
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Fluctuations due to the critical point should be dominated by fluctuations of pions with $p_T \leq 500$ MeV/c

suggestion to do analysis with several upper $p_T$ cuts

- $p_T < 750$ MeV/c
- $p_T < 500$ MeV/c
- $p_T < 250$ MeV/c

![Graphs showing energy dependence of $\Phi_{p_T}$ measure](image)

No significant energy dependence of $\Phi_{p_T}$ measure also when low transverse momenta are selected.

Remark: predicted fluctuations at the critical point should result in $\Phi_{p_T} \approx 20$ MeV/c, the effect of limited acceptance of NA49 reduces them to $\Phi_{p_T} \approx 10$ MeV/c
• Proton number fluctuations (Hatta-Stephenov, PRL 2003)

• Neat idea: directly linked to the baryonic susceptibility which ought to diverge at the critical point. Since diverging $\xi$ is linked to $\sigma$ mode, which cannot mix with any isospin modes, expect $\chi_I$ to be regular.
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• Leads to a ratio $\chi_Q:\chi_I:\chi_B = 1:0:4$

• Assuming protons, neutrons, pions to dominate, both $\chi_Q$ and $\chi_B$ can be shown to be proton number fluctuations only.
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• Isentropic trajectories focus at the critical point (Asakawa-Nonaka, PRC 2005).

• This leads to the emission of high $p_T$ particles at earlier times. (Asakawa-Bass-Nonaka-Müller, INT 2008 workshop).

• Note this is NOT a fluctuations signal but model (EoS) dependent?
Focusing Effect

- Isentropic trajectories on $T$-$\mu_B$ plane

With QCD critical point

Bag Model + Excluded Volume Approximation (No Critical Point)

= Usual Hydro Calculation

Focused

Not Focused

Chiho NONAKA
QCD Critical Point?

steeper $\bar{p}$ spectra at high $P_T$

NA49, PRC73,044910(2006)

Chiho NONAKA
Summary

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So far no signs of a critical point in the experimental results at CERN. Will RHIC deliver it for us?