Results from Lattice QCD

Rajiv V. Gavai
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Some Results from Lattice QCD

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Introduction

QCD Phase Diagram

Speed of Sound

J/ψ

Summary

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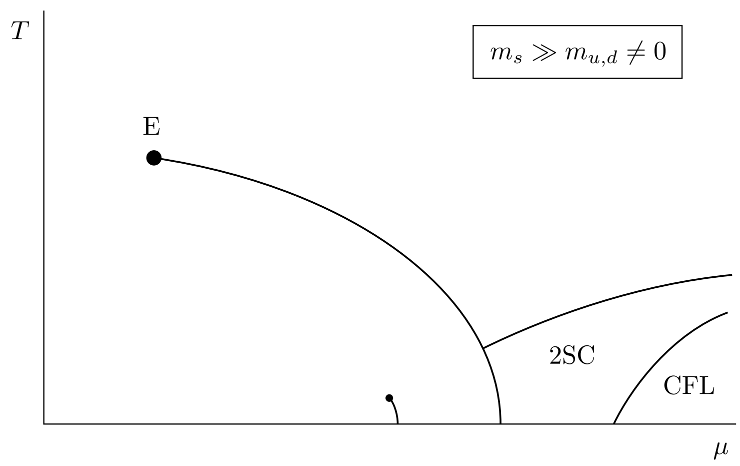
- An interesting theoretical issue – Conformal Invariance and AdS/CFT predictions.

QCD Phase Diagram

Expected QCD Phase Diagram and Lattice Approaches to unravel it.



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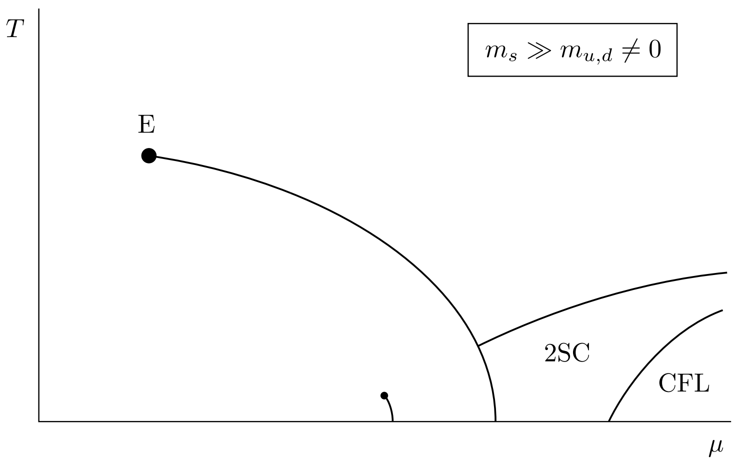

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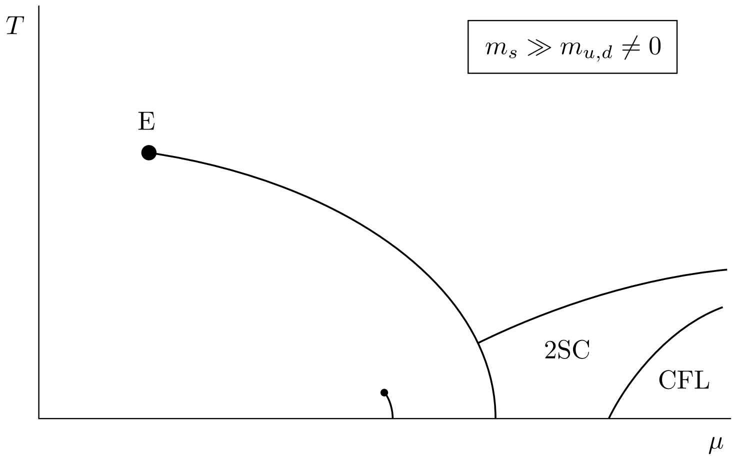

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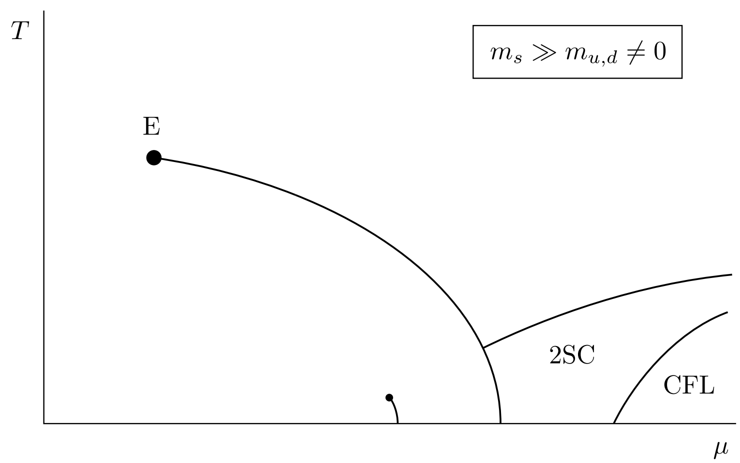

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- Taylor Expansion (C. Allton et al., PR D66 (2002) 074507 & D68 (2003) 014507; R.V. Gavai and S. Gupta, PR D68 (2003) 034506).

Why Taylor series expansion?

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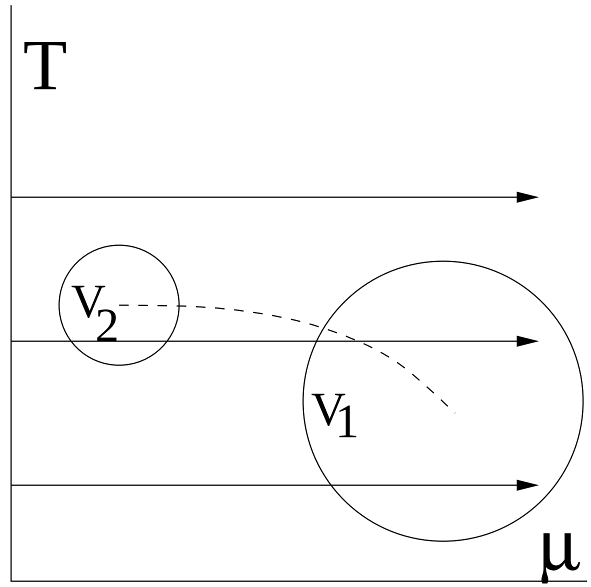

We study volume dependence at several T to i) bracket the critical region and then to ii) track its change as a function of volume.

How Do We Do This Expansion?

Assuming N_f flavours of quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \ \prod_f \mathrm{Det} \ M(m_f,\mu_f) \quad .$$

Canonical definitions then yield various number densities and susceptibilities :

$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i} \text{ and } \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j} \quad .$$

Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure P has the expansion in μ:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d} \frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u} \frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d} \tag{1}$$

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- From this expansion, a series for baryonic susceptibility can be constructed. Its radius of convergence gives the nearest critical point.

- Successive estimates for the radius of convergence can be obtained from these using $\sqrt{\frac{\chi_B^n}{\chi_B^{n+2}}}$. We use terms up to 8th order in μ, i.e., estimates from 2/4, 4/6 and 6/8 terms.

- Coefficients for the off-diagonal susceptibility, χ_{11}, can be constructed similarly.

- The ratio χ_{11}/χ_{20} can be shown to yield the ratio of widths of the measure in the imaginary and real directions at $\mu = 0$.

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Our Simulations & Results

- Lattice used : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24
- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm with traj. length of 1 MD time on $N_s = 8$, scaled $\propto N_s$ on larger ones.
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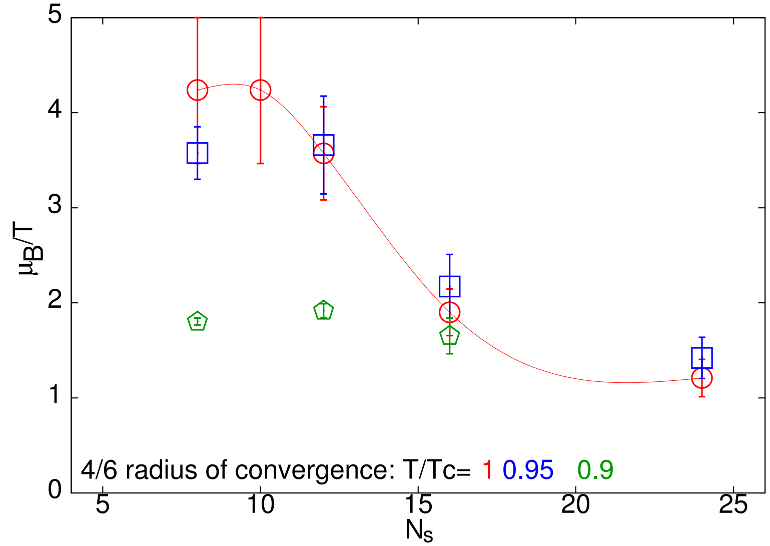
- Simulations made at T/T_c = 0.75(2), 0.80(2), 0.85(1), 0.90(1), 0.95(1), 0.975(10), 1.00(1), 1.05(1), 1.25(1), 1.65(6) and 2.15(10)

- Typical stat. 50-100 in max autocorrelation units.


4/6 radius of convergence: T/Tc= 1 0.95 0.9
μ_B/T
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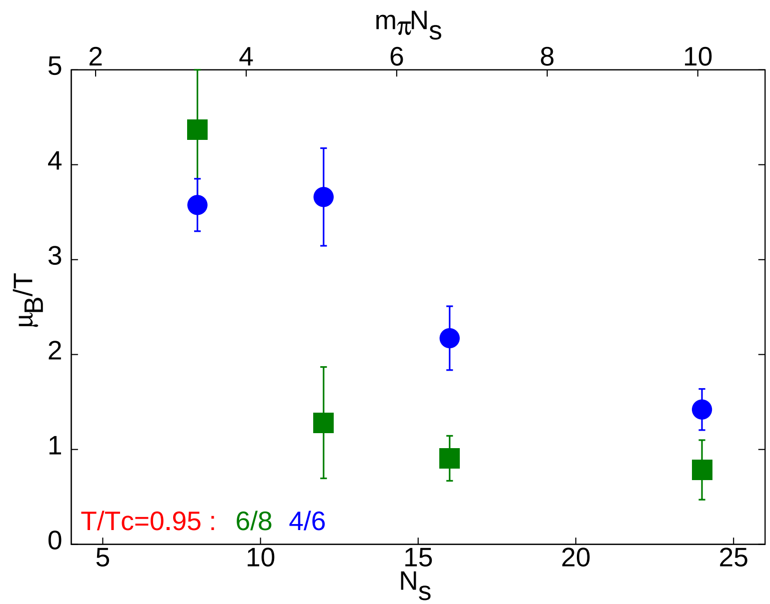
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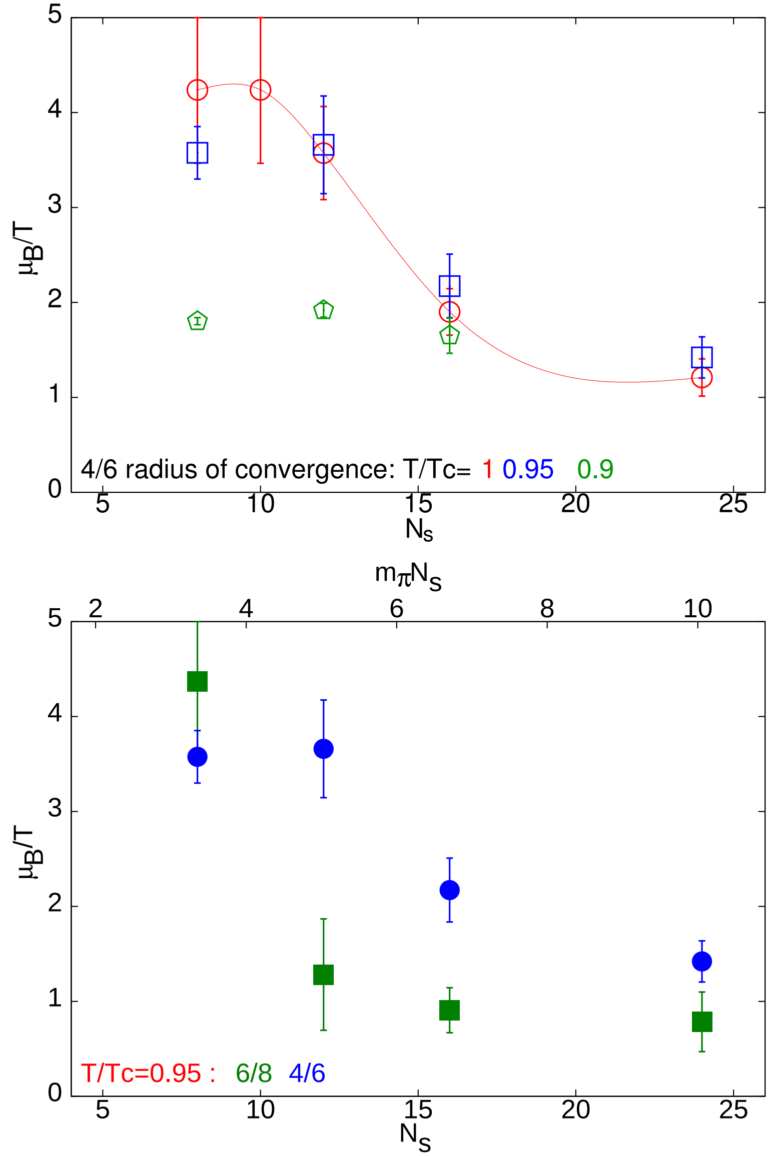

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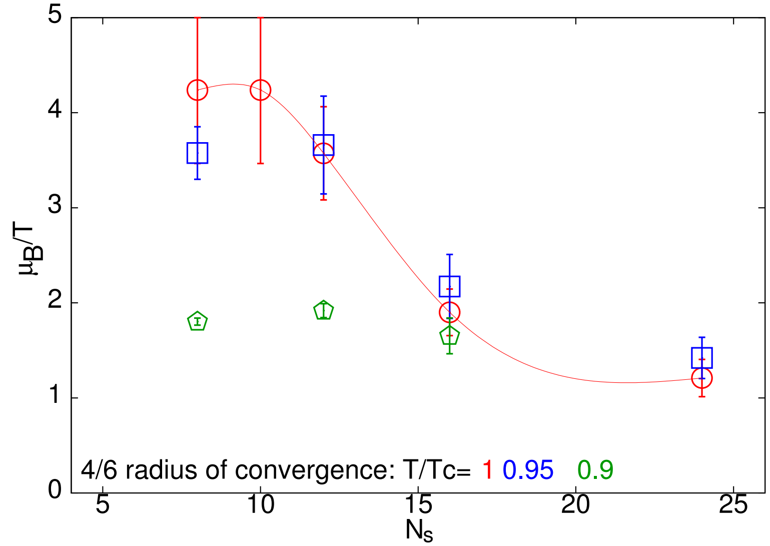

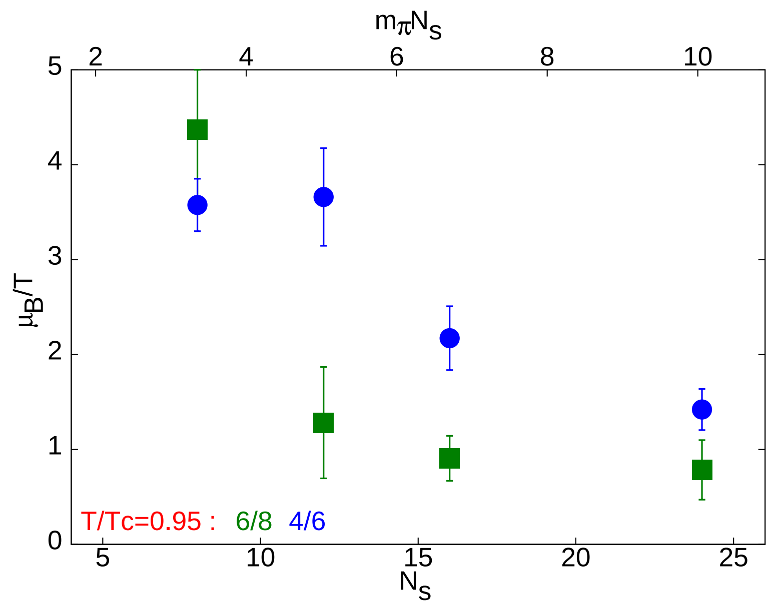

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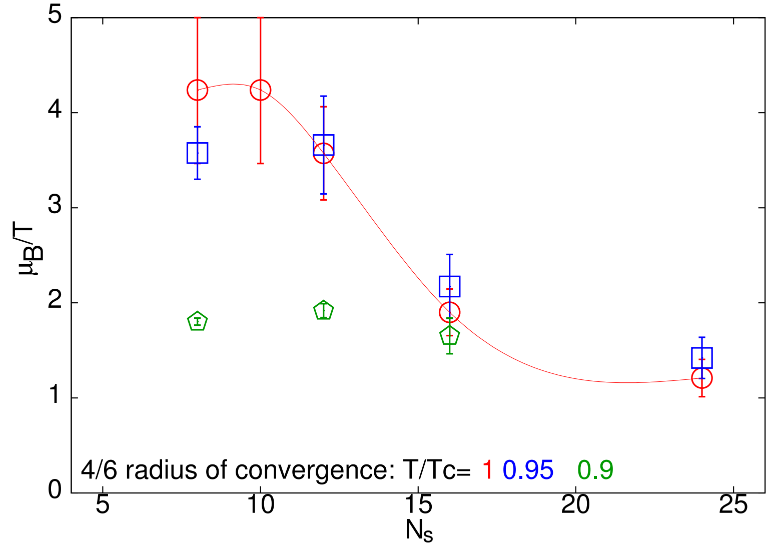

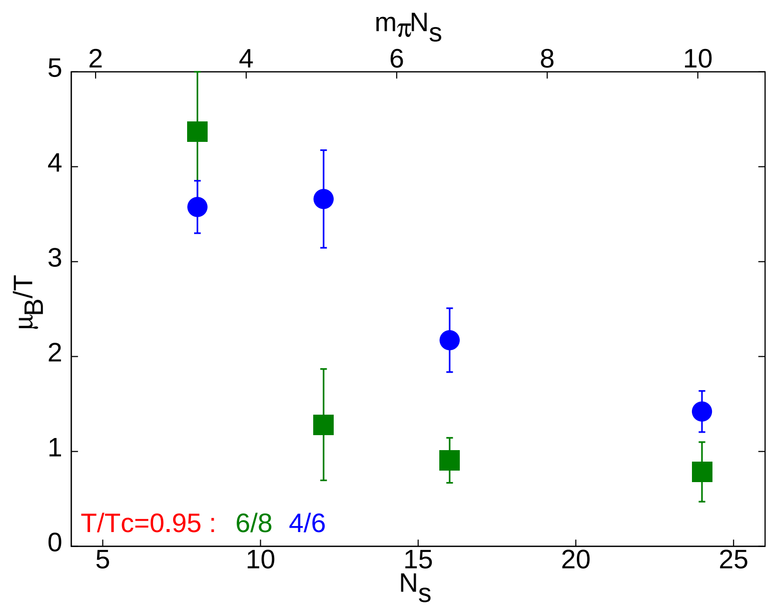

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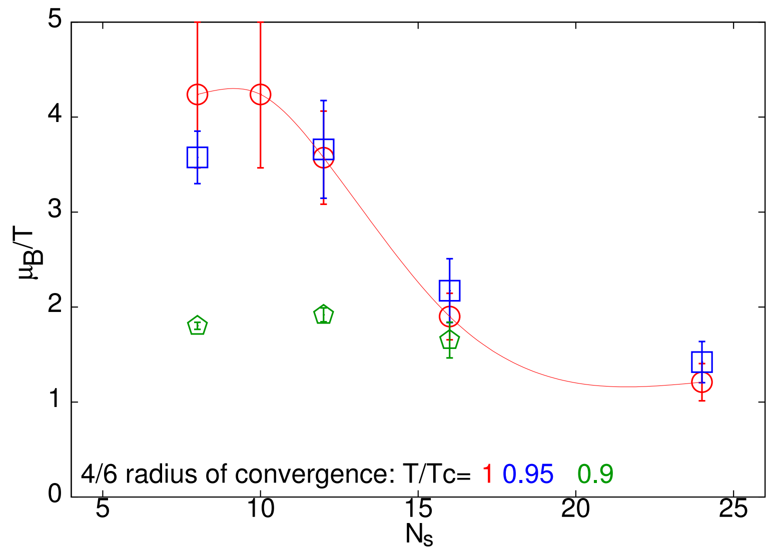

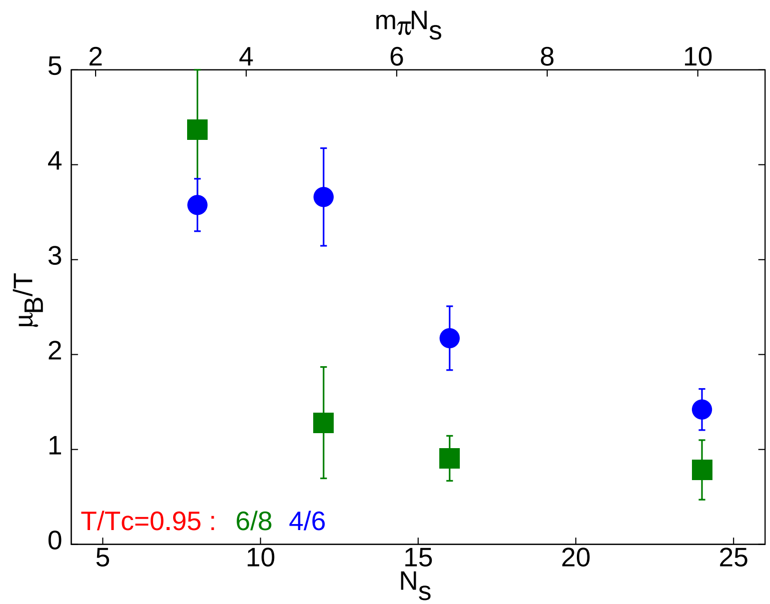

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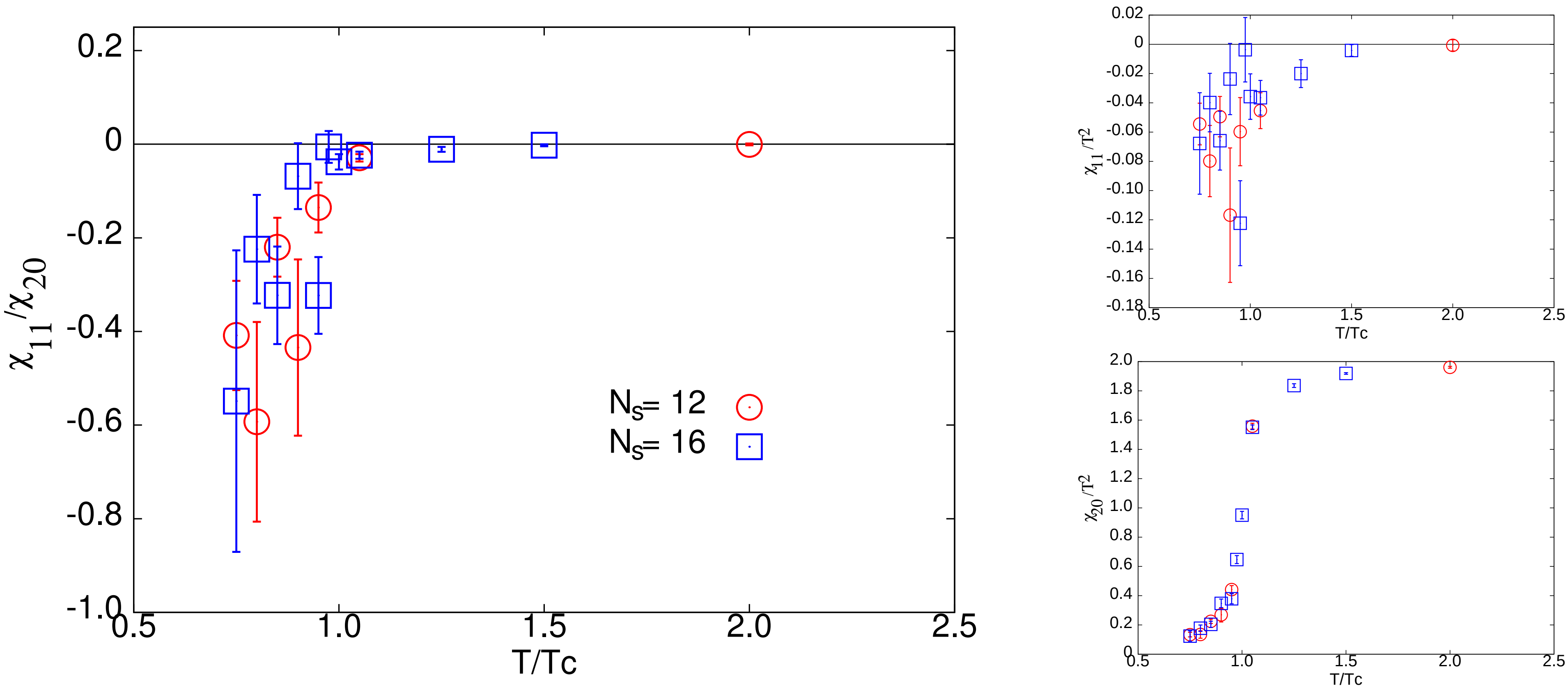
- Bielefeld-Swansea results (hep-lat/0501030) up to 6th order. They use $N_s m_\pi \sim 15$ but have a large $m_\pi/m_\rho \sim 0.7$.

More Details

Measure of the seriousness of sign problem : Ratio χ_{11}/χ_{20}



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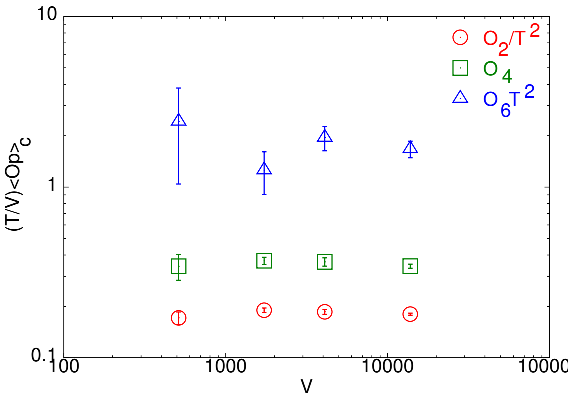
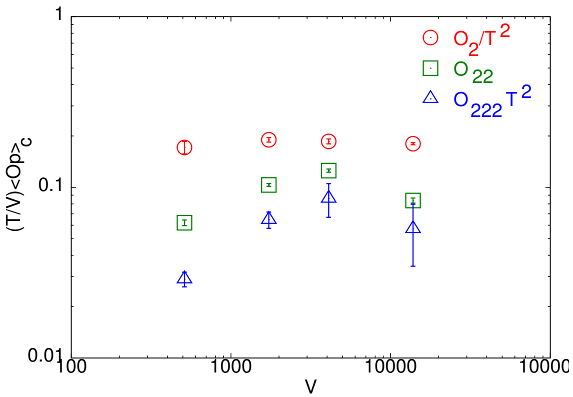
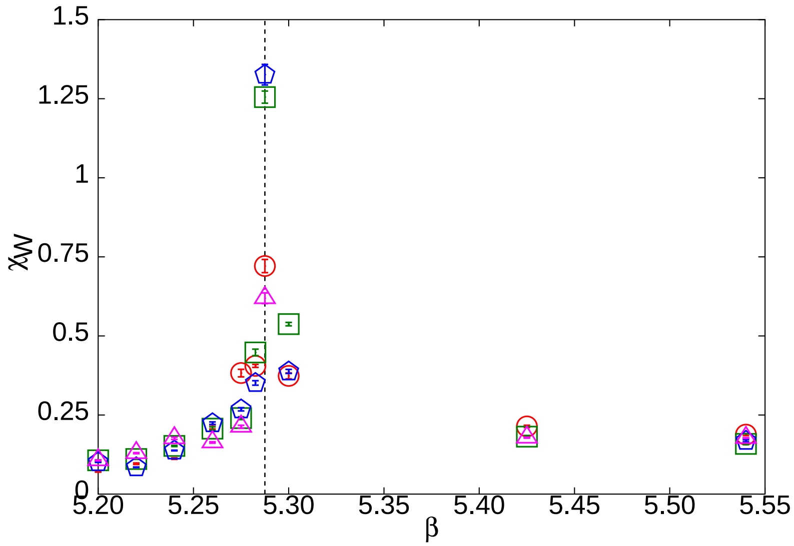
♠ We had earlier suggested to obtain more pairs of diverging terms by taking larger N_f.

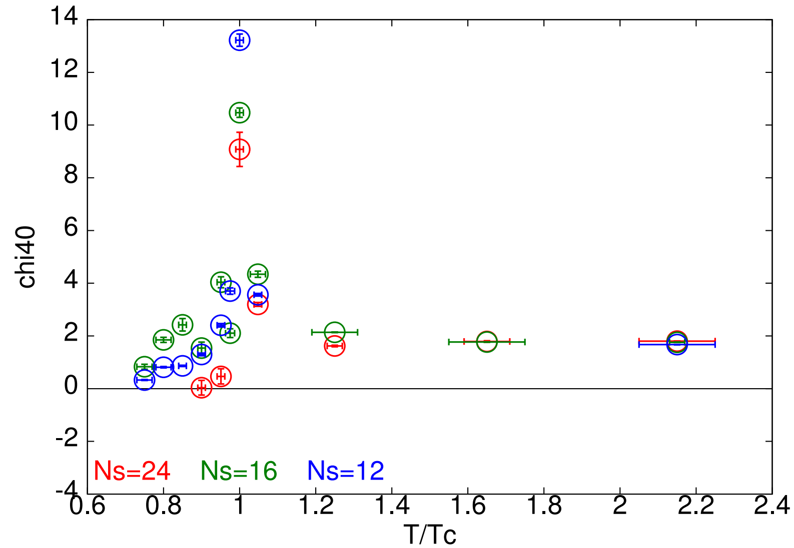
♠ E.g. $T/V\langle\mathcal{O}_{22}\rangle_c$ should be finite as it is a combination of Taylor Coeffs.





♠ Interesting to note that χ_{40} shows the same volume dependence at T_c as χ_L which in turn comes from the $\langle \mathcal{O}_{22} \rangle_c$.





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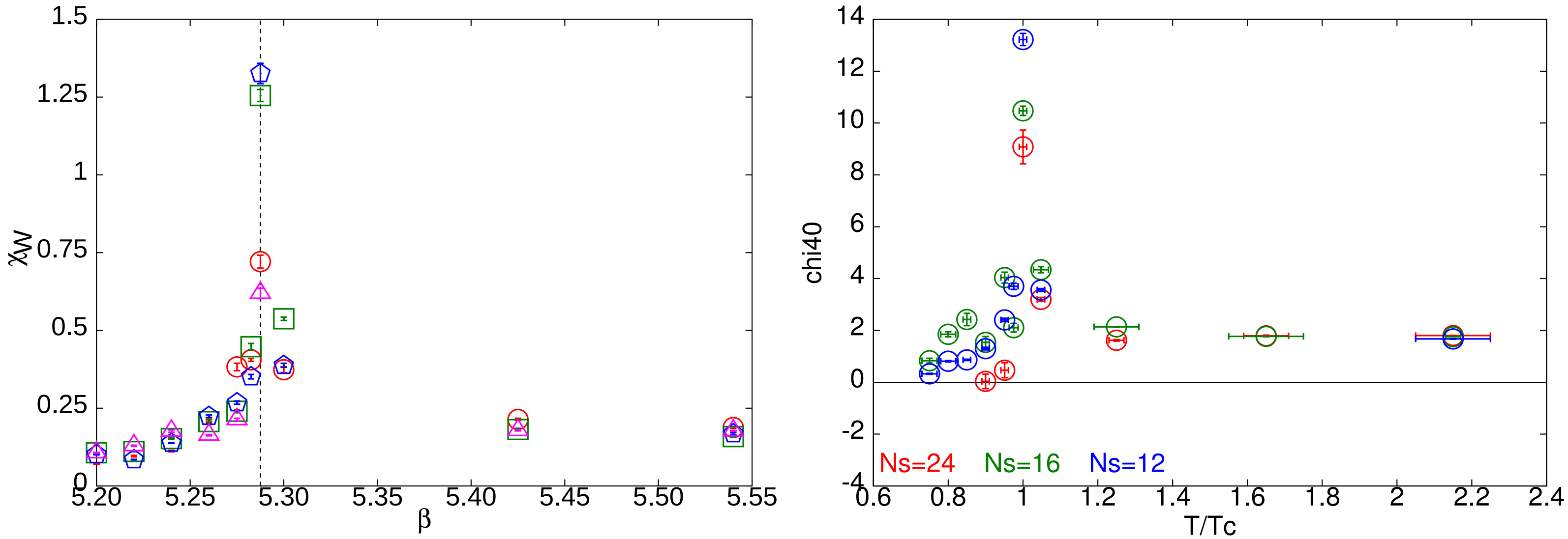

♠ Similar behaviour in higher order terms as well.

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- Using lattices with 8, 10, and 12 temporal sites ($38^3 \times 12$ and 38^4 lattices) and with statistics of 0.5-1 million iterations, ϵ, P, s, C_s^2 and C_v obtained in continuum.

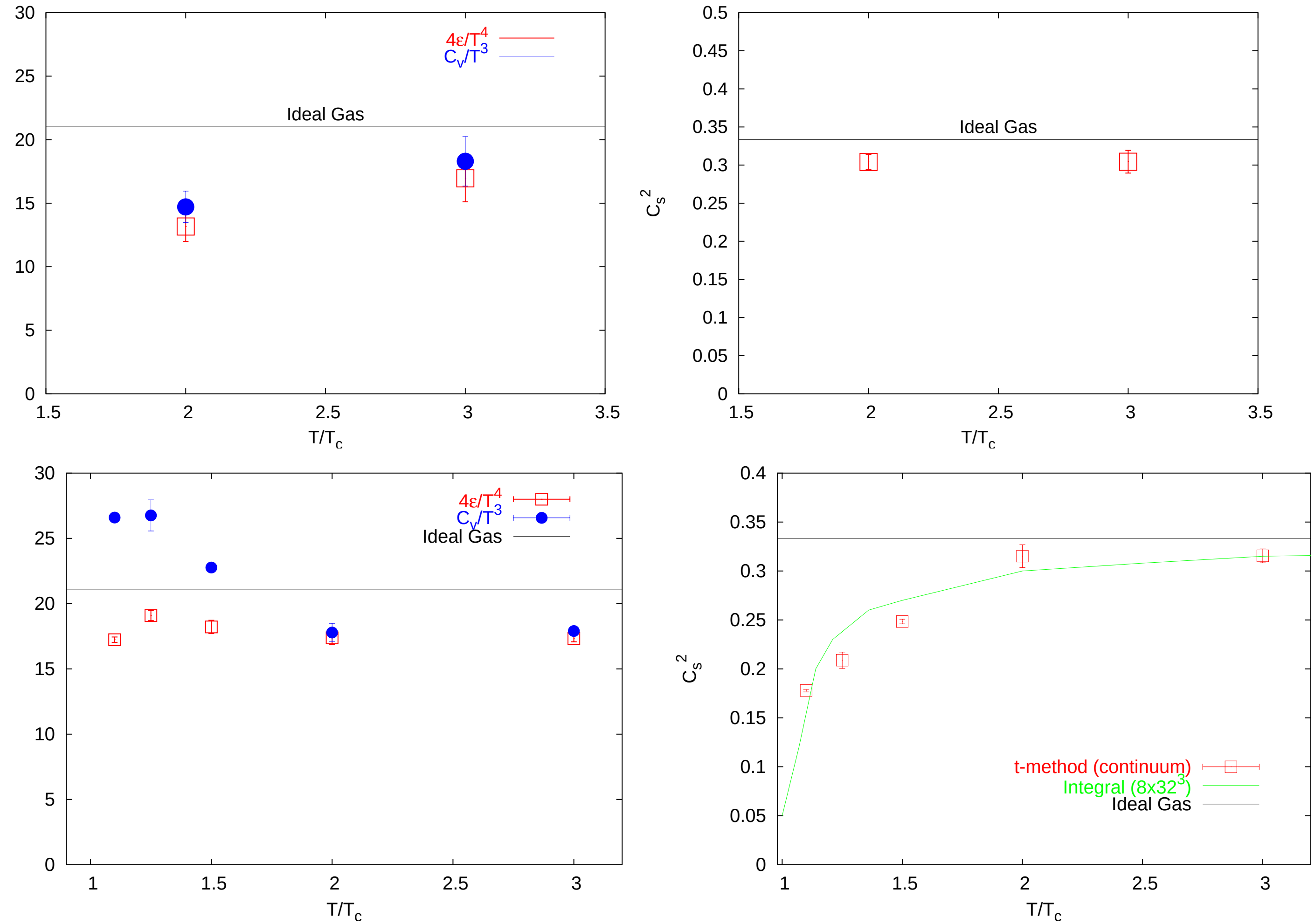
- Entropy agrees with strong coupling SYM prediction

(Gubser, Klebanov & Tseytlin, NPB '98, 202)

$$\begin{aligned} \frac{s}{s_0} &= f(g^2 N_c), \quad \text{where} \\ f(x) &= \frac{3}{4} + \frac{45}{32}\zeta(3)(2x^{-3/2}) + \cdots \quad \text{and} \\ s_0 &= \frac{2}{3}\pi^2 N_c^2 T^3, \end{aligned} \tag{2}$$

for $T = 3T_c$ but fails at $2T_c$, as do various weak coupling schemes.

Results for t = 1 and 0 respectively:



Persistence of J/ψ

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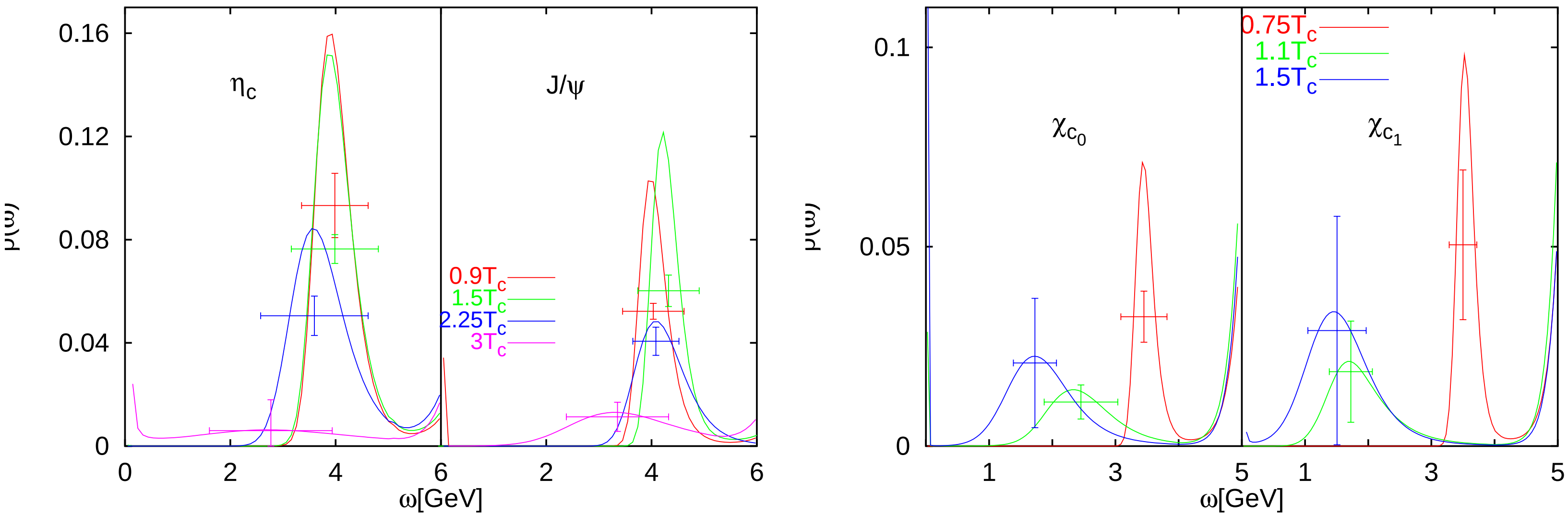
- Caution : nonzero temperature obtained by making temporal lattices shorter.



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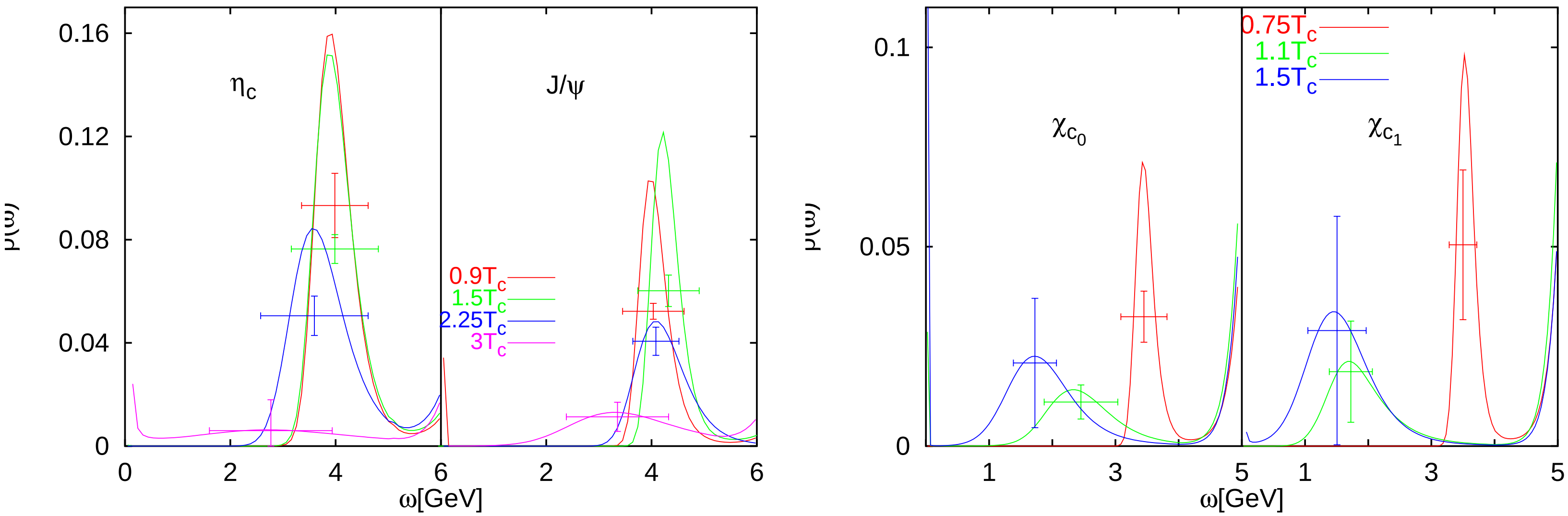

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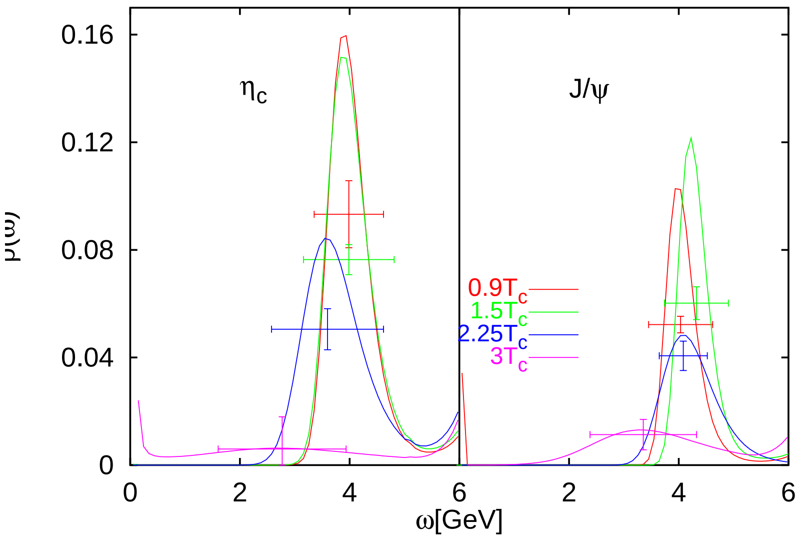

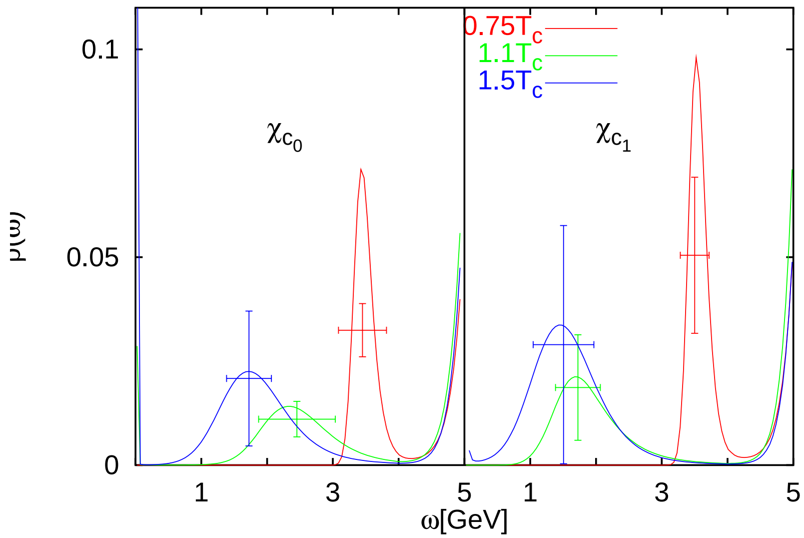

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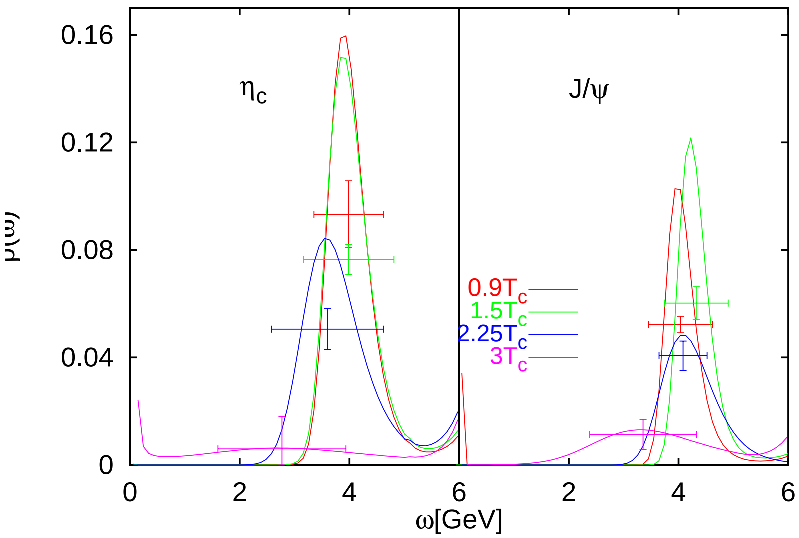

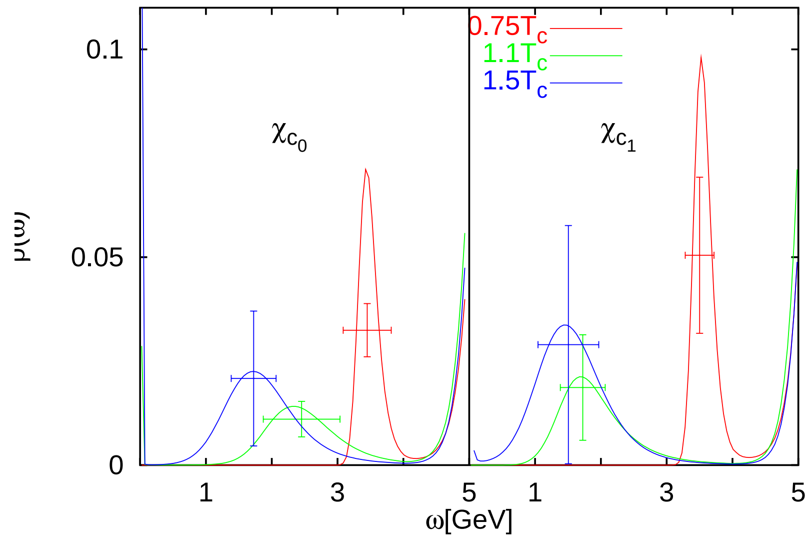

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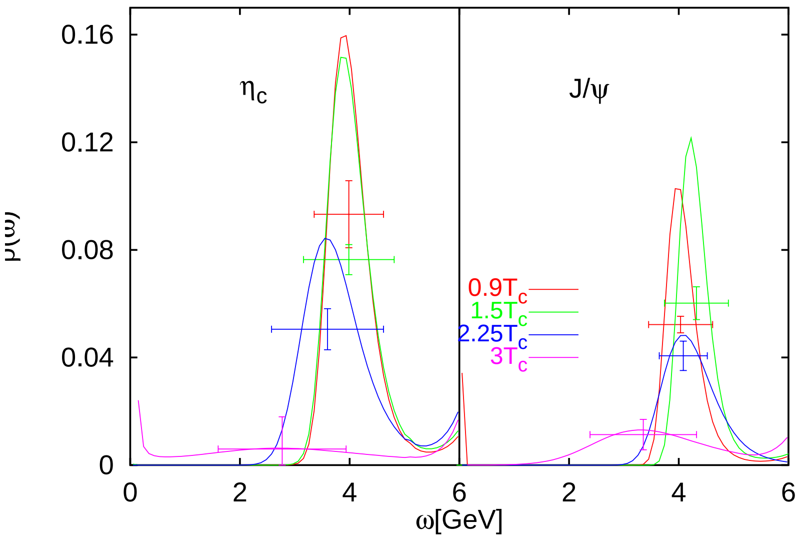
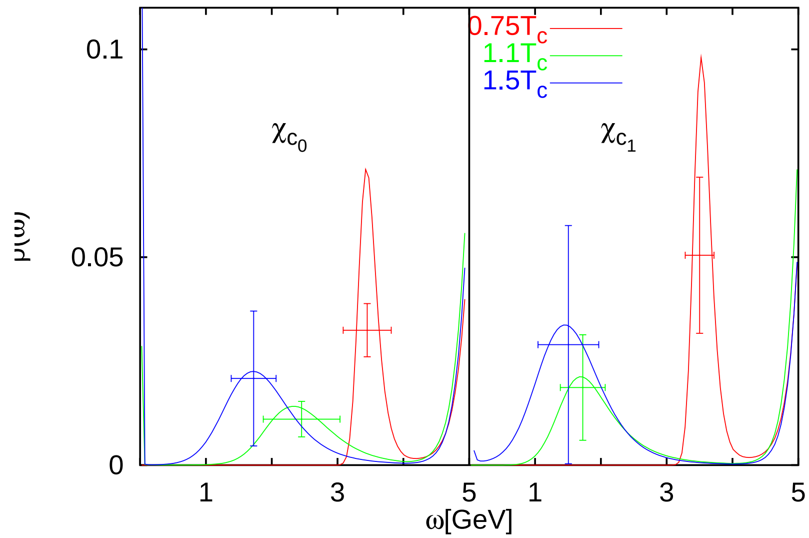
♠ Change of suppression patterns as a function of T or $\sqrt{s}$?





$48^3 \times 12$ to $64^3 \times 24$ Lattices used : (S. Datta et al., Phys. Rev. D 69, 094507 (2004).)

♠ χ_c seems to indeed dissolve by $1.1 T_c$, however, J/ψ and η_c persist up to 2.25 T_c and are gone at $3T_c$; Similar results by Asakawa-Hatsuda and Matsufuru.

♠ Recall that about 60-70 % observed J/ψ come through χ decays.

♠ Change of suppression patterns as a function of T or $\sqrt{s}$?

♠ Effect of inclusion of dynamical fermions ?

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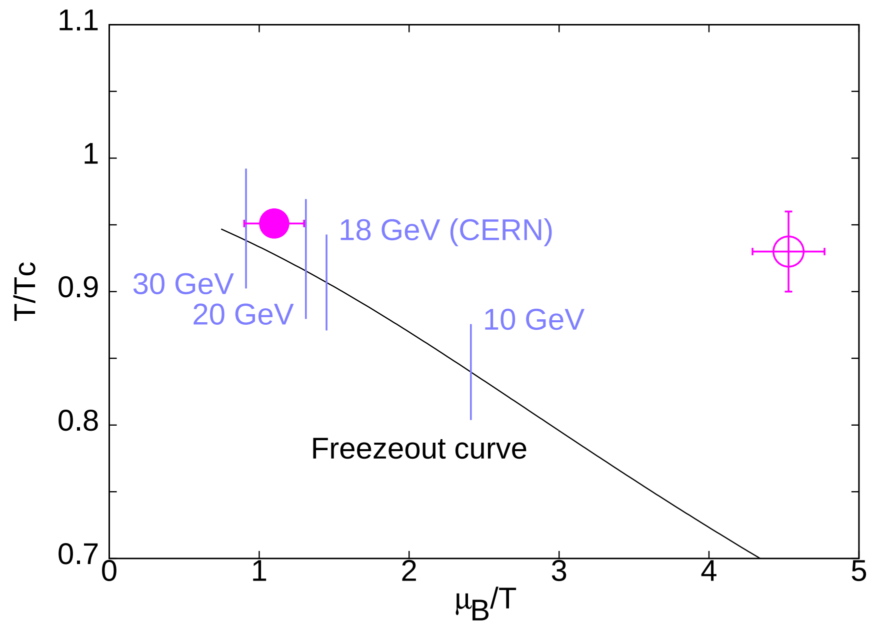

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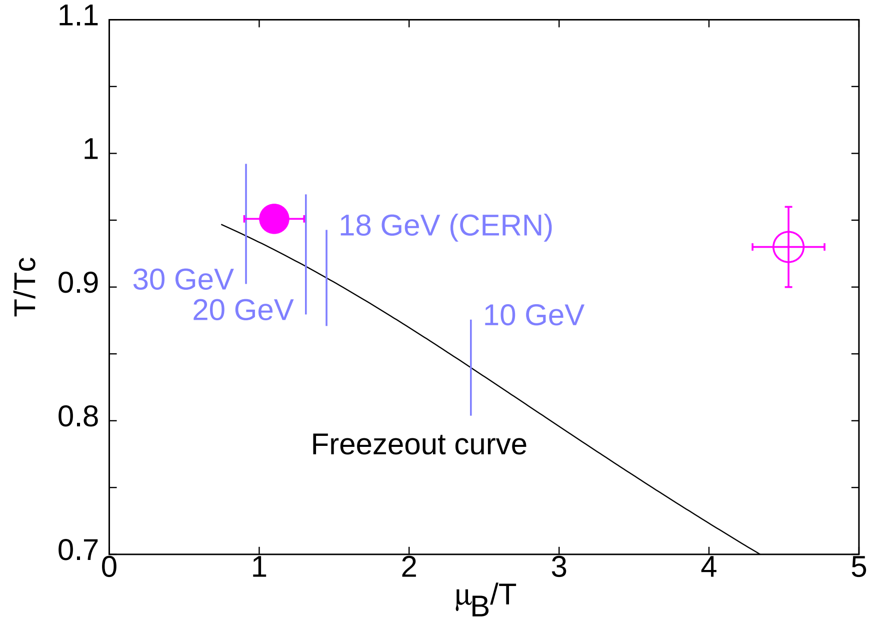

Continuum results on Speed of Sound, and intriguing persistence of J/ψ in QGP.