Excursions in QCD Phase Diagram

Rajiv V. Gavai and Sourendu Gupta

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Motivation

Quark Number Susceptibility

Wroblewski Parameter

ΔP, χ in μ-T plane

Screening Lengths

Summary

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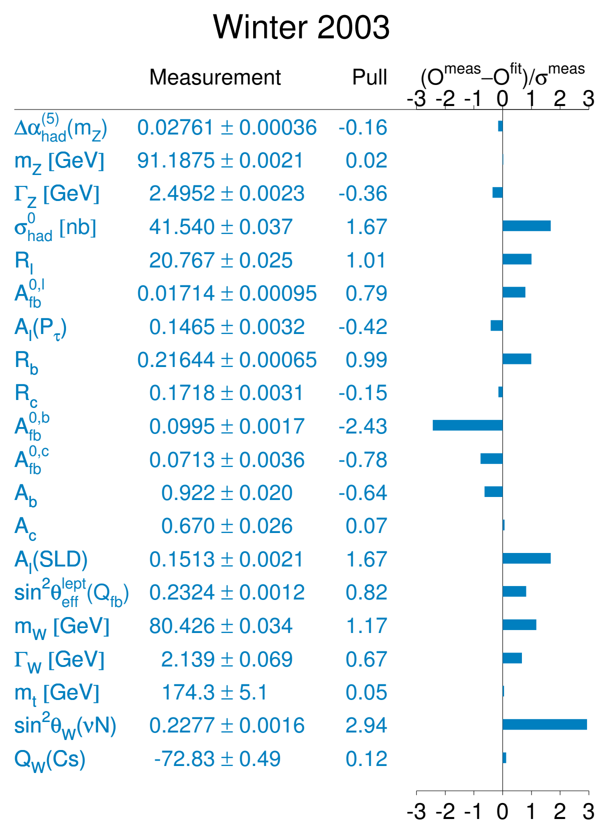
- Standard Model – Very Successful !

- Precision tests from LEP

- All tests based on perturbation theory

- Need to understand non-perturbative QCD to explain baryonic matter in our Universe, i.e., us.

- Lattice QCD – only well-understood, viable tool for this.



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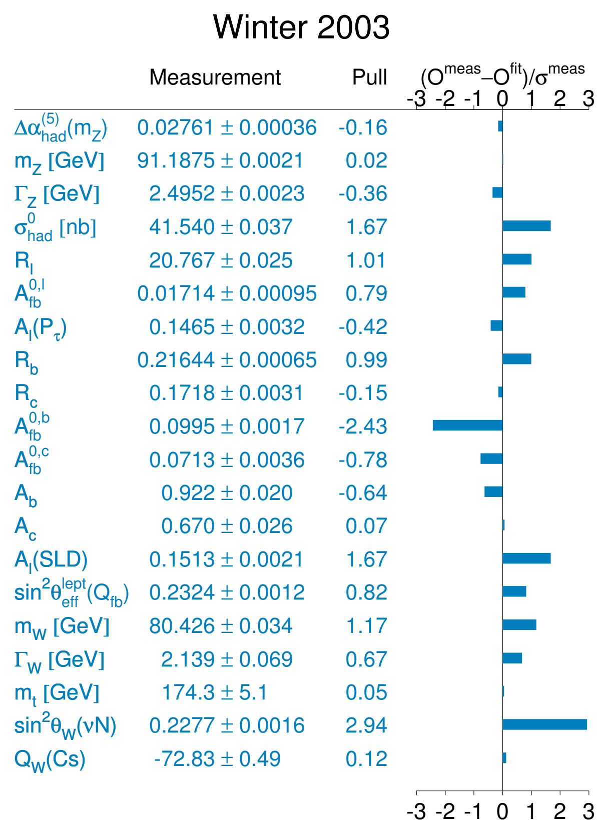

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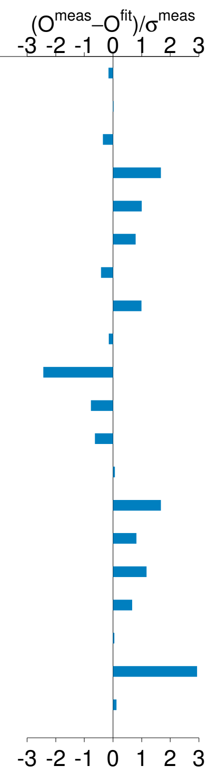
Winter 2003

	Measurement	Pull
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02761 ± 0.00036	-0.16
m_Z [GeV]	91.1875 ± 0.0021	0.02
Γ_Z [GeV]	2.4952 ± 0.0023	-0.36
σ_{had}^0 [nb]	41.540 ± 0.037	1.67
R_l	20.767 ± 0.025	1.01
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.79
$A_l(P_\tau)$	0.1465 ± 0.0032	-0.42
R_b	0.21644 ± 0.00065	0.99
R_c	0.1718 ± 0.0031	-0.15
$A_{fb}^{0,b}$	0.0995 ± 0.0017	-2.43
$A_{fb}^{0,c}$	0.0713 ± 0.0036	-0.78
A_b	0.922 ± 0.020	-0.64
A_c	0.670 ± 0.026	0.07
A_l(SLD)	0.1513 ± 0.0021	1.67
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.82
m_W [GeV]	80.426 ± 0.034	1.17
Γ_W [GeV]	2.139 ± 0.069	0.67
m_t [GeV]	174.3 ± 5.1	0.05
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	2.94
$Q_W(Cs)$	-72.83 ± 0.49	0.12



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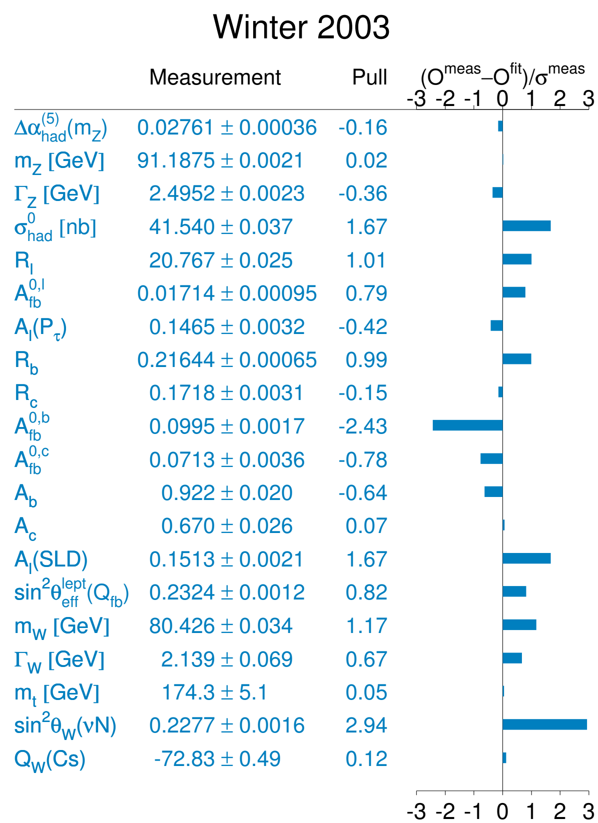
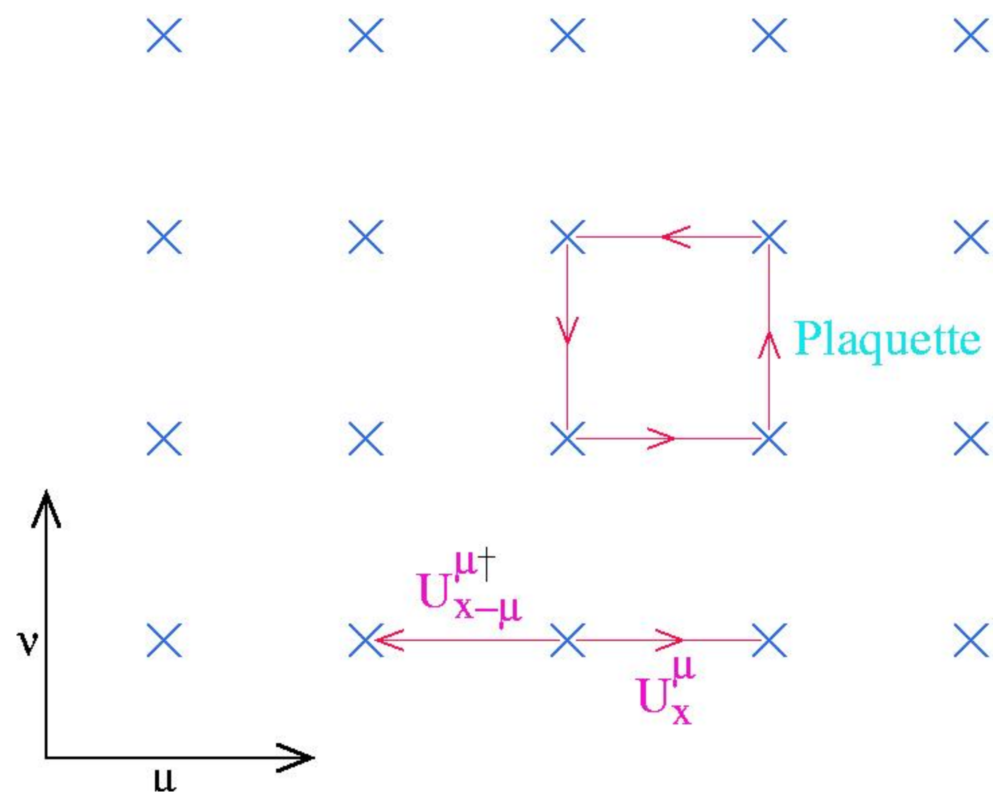


Basic Lattice Gauge Theory

- Discrete space-time : Lattice spacing a UV Cut-off.



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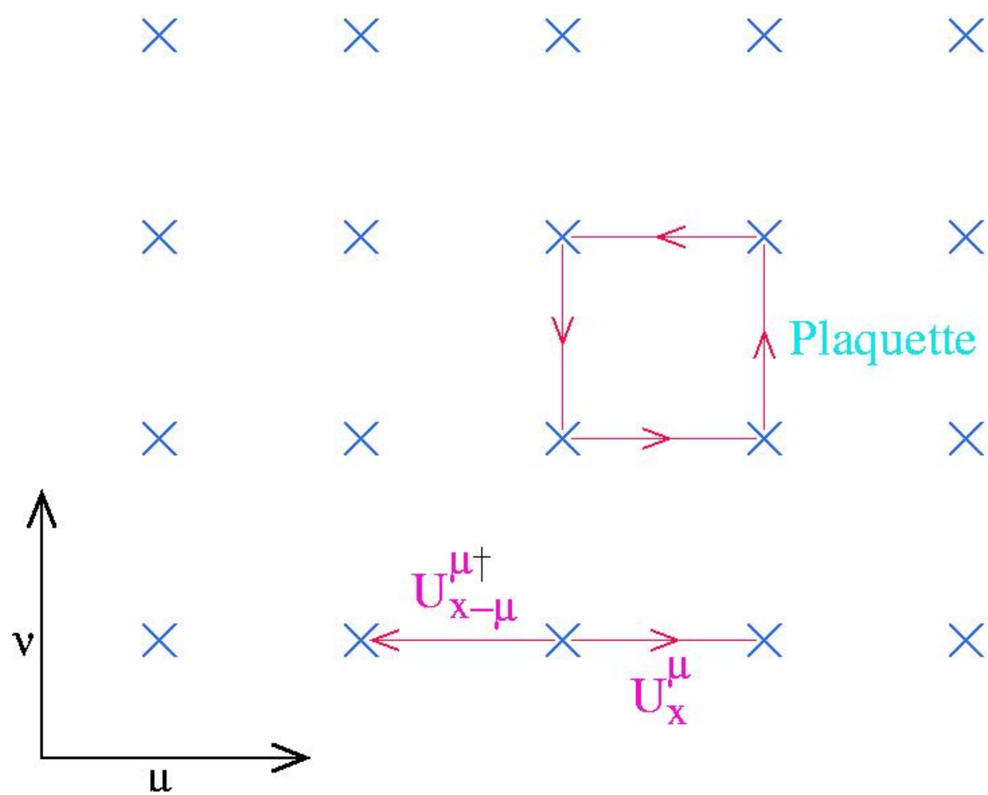
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$



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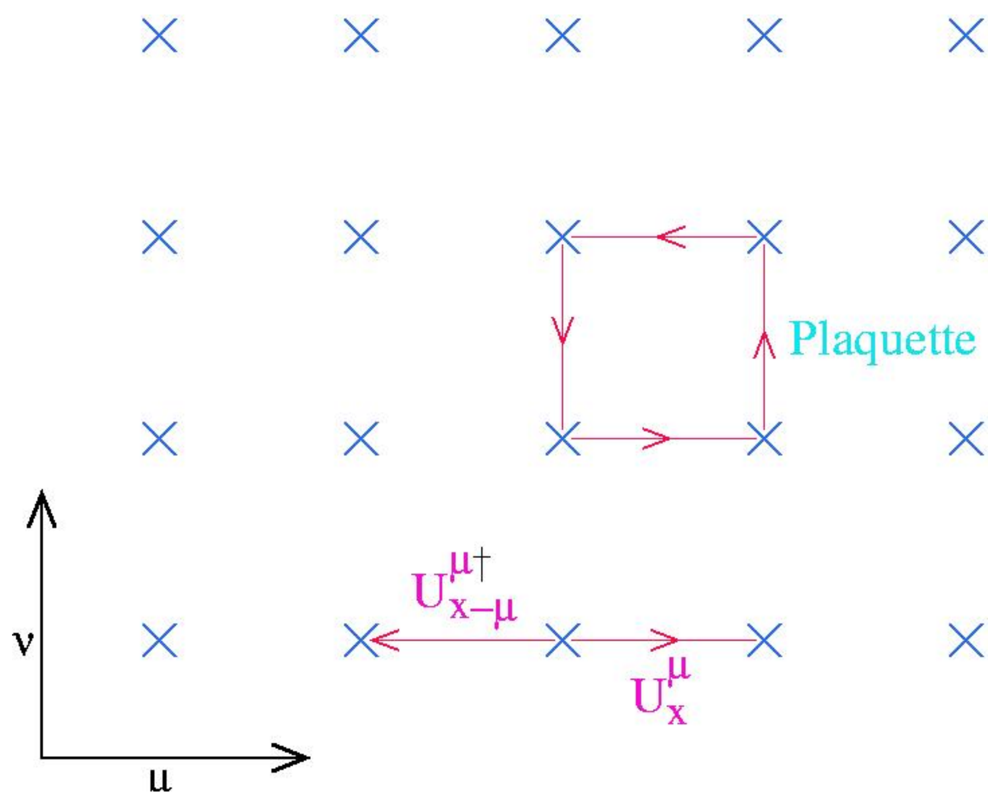
- Gauge transform $V_x \in SU(3)$ $\Rightarrow \psi'(x) = V_x \psi(x)$, $U'_\mu(x) = V_x U_\mu(x) V^{-1}_{x+\hat{\mu}}$.
- Gauge invariance : Actions from Closed Wilson loops, e.g., plaquette.



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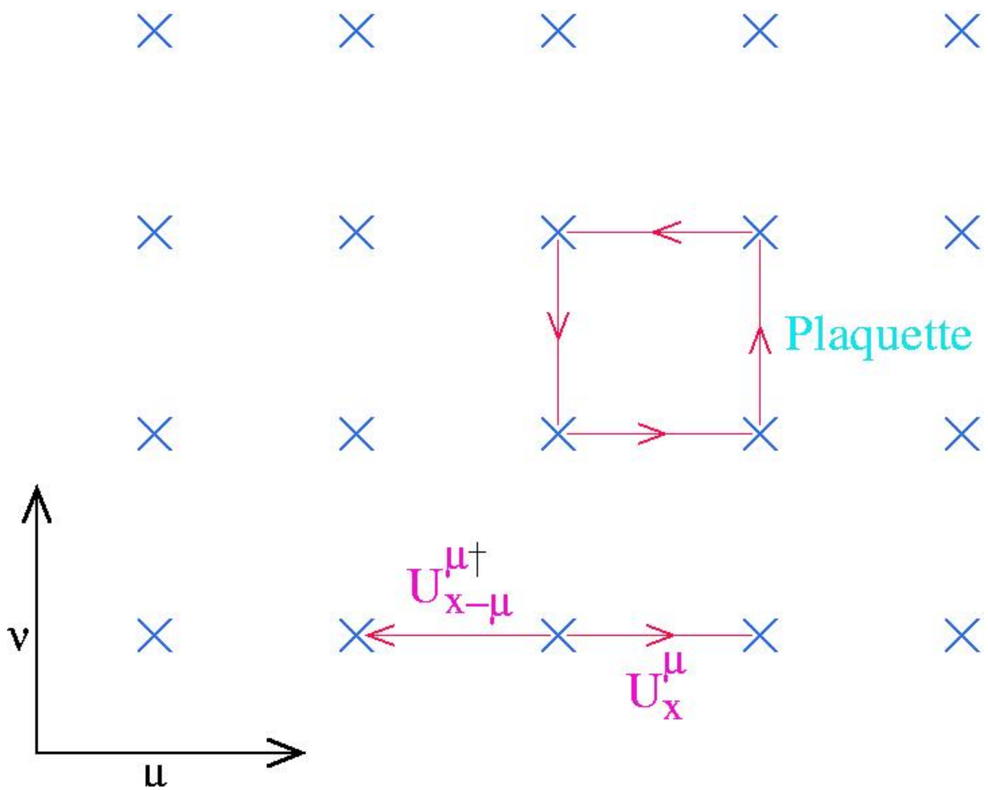
- Fermion Actions : Staggered, Wilson, Overlap..



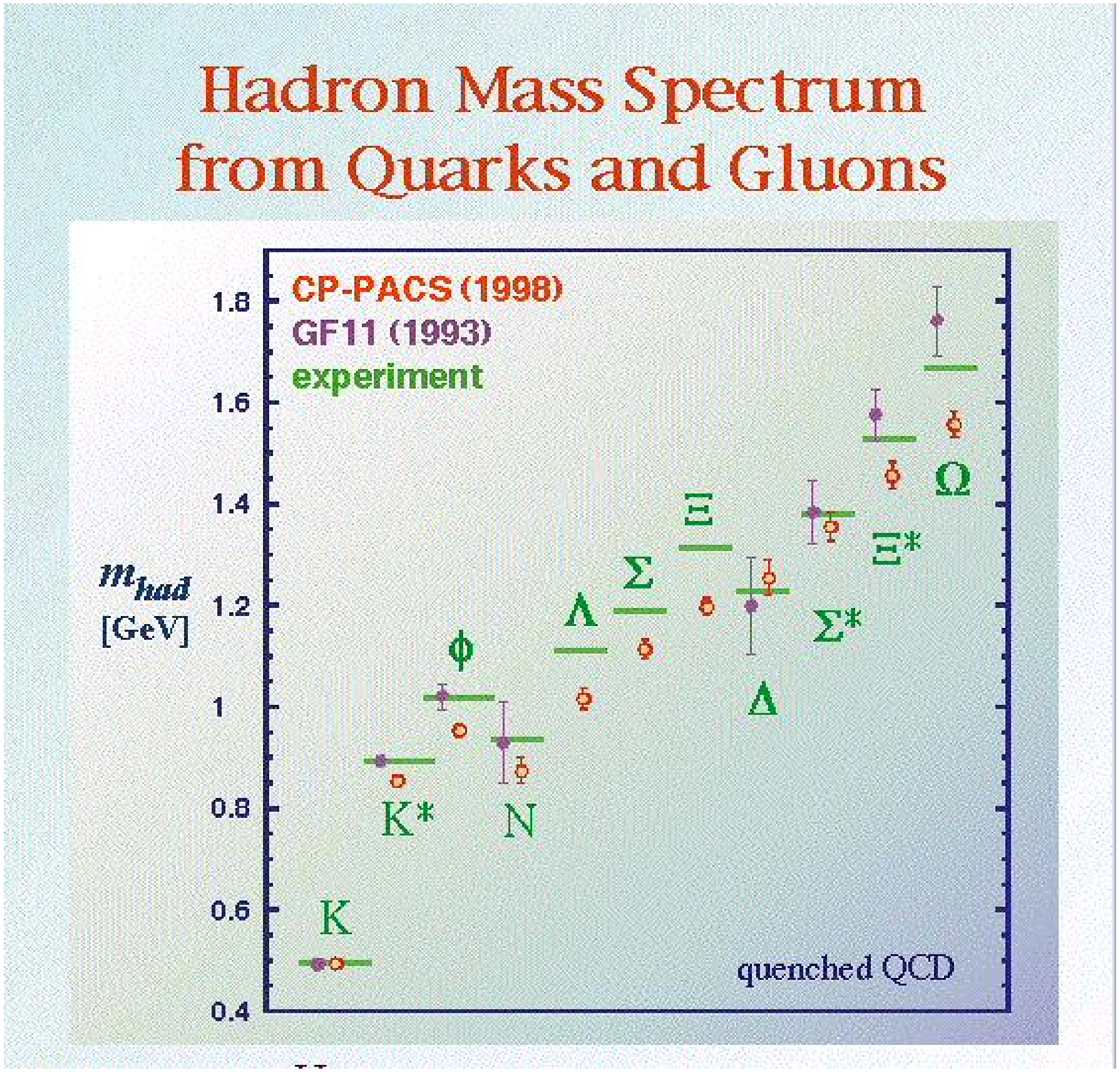
Typically, we need to evaluate

$$\langle \Theta(m_v) \rangle = \frac{\int DU \exp(-S_G)\Theta(m_v) \ \text{Det} \ M(m_s)}{\int DU \exp(-S_G) \ \text{Det} \ M(m_s)} \ , \tag{1}$$

where M is the Dirac matrix in x, colour, spin, flavour space for fermions of mass m_s, S_G is the gluonic action, and the observable Θ may contain fermion propagators of mass m_v.

Since $\langle \Theta \rangle$ is computed by averaging over a set of configurations $\{U_\mu(x)\}$ which occur with probability $\propto \exp(-S_G) \cdot \text{Det}\ M$, the complexity of evaluation of Det $M \implies$ approximations : Quenched ($m_s = \infty$ limit), Partially Quenched (low $m_s = m_u = m_d$), and Full (including a heavier s quark).

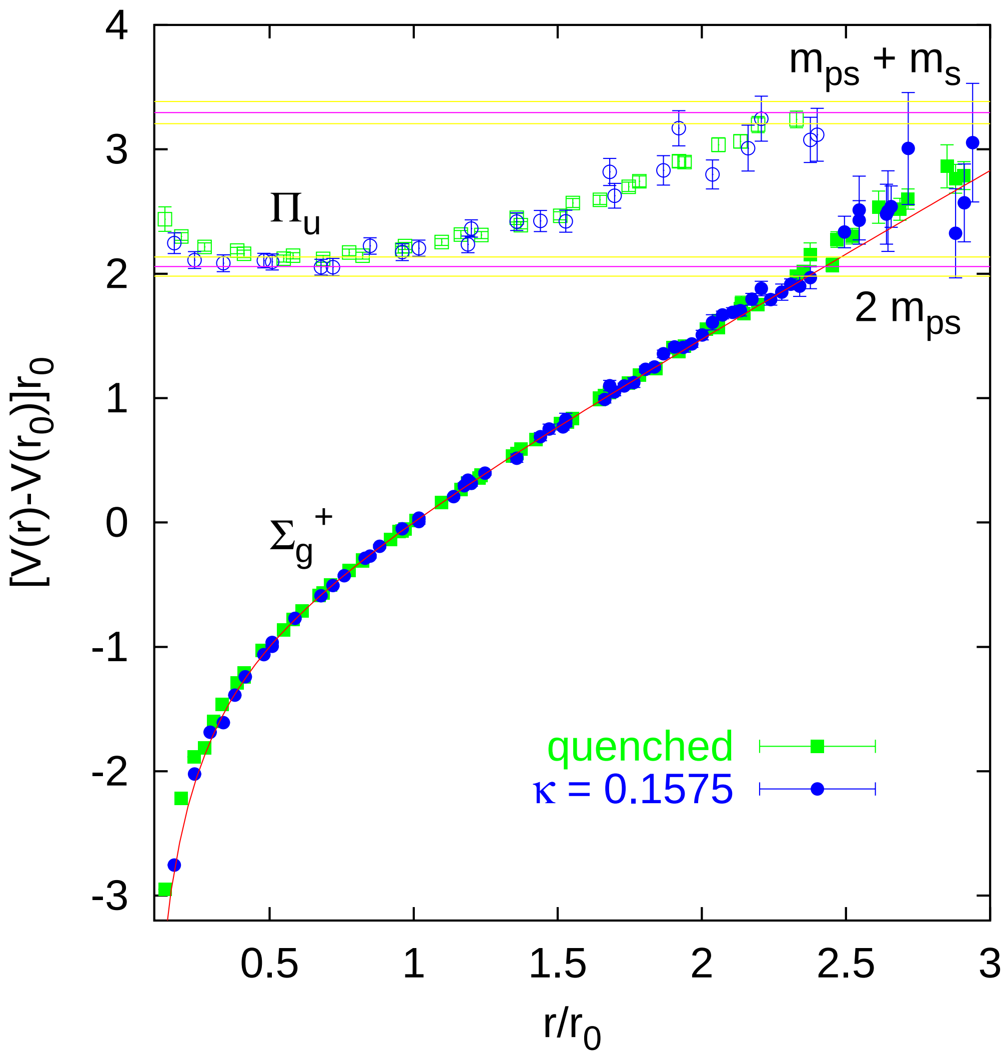
Q $\rightarrow$ PQ $\rightarrow$ Full $\rightsquigarrow$ Computer time $\uparrow$ and Precision $\downarrow$.



Baryon mass comes out (almost) right.

At least in Quenched Approximation

(From CP-PACS Collaboration, Japan)



As does the heavy quark potential $V_{Q\bar{Q}}$.

Here r_0 is roughly 0.5 fm.

(Bali, Phys. Rep. 343 (2001) 1.)

QCD Phase Diagram

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- Theoretically profound : A new critical point ?
- Lattice details :
  - $N_s^3 \times N_t$ Lattice, $N_s \gg N_t$ for $T \neq 0$,
  - Spatial Volume $V = N_s^3 a^3$,
  - Temperature $T = 1/N_t a(\beta)$,
  - Chemical potential: Multiply each $U_4(x)$ by $f(a\mu)$ and $U_4^\dagger(x)$ by $1/f(a\mu)$, where $f(a\mu) = 1 + a\mu + \mathcal{O}(a^2)$. (Gavai, PRD '85)

- Known choices : $f_{HK}(x) = \exp(x)$ and $f_{BG} = (1+x)/\sqrt{1-x^2}$.

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- Order Parameters : Chiral condensate $\langle \bar{\psi}\psi \rangle$,
  Polyakov Loop $\langle L \rangle$, where $L(\vec{x}) = \frac{1}{3} \prod_{t=1}^{N_t} \mathrm{tr}\; U_4(\vec{x}, t)$

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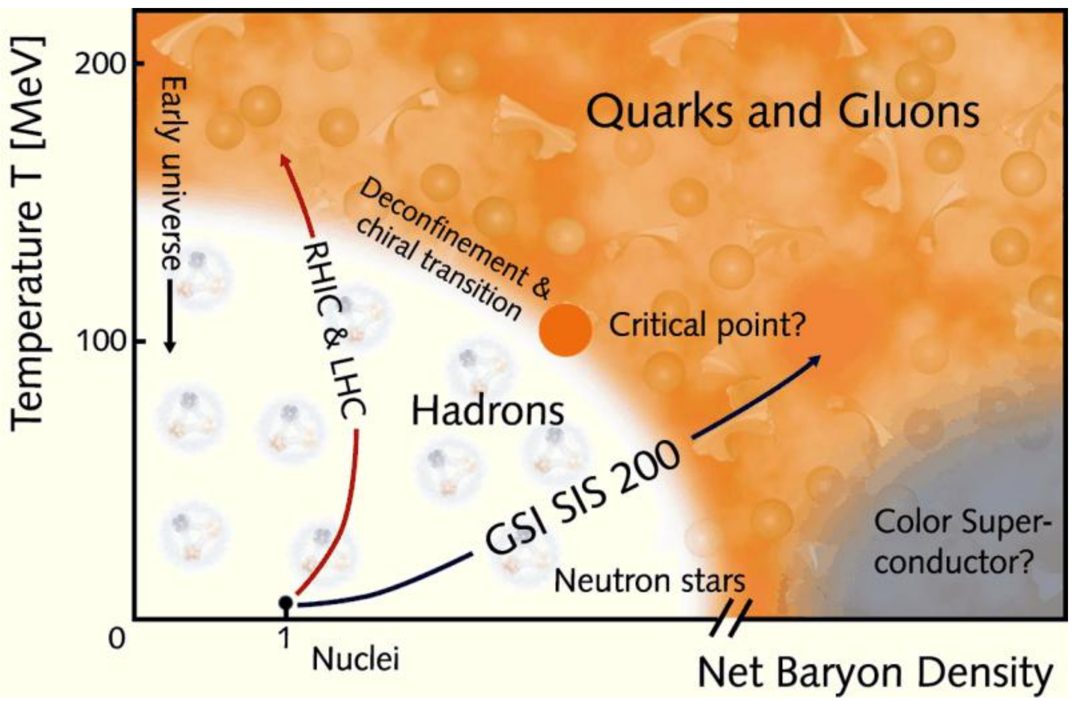

$\mu_{\rm B} \neq 0$

- Phase Problem : Det $M(\mu)$ is complex for $\mu \neq 0$.
- Early results in quenched approximation and $T = 0$:- $\langle\bar\psi\psi\rangle = 0$ at $\mu_{\rm B} \sim m_\pi$!
- Exciting results in recent past for small μ, starting in the $T_c(\mu = 0)$ neighbourhood.
  - Re-weighting Method (Fodor & Katz, JHEP '02)
  - Imaginary μ (de Forcrand & Philipsen, NPB '02, D'Elia & Lombardo, PRD '03)
  - Re-weighting & Taylor Expansion in μ (Allton et al., PRD '02)
- Large μ simulations possible when Det M is real, *e.g.*, 2 colours or $\mu_{I_3} \neq 0$. Show agreement with effective chiral theory (Kogut & Sinclair '02, S. Gupta '02)

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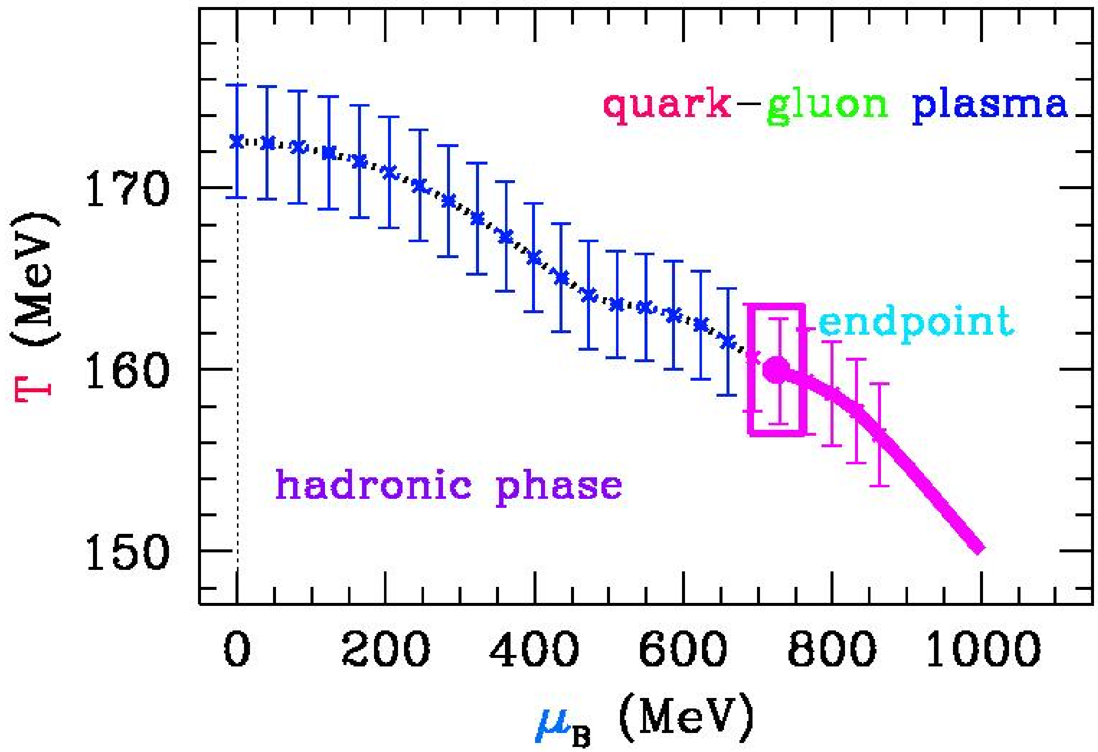
Fodor-Katz Results



$N_s^3 \times 4$ Lattices,
$N_s =$ 4,6,8;
Bit heavy u,d quarks.
Critical End-point :
$T =$ 160(4) MeV,
$\mu =$ 725(35) MeV

How reliable are these results ? Methods, Prescription dependence...

We address some of these issues via Quark Number Susceptibilities.

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Definitions: For u, d, and s quarks, the partition function is

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where μ_f are corresponding chemical potentials. Defining $\mu_0 = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, baryon and isospin density/susceptibilities can be obtained as : (Gottlieb et al. '87, '96, '97, Gavai et al. '89)

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$$n_i = \frac{T}{V} \frac{\partial \ln \mathcal{Z}}{\partial \mu_i}, \qquad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}$$

Setting $\mu_i = 0$, $n_i = 0$ but χ_{ij} are nontrivial. Diagonal χ's are

$$\chi_0 = \frac{T}{2V}[\langle \mathcal{O}_2(m_u) + \frac{1}{2}\mathcal{O}_{11}(m_u)\rangle] \quad (3)$$

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Here $\mathcal{O}_2 = \mathrm{Tr}\ M_u^{-1}M_u'' - \mathrm{Tr}\ M_u^{-1}M_u'M_u^{-1}M_u'$, and $\mathcal{O}_{11}(m_u) = (\mathrm{Tr}\ M_u^{-1}M_u')^2$, and the traces are estimated by a stochastic method:
$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L}(\mathrm{Tr}\ A)_i(\mathrm{Tr}\ A)_j/L(L-1)$,
where R_i is a complex vector from a set of N_v subdivided in L independent sets.

Comparing Full and Quenched QCD

Gavai & Gupta PR D '01; Gavai, Gupta & Majumdar, PR D 2002.

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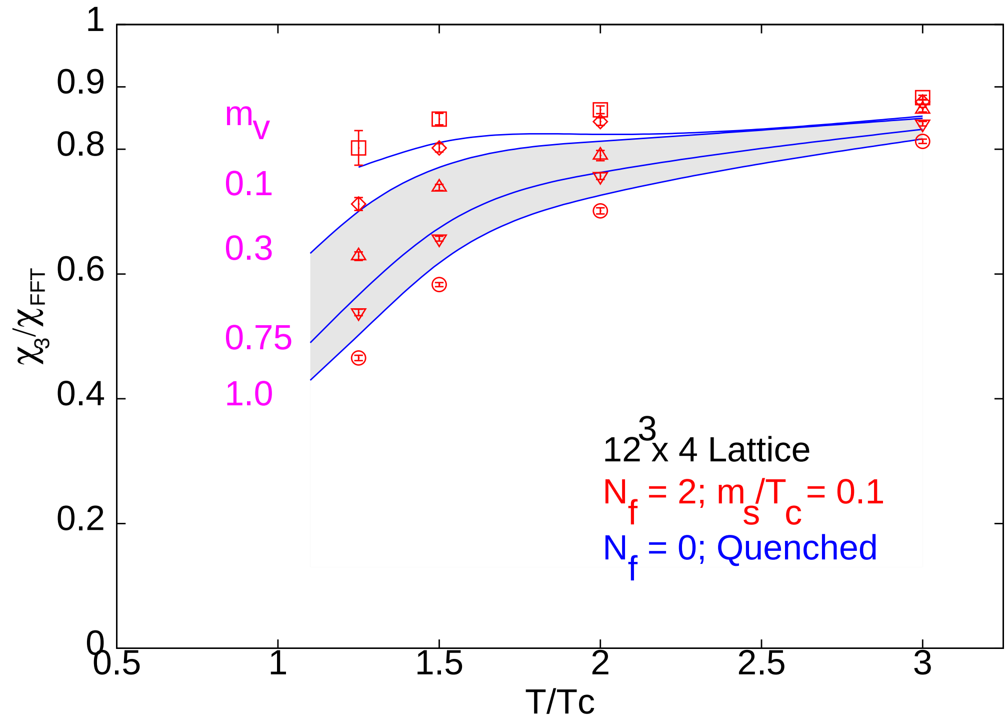

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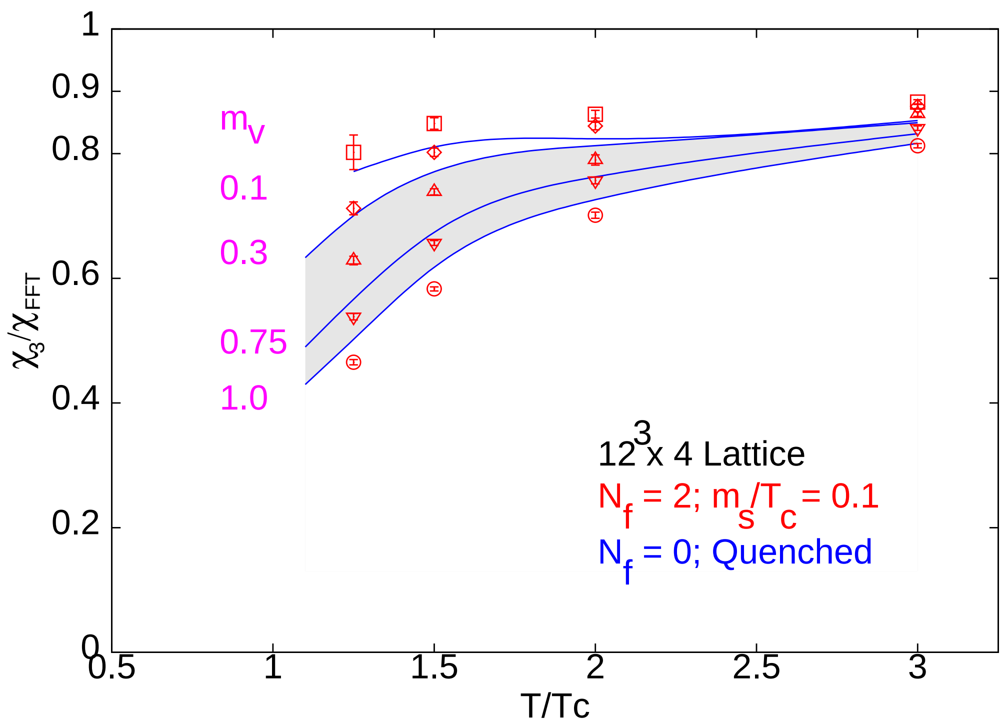

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1) χ_{FFT} — Ideal gas results for same Lattice.

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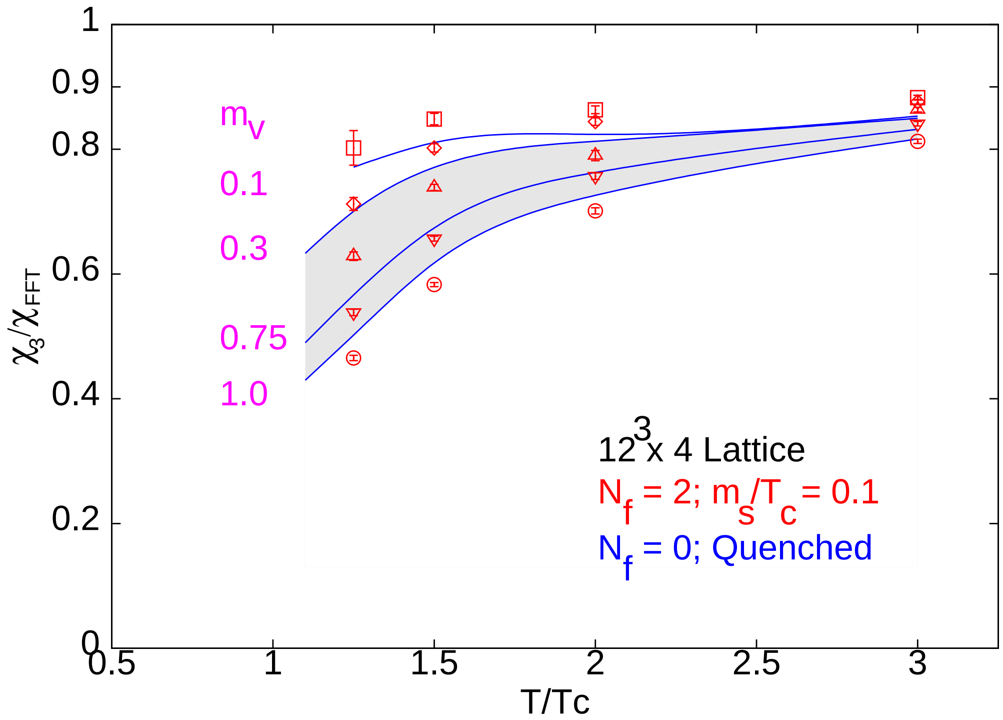

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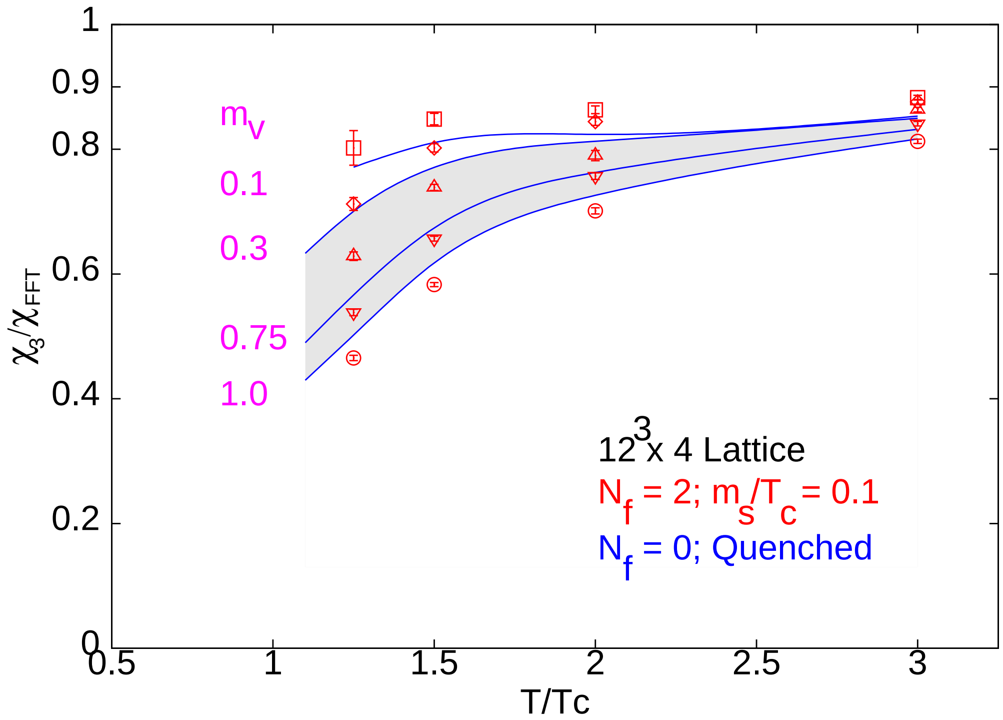

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3) PDG values for strange quark mass $\Longrightarrow m_v^{strange}/T_c$ $\simeq$ 0.3-0.7 (N_f=0); 0.45-1.0(N_f=2).

Perturbation Theory

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Weak coupling expansion gives:
$\frac{\chi}{\chi_{FFT}} = 1 - 2(\frac{\alpha_s}{\pi}) + 8\sqrt{(1 + 0.167N_f)}(\frac{\alpha_s}{\pi})^{\frac{3}{2}}$
(Kapusta 1989).

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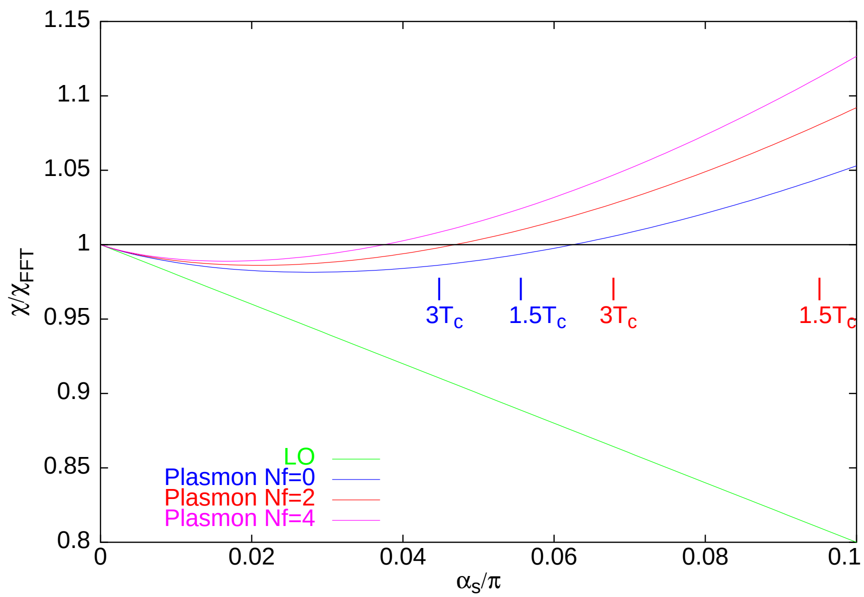

♣ Minm 0.981 (0.986) at 0.03 (0.02) for $N_f = 0$ (2).

♣ For $1.5 \leq T/T_c \leq 3$ pert. theory $\longrightarrow$ 0.99-0.98 (1.08=1.03) for $N_f = 0$ (2).

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Hard Thermal Loop & Self-consistent resummation give :
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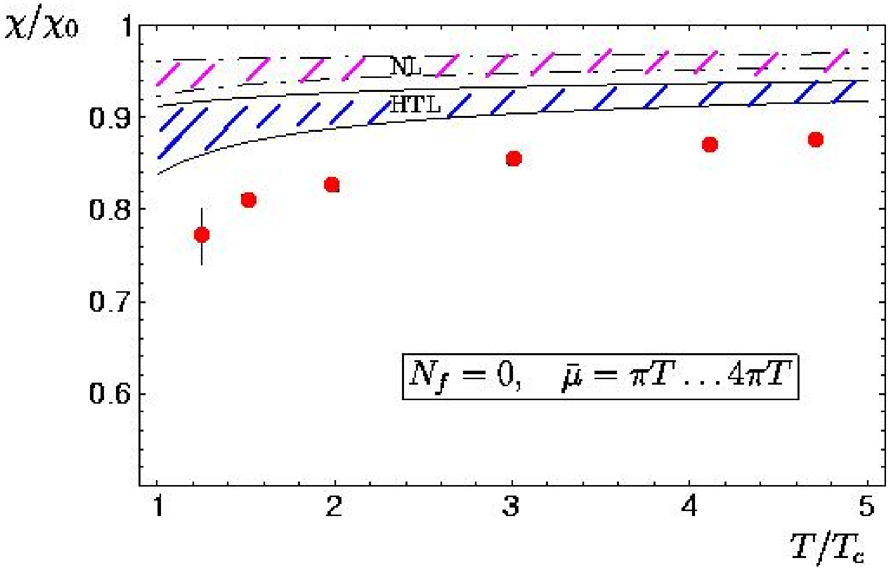

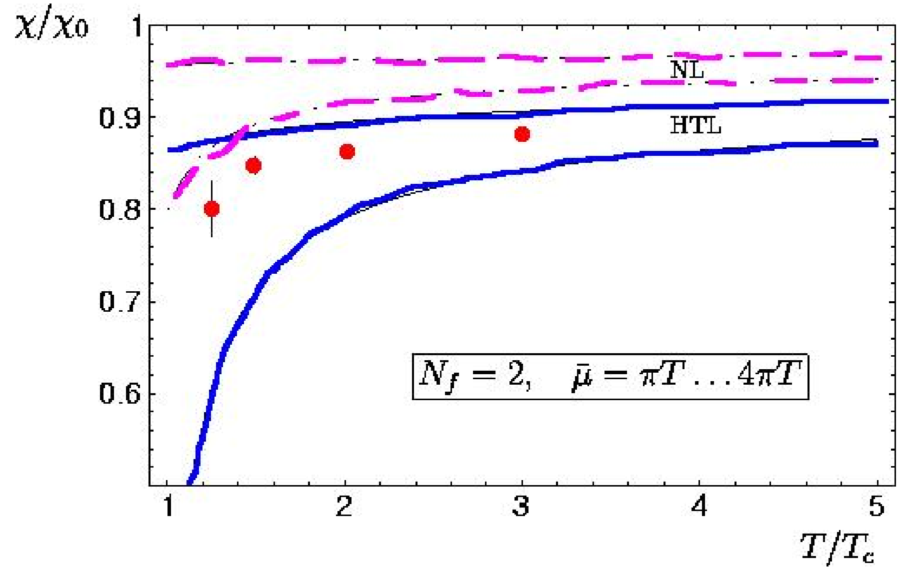

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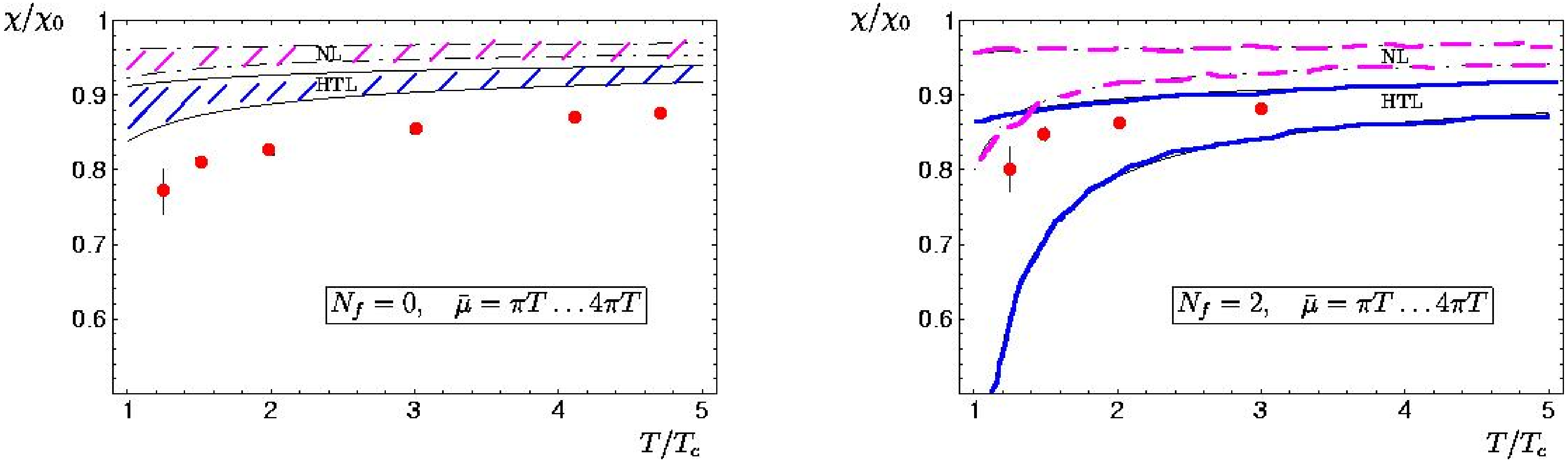

Our results for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Check for larger N_t and improved actions.

Taking Continuum Limit

(Gavai & Gupta, PR D '02 and hep-lat/0211015)

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Taking Continuum Limit

(Gavai & Gupta, PR D '02 and `hep-lat/0211015`)

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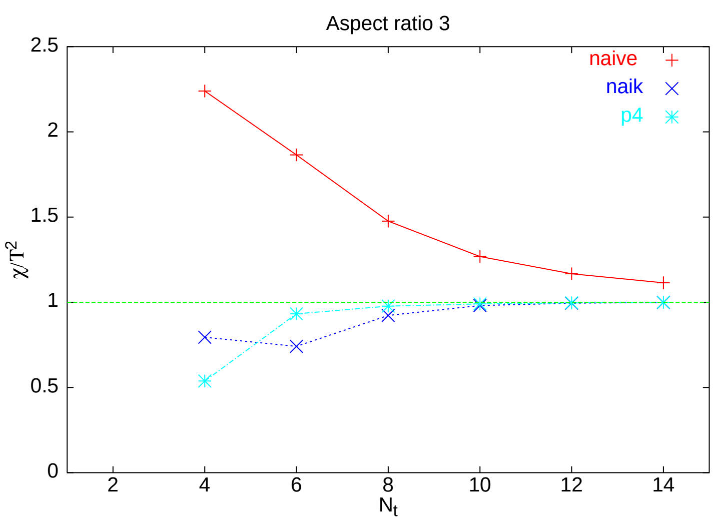
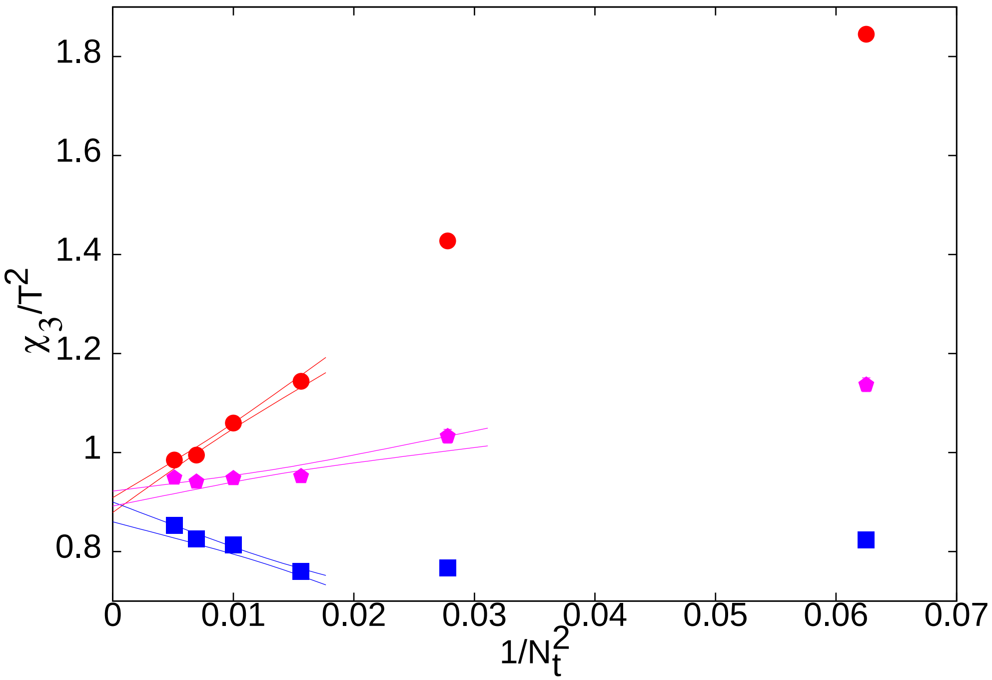


♠ Does improve the N_t-dependence of the free fermions.

Results at $2T_c$:



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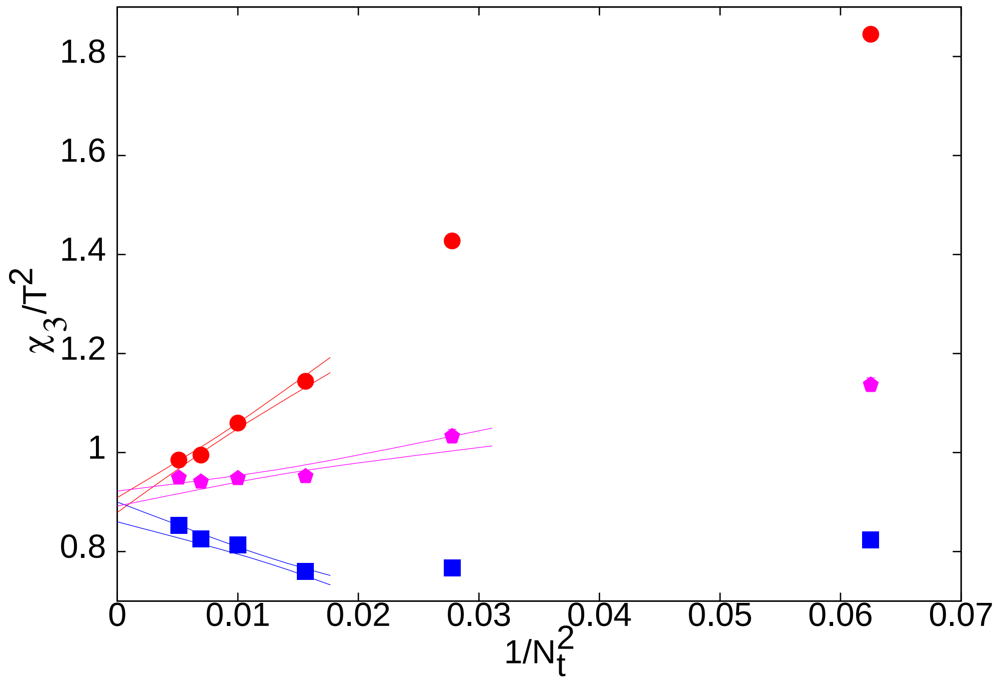

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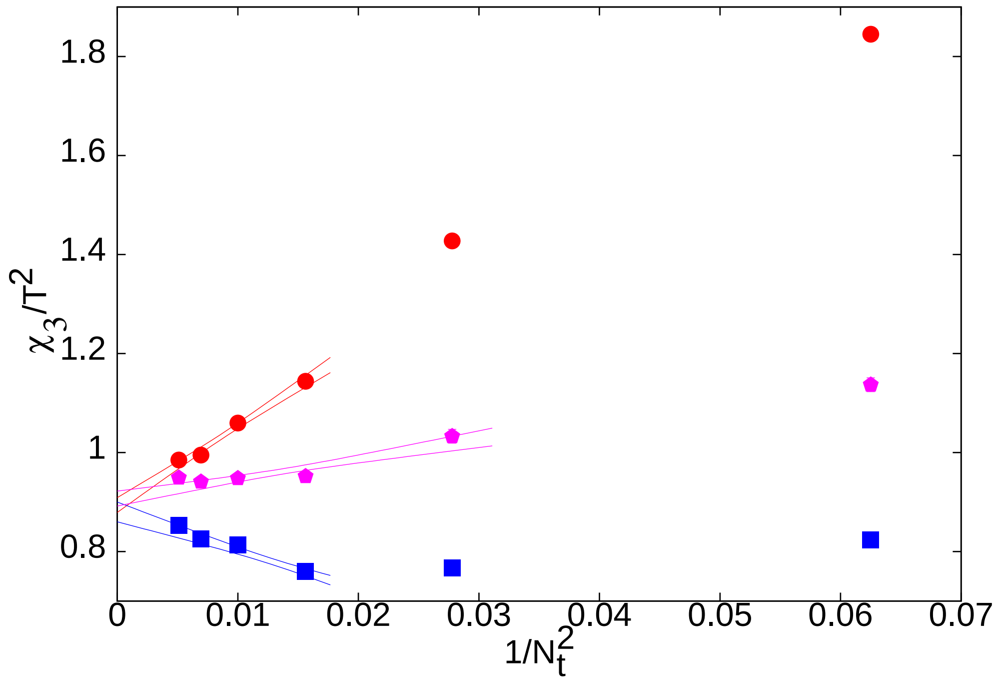

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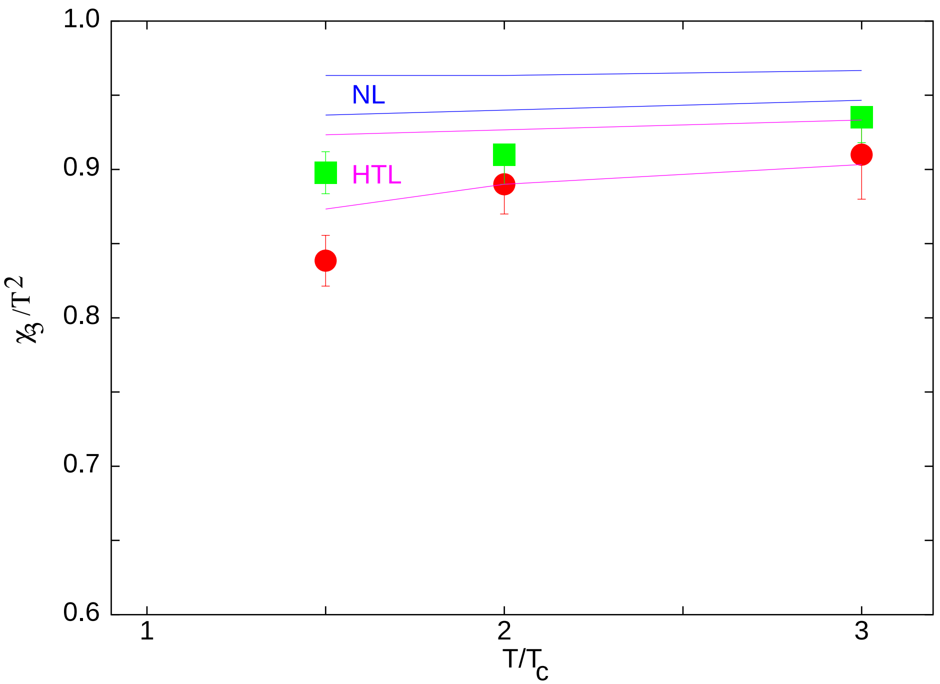

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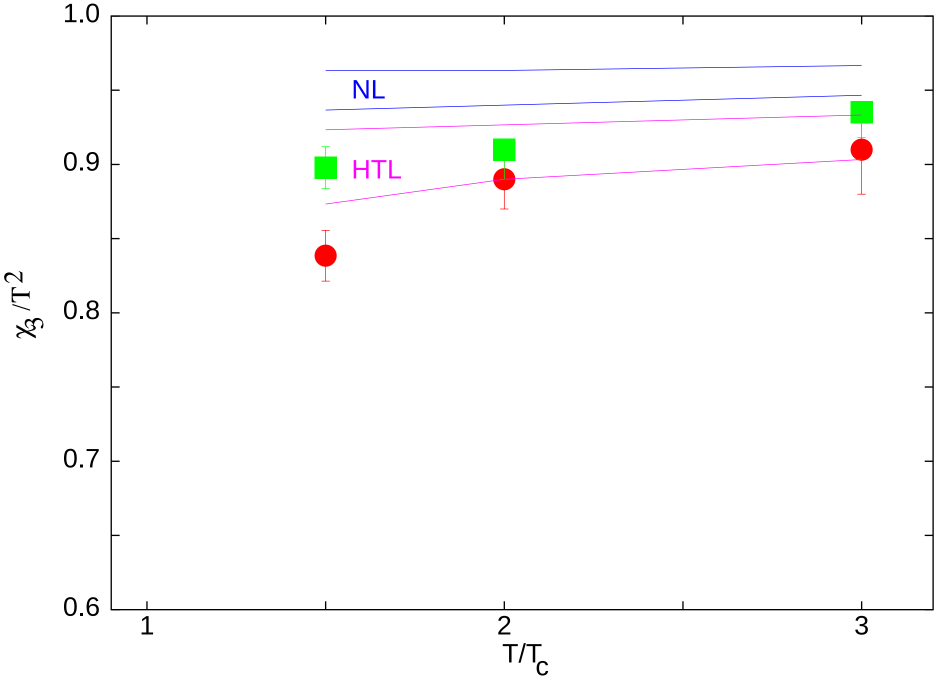

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♡ Also reproduced in dimensional reduction (1 free parameter). Vuorinen, PR D '03.

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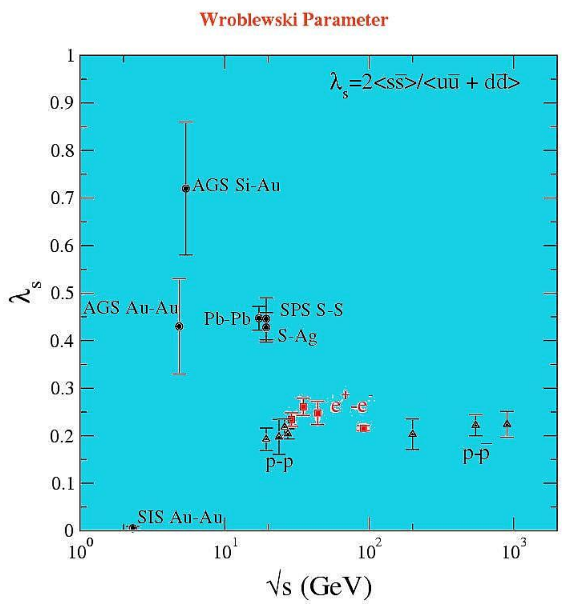
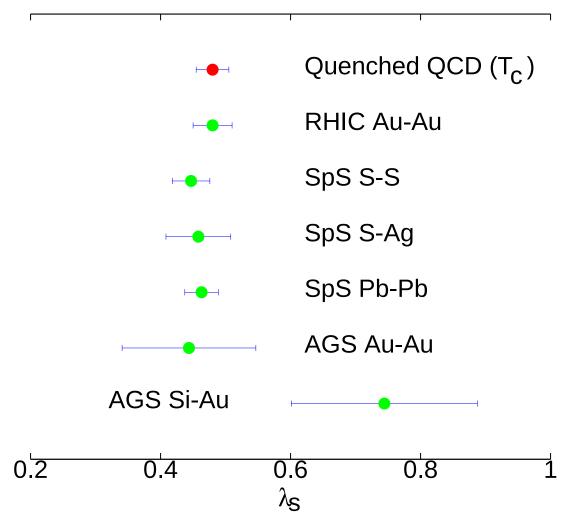
- Ratio of newly created strange quarks to light quarks
- We argued that 1) the ratio χ_s/χ_u estimates it well and 2) obtained it using our continuum QNS for $m/T_c = 0.03$ for u, d and $m/T_c = 1$ for s quark.





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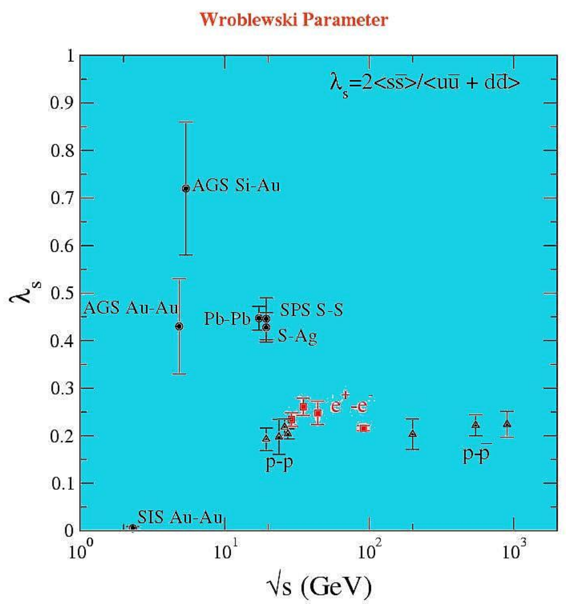

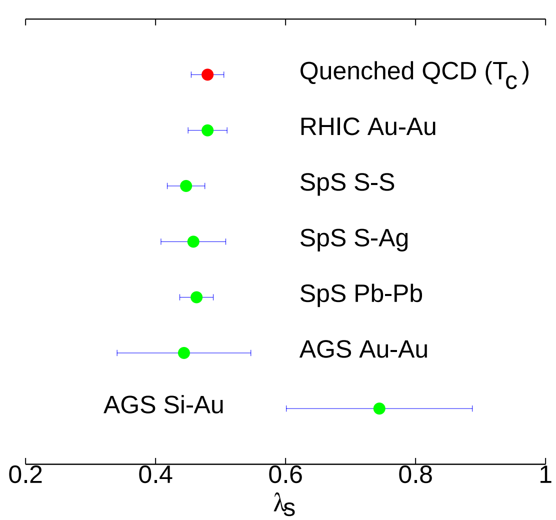

Caveats

- Quenched approximation – Expect a shift of 5-10 % in full QCD.
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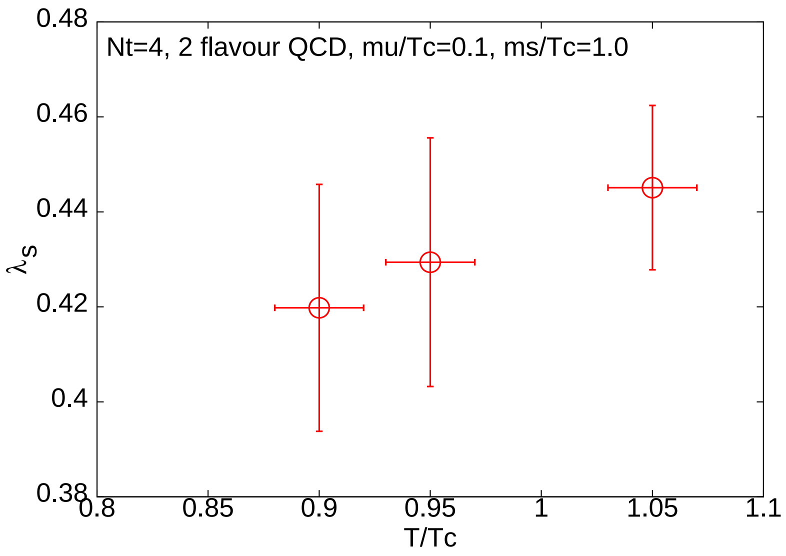

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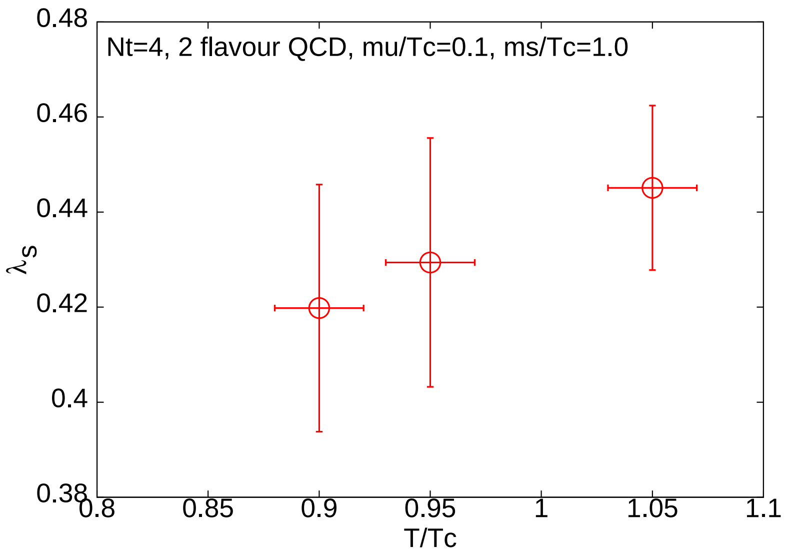

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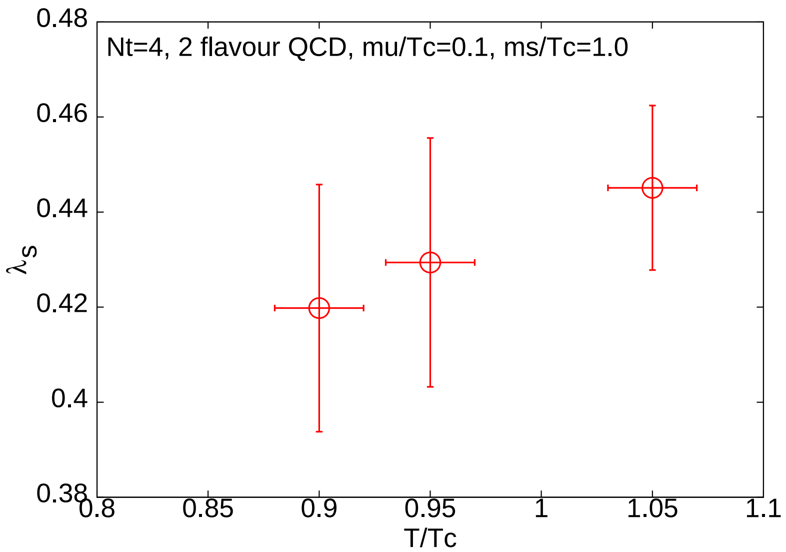

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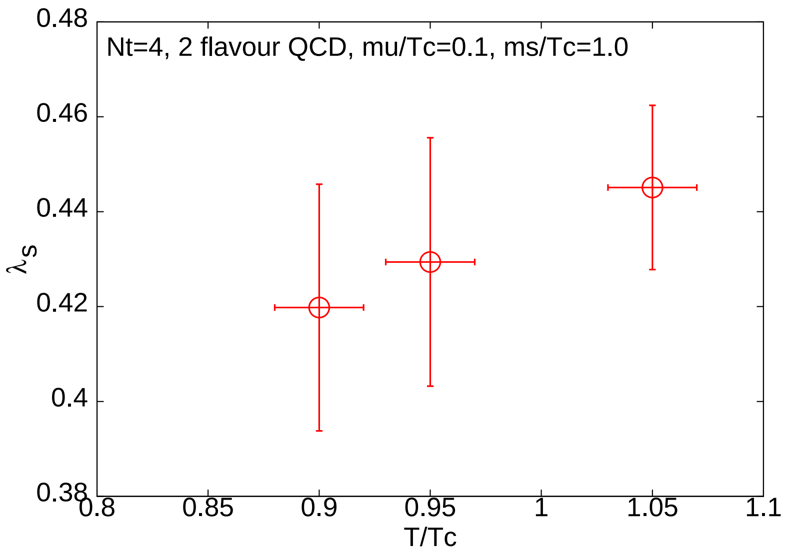

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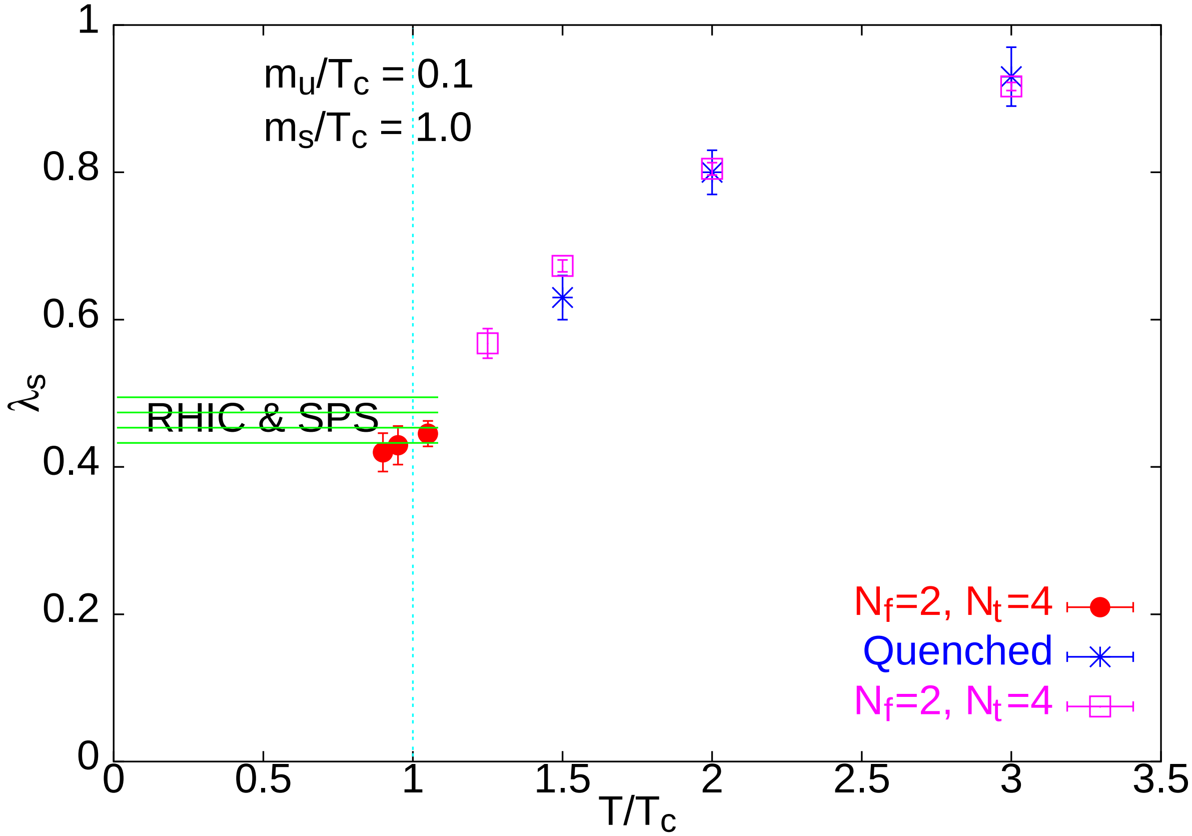
λ_s as a function of T



- At SPS and RHIC, $\mu_{\rm B} \neq 0$; But observed λ_s is insensitive to it. .
  - Theoretically, Screening mass- Susceptibility correlation and μ-dependence results of QCD-TARO on screening masses too suggest such an insensitivity.
  - Needs to be checked explicitly.
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ΔP, χ in μ-T plane

Higher order susceptibilities, defined by

$$\chi_{fg\cdots} = \frac{T}{V}\frac{\partial^n \log Z}{\partial\mu_f\partial\mu_g\cdots} = \frac{\partial^n P}{\partial\mu_f\partial\mu_g\cdots} \ , \tag{6}$$

are Taylor coefficients of the pressure P in its expansion in μ.

Defining

$$\frac{\mu_2^*}{T} = \sqrt{\frac{12\chi_{uu}/T^2}{|\chi_{uuuu}|}} \ , \tag{7}$$

and similarly μ_i^* (i^{th} term $=(i+2)^{th}$ term), the Taylor series expansion for Pressure $\Delta P = P(\mu) - P(\mu = 0)$ for 2 flavours can be re-organized as,

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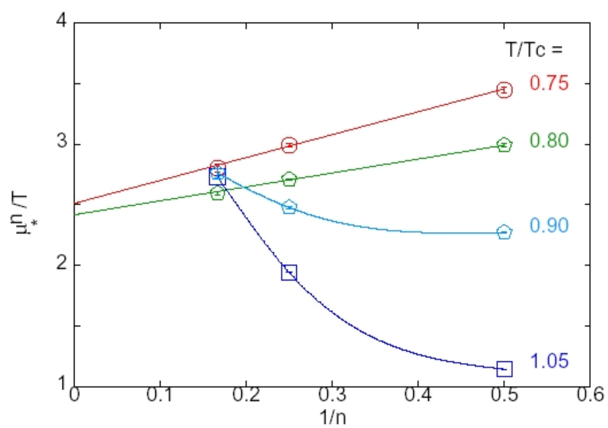

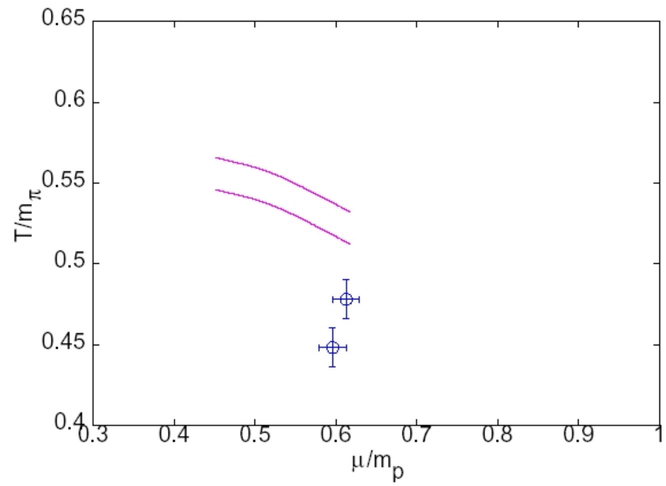

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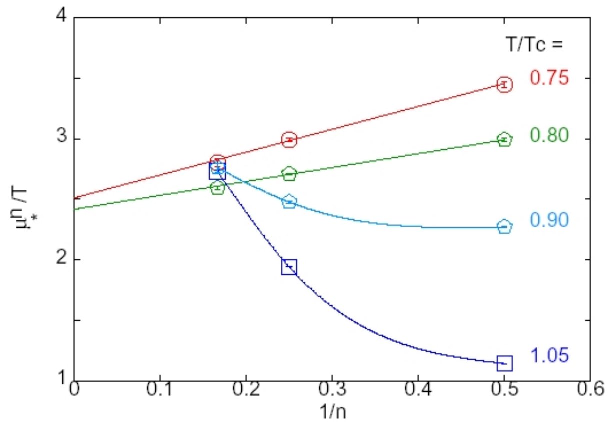

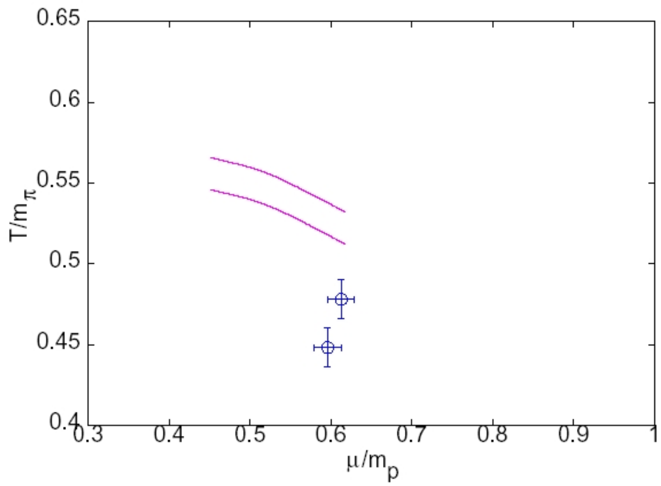

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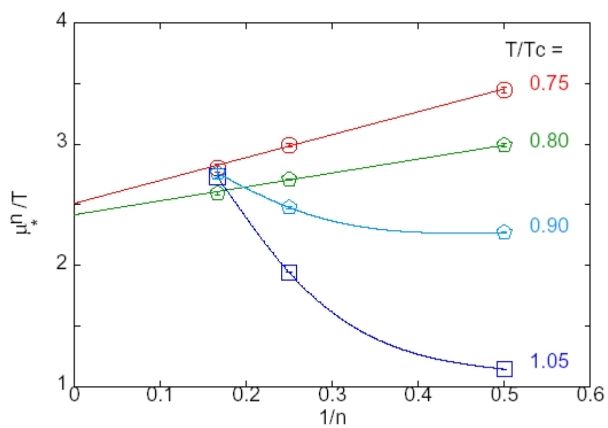

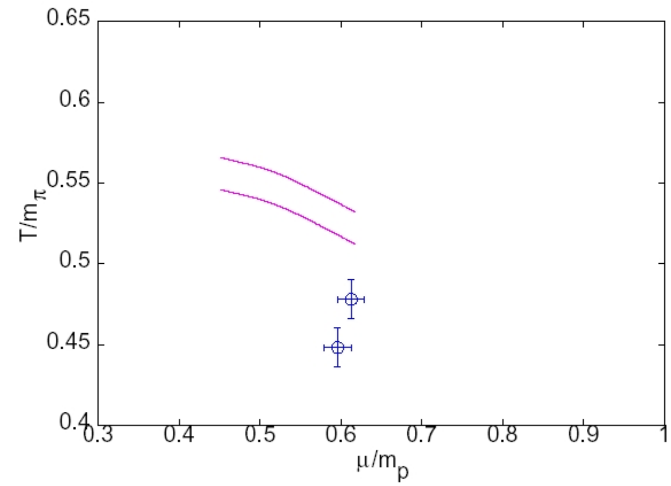

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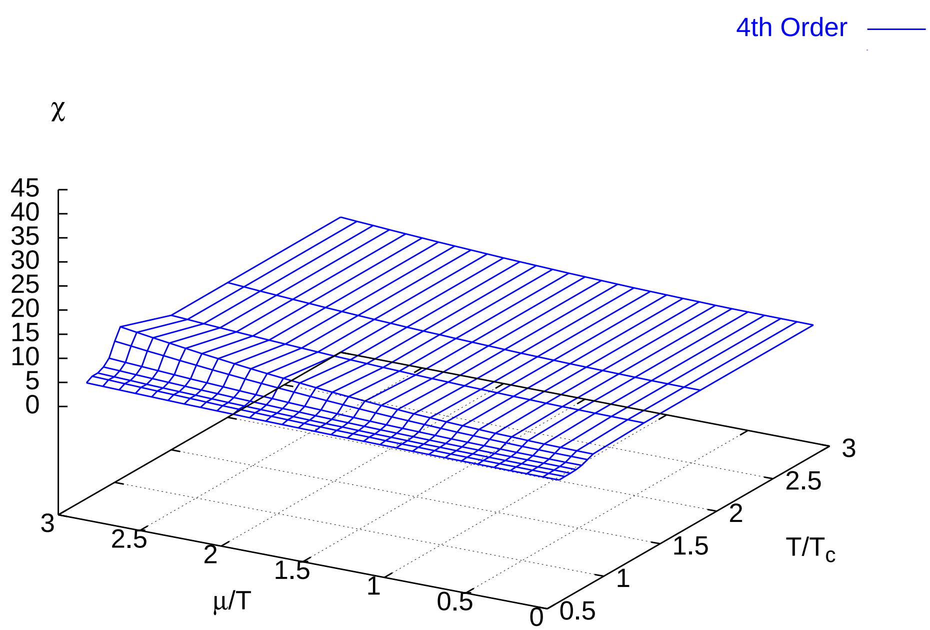

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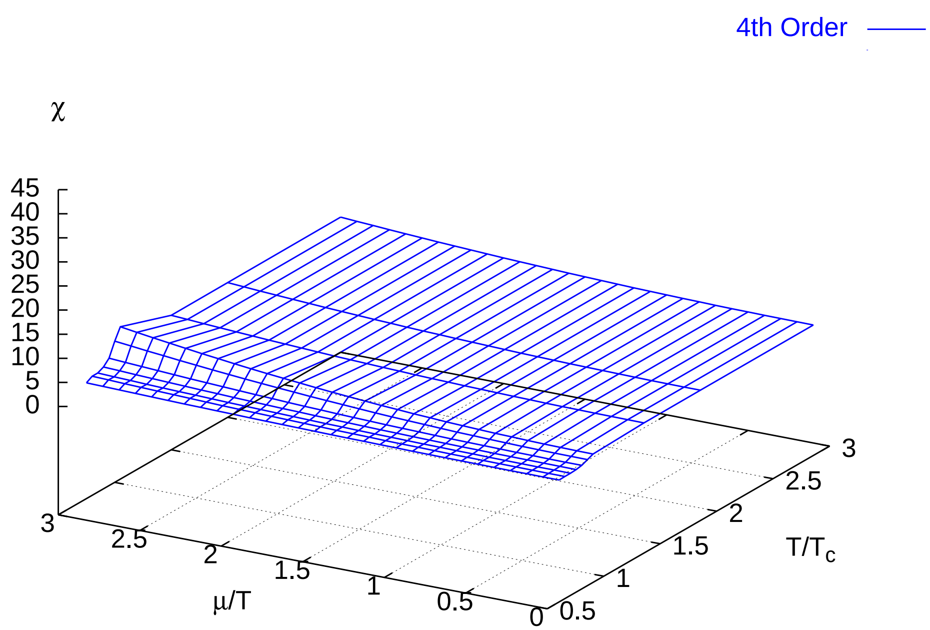

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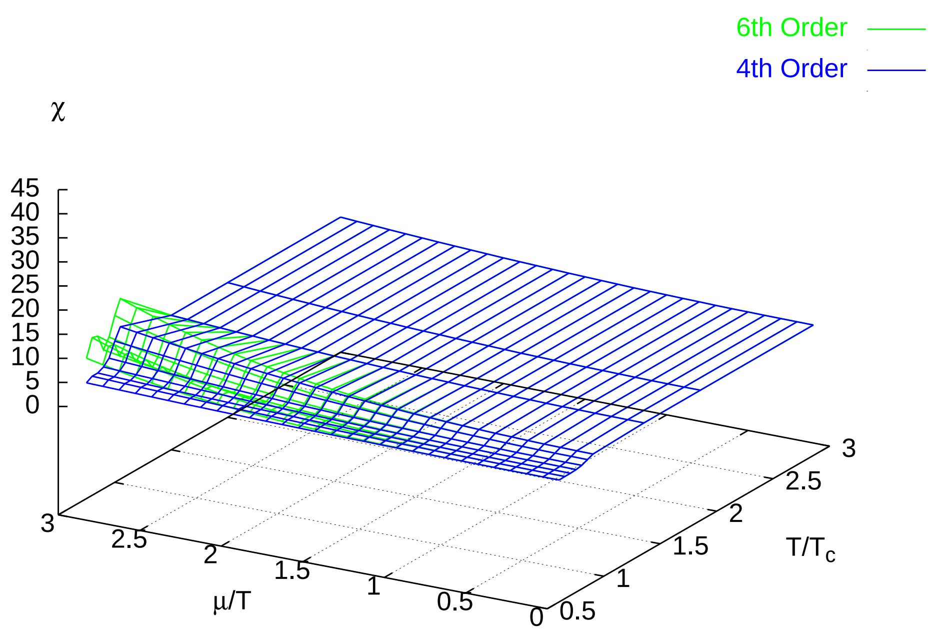

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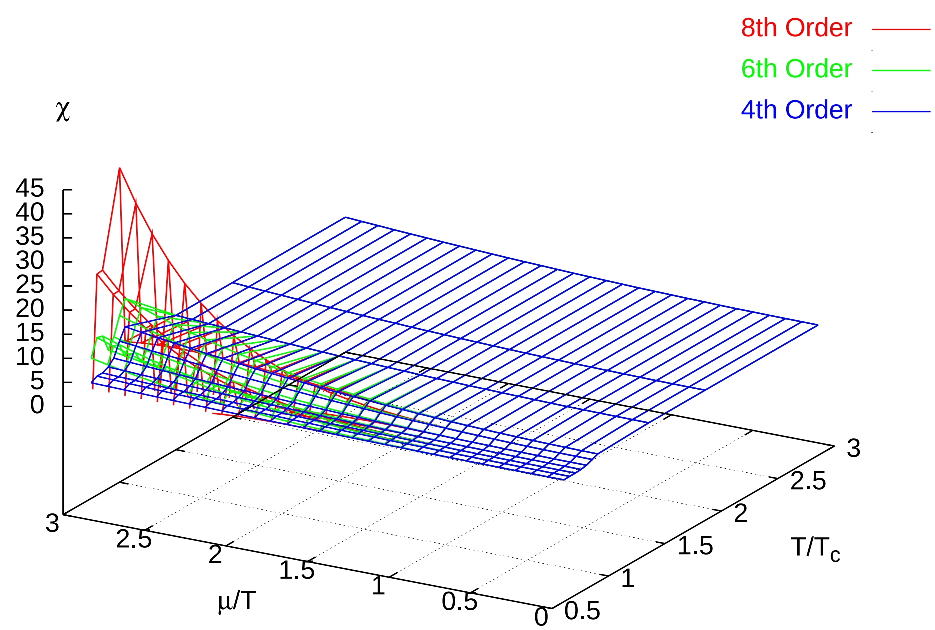

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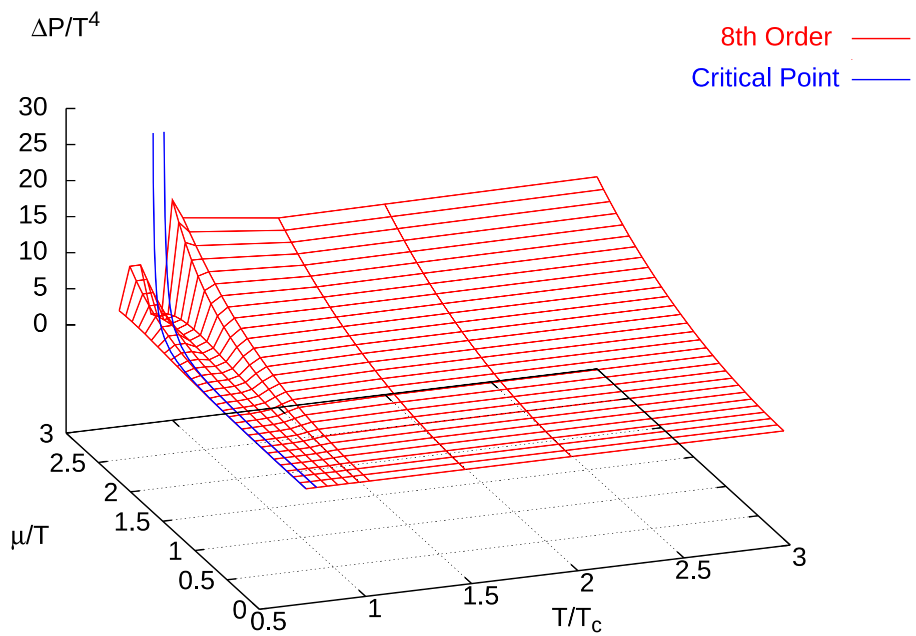

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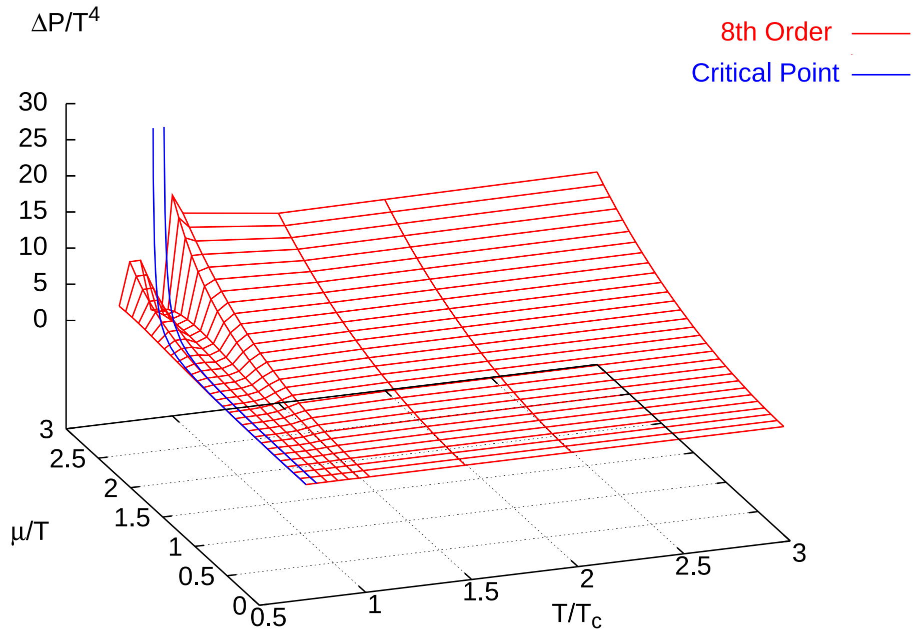

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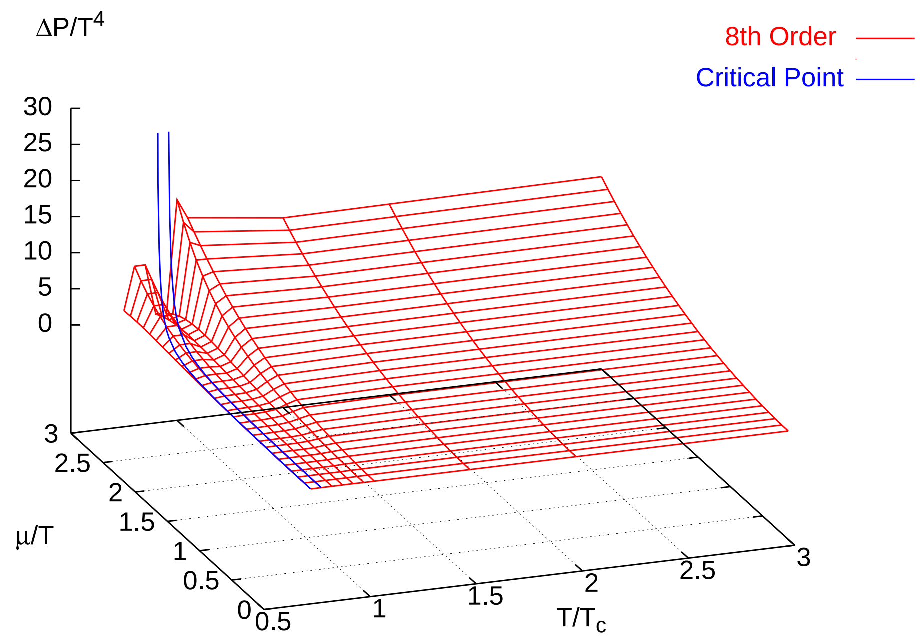

Continuum Limit

Recall,

- Chemical potential on lattice : Multiply each $U_4(x)$ by $f(a\mu)$ and $U_4^\dagger(x)$ by $1/f(a\mu)$, where $f(a\mu) = 1 + a\mu + \mathcal{O}(a^2)$. (Gavai, PRD '85)

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In continuum, $f(a\mu) = 1 + a\mu \rightarrow f''(0) = 0$.
On lattice, in general, all derivatives exist and depend on the nature of function : prescription dependence !

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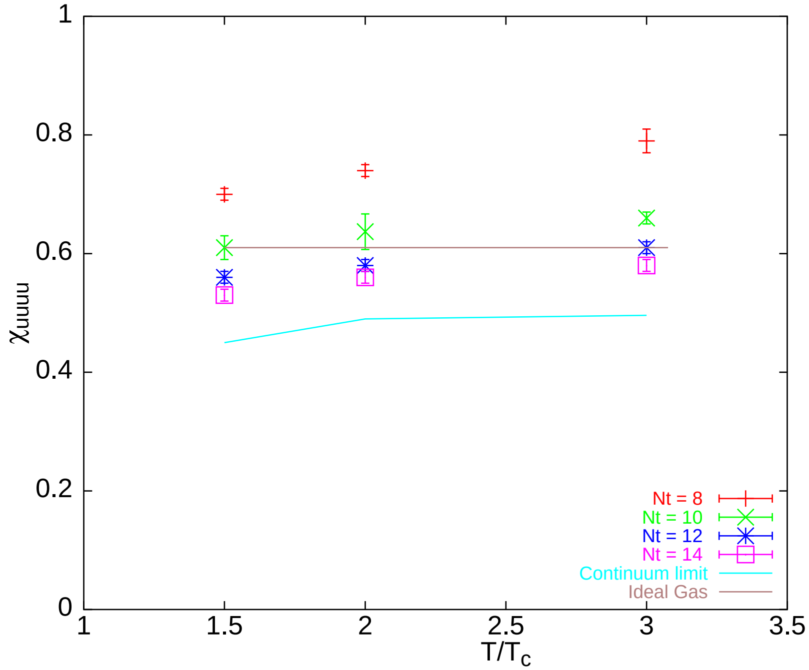
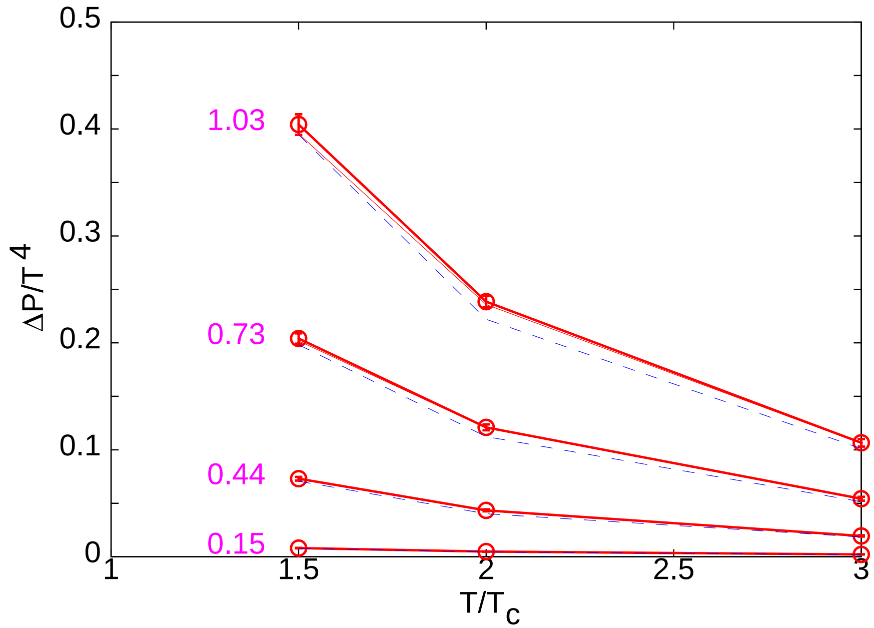
Our Results

Our results for χ_{uuuu} and ΔP: Gavai and Gupta, PR D68, '03





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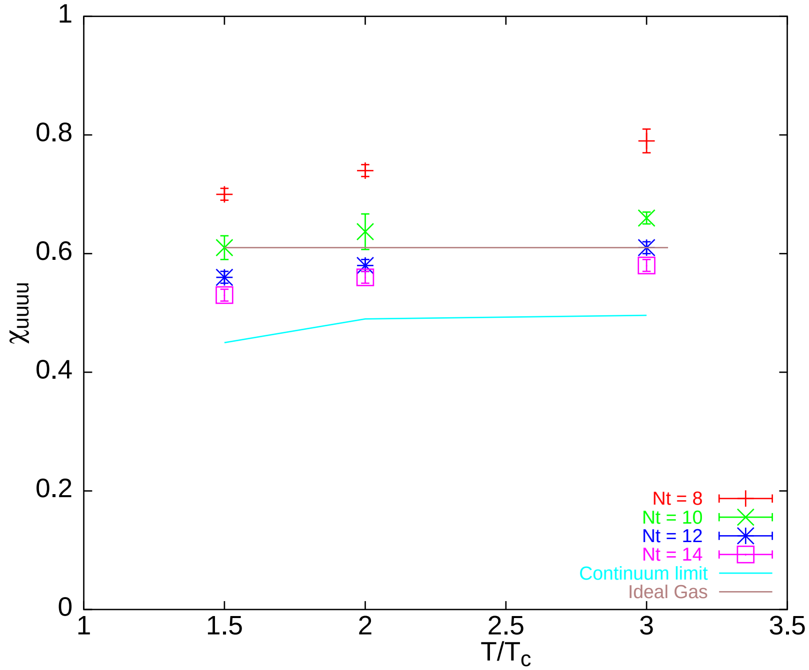

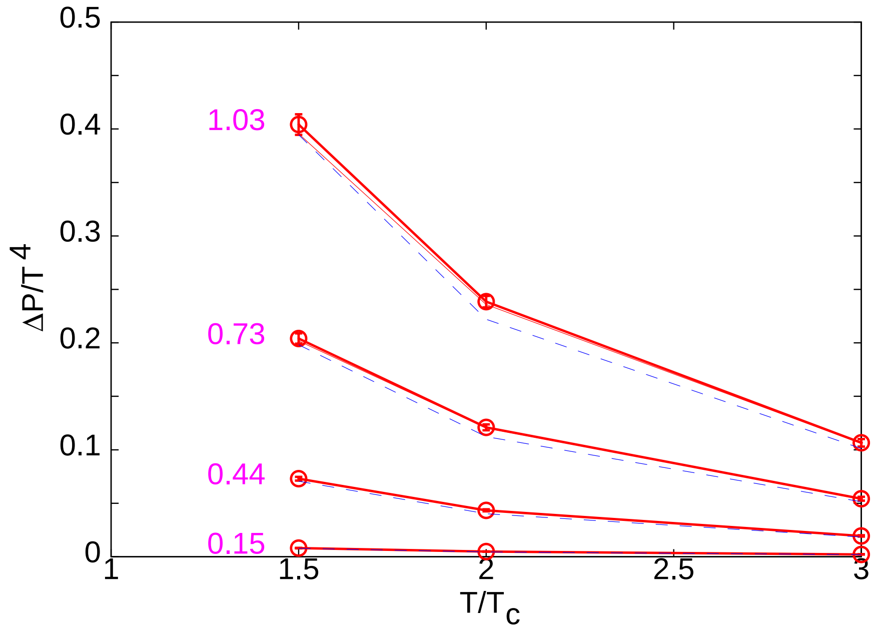

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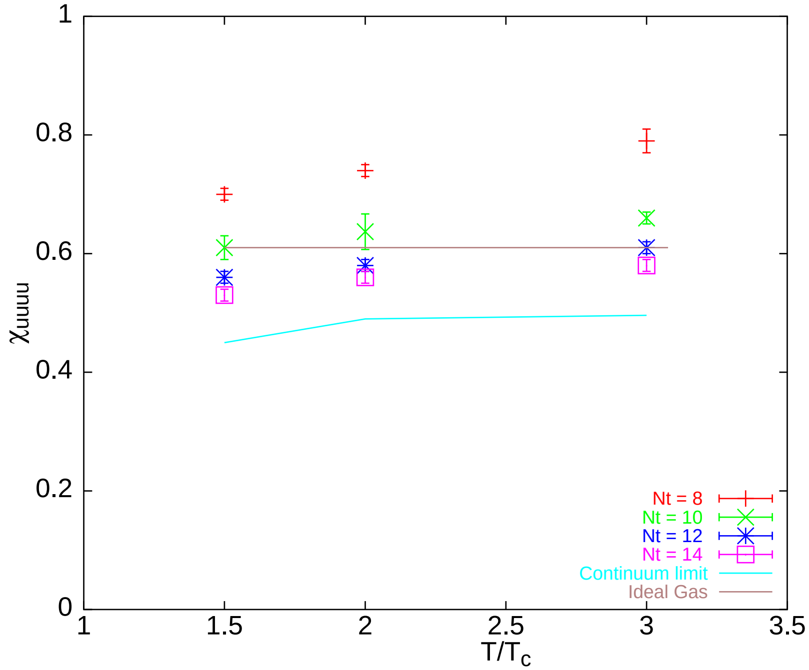

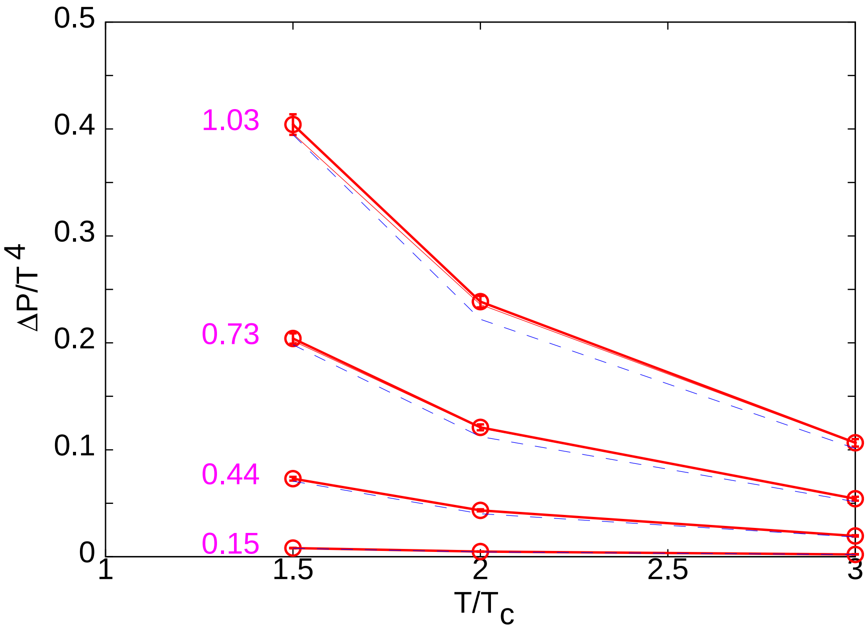

♡ Both reproduced in dimensional reduction (1 free parameter). Vuorinen, PR D68, '03

♡ Our results for P agree with Fodor-Katz (PL B568, '03) and the recent Bielefeld results (PR D68, '03).

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- Obtained from the exponential decay of

$$C_{\Gamma}(z) = \sum_{x,y,t} \langle M^{-1}_{\alpha\beta}(x,y,z,t)\Gamma M^{\dagger -1}_{\beta\alpha}(x,y,z,t)\Gamma\rangle \qquad (10)$$

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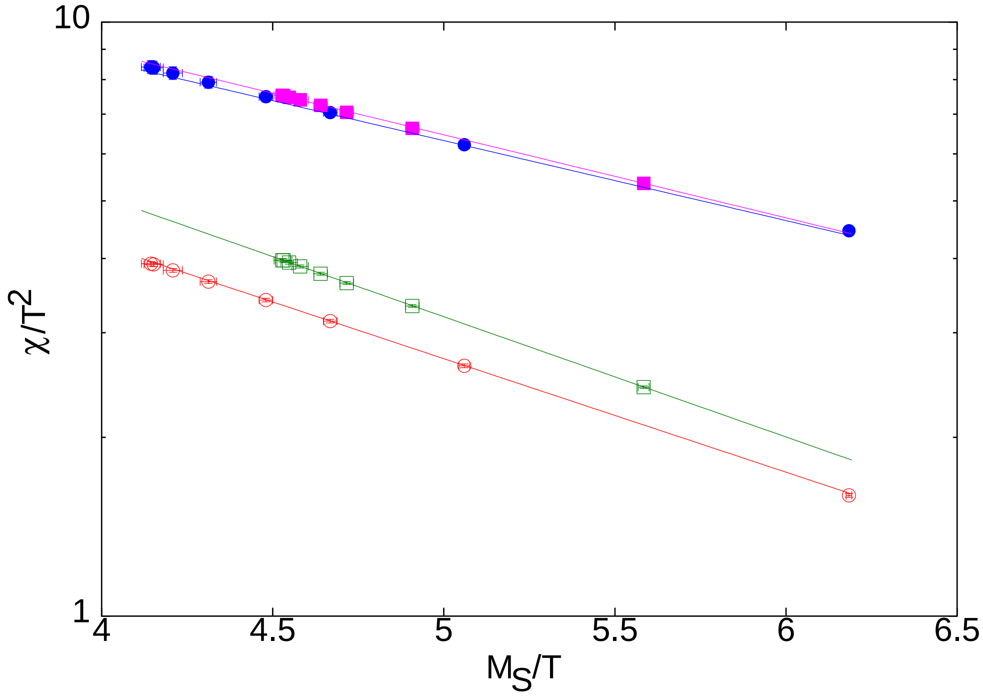
- Summing up the C_Γ for pion $\rightarrow$ Pion susceptibility.

$N_t = 4$ Lattices with $N_z = 16$.
$4\chi_3/T^2$ (open symbols) and $\chi_\pi/10T^2$ (filled)
at $2T_c$ (lower set) and $3T_c$.
(Gavai, Gupta & Majumdar, PR D '02)



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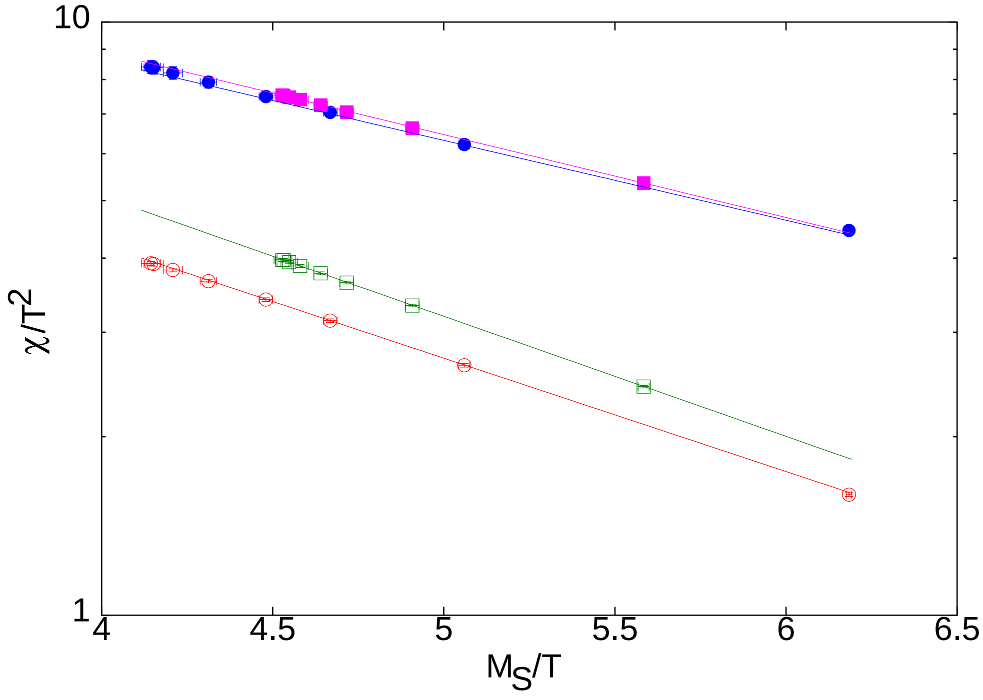

Why ? $\chi_3 \sim \sum$ propagator of nonlocal vector meson.

Taking Continuum Limit

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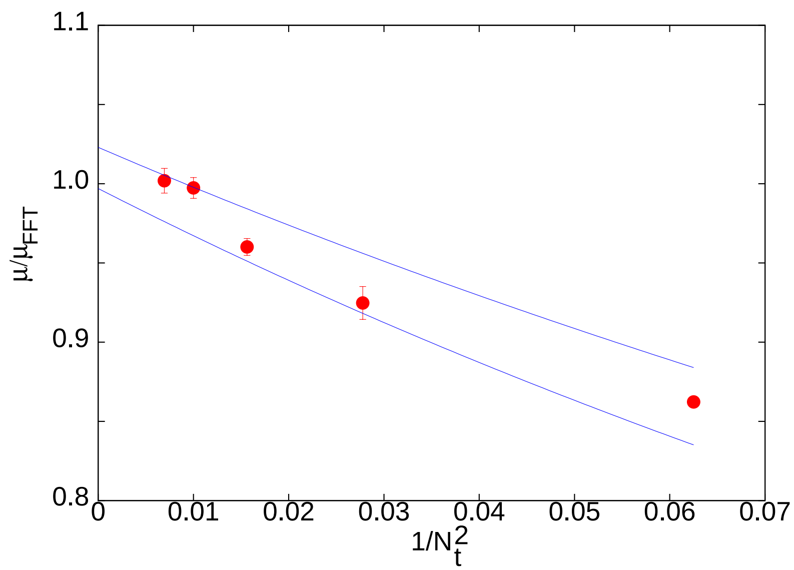
On finer lattices, a $= 1/8T$–$1/12T$, Pion screening lengths become degenerate with those of ρ , i.e, also close to FFT!! (Gavai & Gupta, hep-lat/0211015)



- $m_v/T_c = 0.03$,
- Lattices up to 48×26^2.

Overlap Fermions agree:

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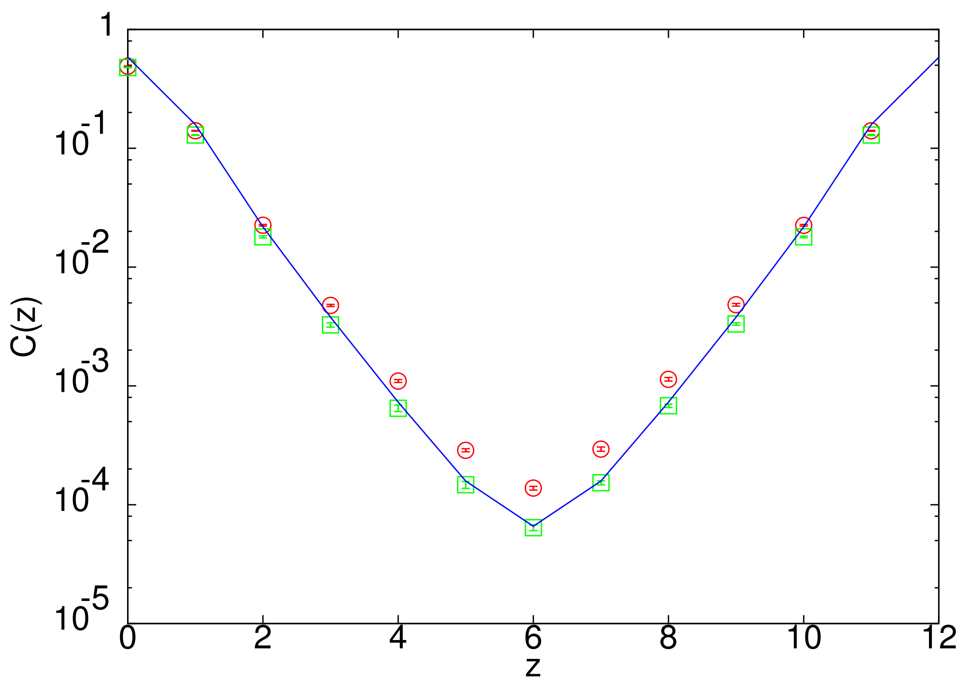

Configurations with zero modes excluded. $12^3 \times 4$ lattice at $T = 1.5T_c$. Quenched Approximation. $m/T_c = 0.006$

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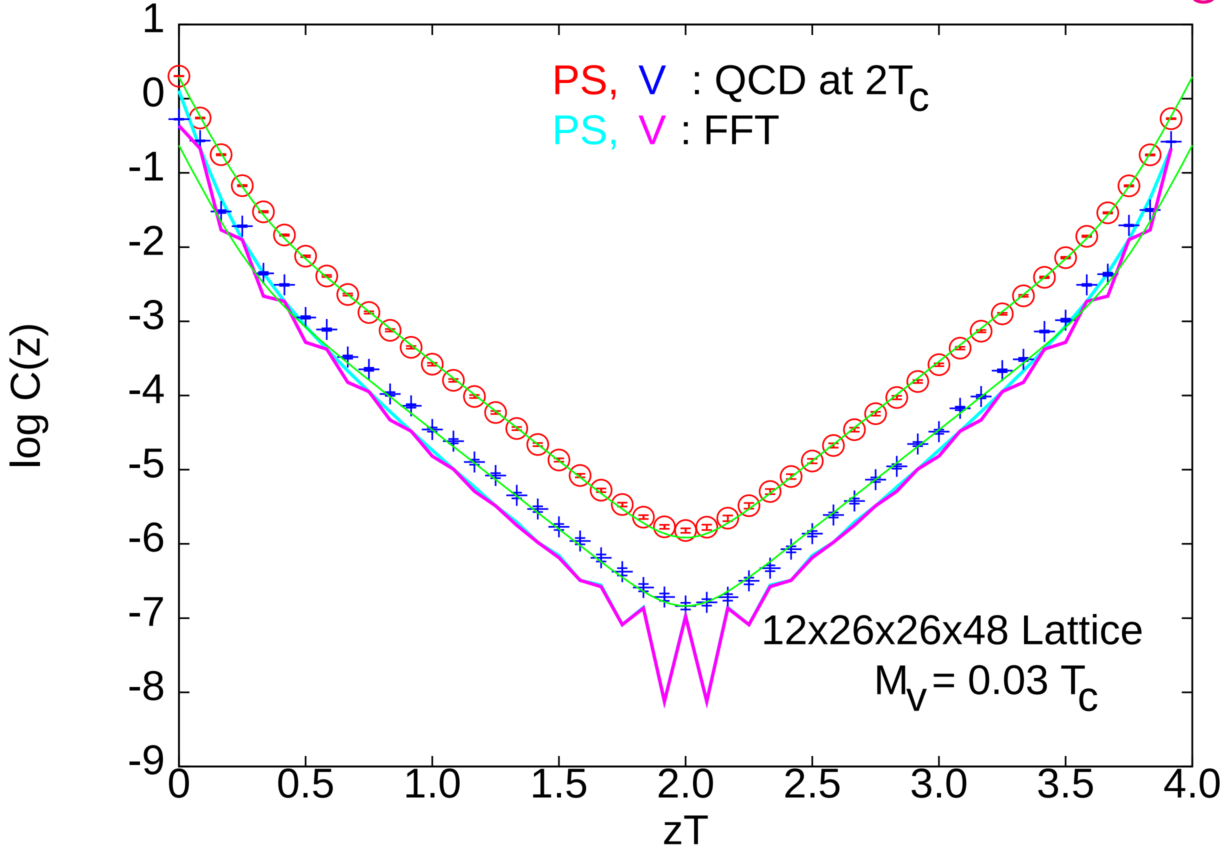
Note that both PS and V have SAME fit with changed normalization.



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- Pressure for nonzero μ obtained. At both SPS and RHIC, χ_{uu} is the major contribution.

- Many questions still for full 2+1 QCD : Order, Large N_t, $\cdots$.