The Wroblewski parameter from lattice QCD

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T. I. F. R., Mumbai, India

Strangeness Spectra Session, QM '04

Introdution

λ_s from Quark Number Susceptibility

Summary

Introduction

- Enhancement of strangeness production as a promising signal of QGP (Rafelski-Müller, Phys. Rev. Lett '82, Phys. Rept '86..).

- Most signal considerations based on Simple Models.

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  - Energy Threshold for $(s\bar{s})$ in QGP $<$ in Hadron Gas.
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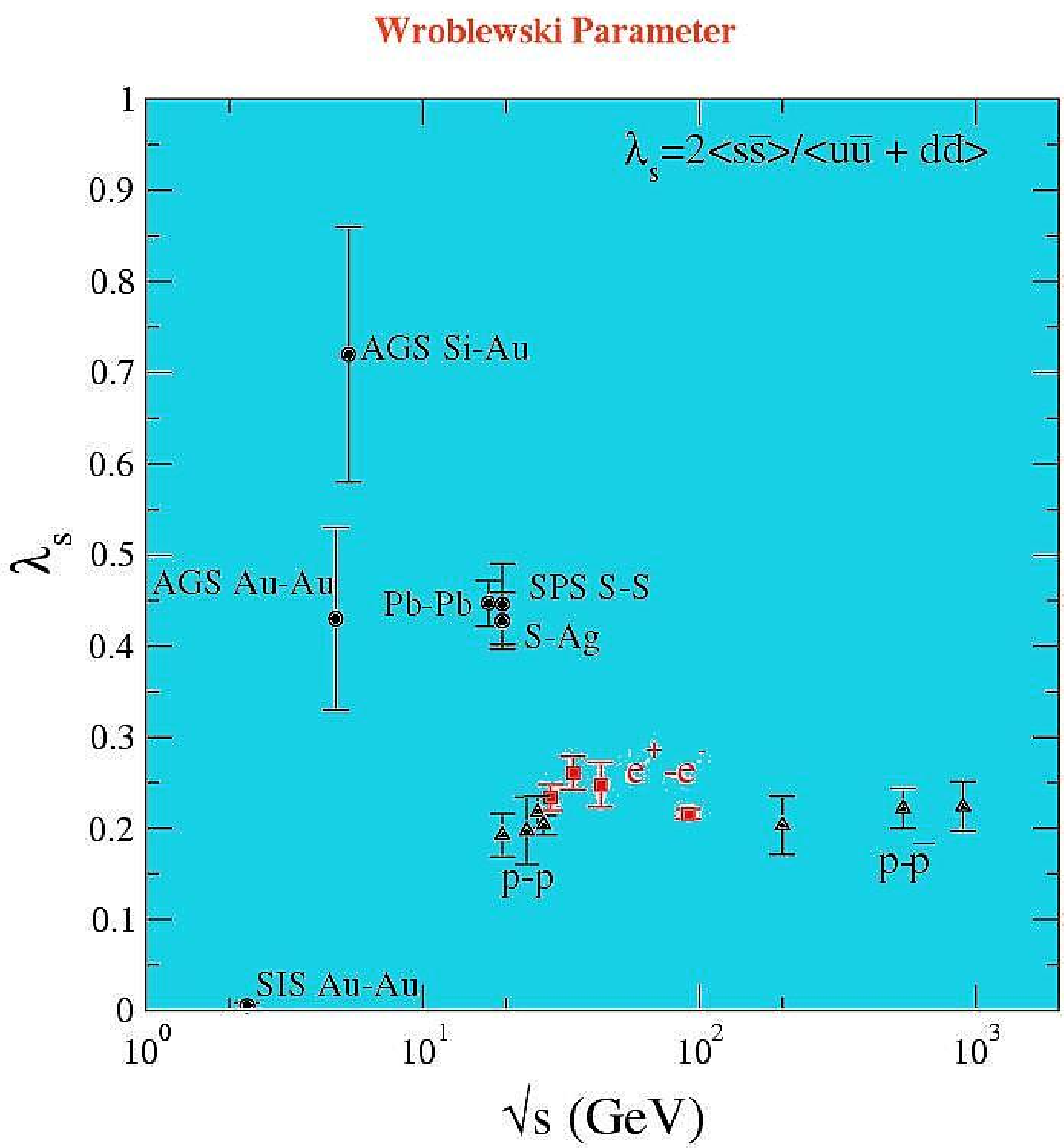

Ratio of newly created strange quarks to light quarks :

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle} \quad (1)$$

Hadron gas fireball model

(Becattini-Heinz '97).

Quark Number Susceptibility

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♠ Our improvement: Fixed m_q/T_c, Continuum limit...

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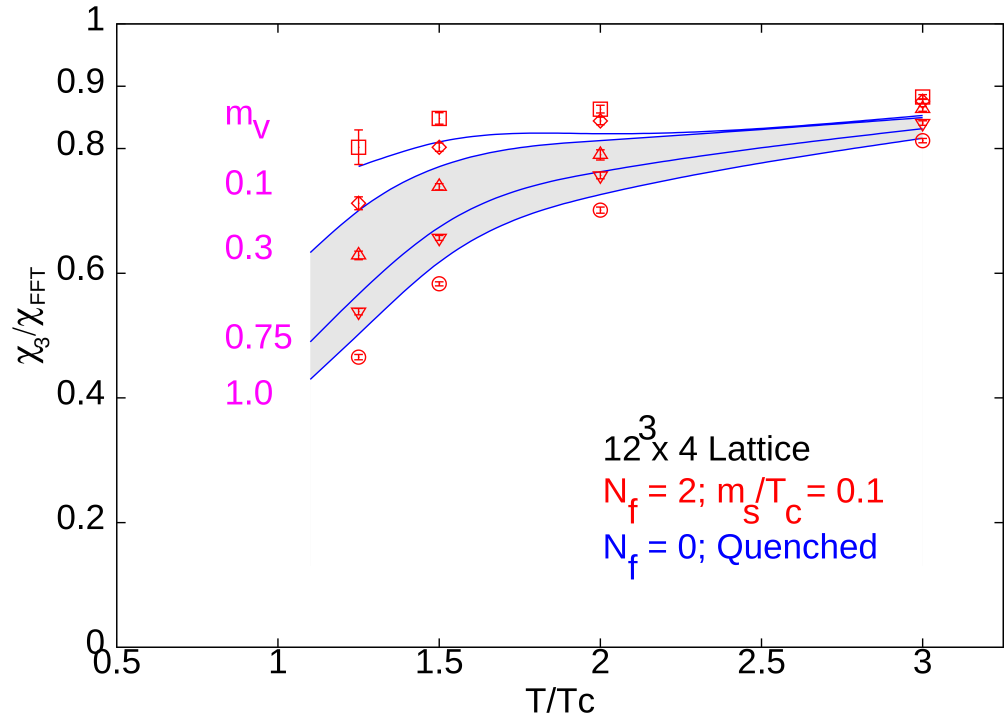

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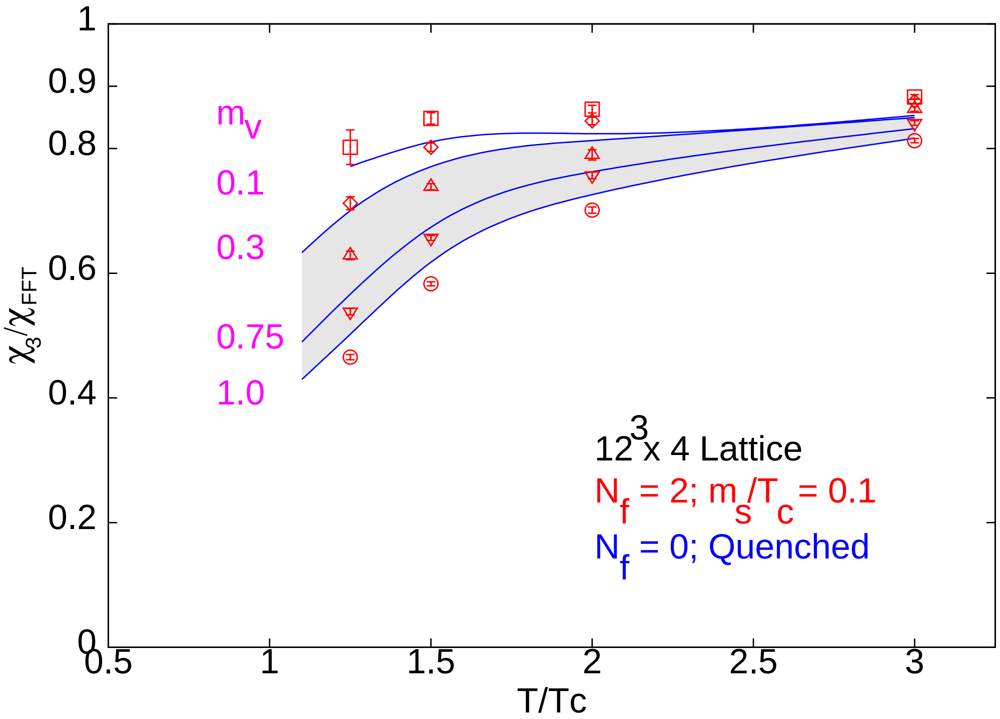

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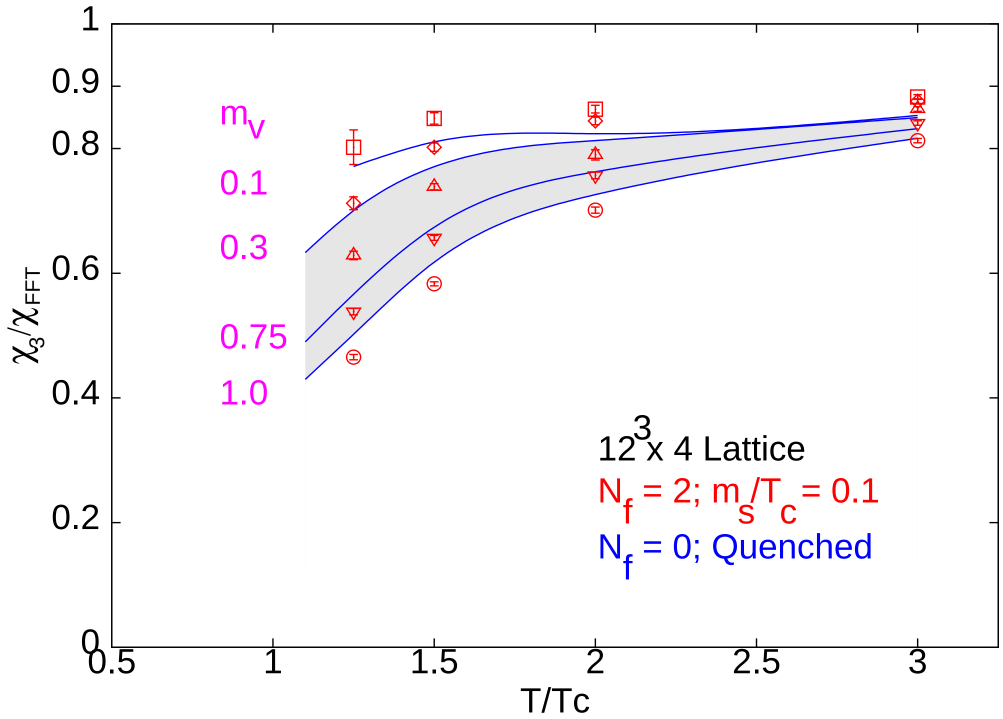

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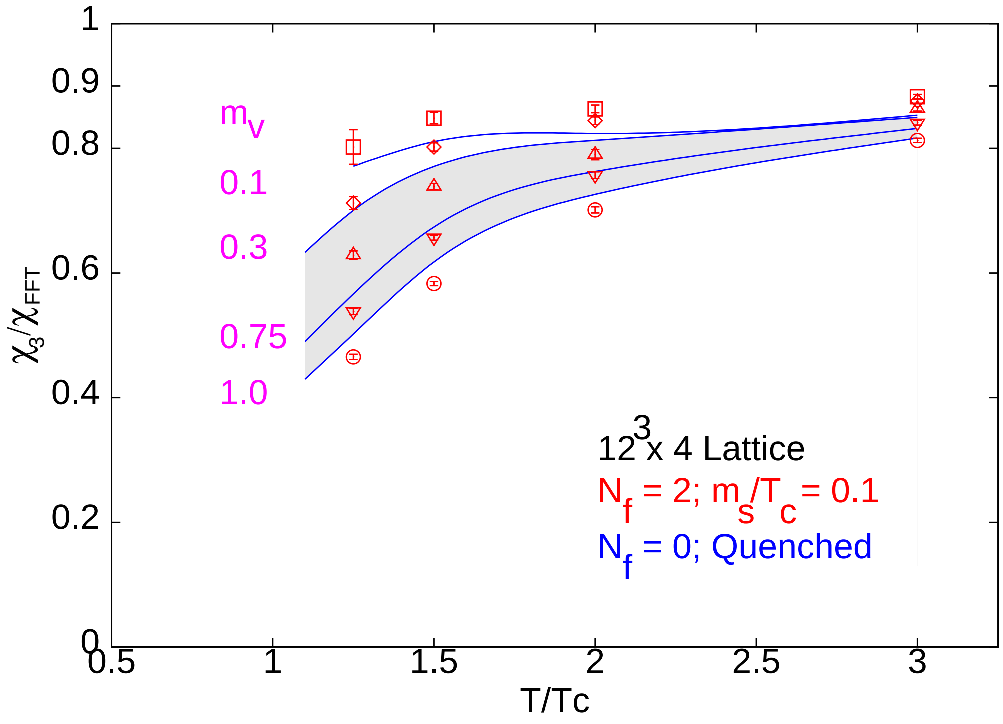

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3) PDG values for strange quark mass $\Longrightarrow m_v^{strange}/T_c$ $\simeq$ 0.3-0.7 (N_f=0);
0.45-1.0(N_f=2).

Taking Continuum Limit

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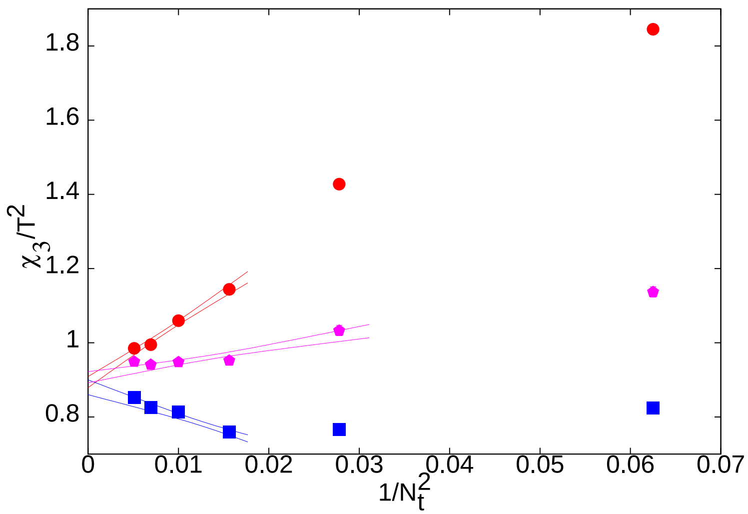

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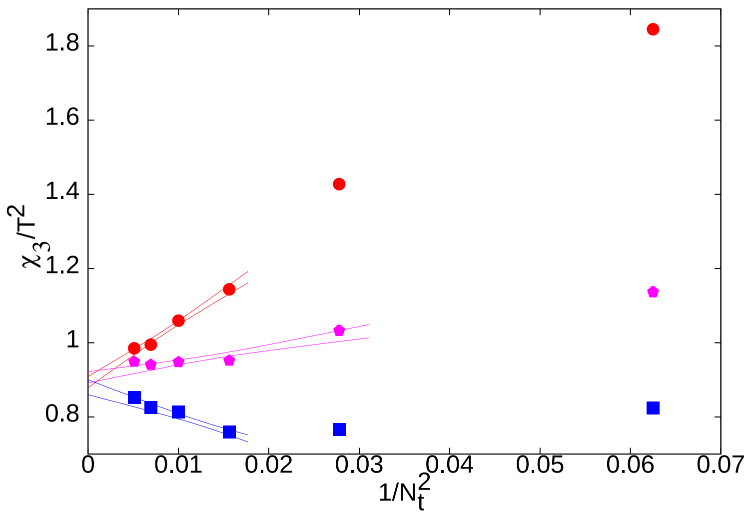

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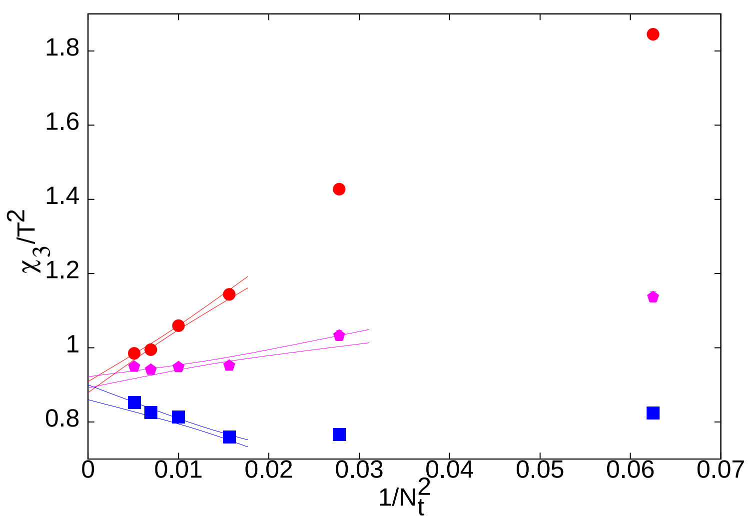

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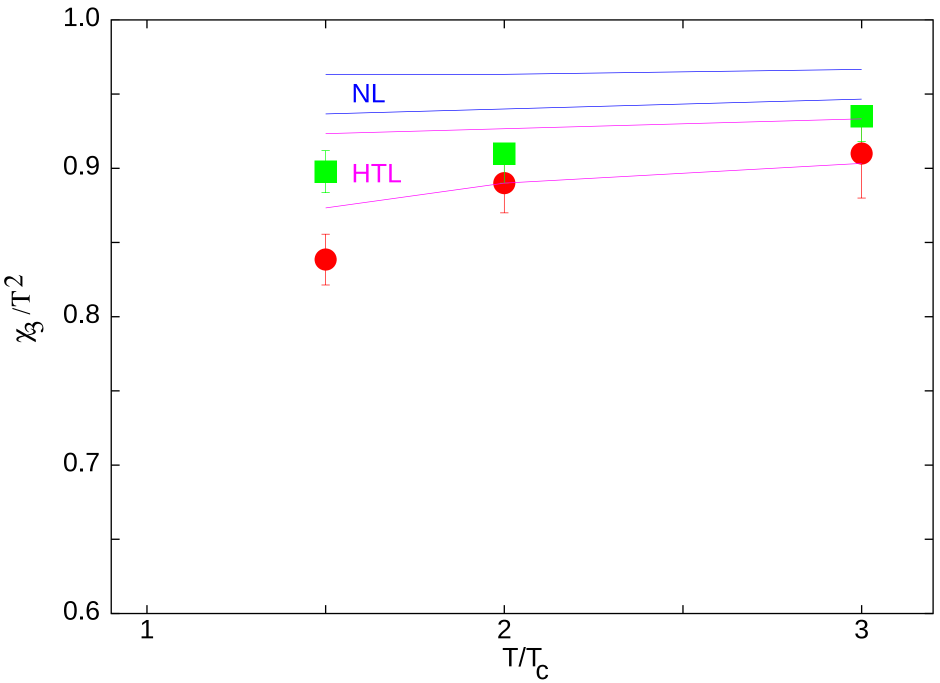

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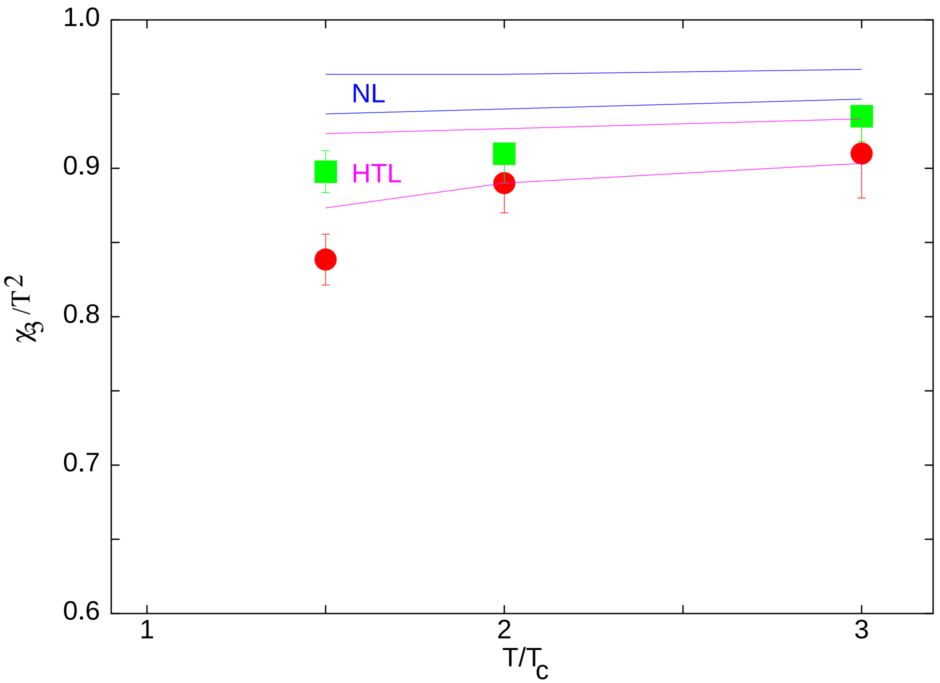

Naik action (Squares) and Staggered action (circles)

♡ Also reproduced in dimensional reduction (1 free parameter). Vuorinen, PR D '03.

♡ Note that χ_{ud} behaves the same way for ALL N_t and both fermions, leading to the same $O(10^{-6})$ values in continuum too.

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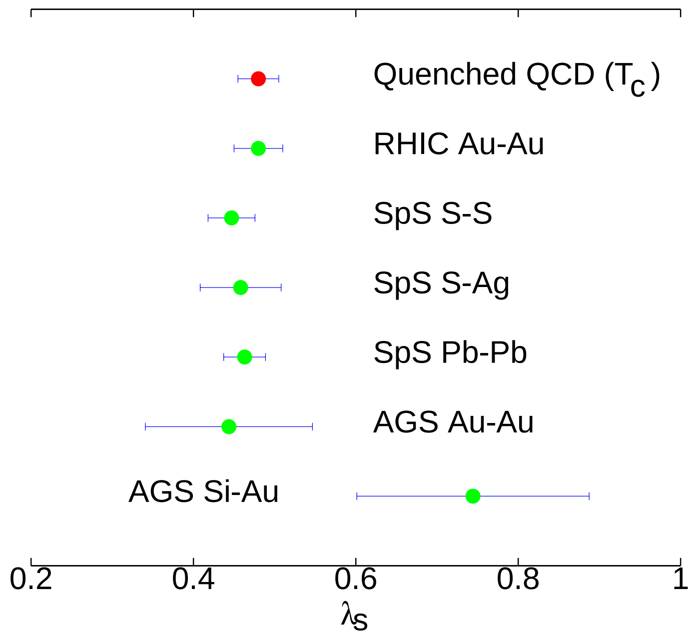

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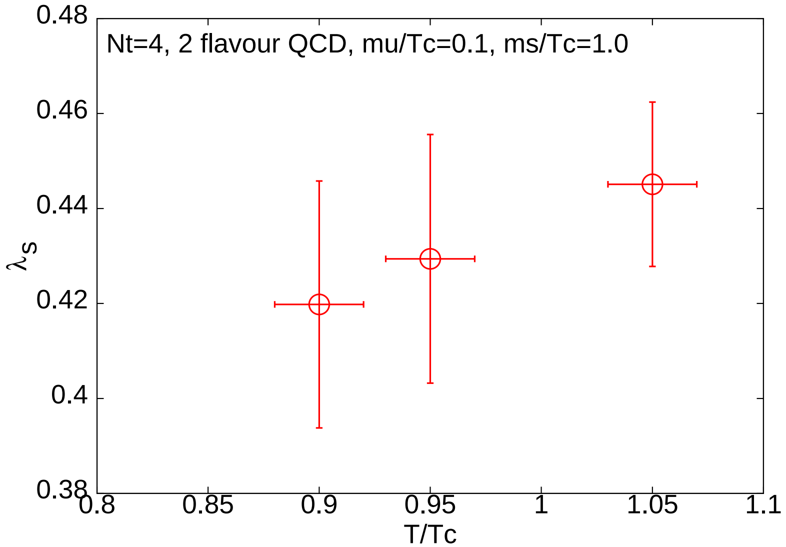

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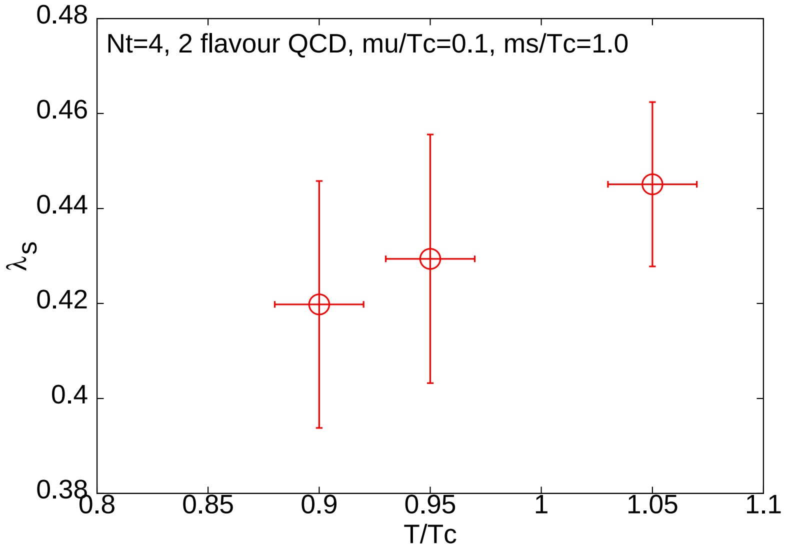

♣ Large finite volume effects below T_c
♣ Up to 12^3 Lattices used.
♣ Strong dependence on m_s expected.
♣ Large finite a effects.

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- Assumed : Chemical equilibration in the plasma.

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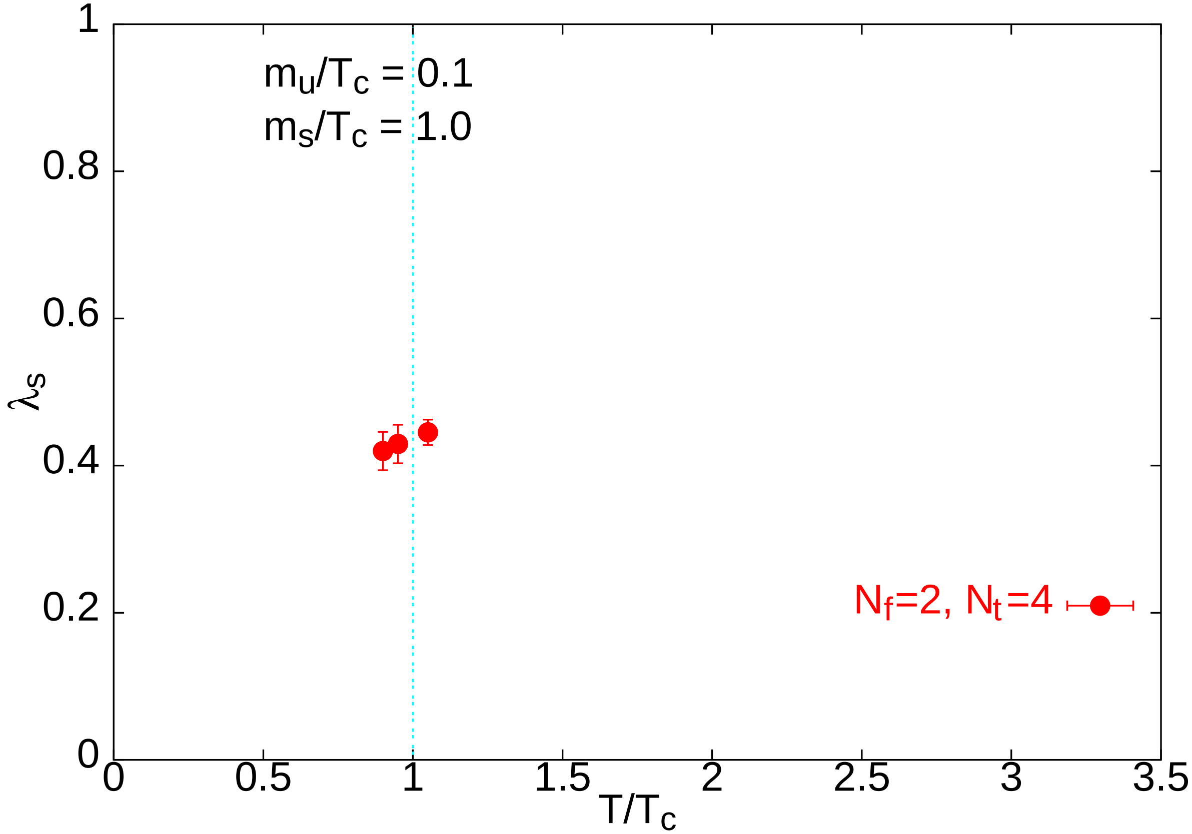

$m_u/T_c = 0.1$
$m_s/T_c = 1.0$
N_f=2, N_t=4
λ_s
T/T_c
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5

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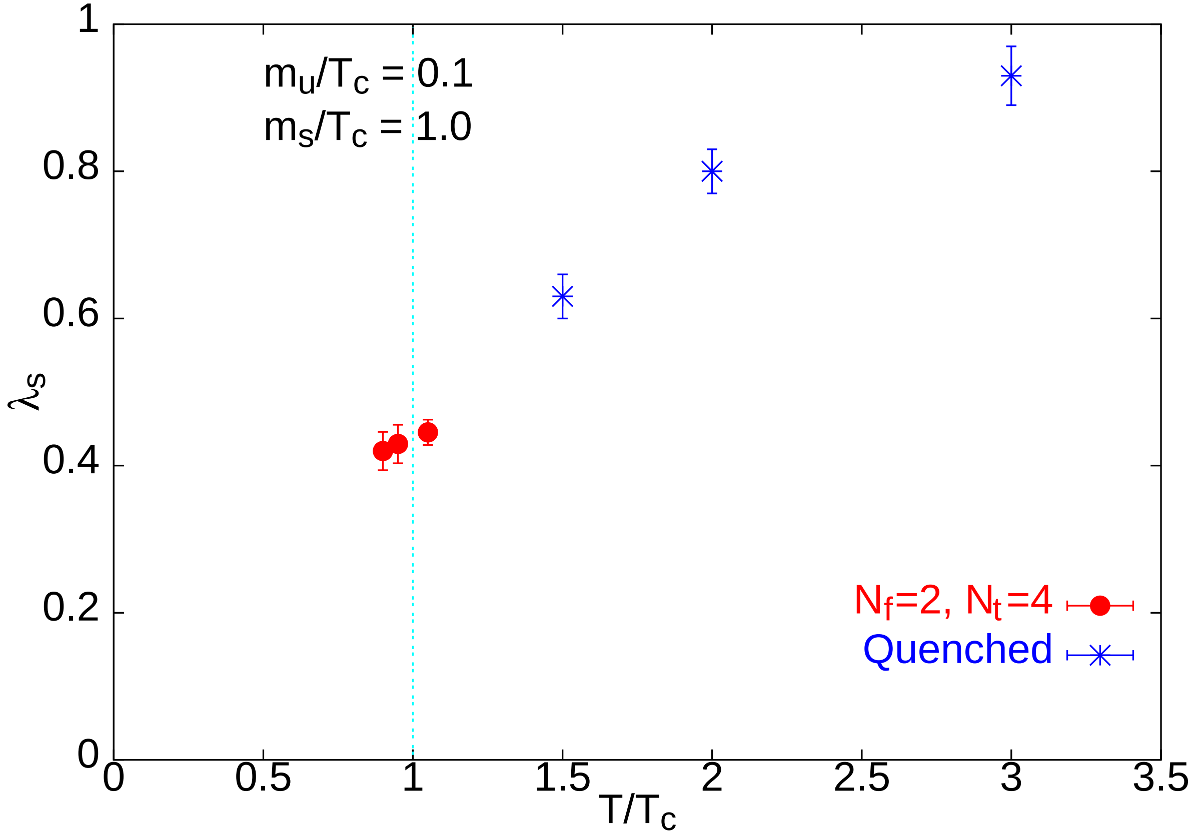

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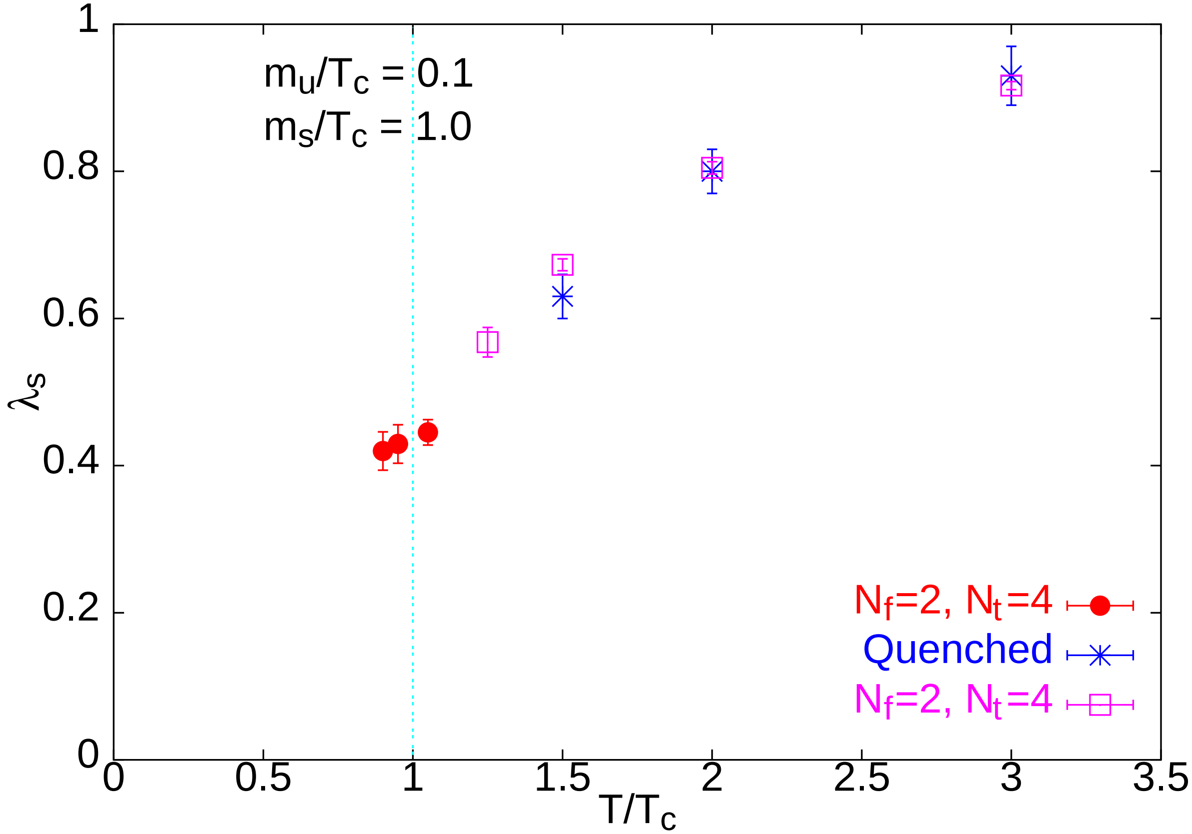

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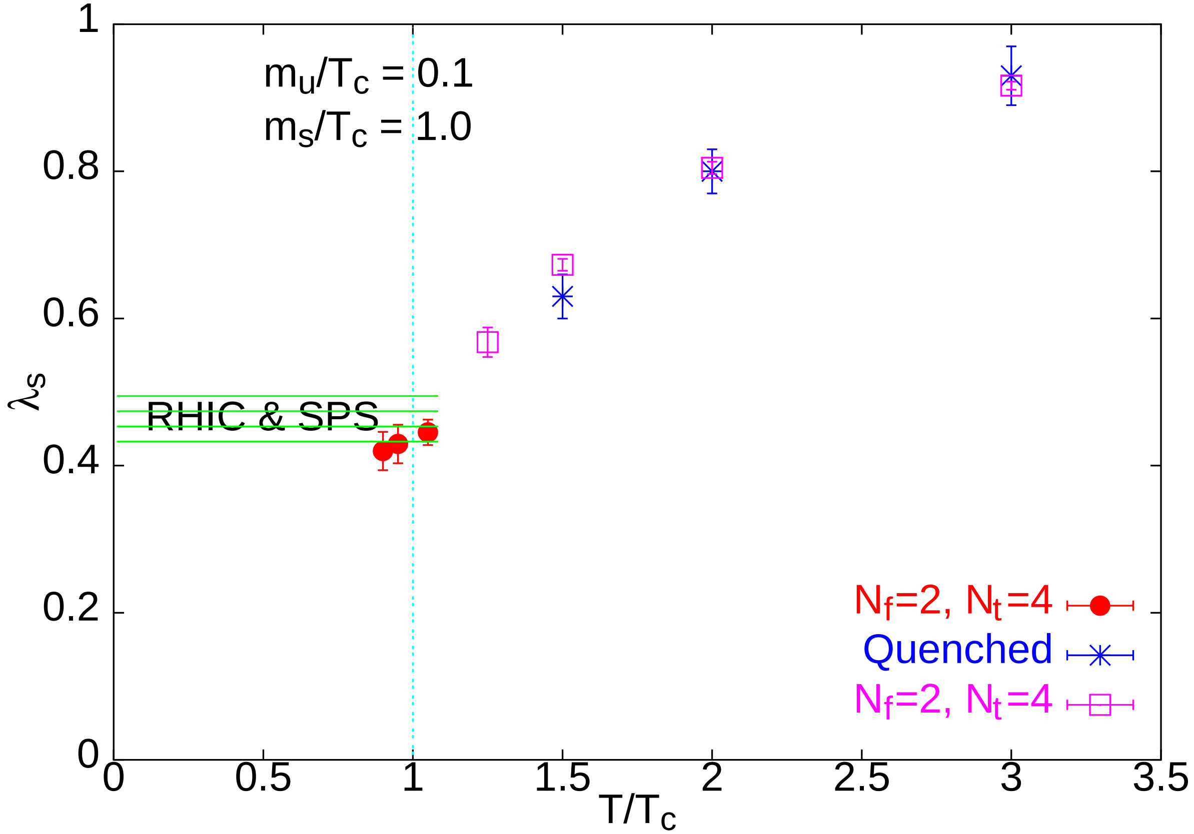

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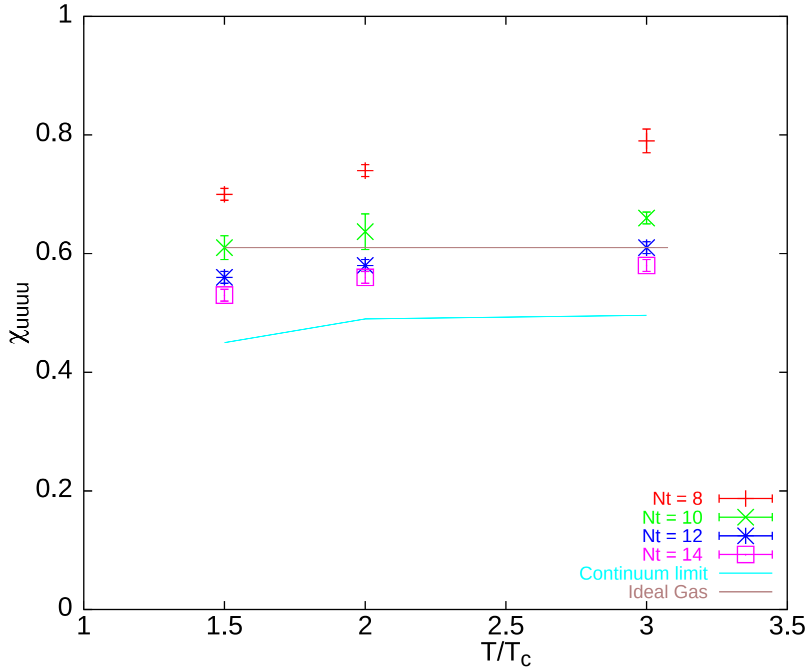
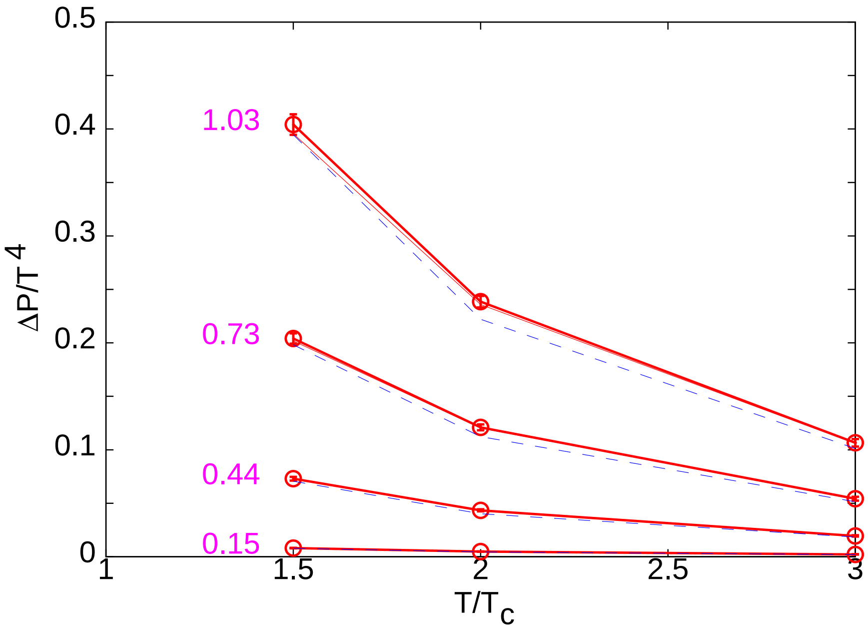
- Continuum limit of χ_{uu} yields λ_s in agreement with RHIC and SPS results after extrapolation to T_c. First full QCD investigations show intersting trend.
- Pressure for nonzero μ obtained in continuum. At both SPS and RHIC, χ_{uu} is the major contribution. Need to extend to Full QCD.

Our Results

Our results for χ_{uuuu} and ΔP: Gavai and Gupta, PR D68, '03





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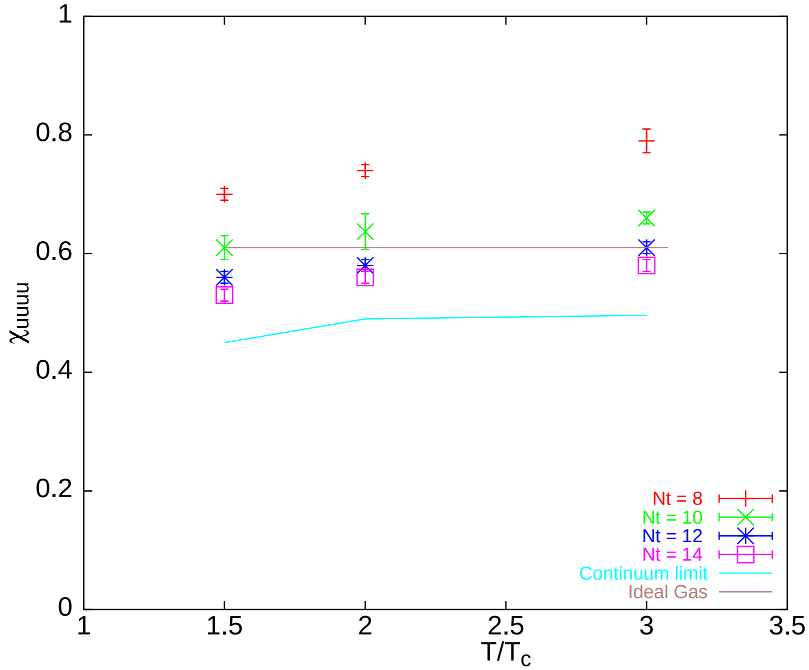

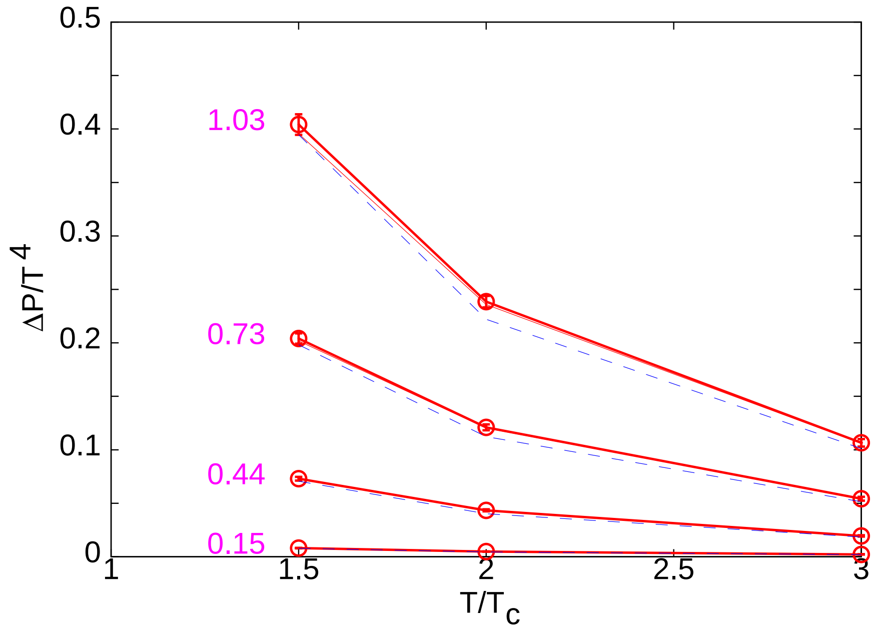

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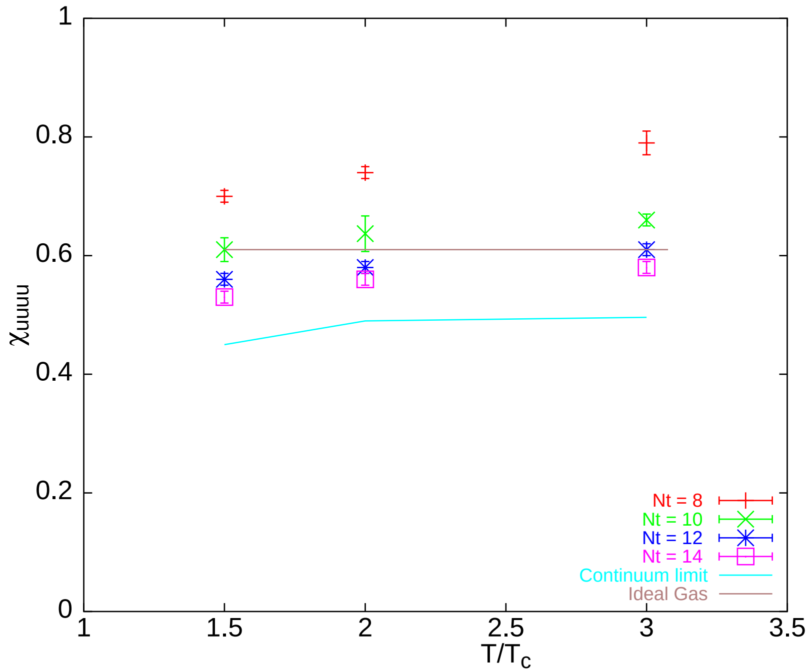

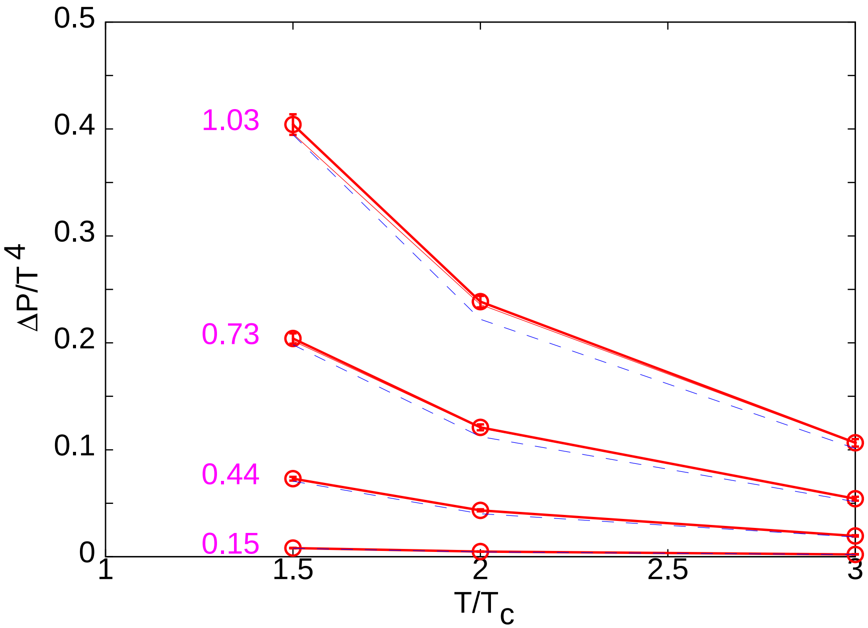

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♡ Our results for P agree with Fodor-Katz (PL B568, '03) and the recent Bielefeld results (PR D68, '03).

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$$\chi_0 = \frac{T}{2V}[\langle \mathcal{O}_2(m_u) + \frac{1}{2}\mathcal{O}_{11}(m_u)\rangle] \quad (5)$$

$$\chi_3 = \frac{T}{2V} \langle \mathcal{O}_2(m_u)\rangle \quad (6)$$

$$\chi_s = \frac{T}{4V}[\langle \mathcal{O}_2(m_s) + \frac{1}{4}\mathcal{O}_{11}(m_s)\rangle] \quad (7)$$

All of these can be written as traces of products of M^{-1} and various derivatives of M.

Setting $\mu_i = 0$, n_i =0 but χ are nontrivial. Diagonal χ_{ii}'s are

$$\chi_0 = \frac{T}{2V}[\langle \mathcal{O}_2(m_u) + \frac{1}{2}\mathcal{O}_{11}(m_u)\rangle] \quad (5)$$

$$\chi_3 = \frac{T}{2V} \langle \mathcal{O}_2(m_u)\rangle \quad (6)$$

$$\chi_s = \frac{T}{4V}[\langle \mathcal{O}_2(m_s) + \frac{1}{4}\mathcal{O}_{11}(m_s)\rangle] \quad (7)$$

Here $\mathcal{O}_2 = \mathrm{Tr}\ M_u^{-1}M_u'' - \mathrm{Tr}\ M_u^{-1}M_u'M_u^{-1}M_u'$, and $\mathcal{O}_{11}(m_u) = (\mathrm{Tr}\ M_u^{-1}M_u')^2$, and the traces are estimated by a stochastic method:
$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L}(\mathrm{Tr}\ A)_i(\mathrm{Tr}\ A)_j/L(L-1)$,
where R_i is a complex vector from a set of N_v subdivided in L independent sets.