Lattice QCD

Lattice QCD : Some Topics

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Basic Lattice Gauge Theory

Phase Diagram

Quark Number Susceptibility

Screening Lengths

Summary

Basic Lattice Gauge Theory

- Discrete space-time : Lattice spacing a UV Cut-off.
- Matter fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gauge transformation : $\psi'(x) = V_x \psi(x)$, $V_x \in SU(3)$.
- Gauge Fields on links $U'_\mu(x) = V_x U_\mu(x) V^{-1}_{x+\hat{\mu}}$.
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  - Wilson fermions (only flavour symmetry),

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Typically, we need to evaluate

$$\langle \Theta(m_v) \rangle = \frac{\int DU \exp(-S_G)\Theta(m_v) \ \mathrm{Det} \ M(m_s)}{\int DU \exp(-S_G) \ \mathrm{Det} \ M(m_s)} \ , \tag{1}$$

where M is the Dirac matrix in x, color, spin, flavour space for fermions of mass m_s, S_G is the gluonic action, and the observable Θ may contain fermion propagators of mass m_v.

Since $\langle \Theta \rangle$ is computed by averaging over a set of configurations $\{U_\mu(x)\}$ which occur with probability $\propto \exp(-S_G) \cdot \mathrm{Det}\ M$, the complexity of evaluation of Det $M \Longrightarrow$ approximations : Quenched ($m_s = \infty$ limit), Partially Quenched (low $m_s = m_u = m_d$), and Full (including a heavier s quark).

Q $\rightarrow$ PQ $\rightarrow$ Full $\rightsquigarrow$ Computer time $\uparrow$ and Precision $\downarrow$.

Phase Diagram

- Lattice details :
  - $N_s^3 \times N_t$ Lattice, $N_s \gg N_t$ for $T \neq 0$,
  - Spatial Volume $V = N_s^3 a^3$,
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- Order Parameters : Chiral condensate $\langle \bar{\psi}\psi \rangle$,
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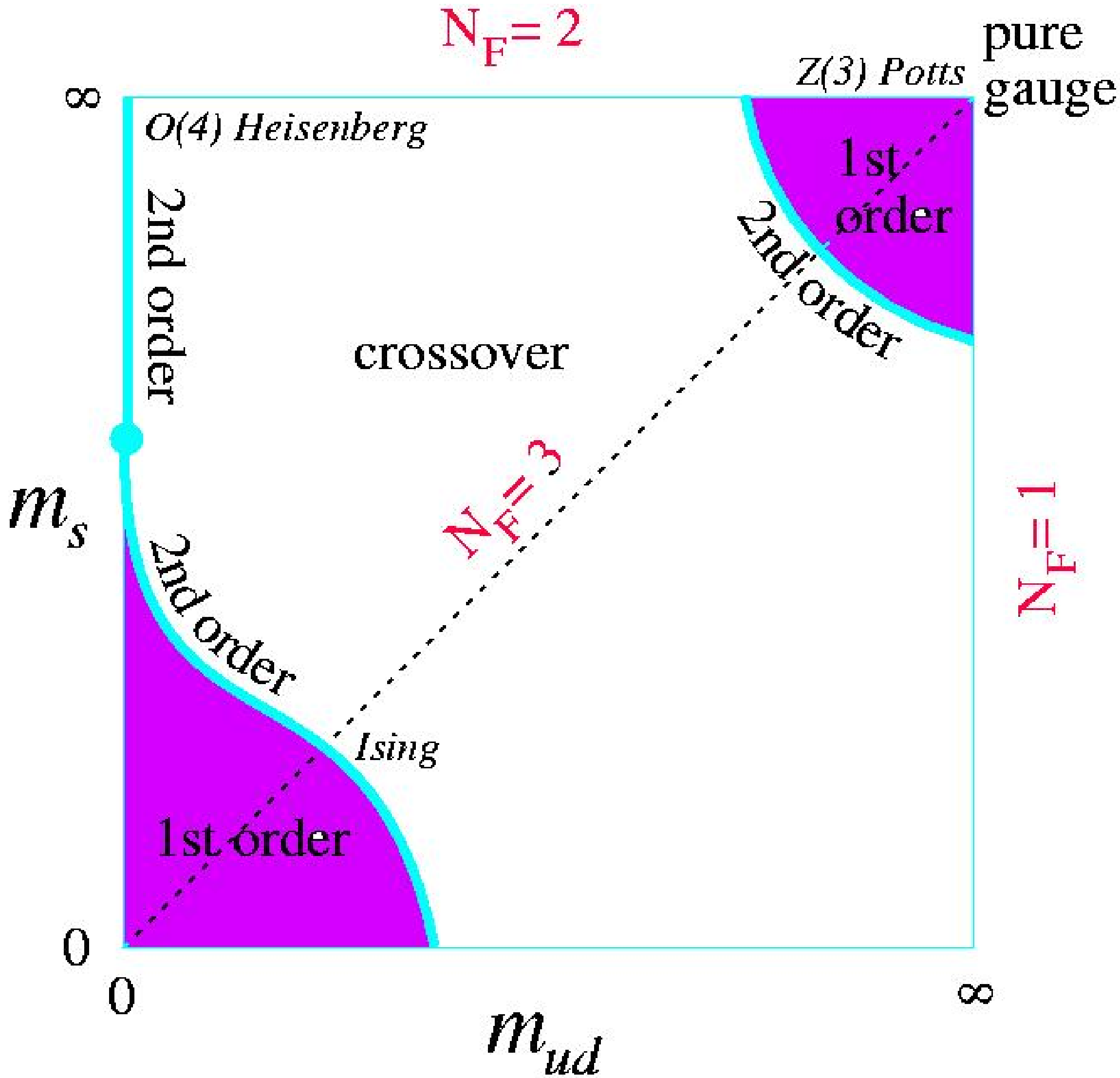
- Theoretical expectations based on effective models :



$N_F = 2$
pure gauge
Z(3) Potts
∞
O(4) Heisenberg
2nd order
1st order
2nd order
crossover
m_s
$N_F = 3$
$N_F = 1$
2nd order
Ising
1st order
0
0
∞
m_{ud}

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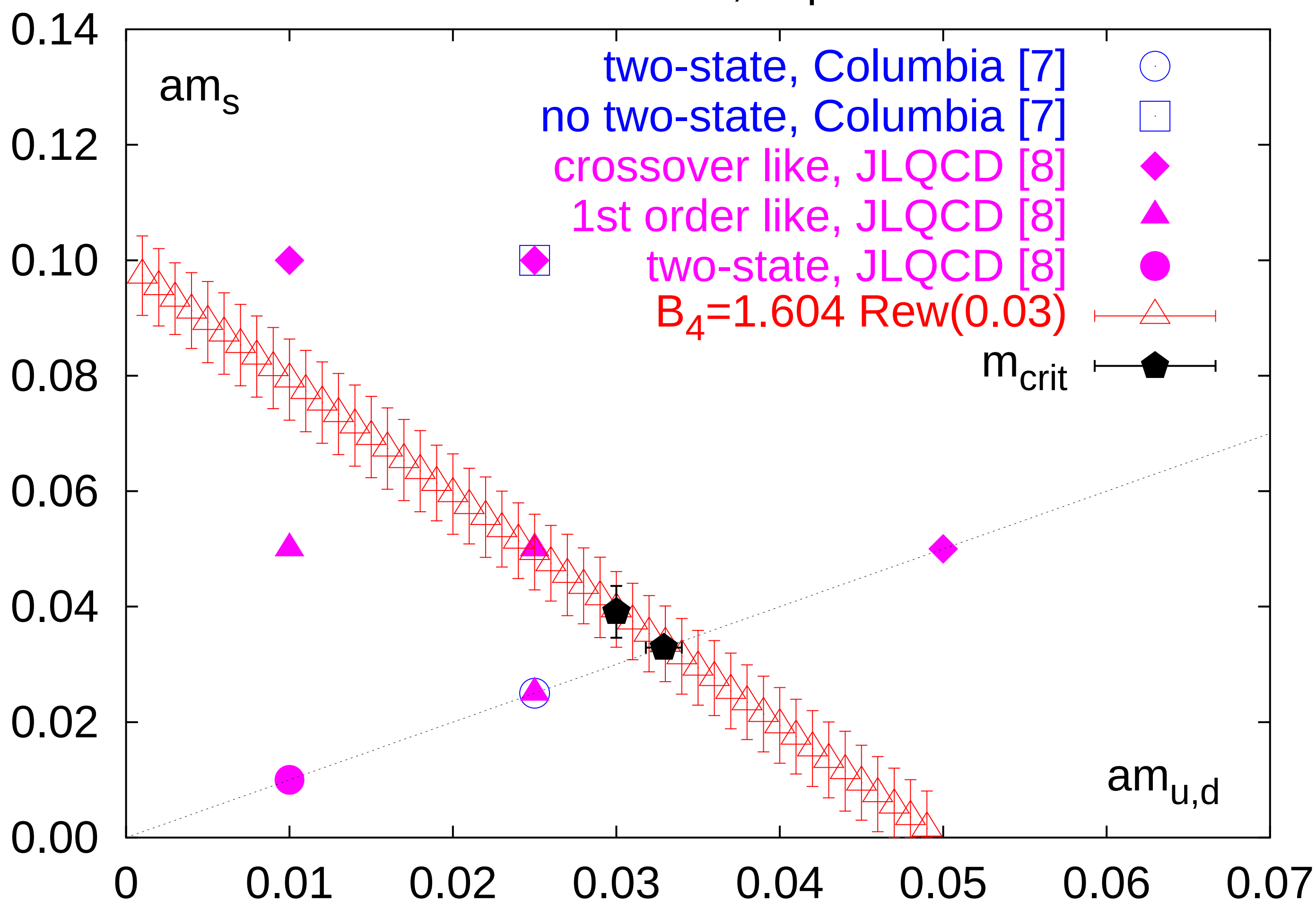
- Theoretical expectations on the Phase diagram work out too.

Lattice'02 : Schmidt et al., hep-lat/0209009



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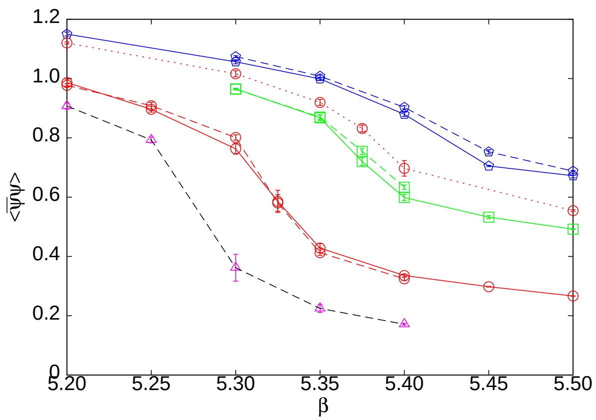
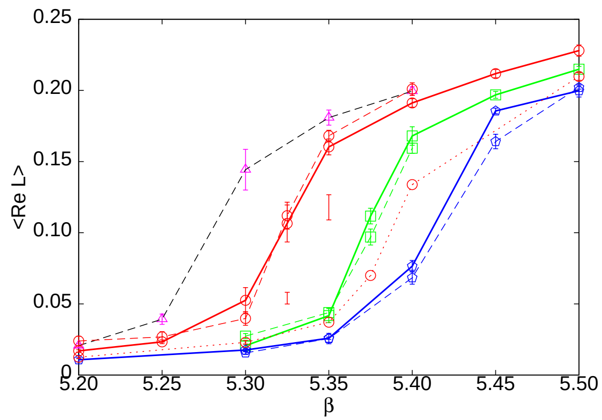
$8^3 \times 4$ Lattice; $m_d/m_u = 1$ (dashed), 2(full) and 10 (dotted).





$\mu_{\rm B} \neq 0$

- QCD at nonzero baryon density may $\longrightarrow$ Color Superconductivity, (Strange) Quark Stars, etc.

- Phase Problem : Det $M(\mu)$ is complex for $\mu \neq 0$.

- Exciting results in recent past for small μ.
  - Re-weighting Method (Fodor & Katz, JHEP '02)
  - Imaginary μ (de Forcrand & Philipsen, May '02, D'Elia & Lombardo, May '02)
  - Taylor Expansion in μ (Allton et al., April '02)

- Large μ simulations possible when Det M is real, *e.g.*, 2 colours or $\mu_{I_3} \neq 0$. Show agreement with effective chiral theory (Kogut & Sinclair '02, S. Gupta '02)

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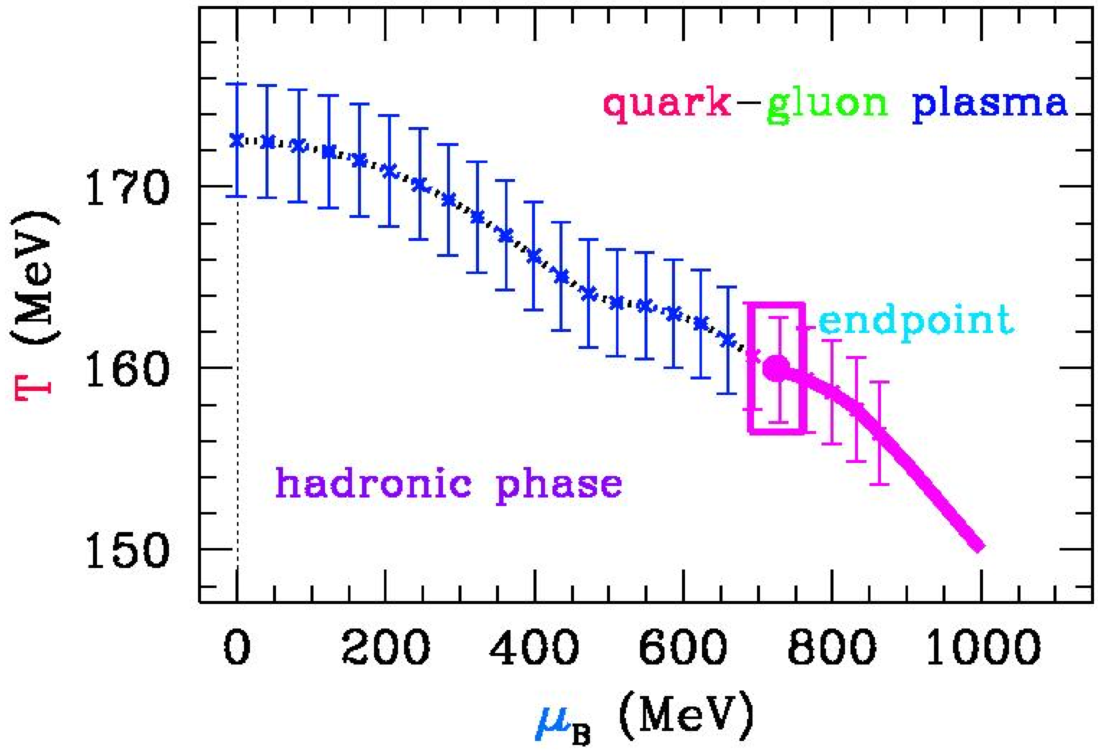
Fodor-Katz Results



$N_s^3 \times 4$ Lattices,
$N_s =$ 4,6,8;
Bit heavy u,d quarks.
Critical Endpoint :
$T =$ 160(4) MeV,
$\mu =$ 725(35) MeV

As $m_{ud} \downarrow$, does $\mu_E \downarrow$? Larger N_t ??

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Definitions: For u, d, and s quarks, the partition function is

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$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}, \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}$$

Setting $\mu_i = 0$, n_i =0 but χ_{ij} are nontrivial. Diagonal χ's are

$$\chi_0 = \frac{1}{2}[\mathcal{O}_1(m_u) + \frac{1}{2}\mathcal{O}_2(m_u)] \tag{3}$$

$$\chi_3 = \frac{1}{2}\mathcal{O}_1(m_u) \tag{4}$$

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Here $\mathcal{O}_i$ trace of products of M^{-1}, M' and M'' and are estimated by a stochastic method:
$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L} (\mathrm{Tr}\ A)_i (\mathrm{Tr}\ A)_j / L(L-1)$,
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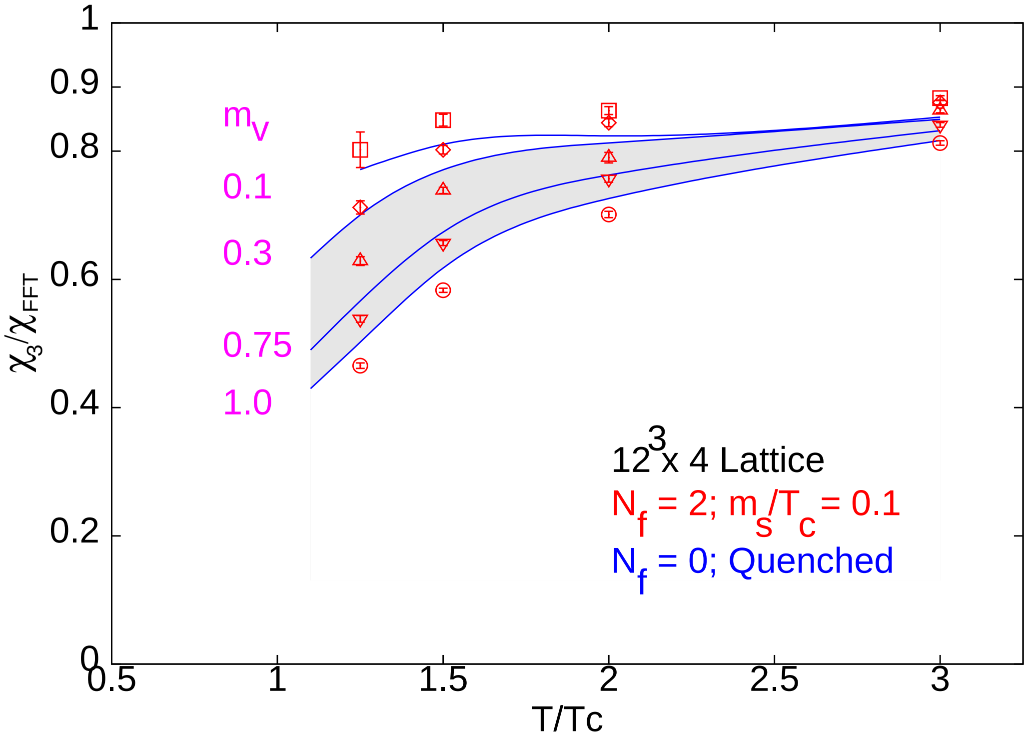
where R_i is a complex vector from a set of N_v, subdivided in L independent sets.
♠ Earlier results : Only close to T_c & for fixed ma.

Gavai & Gupta PR D '01; Gavai, Gupta & Majumdar, PR D 2002

χ_{FFT} — Ideal gas results for same Lattice.



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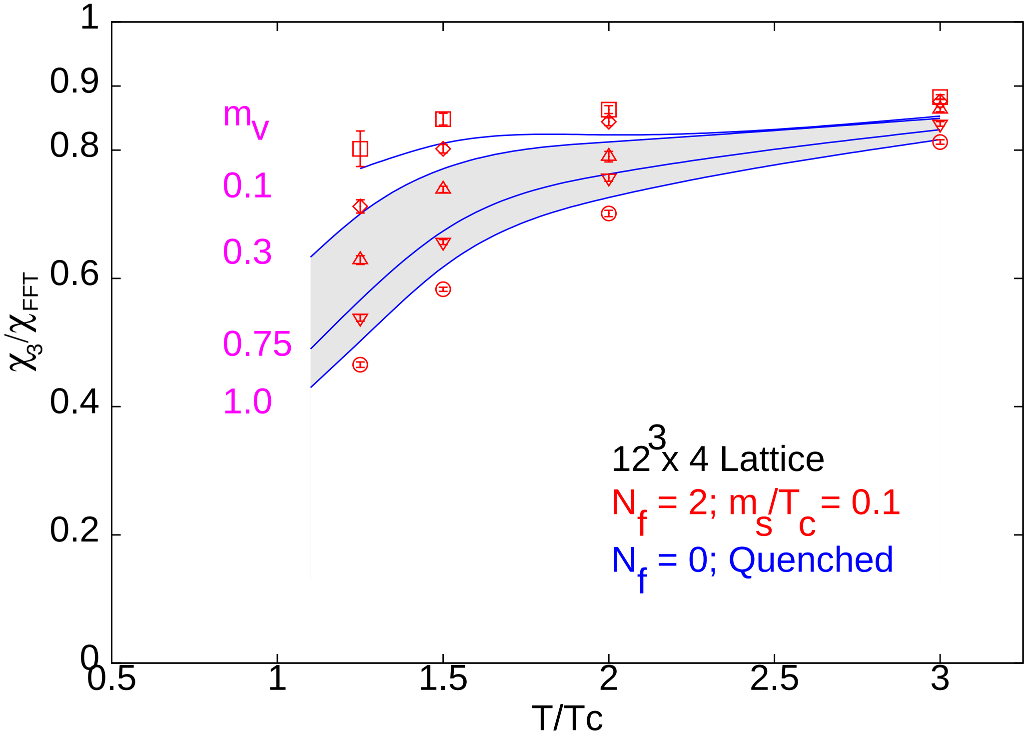

Note that PDG values for strange quark mass $\Longrightarrow$

$m_v^{strange}/T_c \simeq$ 0.3-0.7 (N_f=0); 0.45-1.0(N_f=2).

Perturbation Theory

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Weak coupling expansion gives:
$\frac{\chi}{\chi_{FFT}} = 1 - 2(\frac{\alpha_s}{\pi}) + 8\sqrt{(}1 + 0.167N_f)(\frac{\alpha_s}{\pi})^{\frac{3}{2}}$
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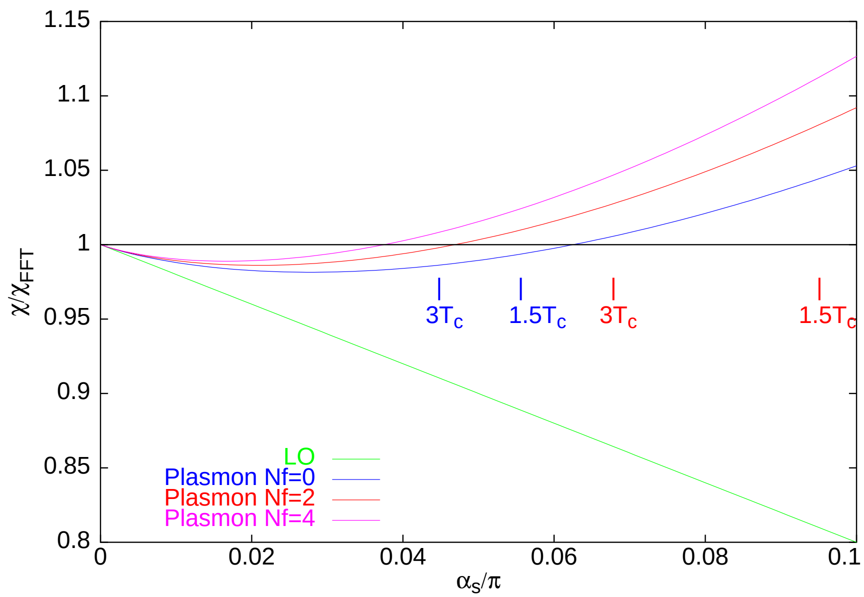

♣ Minm 0.981 (0.986) at 0.03 (0.02) for $N_f = 0$ (2).

♣ For $1.5 \leq T/T_c \leq 3$ pert. theory $\longrightarrow$ 0.99-0.98 (1.08=1.03) for $N_f = 0$ (2).

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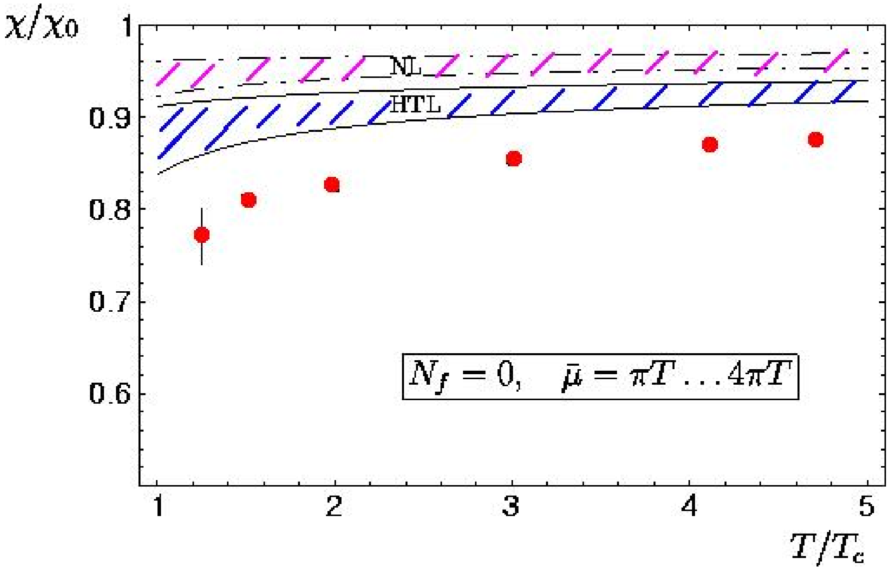

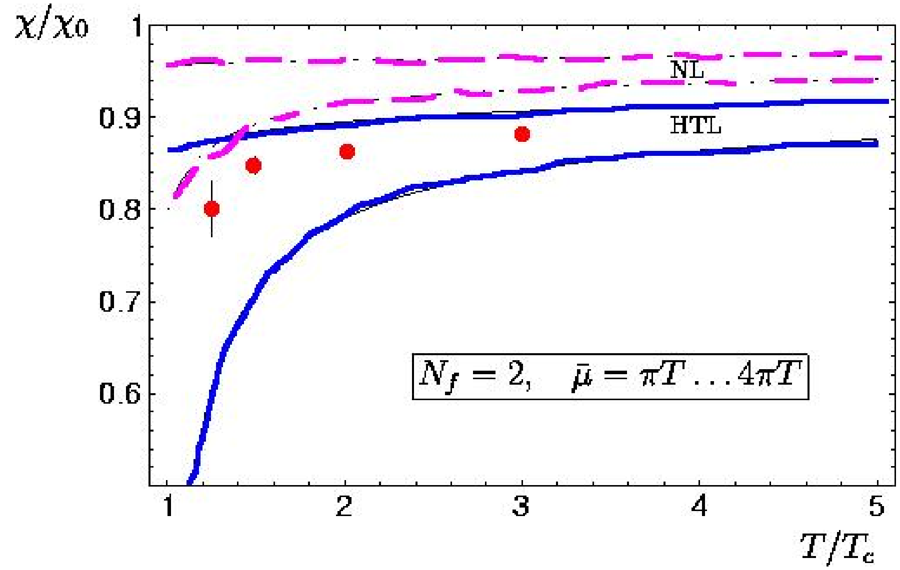

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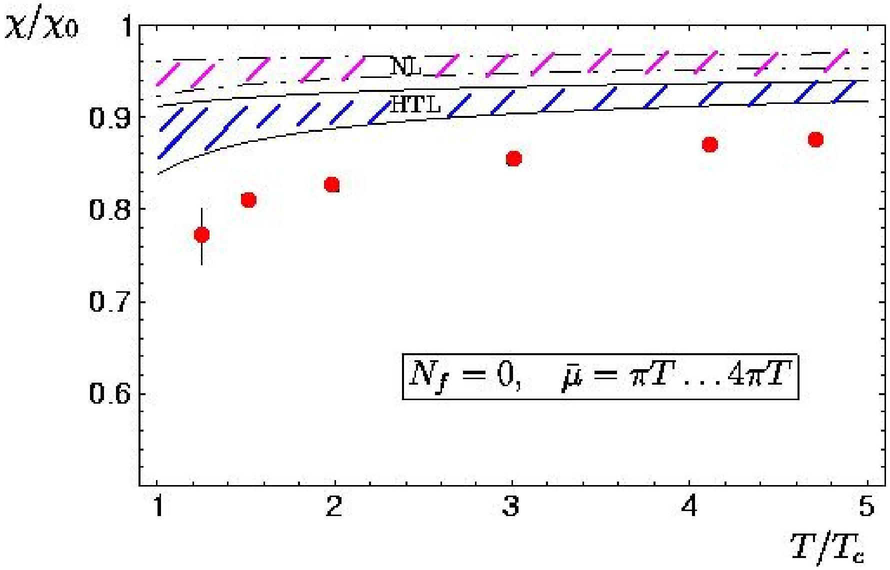

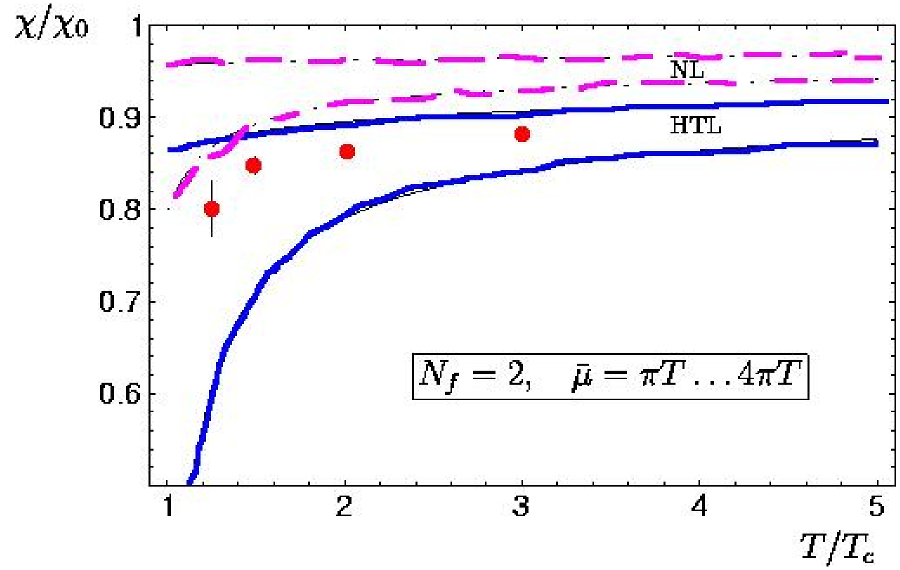

Our results for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Check for larger N_t and improved actions.

χ_{ud}

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♡ Identically zero for Ideal gas but $O(\alpha_s^3)$ in P.T.
Using the same scale and α_s as for $\chi_3 \longrightarrow \chi_{ud} \sim O(10^{-4})$!!

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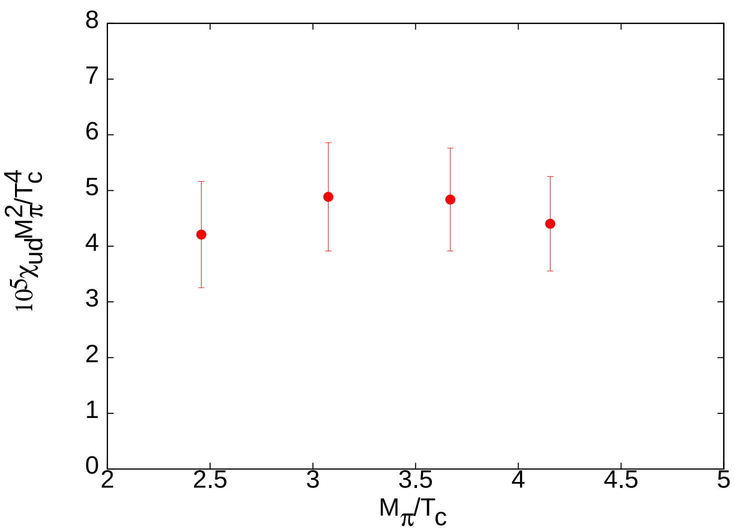

♣ $12^3 \times 4$ Lattice; Quenched.
♣ $T = 0.75 T_c$
♣ Gavai, Gupta & Majumdar, PR D 2002

Taking Continuum Limit

(Gavai & Gupta, PR D '02 and hep-lat/0211015)

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♠ Investigate larger N_t : 6, 8, 10, 12 and 14.

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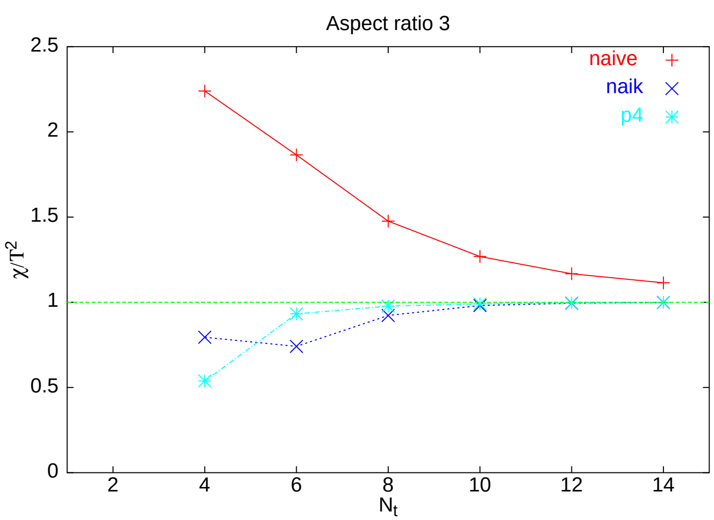
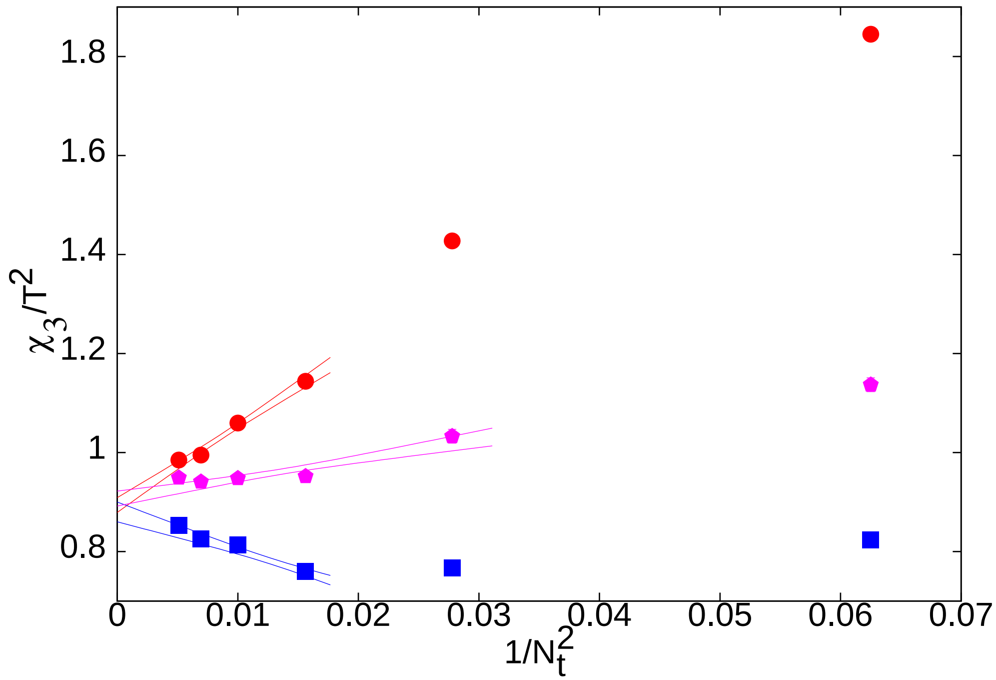


♠ Does improve the N_t-dependence of the free fermions.

Results at $2T_c$:



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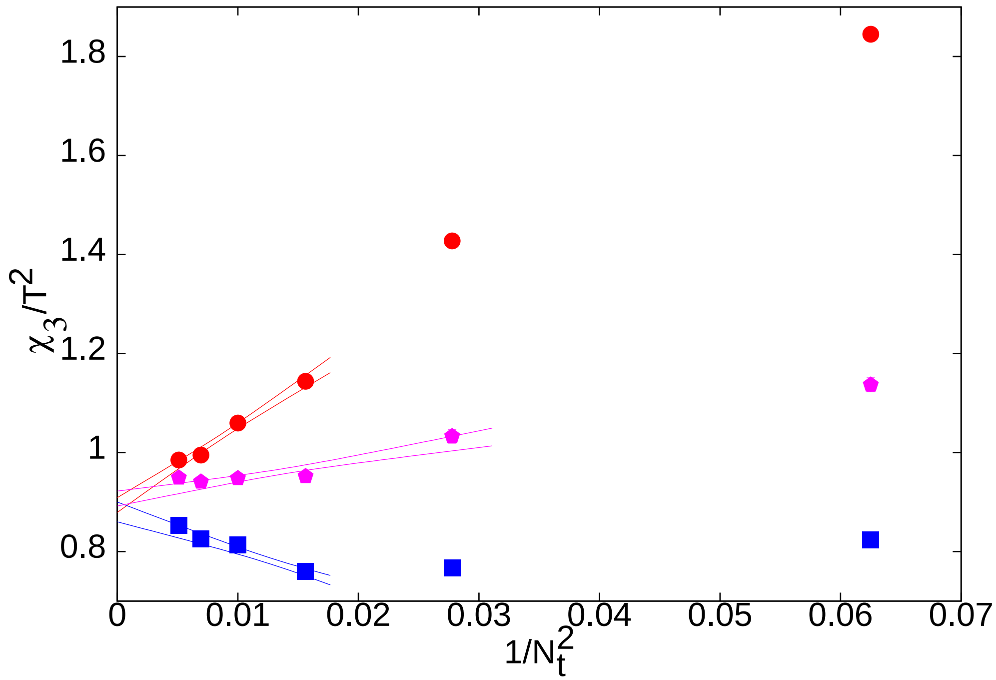

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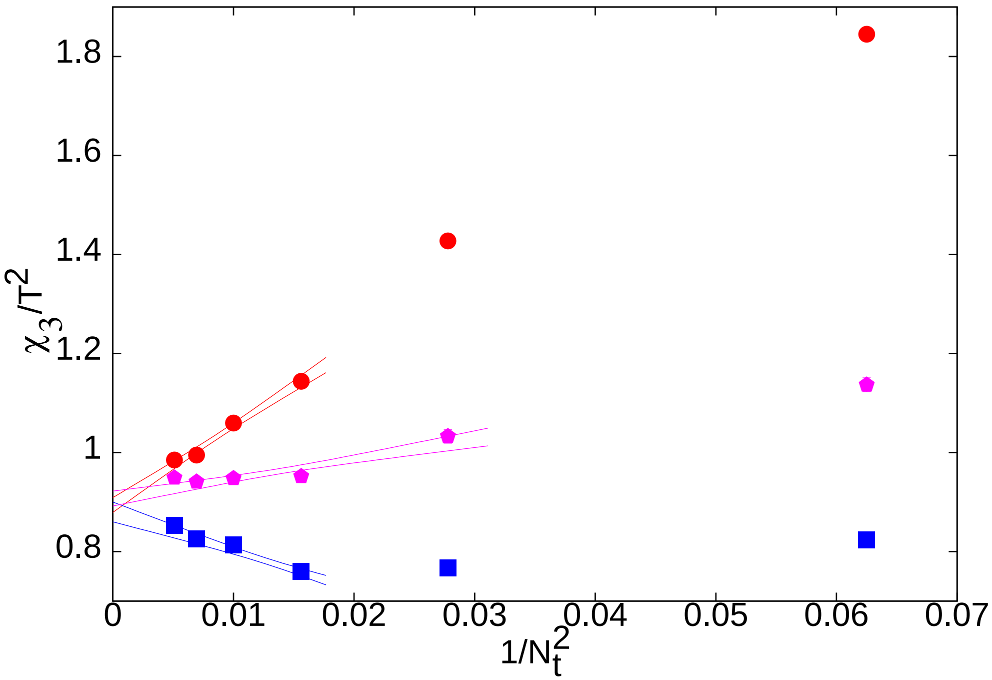

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◇ Milder $N_t^{-2} \sim a^2$-dependence for Naik fermions.

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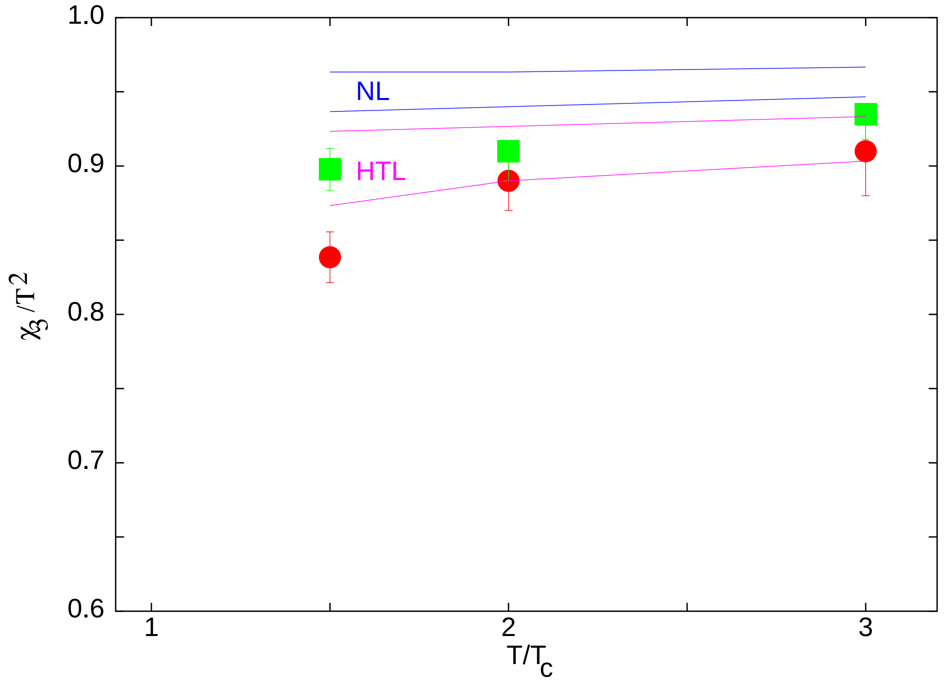

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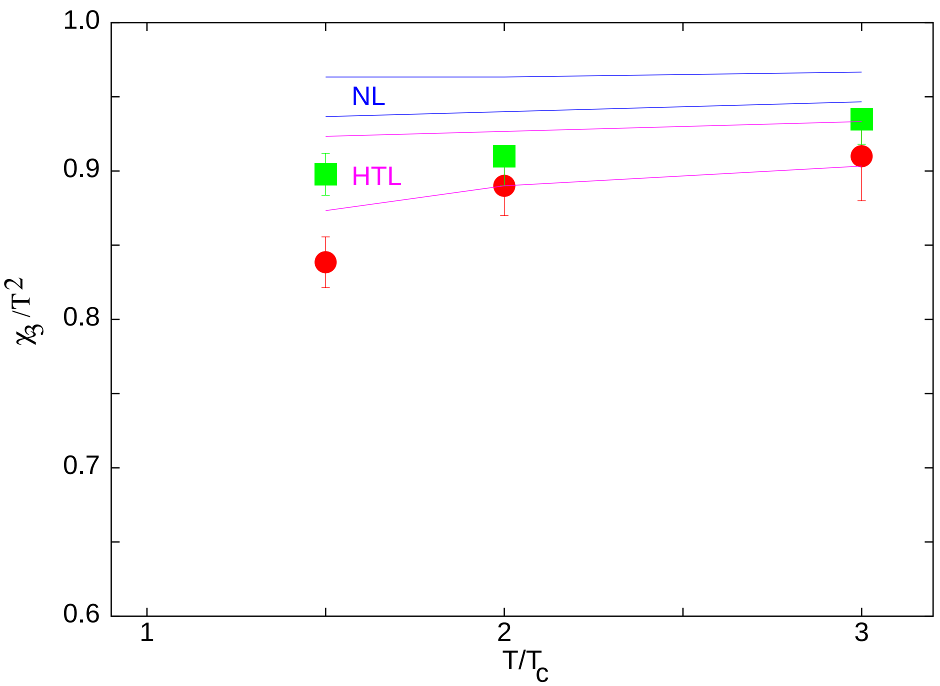

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♡ $\lambda_s(T_c) = 2\chi_s/(\chi_u + \chi_d) \approx 0.4 - 0.5$ vis-a-vis RHIC value 0.47 ± 0.4.

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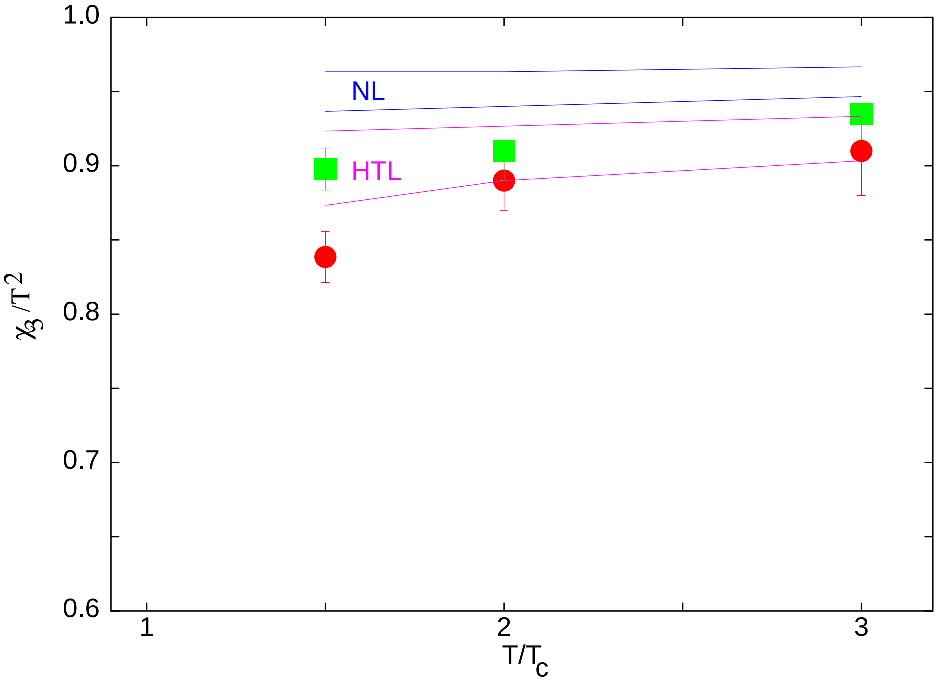

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♡ χ_{ud} behaves the same way for ALL N_t and both fermions, leading to the same $O(10^{-6})$ values in continuum too.

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$$C_\Gamma(z) = \sum_{x,y,t} \langle M^{-1}_{\alpha\beta}(x,y,z,t)\Gamma M^{\dagger -1}_{\beta\alpha}(x,y,z,t)\Gamma\rangle \tag{6}$$

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- Summing up the C_Γ for pion $\rightarrow$ Pion susceptibility.

$N_t = 4$ Lattices with $N_z = 16$.
$4\chi_3/T^2$ (open symbols) and $\chi_\pi/10T^2$ (filled)
at $2T_c$ (lower set) and $3T_c$.
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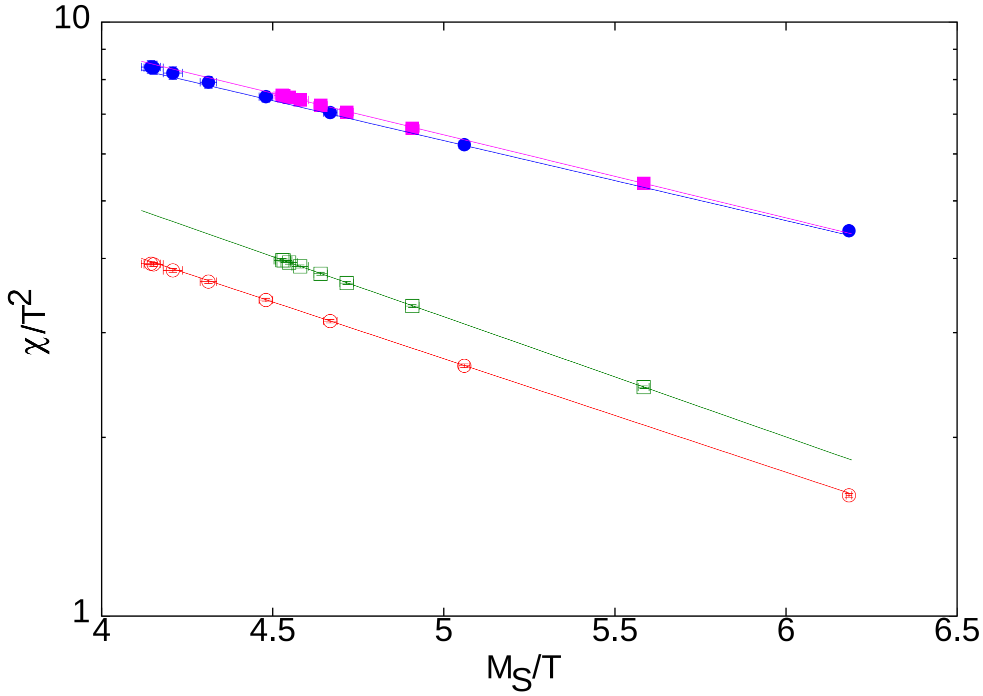

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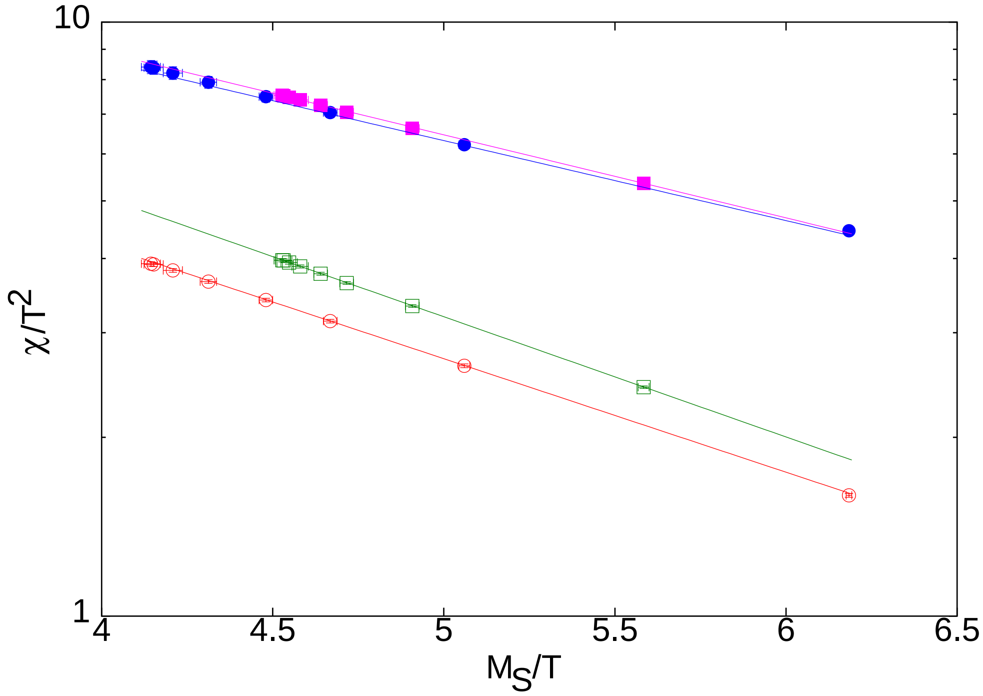

Why ? $\chi_3 \sim \sum$ propagator of nonlocal vector meson.

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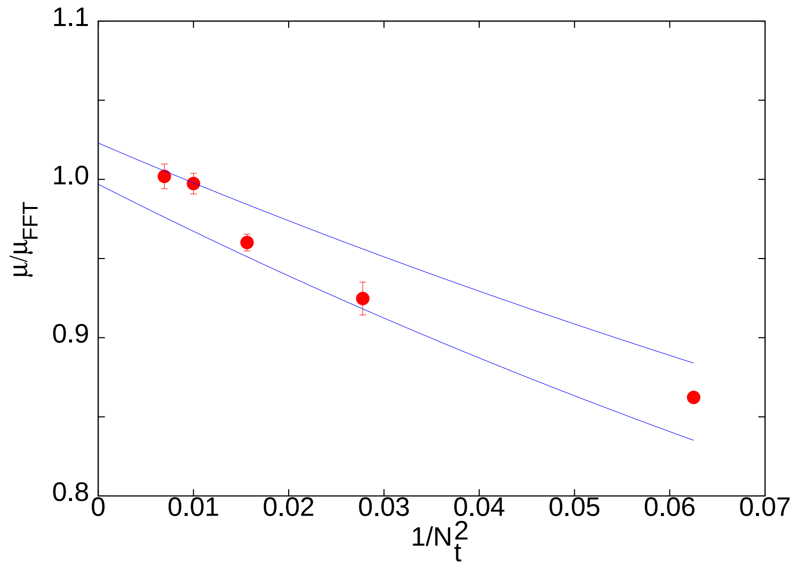

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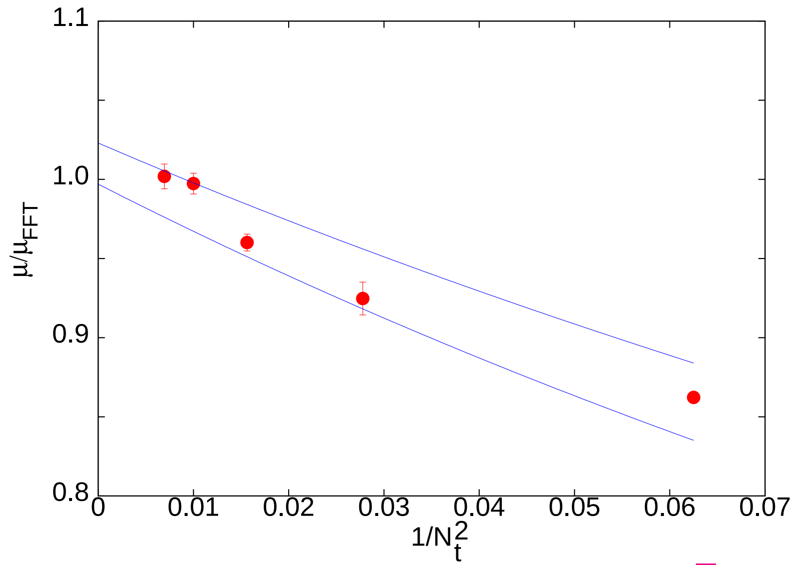

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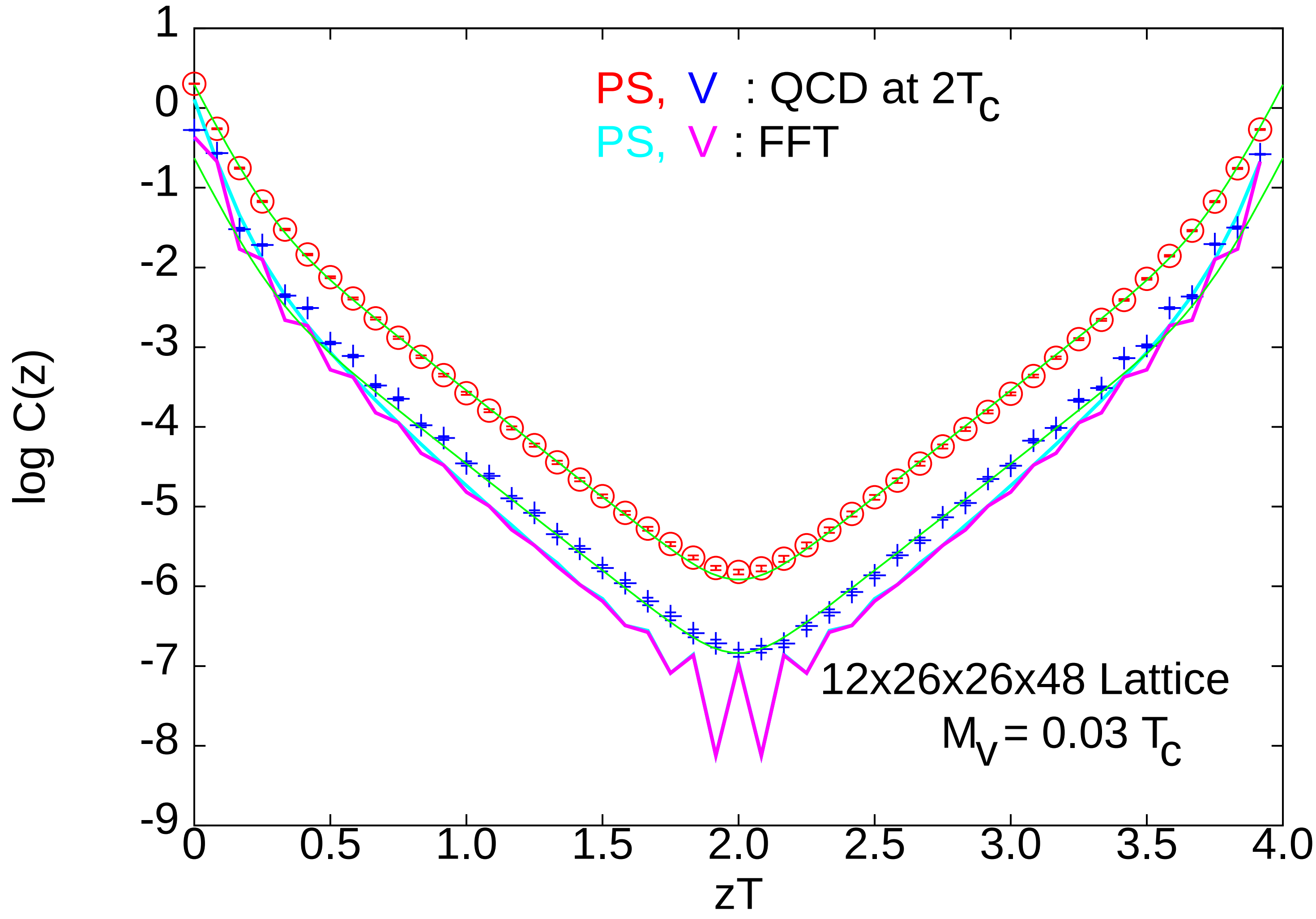
However, chiral condensate, $\langle\bar{\psi}\psi\rangle$ differs from FFT by 2, as do the detailed shapes of the correlators.

Note that both PS and V have SAME fit (green line)with changed normalization.



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- Many questions still for full 2+1 QCD : Order, Large N_t, $\cdots$.