

Charm Flow at PHENIX : A New Milestone for Lattice QCD ?

*Rajiv V. Gavai**
T. I. F. R., Mumbai

** With Debasish Banerjee, Saumen Datta & Pushan Majumdar, arXiv:1109.5738, Phys. Rev. D in press*

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Introduction

Formalism

Our Lattice Results

Summary

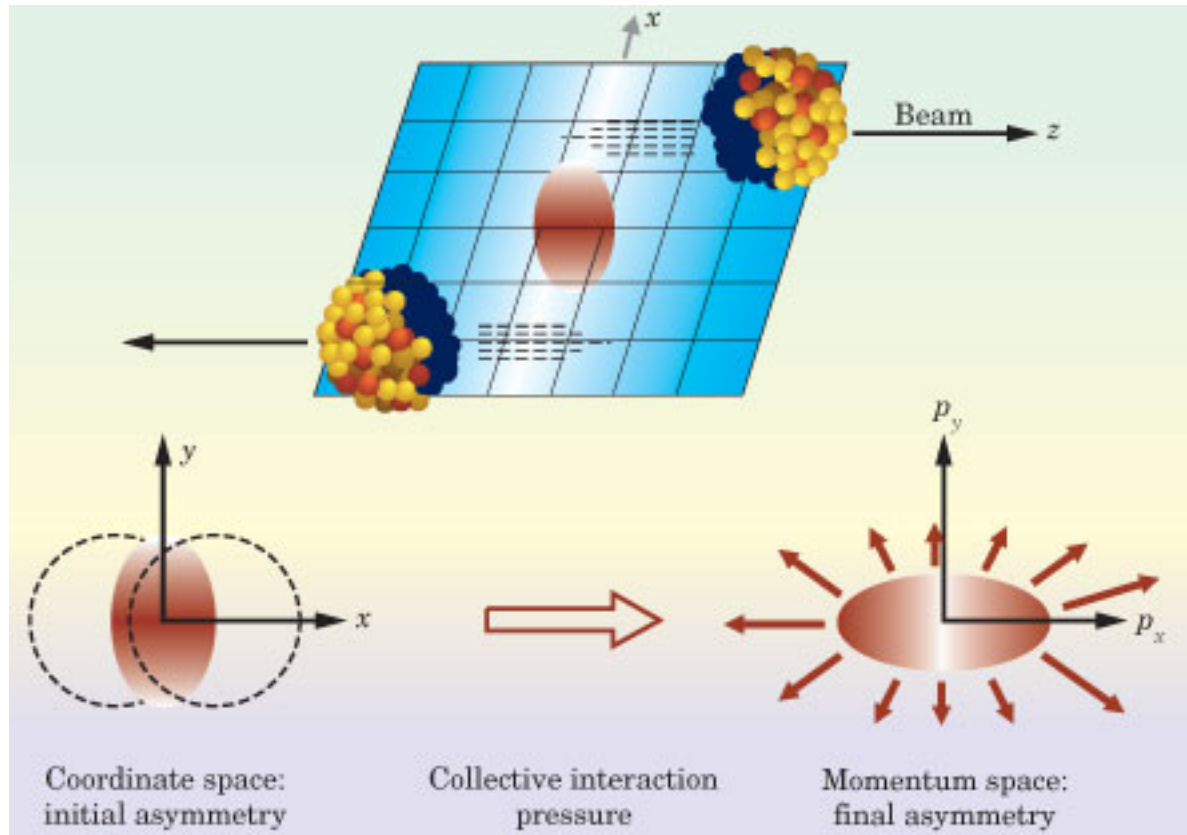
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Introduction : Anisotropic Flow

- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.

Introduction : Anisotropic Flow

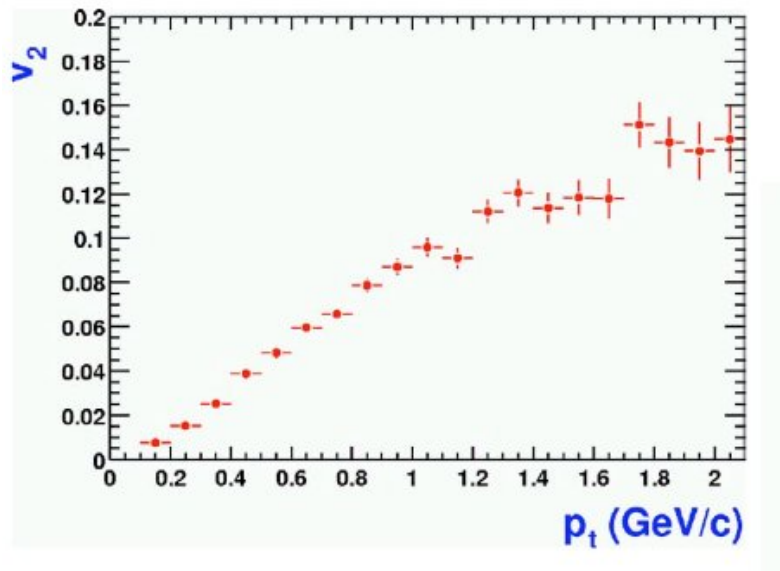
- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.
- Obtained from asymmetric collisions of two nuclei, with their centres not aligned.



- $$v_2(y, p_T) = \frac{\int d\phi dN/(p_T dP_T d\phi dy) \cos(2\phi)}{\int d\phi dN/(p_T dP_T d\phi dy)} \quad (1)$$

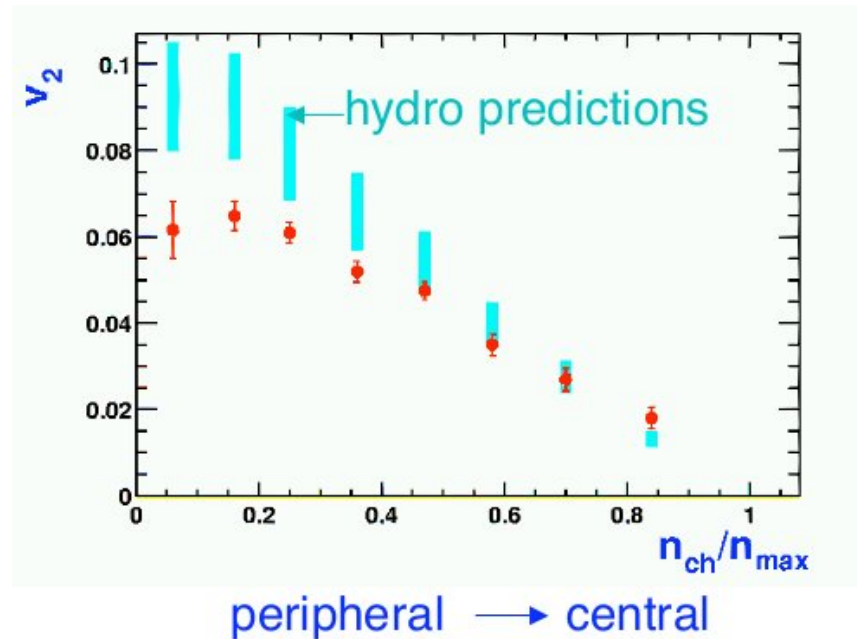
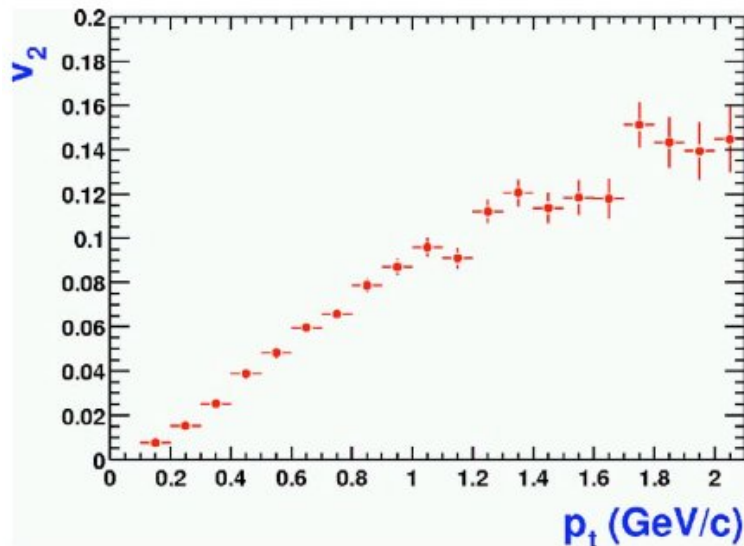
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- (STAR Collaboration, Ackermann et al., PRL 86 (2001) 402.)



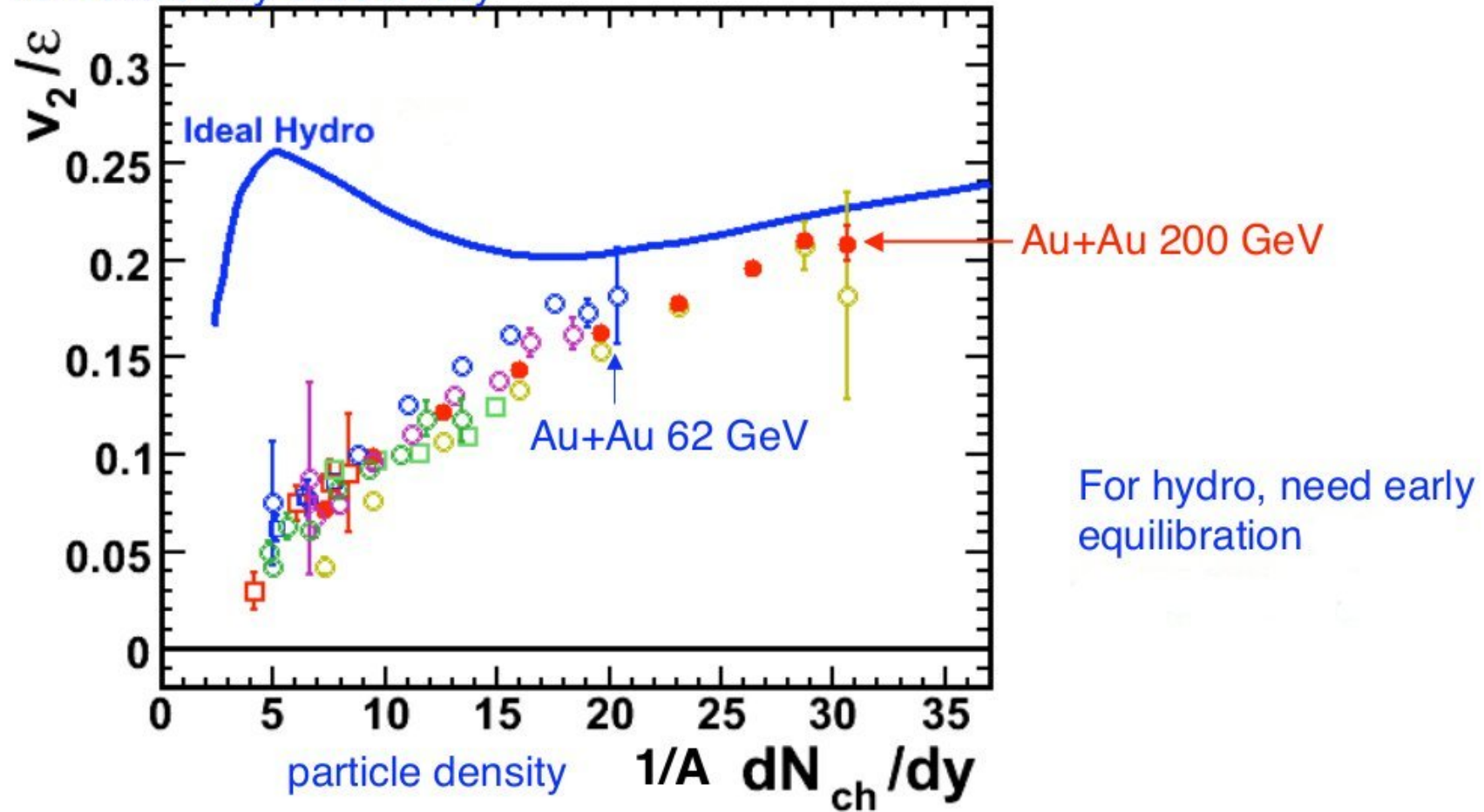
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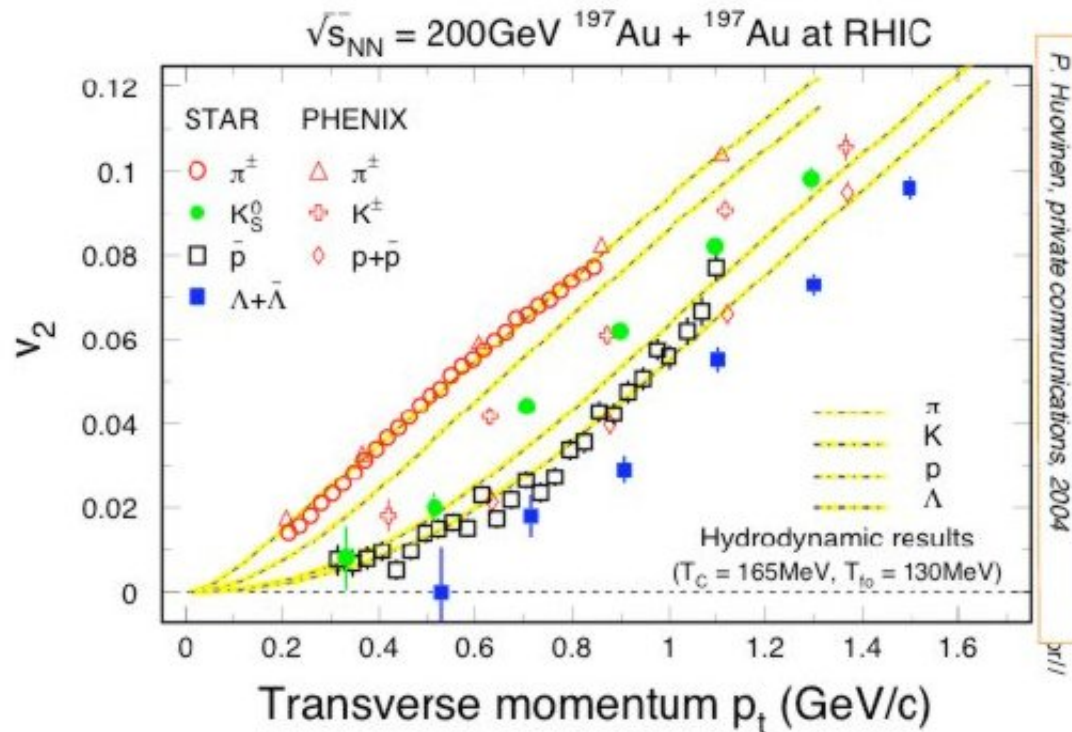
- Good agreement with ideal hydro: Suggesting early thermalization and perfect fluid and many more interesting things.

flow scaled by eccentricity



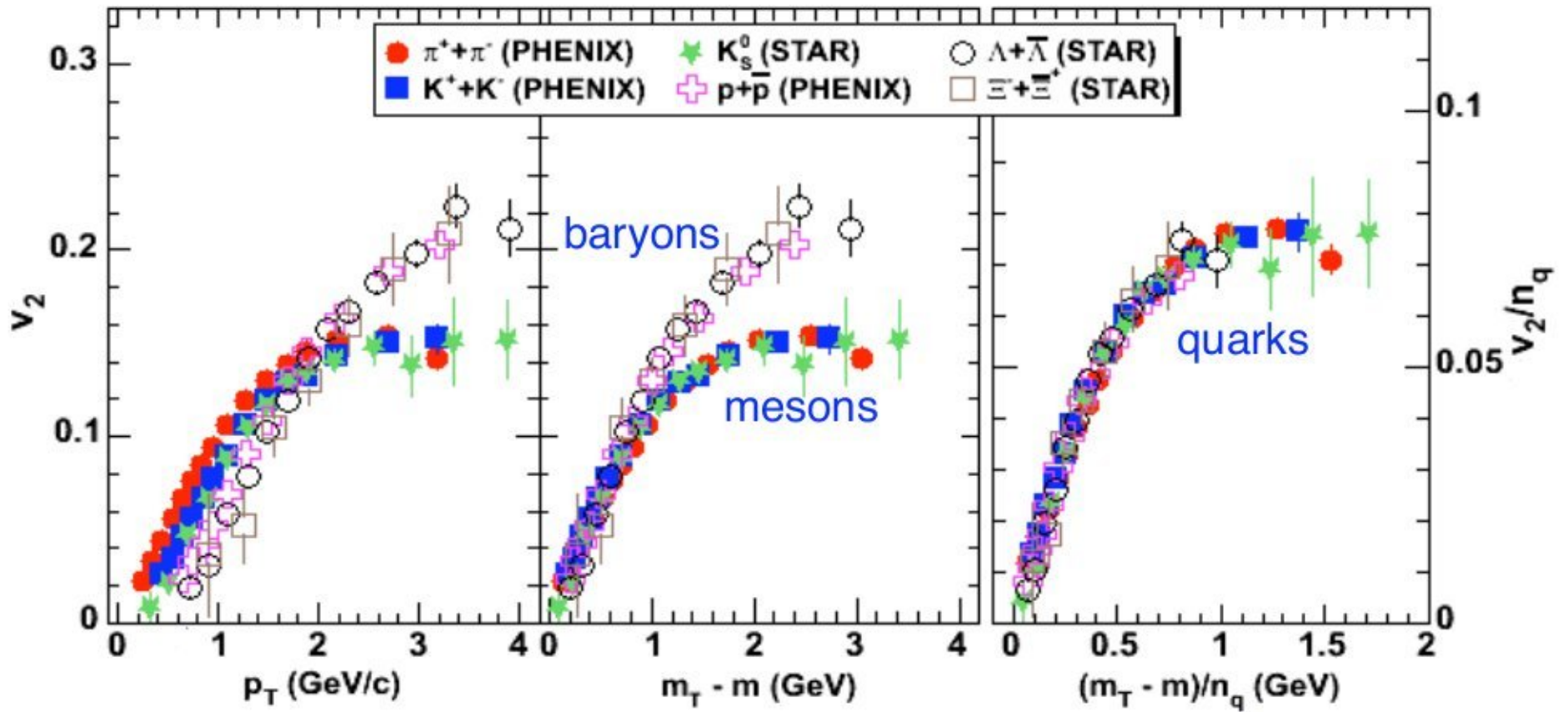
(S. Voloshin, QM06, JPG 31 (2007) S883 & Hydro Curve: Kolb-Sollfrank-Heinz, PRC 62 (2000) 054909.)

v_2 at Low p_T Region



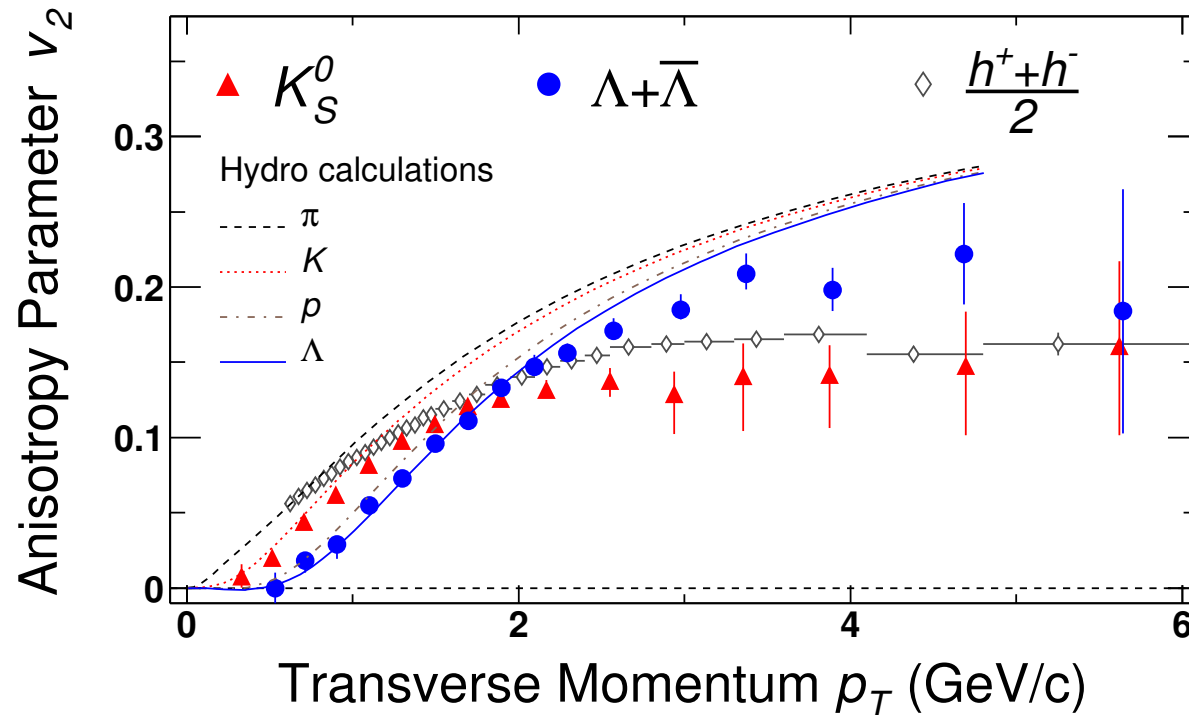
(STAR Collaboration, JPG 31 (2005) S437 & P. Huovinen.)

- Mass Pattern as expected by Hydrodynamics Models. Quantitative agreement depends on the equation of state.



(S. Voloshin, QM02, STAR PRL 95 (2005) & PHENIX PRL 98 (2007))

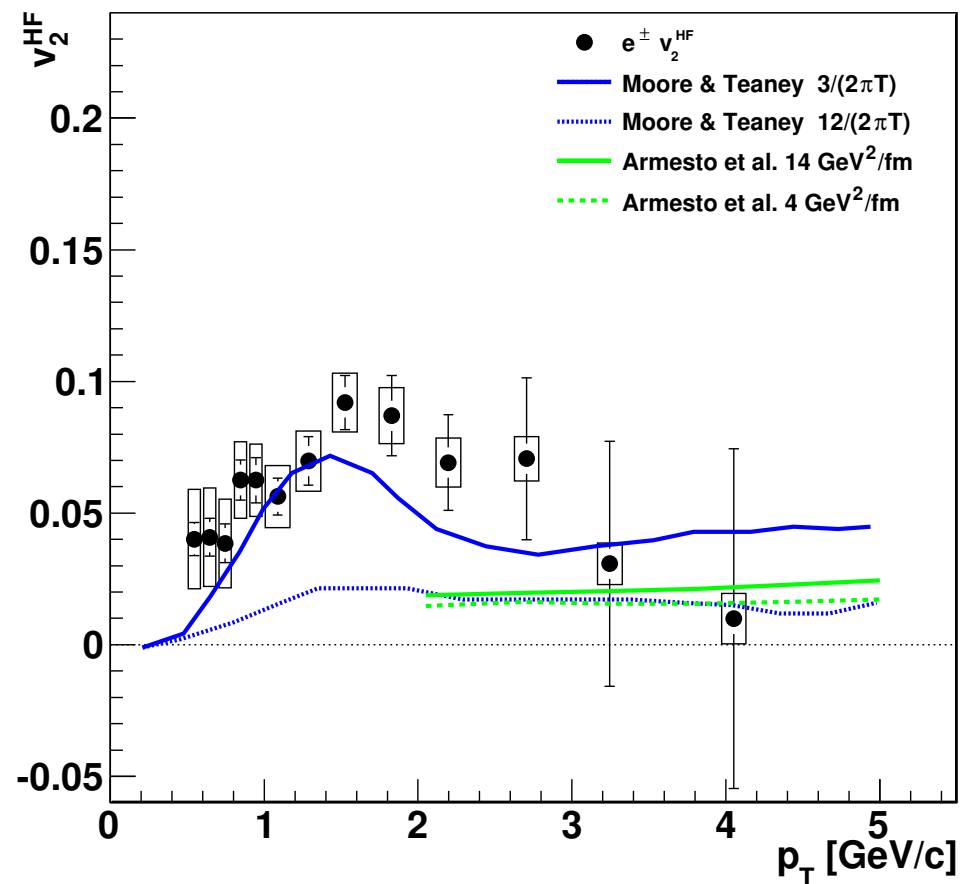
- v_2 scales as number of quarks. Thus, hadrons appear to follow the ‘underlying’ quark flow as Recombination Model would suggest.



(STAR Collaboration, Adams et al., PRL 92 (2004) 052302.)

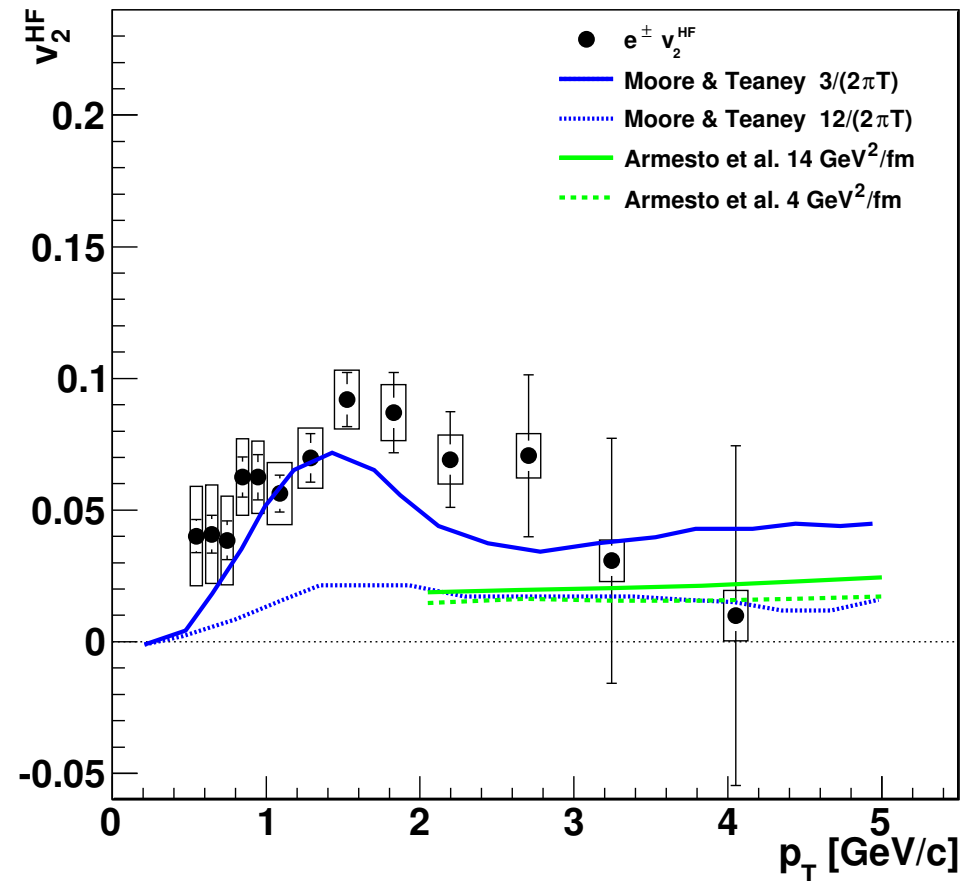
♡ Minimum Bias Au+Au Collisions at 200 GeV/c : Strangeness flows like normal hadrons.

- Naively expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.

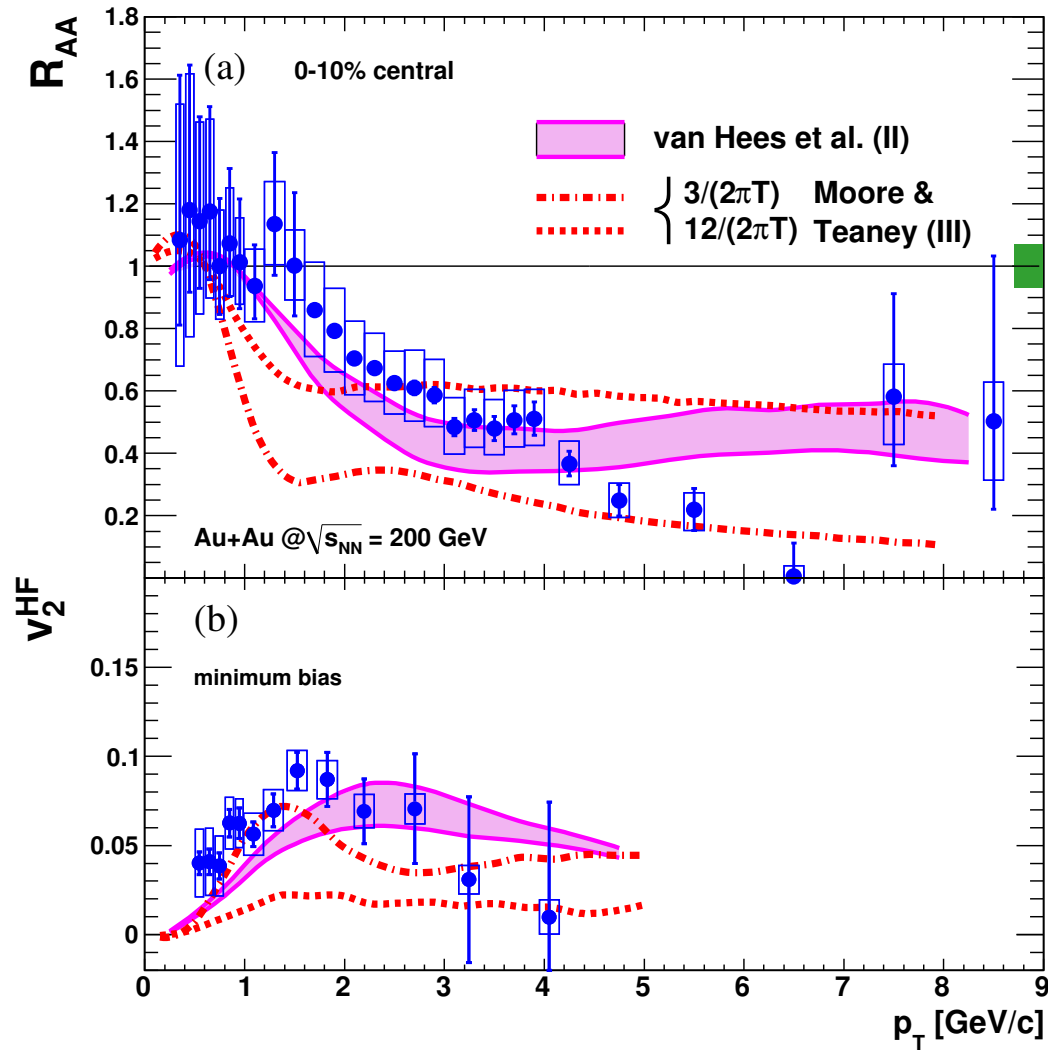


(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

- Naively expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.
- In models (Moore-Teaney, PRC 71, 2005), heavy quark diffusion coefficients governs its elliptic flow **and** suppression.



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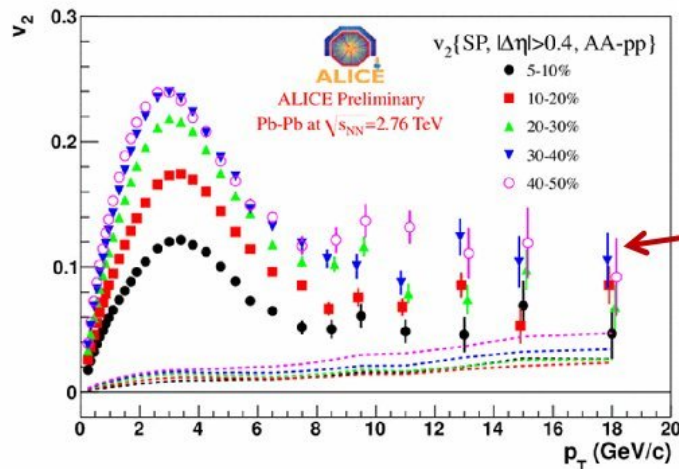


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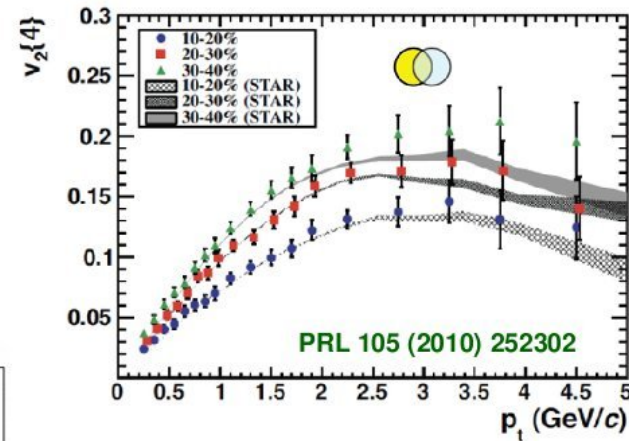
p_T differential elliptic flow

- v_2 vs. p_T does not change within uncertainties between $\sqrt{s_{NN}}=200$ GeV and 2.76 TeV

⇒ 30% increase of p_T integrated flow explained by higher mean p_T due to stronger radial flow at higher energies



ALI-PREL-439



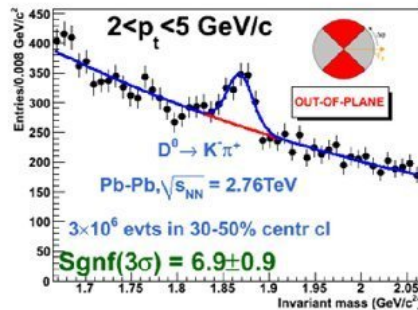
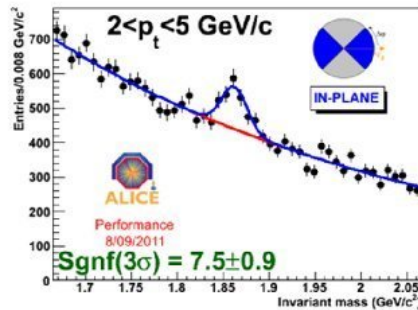
- Significant values of v_2 for $p_T > 8$ GeV/c

⇒ Due to path length dependence of parton energy loss

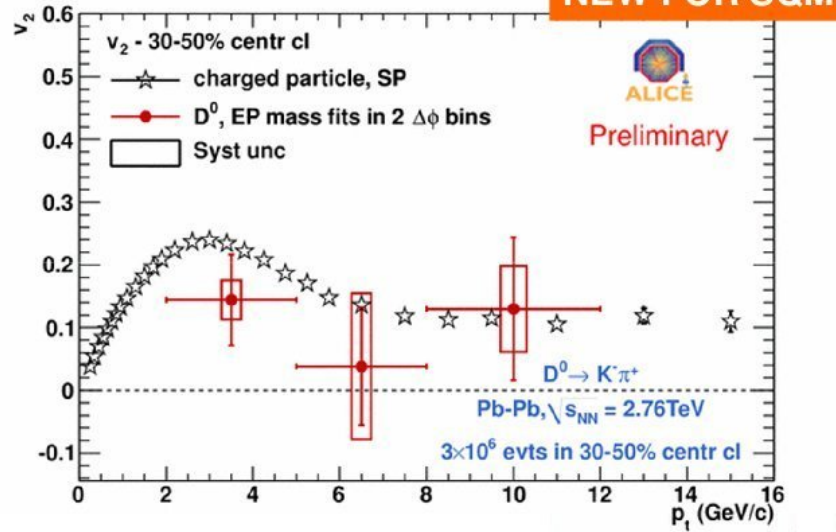
ALICE overview at SQM11, Krakow, Poland by Francesco Prino.

D⁰ elliptic flow

- First direct measurement of D flow in heavy-ion collisions
- Yield extracted from invariant mass spectra of $K\pi$ candidates in 2 bins of azimuthal angle relative to the event plane



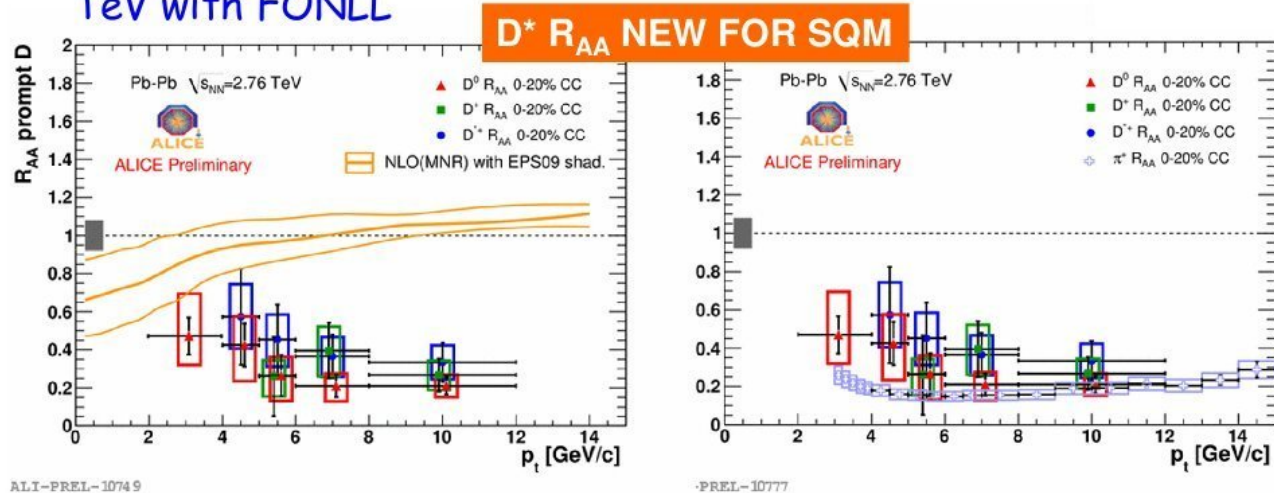
$$v_2 = \frac{\pi}{4} \frac{N_{IN} - N_{OUT}}{N_{IN} + N_{OUT}}$$



ALICE overview at SQM11, Krakow, Poland by Francesco Prino.

D meson R_{AA}

- pp reference from measured D^0 , D^+ and D^* p_T differential cross-sections at 7 TeV scaled to 2.76 TeV with FONLL



- Suppression of prompt D mesons in central (0-20%) PbPb collisions by a factor 4-5 for $p_T > 5$ GeV/c
 - ⇒ Little shadowing at high p_T → suppression is a hot matter effect
 - ⇒ Similar suppression for D mesons and pions
 - ⇒ Maybe a hint of $R_{AA}^D > R_{AA}^\pi$ at low p_T

ALICE overview at SQM11, Krakow, Poland by Francesco Prino.

- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a 'perturbative' estimate, seems to under-predict v_2 substantially.
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- Other models, e.g. van Hees-Greco-Rapp, seem to suggest the same: Heavy Quark Diffusion coefficient is much smaller than perturbative estimates.
- Is it non-perturbative ? **Strong coupling models — AdS/CFT based — do lead to values in the desired range under “suitable” assumptions** [Casalderrey-Solana & Teaney (2006), Gubser(2007)]
- **Can Lattice QCD shed some light on the Charm Flow ?**

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.
- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

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$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t) \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t') \quad (2)$$

- η_D – momentum drag coefficient and 3κ is mean-squared momentum transfer per unit time, $\kappa = \frac{1}{3} \int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t) \xi_i(0) \rangle$.

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- Diffusion constant D can be found to be $2T^2/\kappa$ with $\eta_D = \kappa/2MT$.

- Moore-Teaney also showed that an initial power-law (LO pQCD) transverse momentum distribution of a heavy Q in an expanding QGP at $T_0 = 300$ MeV by $T_f = 165$ MeV approximates a thermal one **provided** $D \leq 3/2\pi T$, assuming an ideal Bjorken expansion of the plasma.
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- Casalderrey-Solana & Teaney (PRD 74 (2006) 085012) suggested to obtain κ from a correlator of the (colour) force exerted on a heavy Q by the (deconfined & coloured) medium.
- Caron-Huot, Laine & Moore (JHEP 0904, 053) provided a suitable definition for κ for a lattice evaluation: The force acting on the heavy quark is given by $M dJ^i/dt$, where $J^\mu(\vec{x}, t) = \bar{\psi}(\vec{x}, t)\gamma^\mu\psi(\vec{x}, t)$ is the conserved current for the heavy quark.

- Using Heavy Quark Effective Theory, they narrowed it down to studying

$$G_E^{\text{Lat}}(\tau) = -\frac{1}{3L} \sum_{i=1}^3 \left\langle \text{Re tr} \left[U(\beta, \tau) E_i(\tau, \vec{0}) U(\tau, 0) E_i(0, \vec{0}) \right] \right\rangle, \text{ where } L$$

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- The spectral function, $\rho(\omega)$, is obtained from the $G_E(\tau)$, as usual, by

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh \omega(\tau - \frac{1}{2T})}{\sinh \frac{\omega}{2T}}. \quad (3)$$

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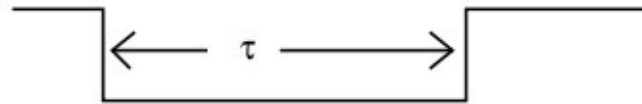
$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh \omega(\tau - \frac{1}{2T})}{\sinh \frac{\omega}{2T}}. \quad (3)$$

- Then momentum diffusion coefficient $\kappa = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \rho(\omega)$.
- They also suggested a suitable discrete version for Lattice QCD :
 $E_i(\vec{x}, \tau) = U_i(\vec{x}, \tau) U_4(\vec{x} + \hat{i}, \tau) - U_4(\vec{x}, \tau) U_i(\vec{x} + \hat{4}).$

- Using this, the numerator can be written as a derivative of an extended (by spatial detour of a) Polyakov loop.

$$G_{E,\text{num}}^i(\tau) = C^i(\tau + 1) + C^i(\tau - 1) - 2C^i(\tau)$$

$$C^i(\tau) = \prod_{x_4=0}^{t-1} U_4(x_4) \cdot U_i(t) \cdot \prod_{x_4=t}^{t+\tau-1} U_4(x_4) \cdot U_i^\dagger(t + \tau) \cdot \prod_{x_4=t+\tau}^{\beta-1} U_4(x_4).$$



Graphical Representation of $C(\tau)$.

Our Lattice Results

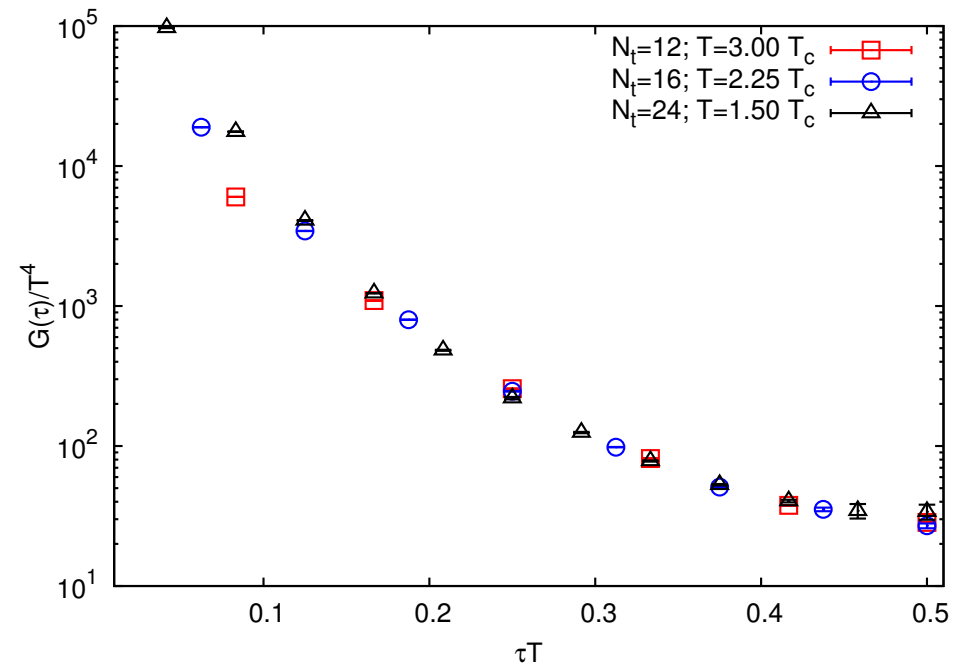
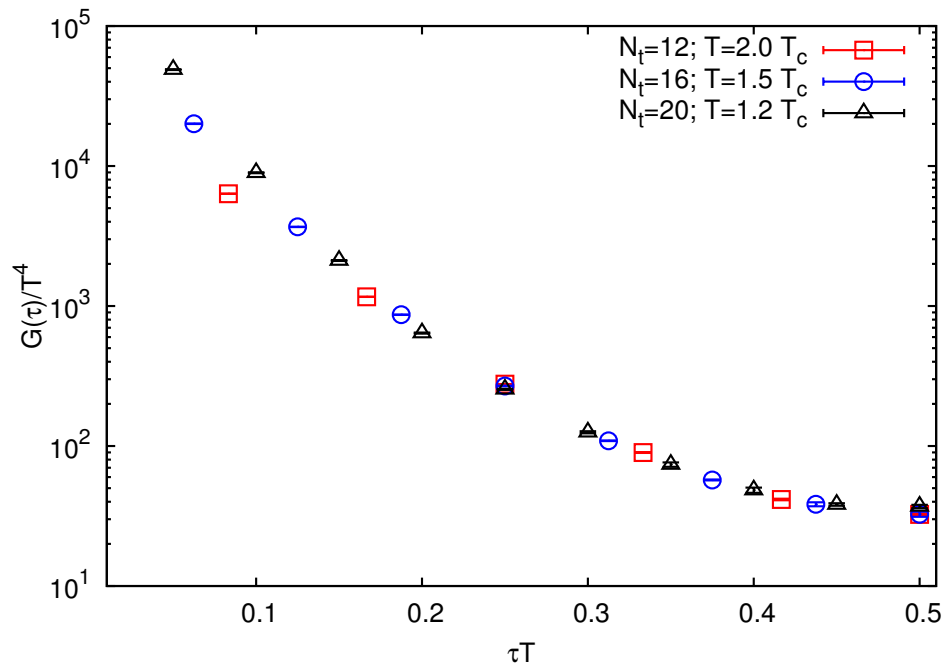
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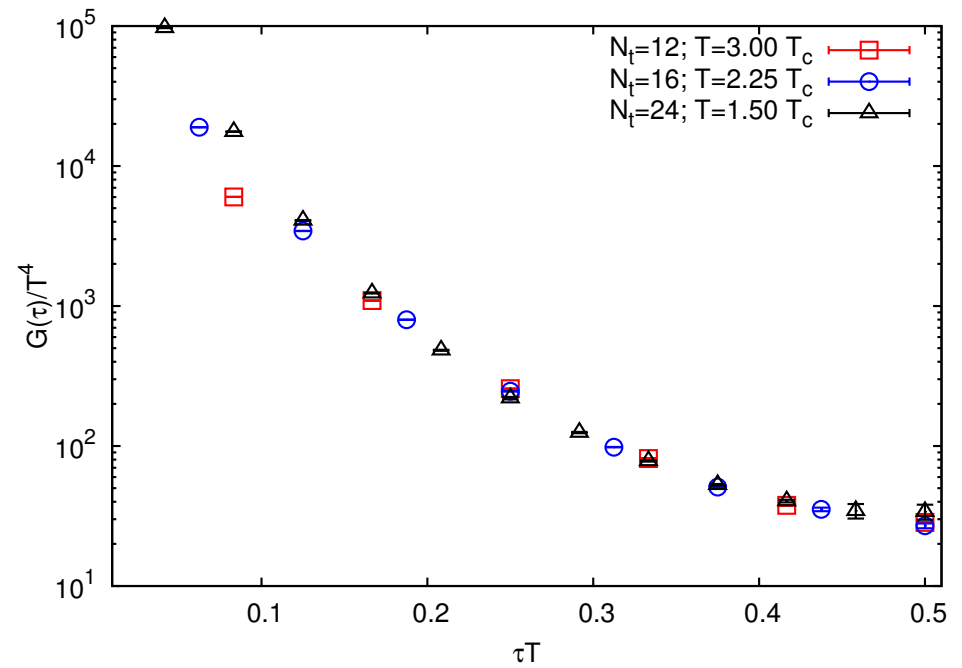
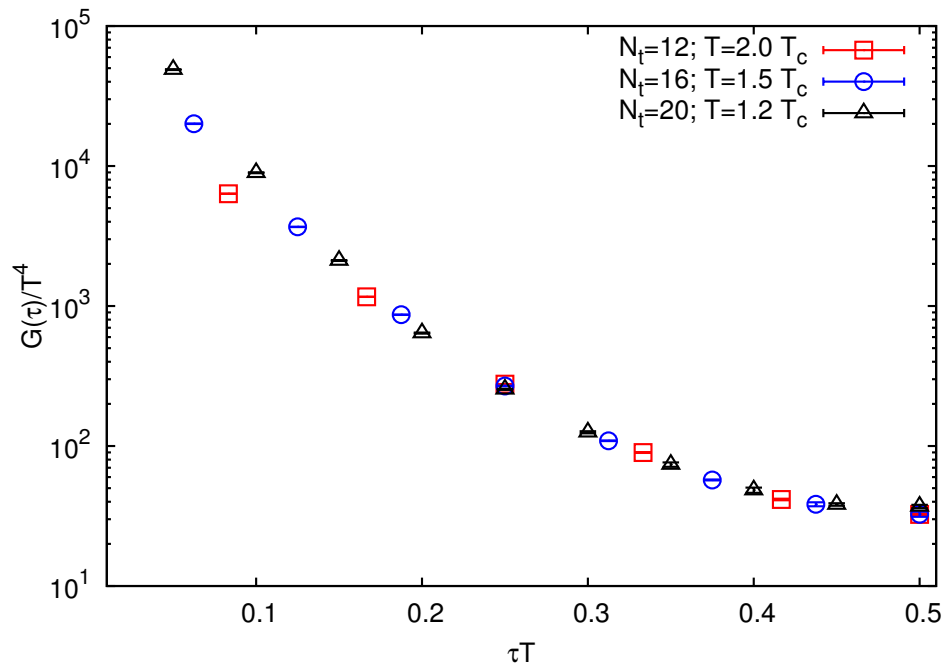
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- We attempted $N_\tau = 12, 16, 20$ and 24 for quenched QCD. Multilevel algorithm (Lüscher-Weisz, JHEP 0109 & 0207) was suitably adopted.
- For the same size error on $G(10)[G(3)]$ on $N_\tau = 20$ lattices, it was found to be $\sim 2500[200]$ times more efficient: Very crucial in getting κ .

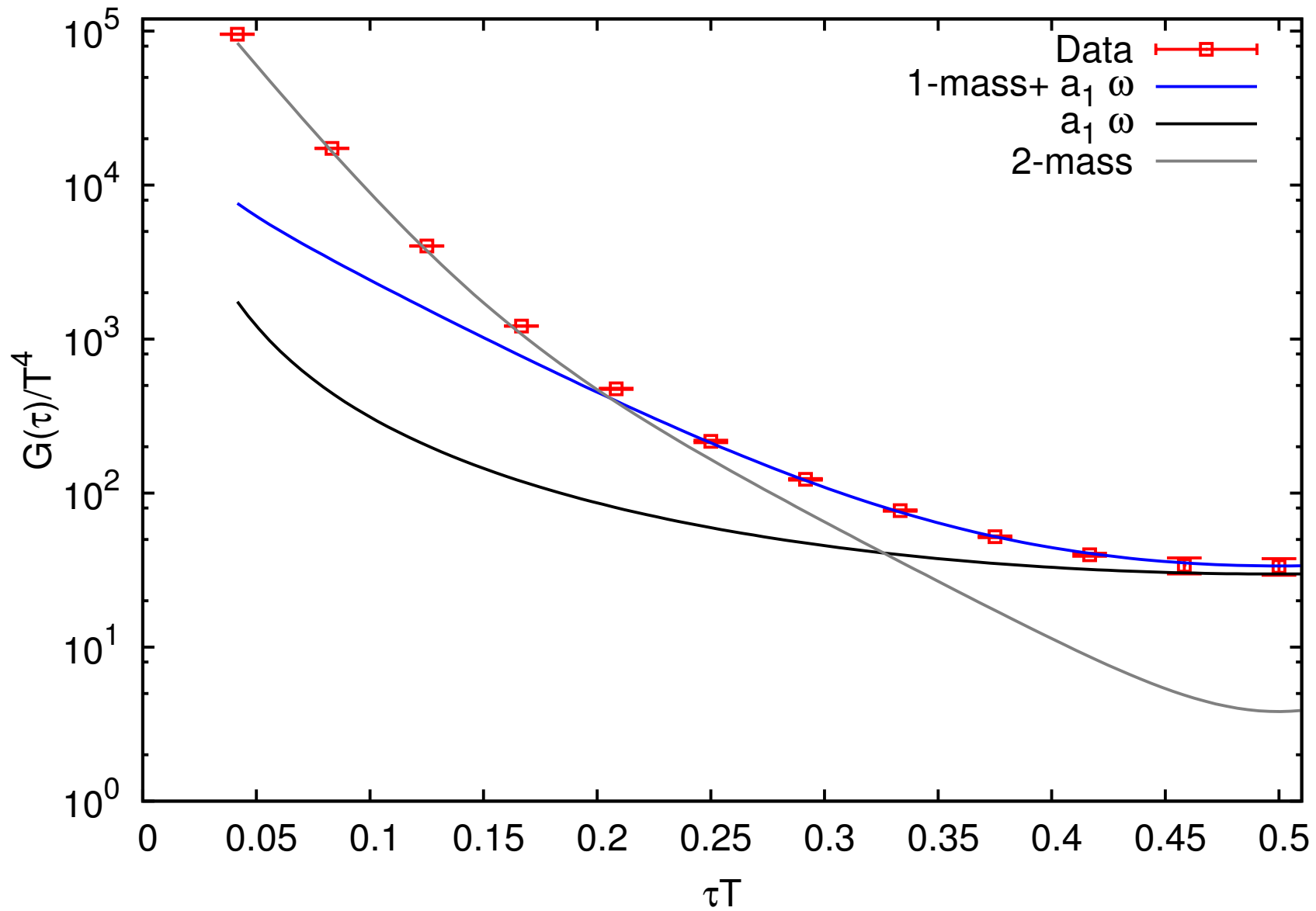
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- Spatial volumes are such that $N_s \geq 2N_\tau$.
- Couplings were chosen suitably to make simulations at $T/T_c = 1.04, 1.09, 1.24, 1.5$ and 1.96 for the two largest N_τ .
- Typical Statistics : Few hundred Independent Configurations, with a few thousand multilevel updates.





- Large τ region shows scaling.
- Low τ region, on the other hand, has only lattice artifacts.

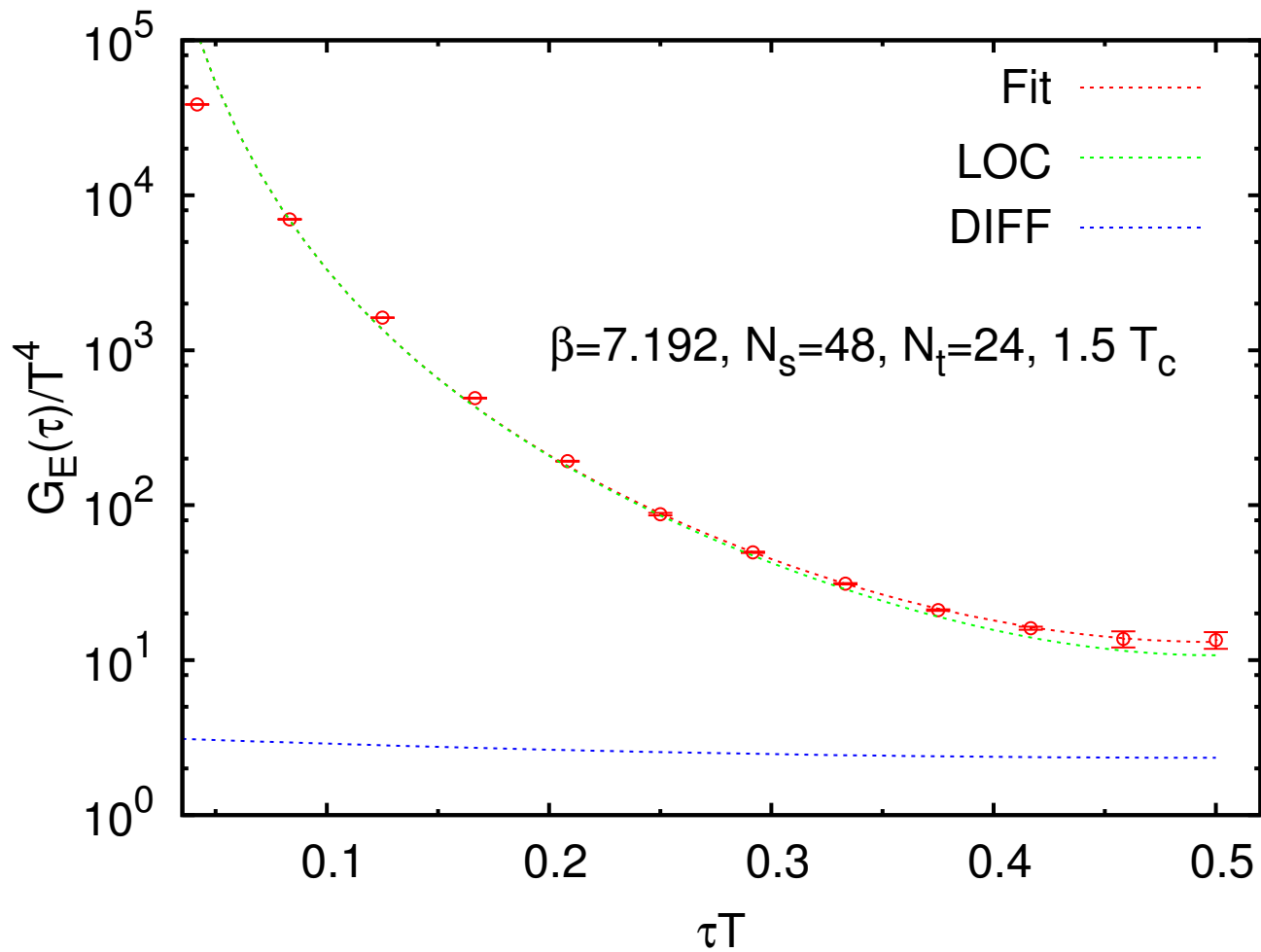


Extracting D

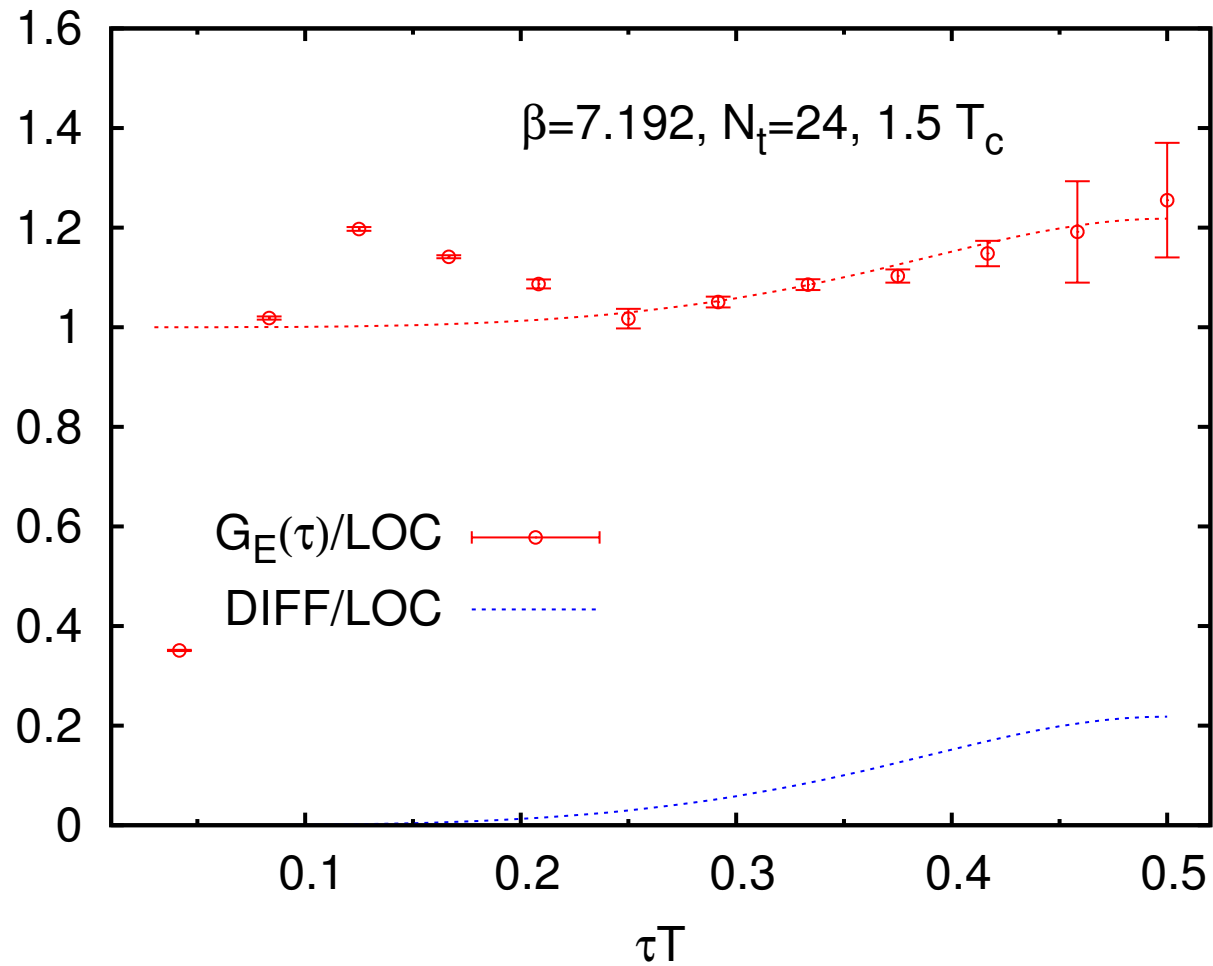
- Getting to the spectral function ρ , an ill-posed problem, has attracted a lot of attention. Many methods can be tried.
- We use an *ansatz* for ρ , obtain G from it, and then fit in the large τ range $[N_\tau/4, N_\tau/2]$

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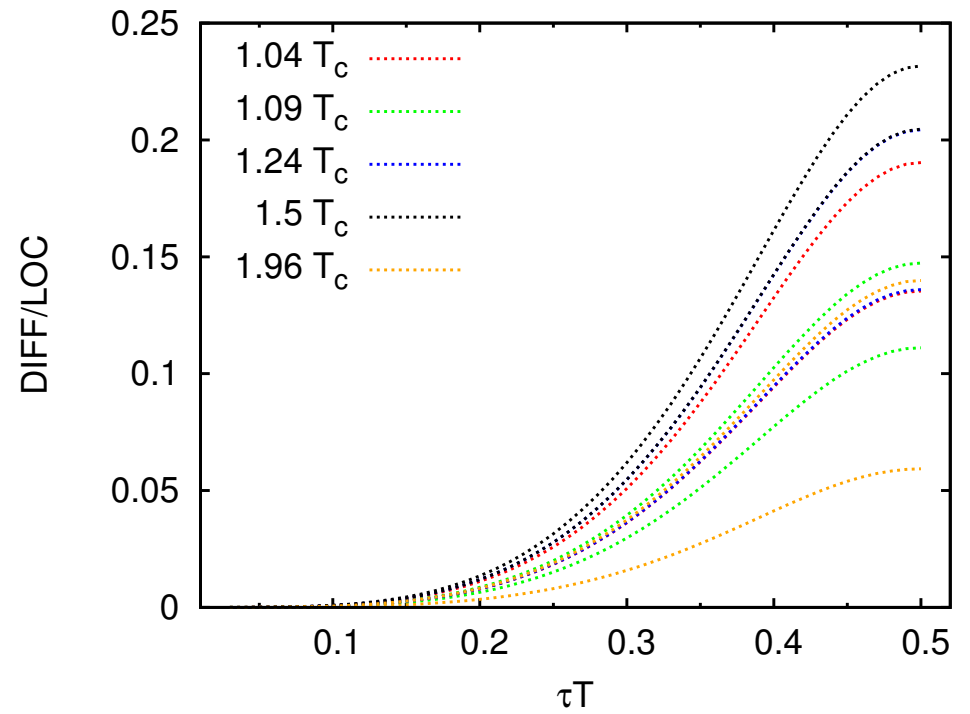
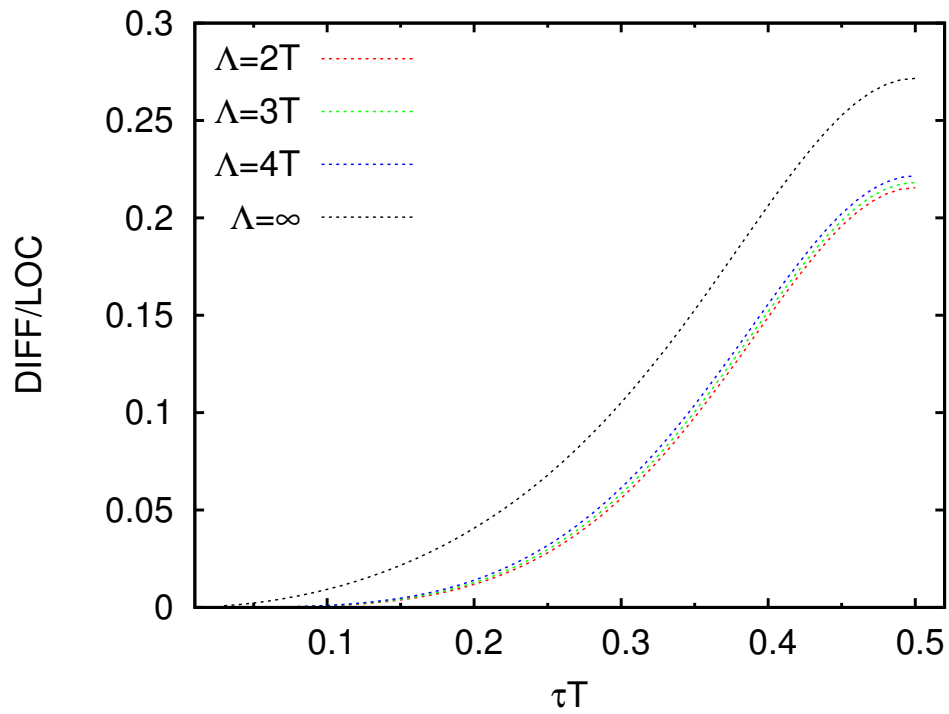
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- We use an *ansatz* for ρ , obtain G from it, and then fit in the large τ range $[N_\tau/4, N_\tau/2]$
- $\rho(\omega) = a\omega \Theta(\omega - \Lambda) + b\omega^3$
First term is the due to the expected DIFFusion constant, and the second is motivated by leading perturbation theory (LOC)
- $\Lambda = 3T$ used; varied from 2 to ∞ for systematic error.



♠ Contribution of the two terms shown as DIFF and LOC.



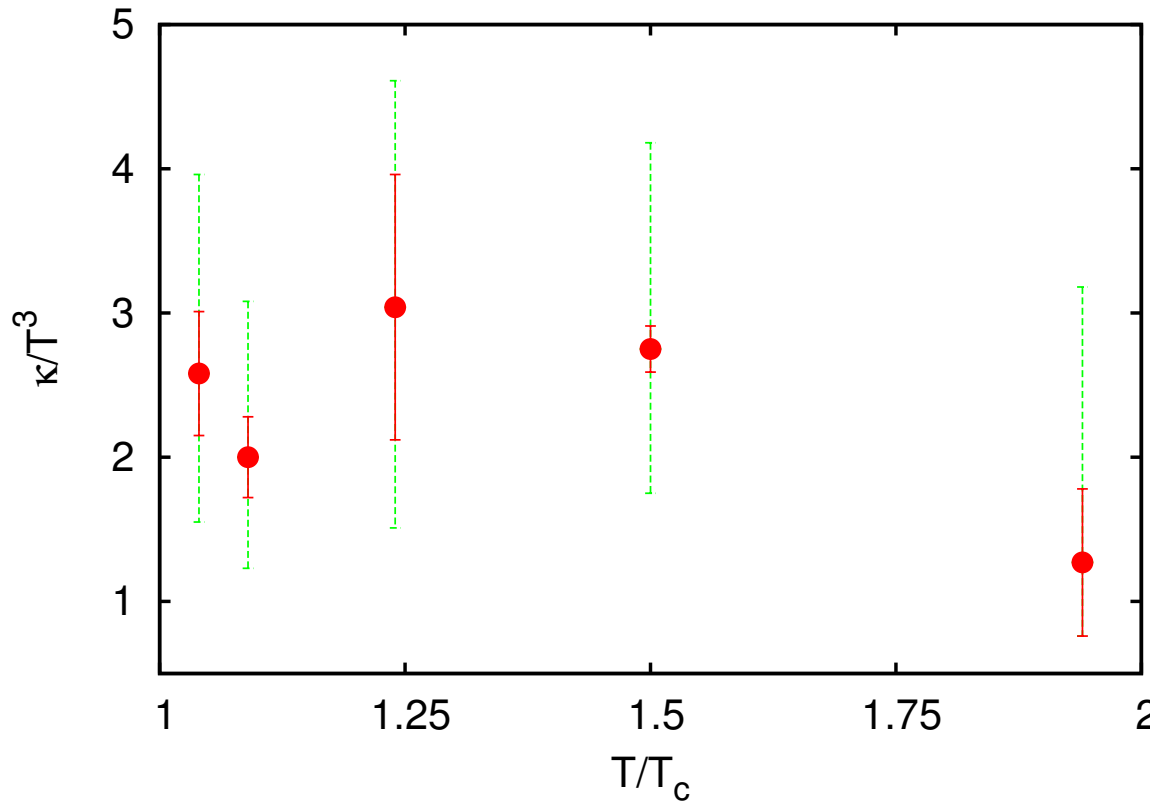
♠ Comparing the DIFF fit with the data after eliminating the LOC.



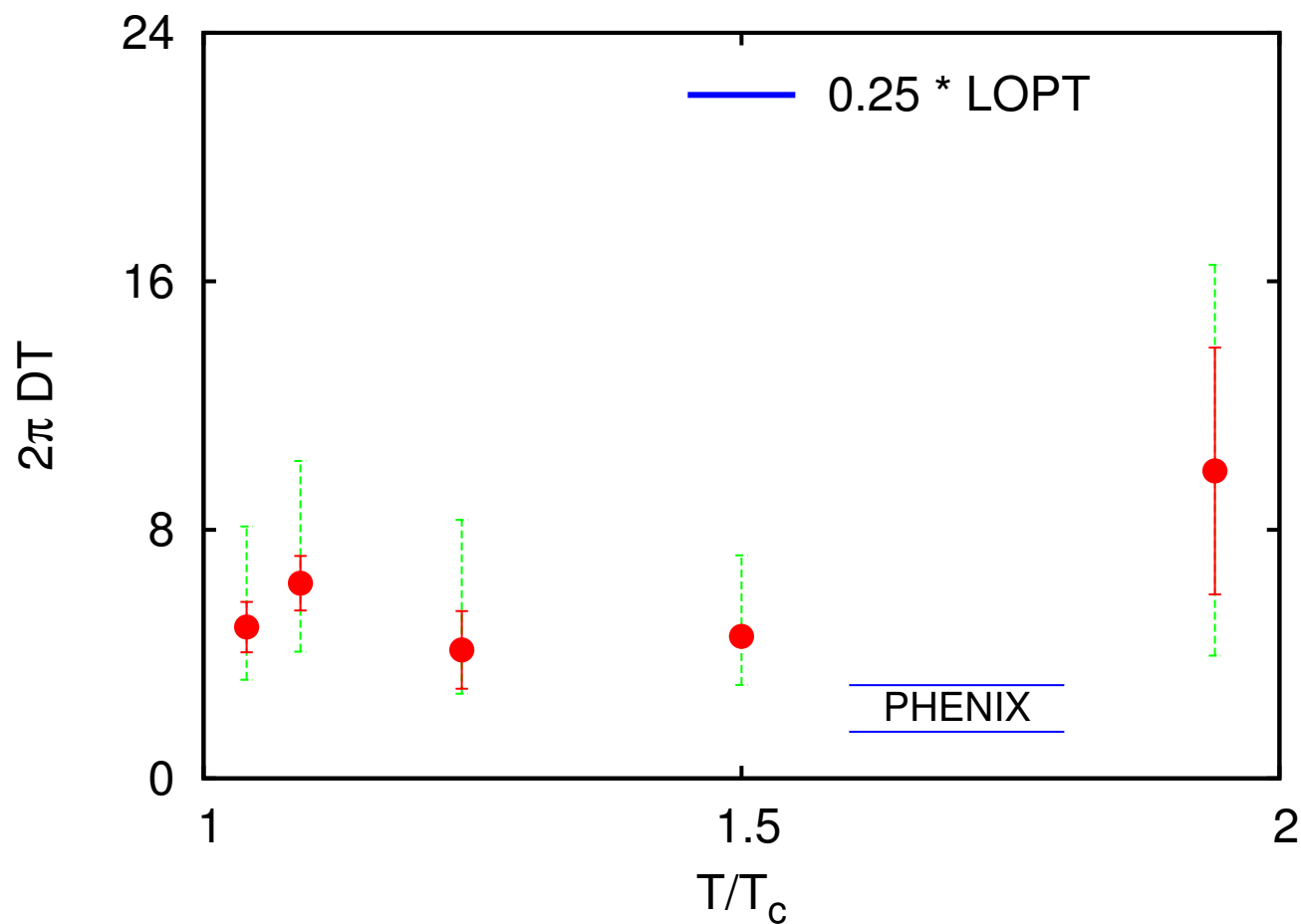
♠ Variation of a with the cut-off Λ and the temperature.

- ♠ Our fit parameter $a \rightsquigarrow \kappa$ modulo the renormalization factor for the electric fields.
- ♠ We use the tadpole factor. It is ~ 1.2 as evaluated from our plaquette values.

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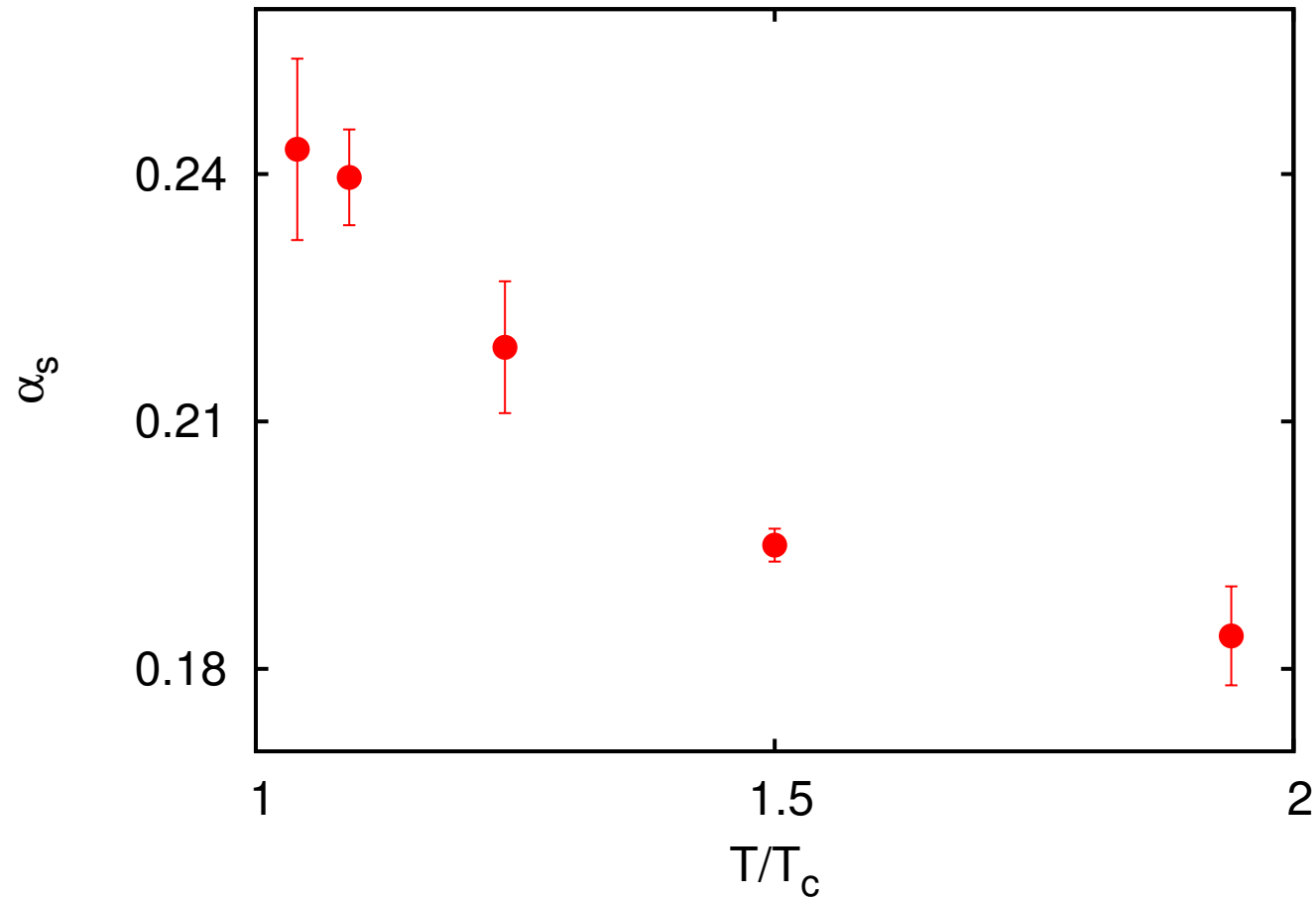


♠ Multiplying by T , obtain D , the quantity used by Moore-Teaney and PHENIX.



♡ In agreement with preliminary Bielefeld estimates (Ding et al. 1107.0311; Francis et al. 1109.3941).

♠ The ω^3 term comes with g^2 . Use as a scheme to define α_s non-perturbatively.



♡ In agreement with other similar estimates (Ding et al. PRD 83 (2011) 034504).

J/ψ : Flows or not ?

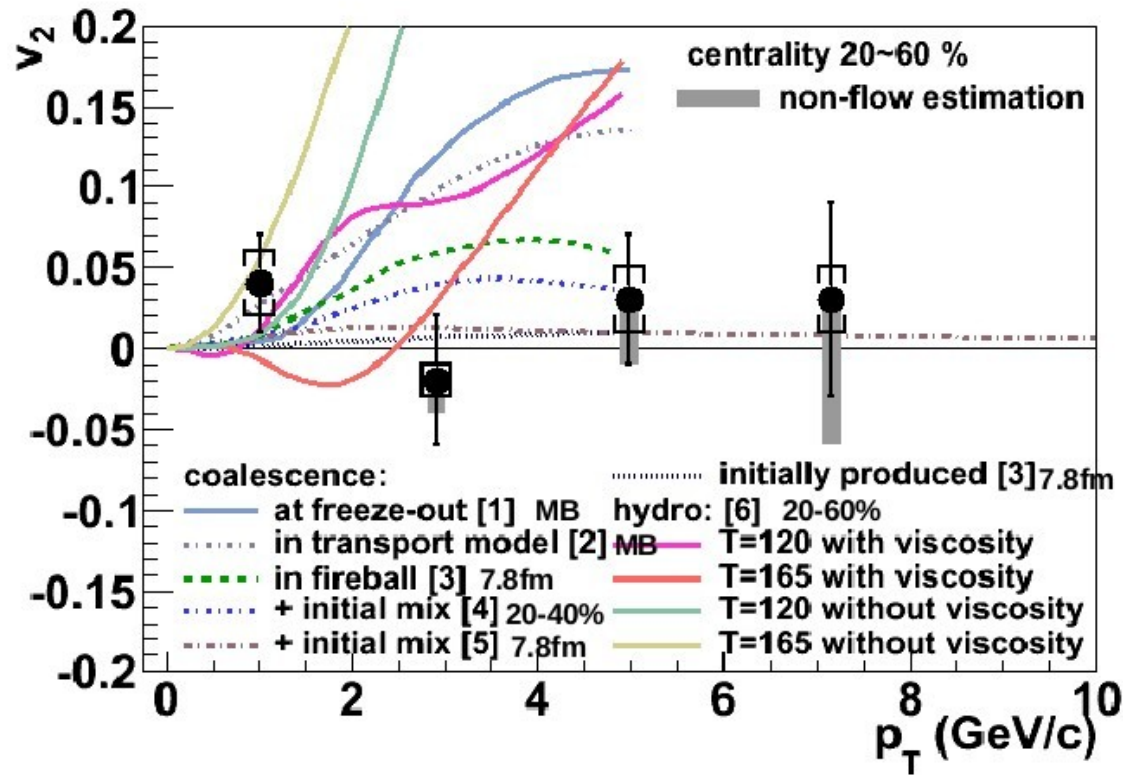
- ♣ The diffusion coefficient D results from *colour* interactions. Expect it to be zero for the colourless J/ψ , leading to very small flow for it due to its large mass.
- ◇ But the thermal charm may be in abundance and may also obey the n_q -scaling.

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- ◇ But the thermal charm may be in abundance and may also obey the n_q -scaling.
- ♠ If thermal charm ‘recombines’ to produce many J/ψ , then one expects J/ψ to flow still.
- ♡ The STAR collaboration presented results for J/ψ flow in the recent Quark Matter 2011.



J/ψ elliptic flow v_2



Consistent with zero
Disfavor coalescence from thermalized charm quarks

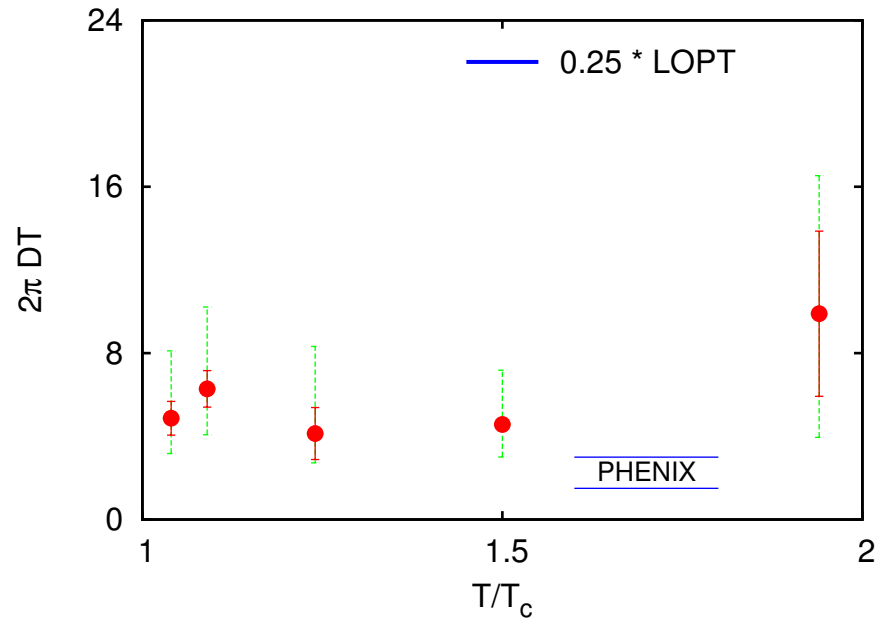
[1] C.M. Ko, R. Rapp, PLB 595, 202.
 [2] R. Rapp, PLB 655, 126.
 [3] P. Zhuang, N. Xu, PRL 97, 232301.
 [4] R. Rapp, 24th WWND, 2008.
 [5] Y. Liu, N. Xu, P. Zhuang, Nucl. Phys. A, 834, 317
 [6] U. Heinz, C. Shen, private communication.

Summary

- We have obtained the diffusion constant D as a function of T/T_c in quenched QCD in the temperature range of interest to RHIC and LHC.
- Our results for DT are almost constant in the range studied.

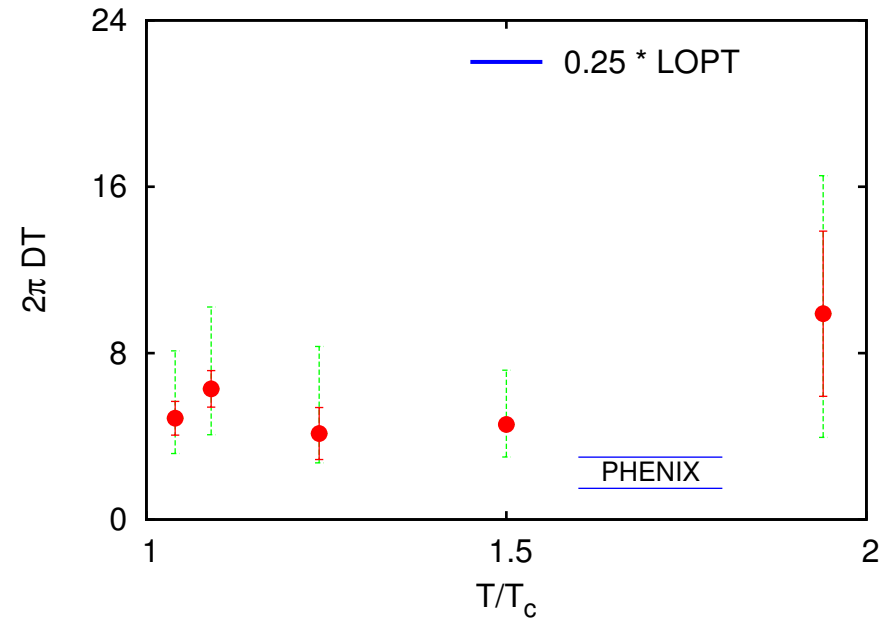
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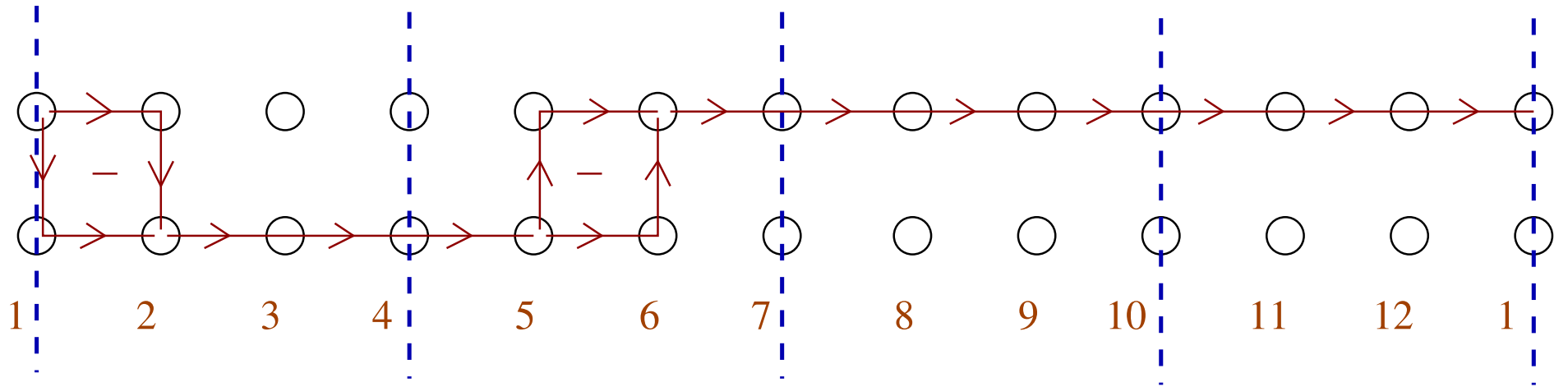
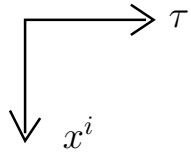


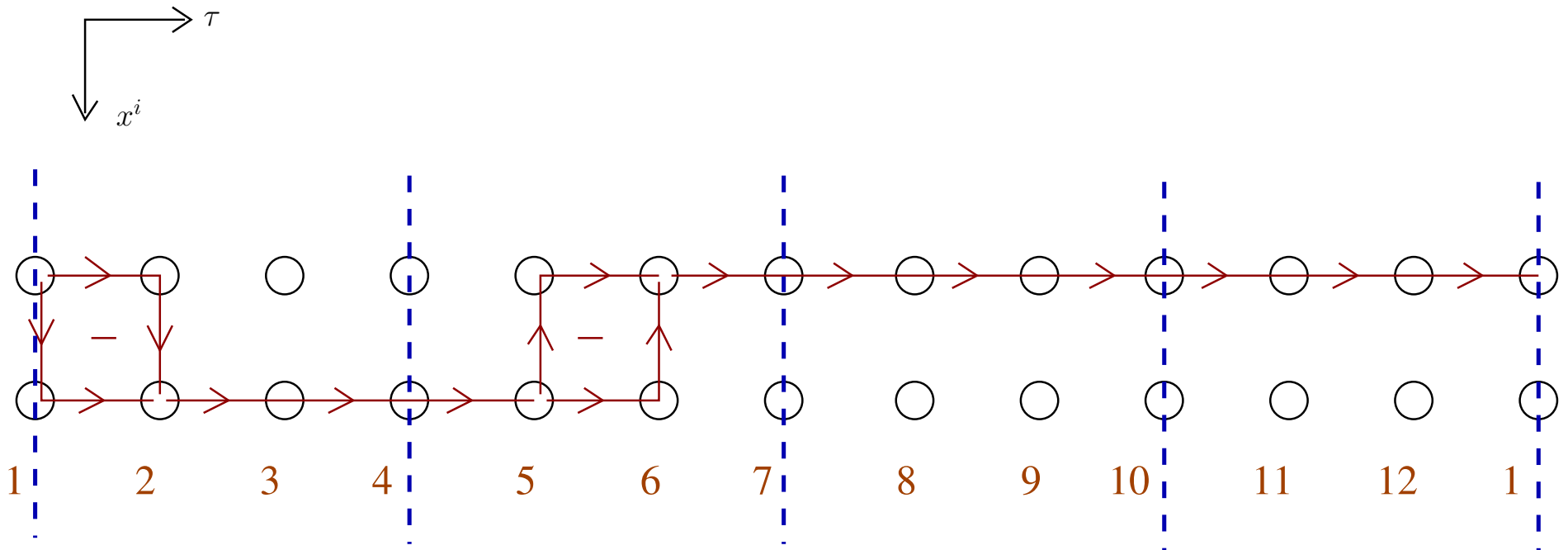
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It would be interesting to see if DT vs. T/T_c exhibits similar flavour independence as the pressure.





β	6.76	6.80	6.90	7.192	7.255
N_τ	20	20	20	24	20
T/T_c	1.04	1.09	1.24	1.5	1.96

Table 1: List of lattices on which diffusion coefficients were extracted, and the temperatures.