Heavy Flavor Flow at PHENIX : A New Pasture for Lattice QCD

Rajiv V. Gavai[*]
T. I. F. R., Mumbai, India

[*] *With Debasish Banerjee, Saumen Datta & Pushan Majumdar, Phys. Rev. D85, 014510 (2012), arXiv:1109.5738*

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Introduction

Formalism

Our Lattice Results

Summary

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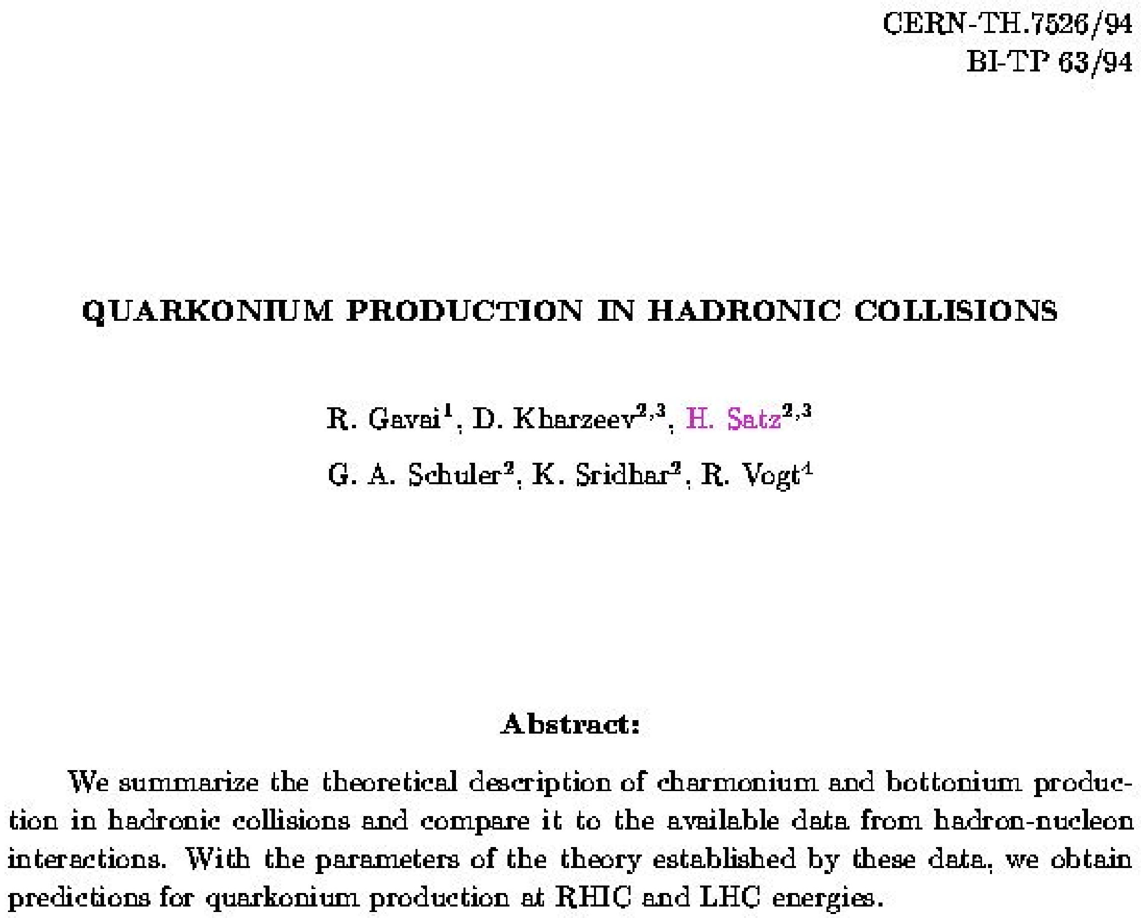
The Iconic Connection

At the Quark Matter 1984, Helsinki.

The Iconic Connection



CERN-TH.7526/94
BI-TP 63/94

QUARKONIUM PRODUCTION IN HADRONIC COLLISIONS

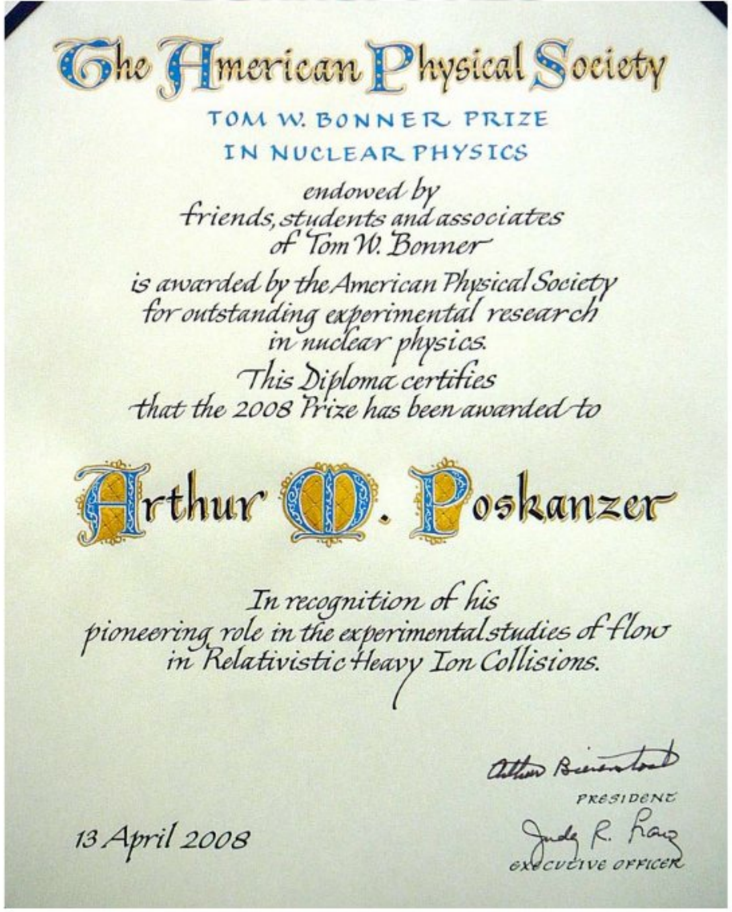
R. Gavai[1], D. Kharzeev[2,3], H. Satz[2,3]
G. A. Schuler[2], K. Sridhar[2], R. Vogt[4]

Abstract:

We summarize the theoretical description of charmonium and bottonium production in hadronic collisions and compare it to the available data from hadron-nucleon interactions. With the parameters of the theory established by these data, we obtain predictions for quarkonium production at RHIC and LHC energies.

The Berkeley Connection

Bonner Prize



The American Physical Society

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endowed by
friends, students and associates
of Tom W. Bonner
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in nuclear physics.
This Diploma certifies
that the 2008 Prize has been awarded to

Arthur M. Poskanzer

In recognition of his
pioneering role in the experimental studies of flow
in Relativistic Heavy Ion Collisions.

Arthur Bienenstock
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13 April 2008

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Introduction : Anisotropic Flow

- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.

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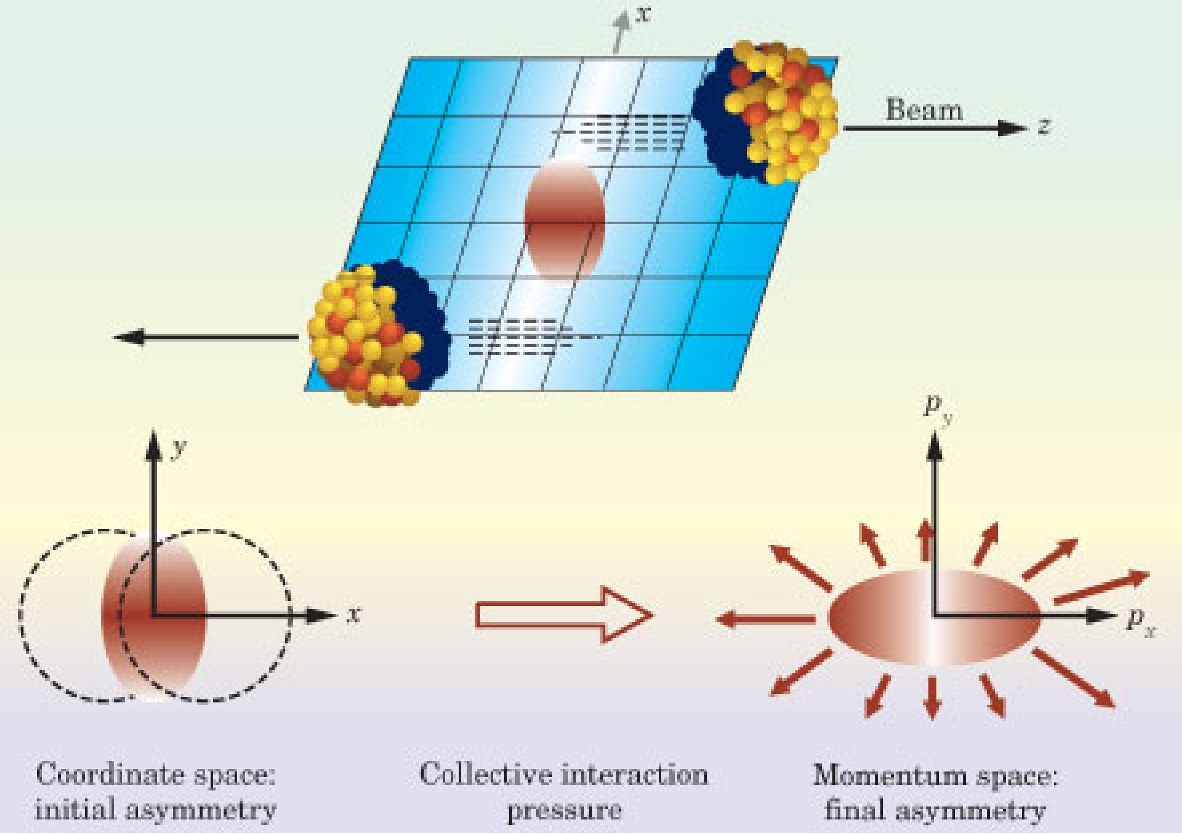
- Obtained from asymmetric collisions of two nuclei, with their centres not aligned.



- $$v_2(y, p_T) = \frac{\int d\phi\ dN/(p_T dP_T d\phi dy)\ \ \cos(2\phi)}{\int d\phi\ dN/(p_T dP_T d\phi dy)} \tag{1}$$

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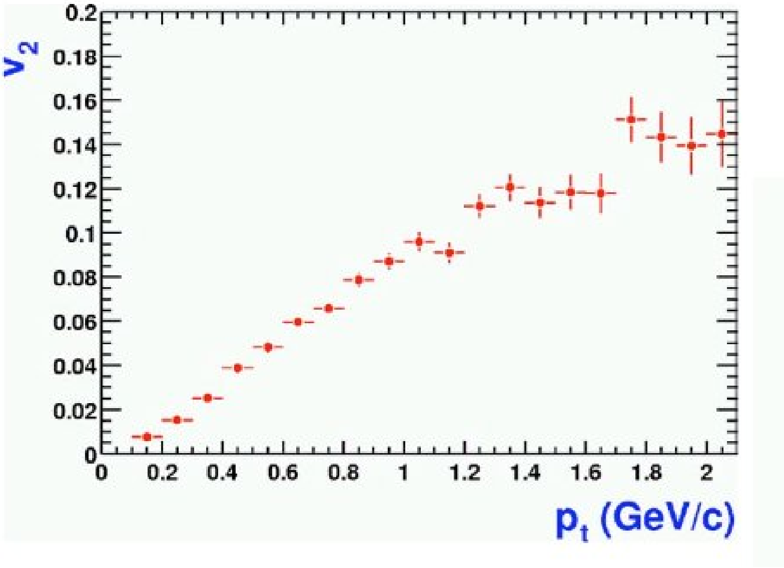
- (STAR Collaboration, Ackermann et al., PRL 86 (2001) 402.)



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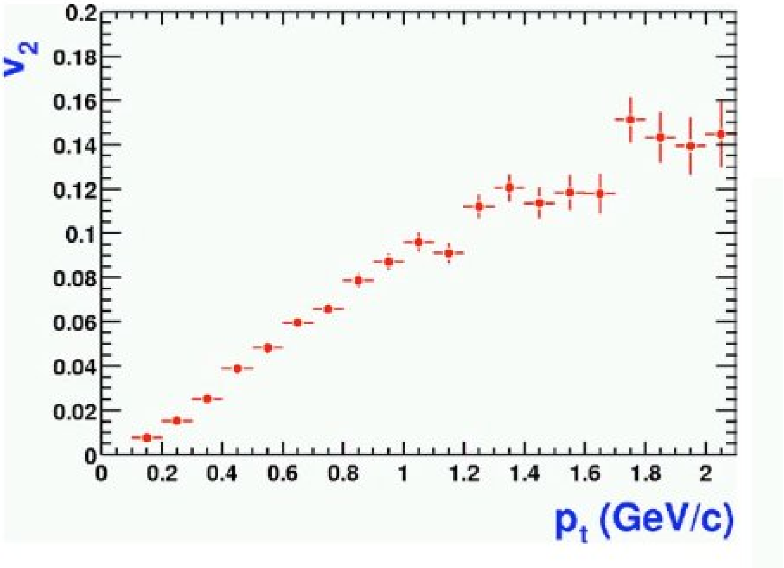
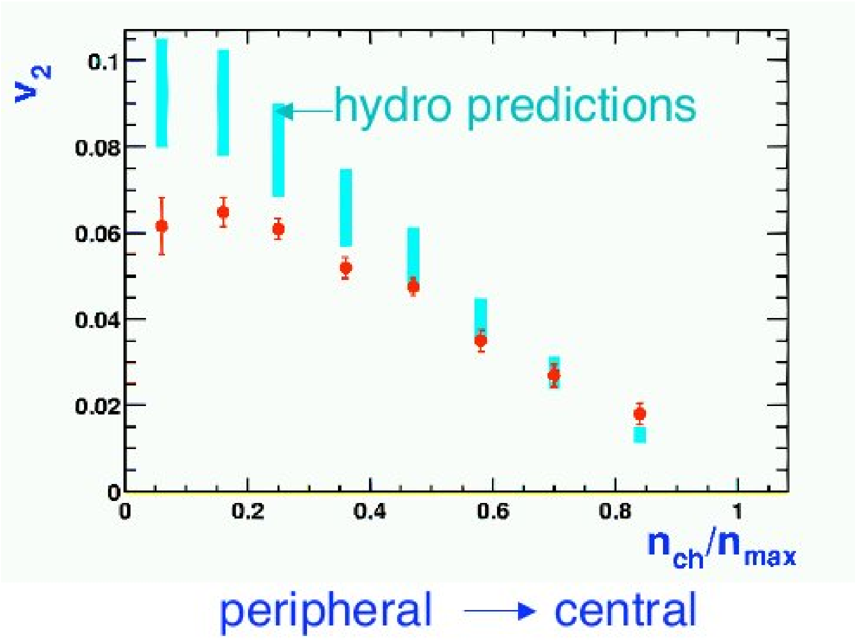
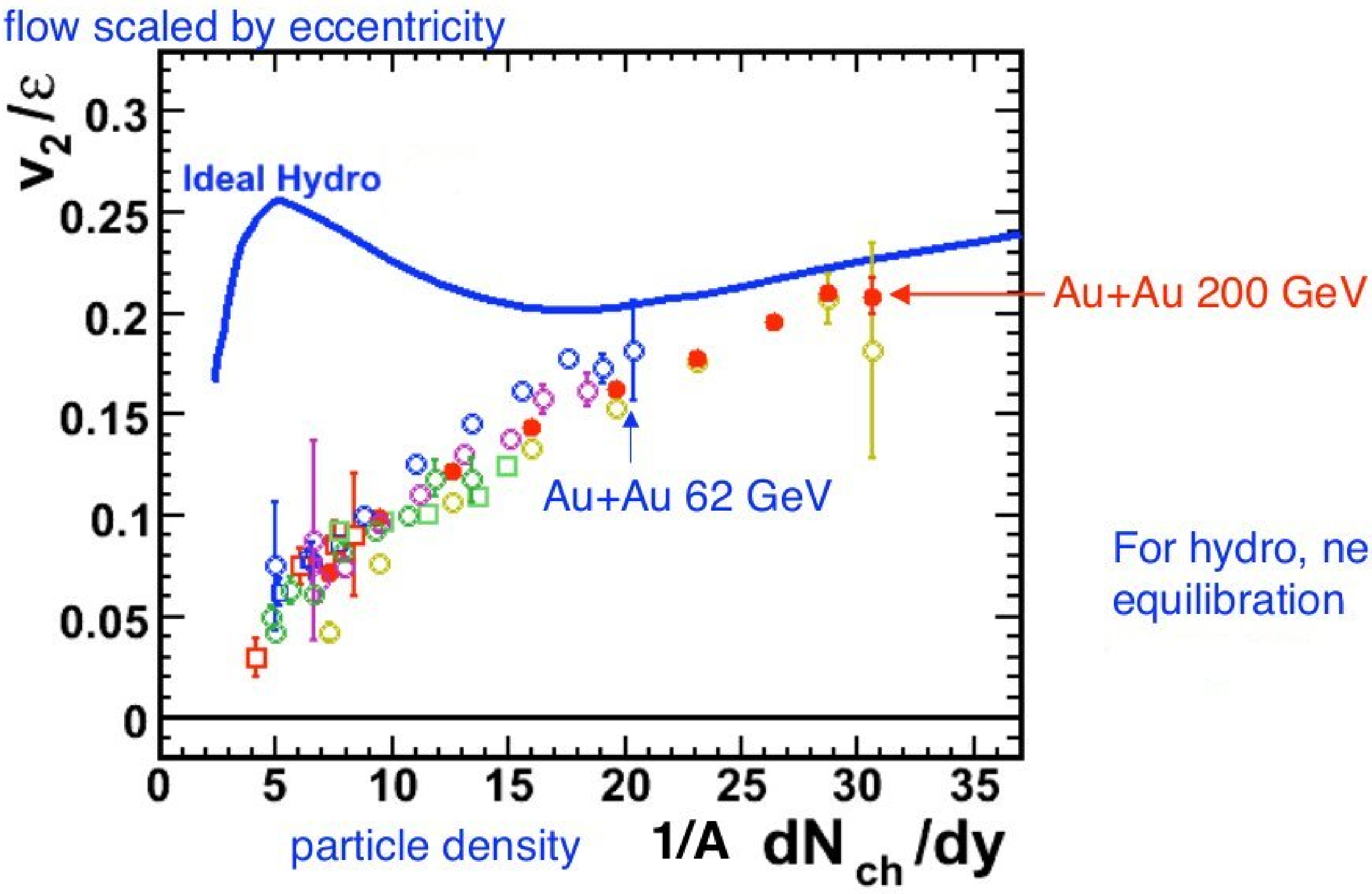




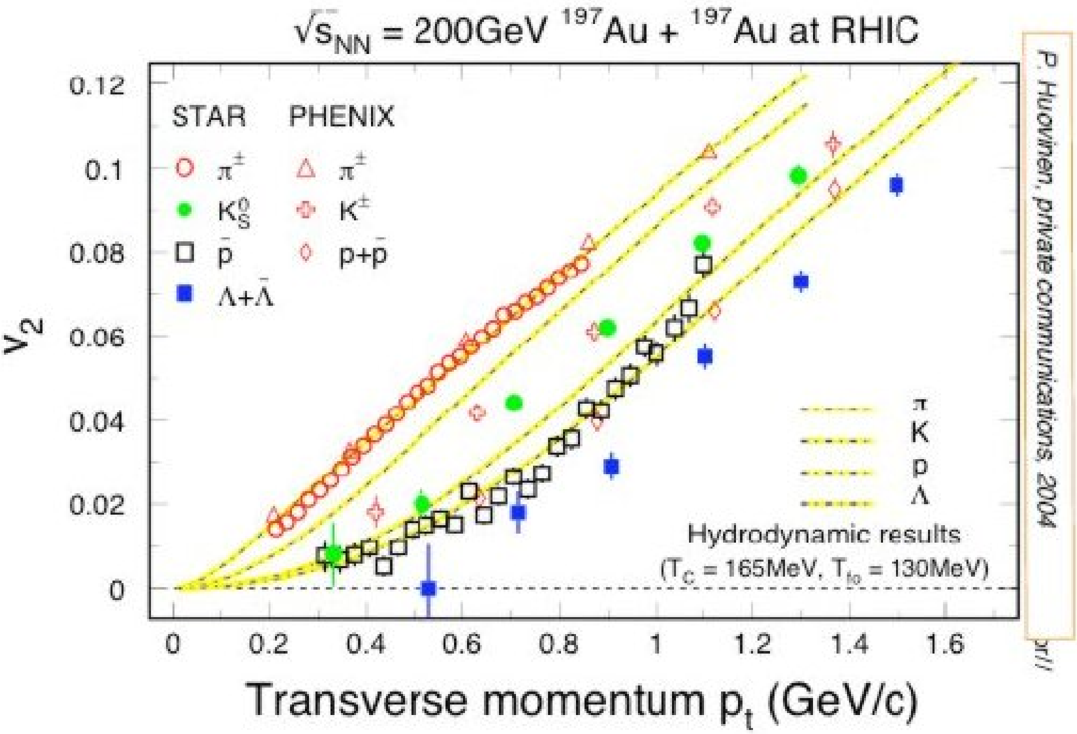
- Good agreement with ideal hydro: Suggesting early thermalization and perfect fluid and many more interesting things.



For hydro, need early equilibration

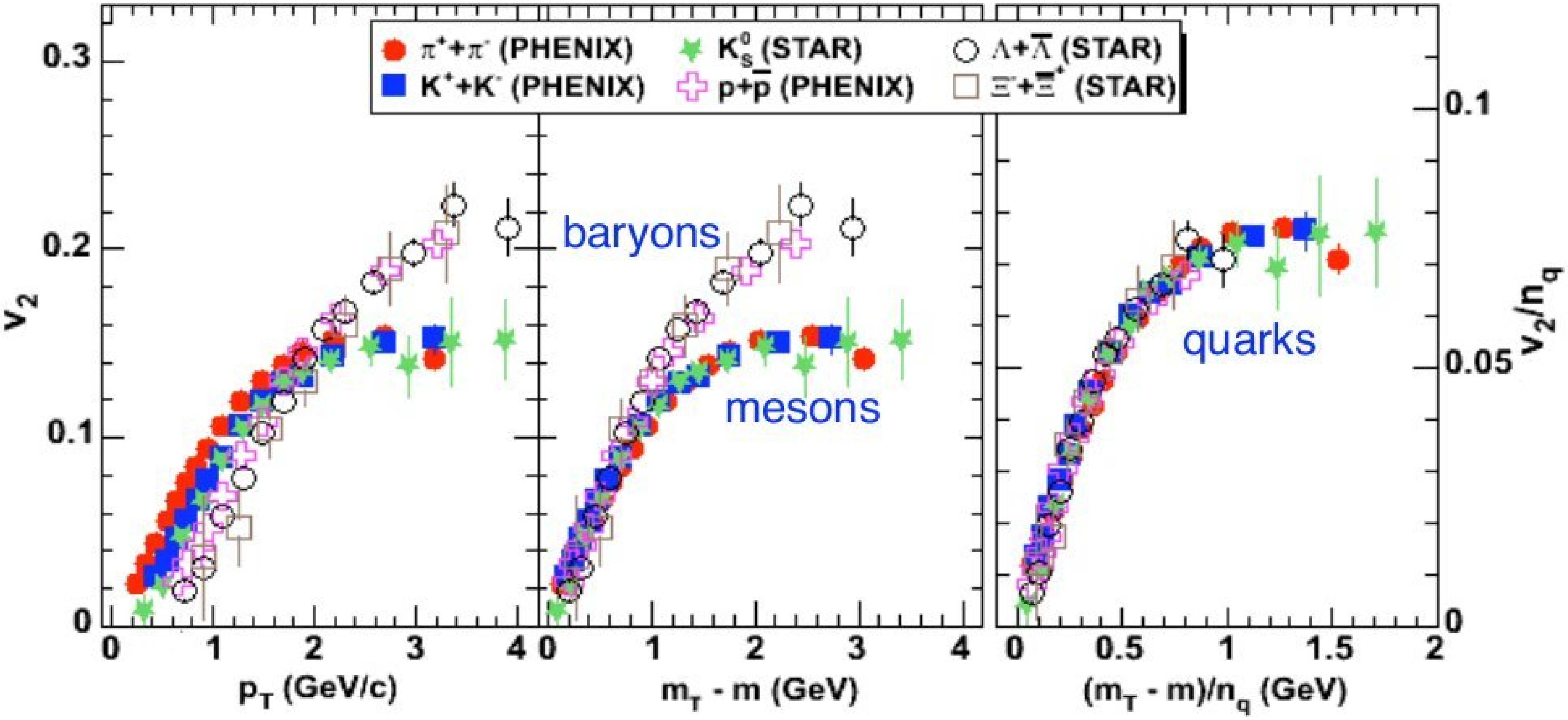
(S. Voloshin, QM06, JPG 31 (2007) S883 & Hydro Curve: Kolb-Sollfrank-Heinz, PRC 62 (2000) 054909.)

v_2 at Low p_T Region



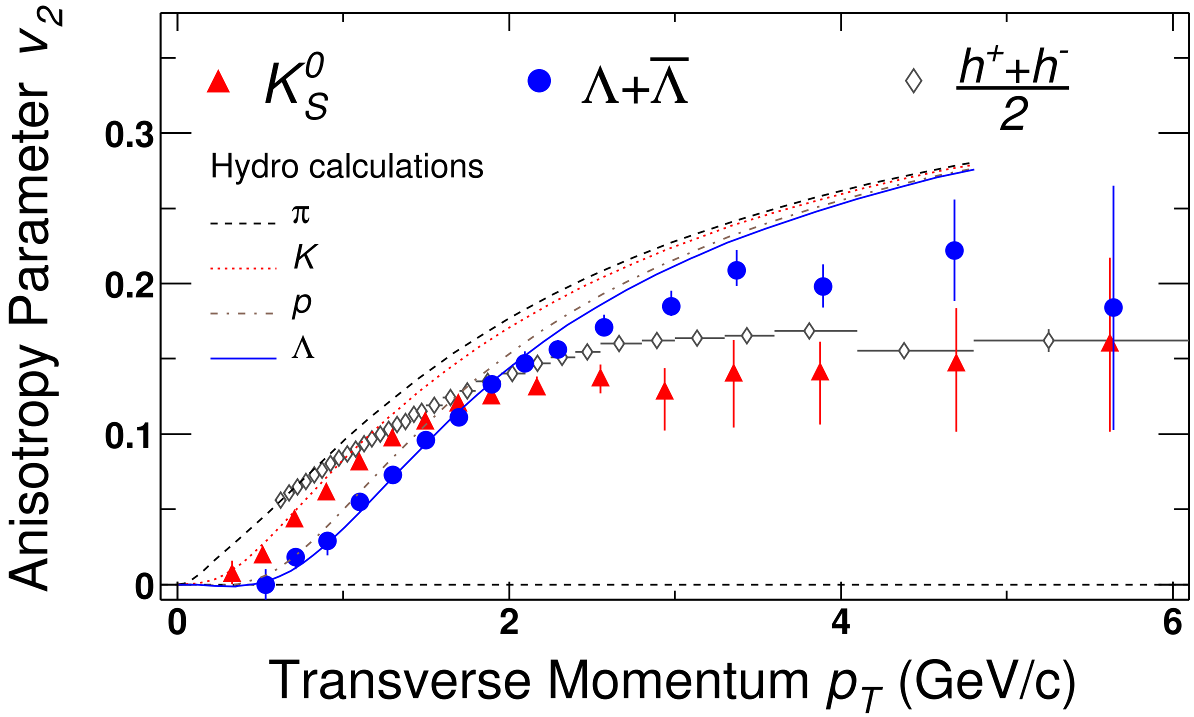
(STAR Collaboration, JPG 31 (2005) S437 & P. Huovinen.)

- Mass Pattern as expected by Hydrodynamics Models. Quantitative agreement depends on the equation of state.



(S. Voloshin, QM02, STAR PRL 95 (2005) & PHENIX PRL 98 (2007))

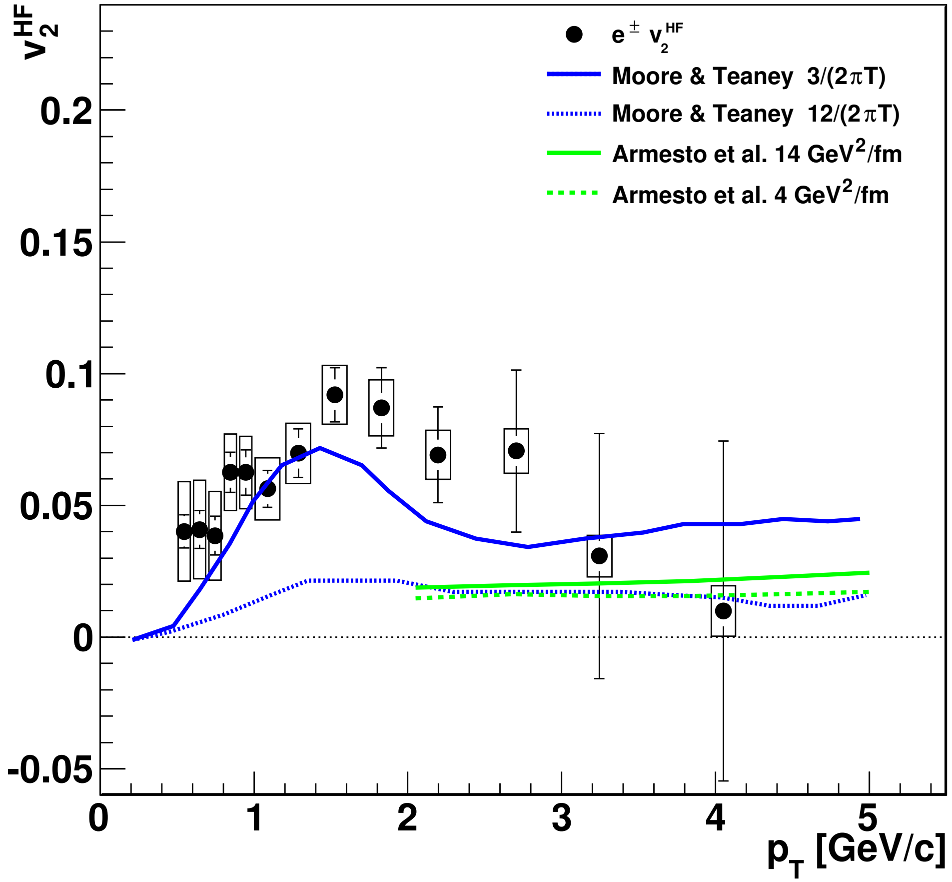
- v_2 scales as number of quarks. Thus, hadrons appear to follow the ‘underlying’ quark flow as Recombination Model would suggest.



(STAR Collaboration, Adams et al., PRL 92 (2004) 052302.)

♡ Minimum Bias Au+Au Collisions at 200 GeV/c : Strangeness flows like normal hadrons.

- Naıvely expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.



(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

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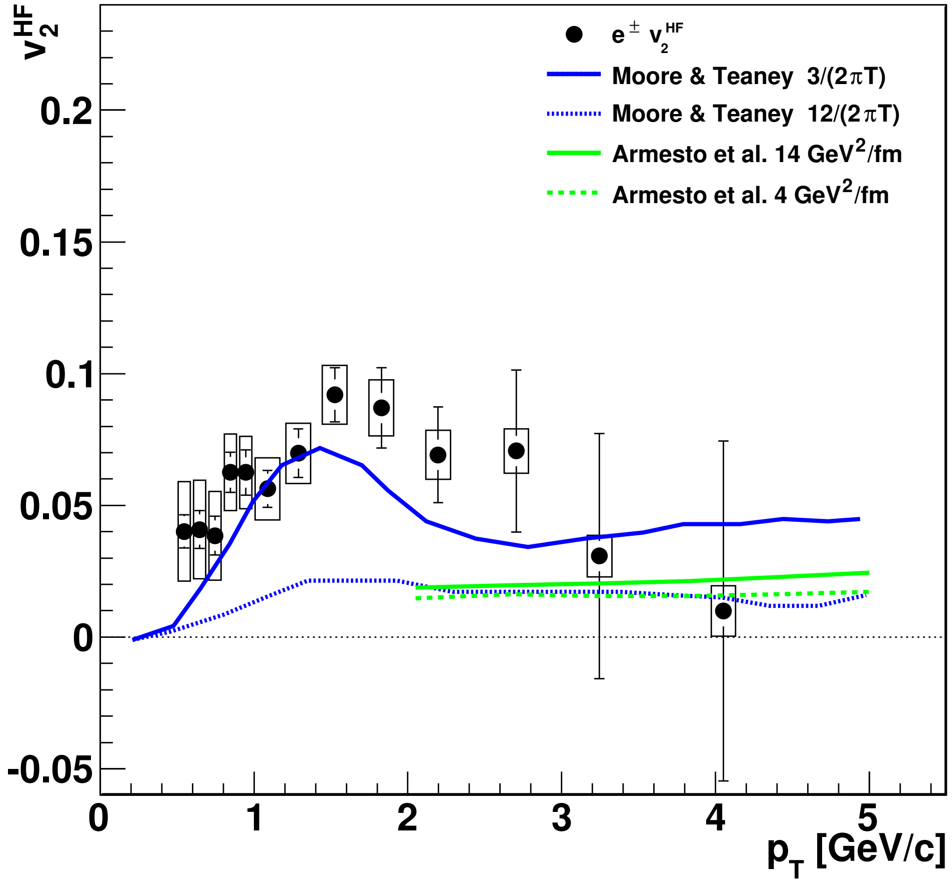
- In models (Moore-Teaney, PRC 71, 2005), heavy quark diffusion coefficients governs its elliptic flow **and** suppression.



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- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a ‘perturbative’ estimate, seems to under-predict v_2 substantially.

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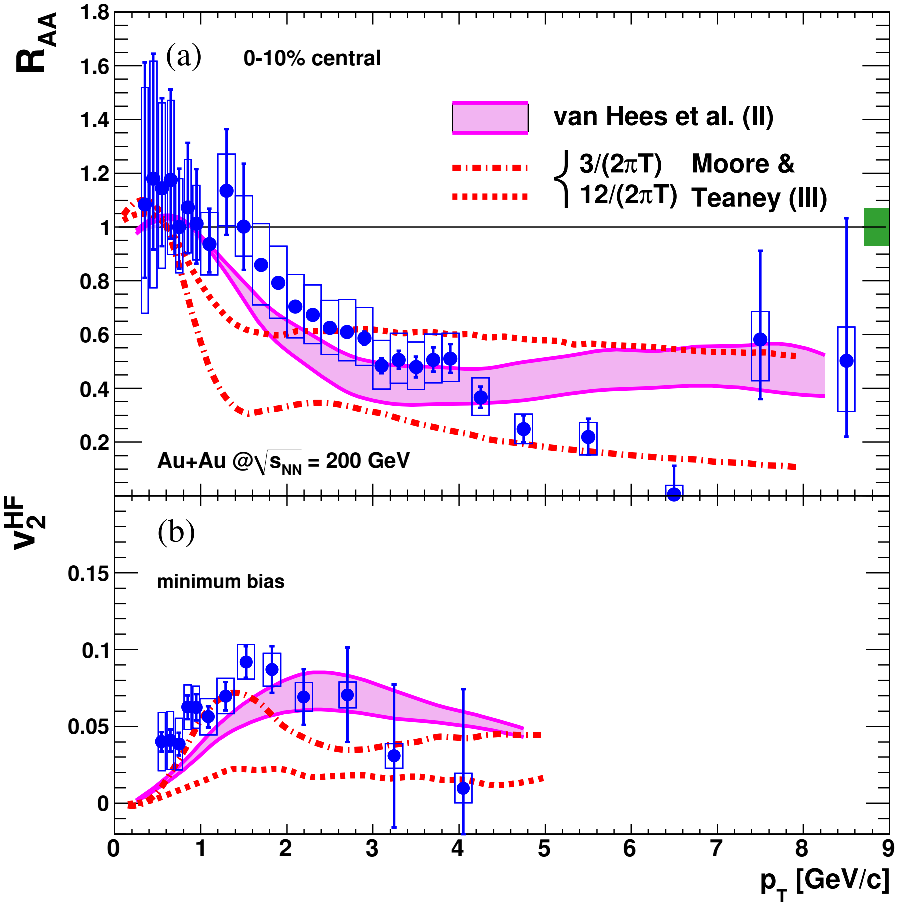
- Similar value also explains the suppression in the PHENIX R_{AA} for heavy quarks at RHIC.

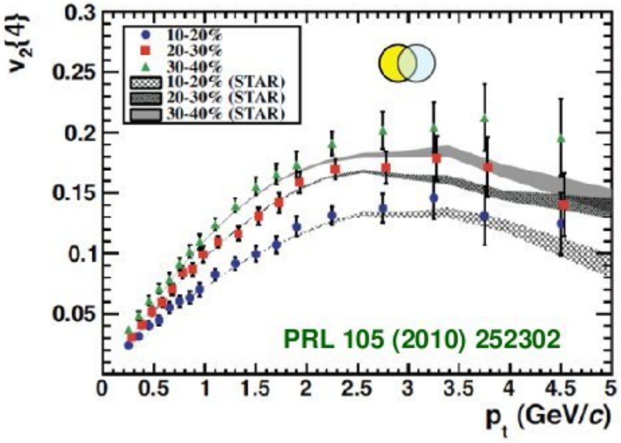
- Other models, e.g. van Hees-Greco-Rapp, seem to suggest the same.

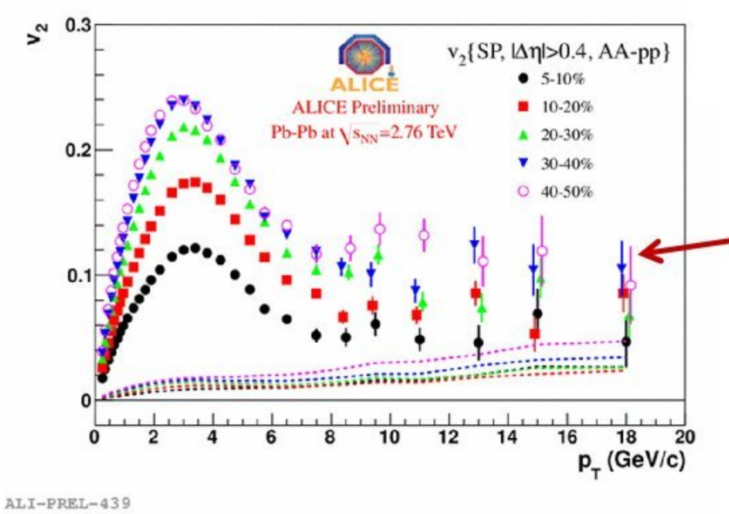


(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

p_t differential elliptic flow

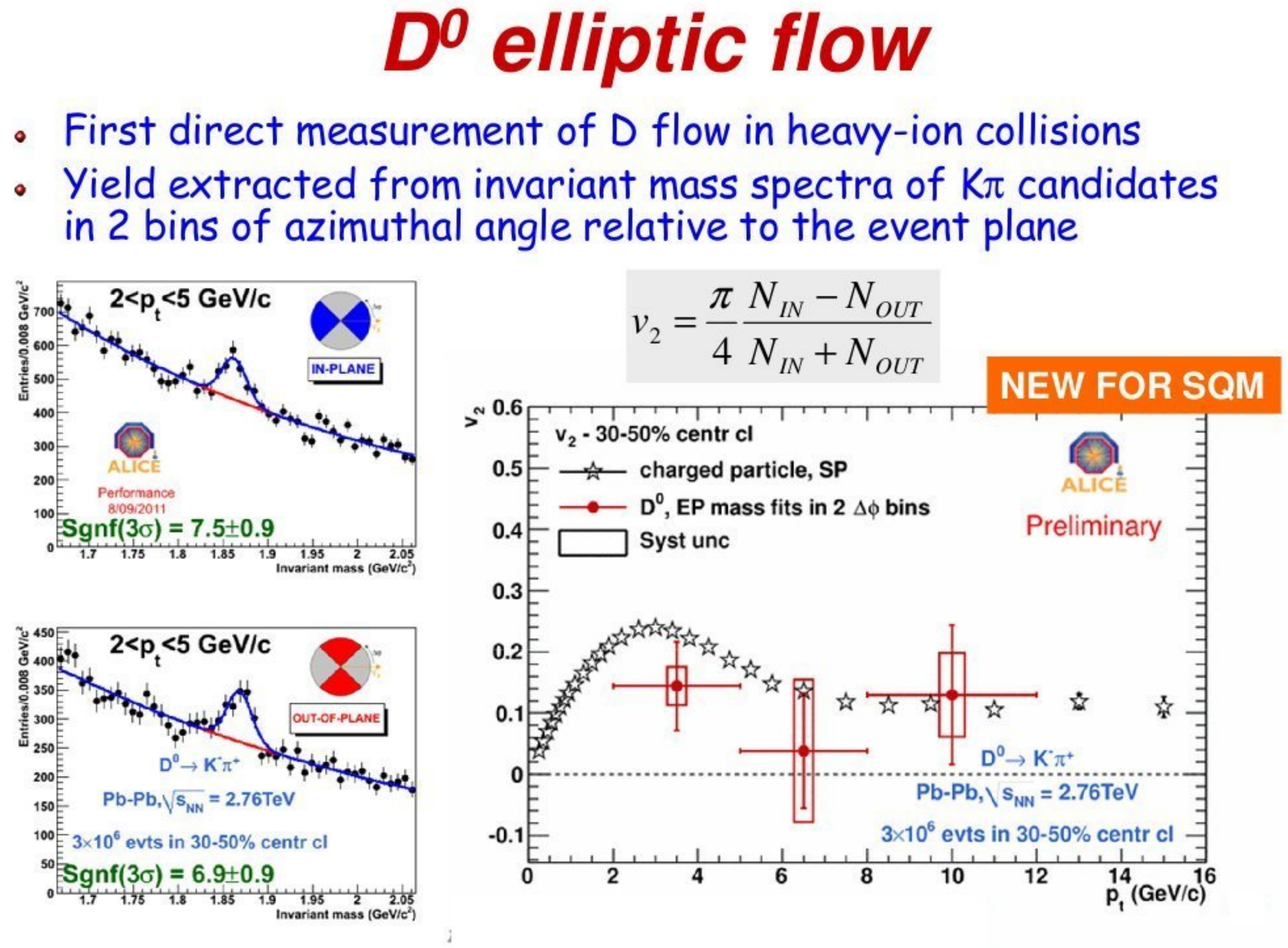
- v_2 vs. p_T does not change within uncertainties between $\sqrt{s_{NN}}$=200 GeV and 2.76 TeV
  - ⇨ 30% increase of p_T integrated flow explained by higher mean p_T due to stronger radial flow at higher energies





- Significant values of v_2 for p_T>8 GeV/c
  - ⇨ Due to path length dependence of parton energy loss

ALICE overview at SQM11, Krakow, Poland by Francesco Prino.



D^0 elliptic flow

- First direct measurement of D flow in heavy-ion collisions
- Yield extracted from invariant mass spectra of Kπ candidates in 2 bins of azimuthal angle relative to the event plane

$$v_2 = \frac{\pi}{4}\frac{N_{IN}-N_{OUT}}{N_{IN}+N_{OUT}}$$

NEW FOR SQM

ALICE overview at SQM11, Krakow, Poland by Francesco Prino.

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- Is it non-perturbative ? Strong coupling models — AdS/CFT based — do lead to values in the desired range under "suitable" assumptions [Casalderrey-Solana & Teaney (2006), Gubser(2007)]

- Can Lattice QCD shed some light on the Charm Flow ?

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.

- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

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$$\frac{dp_i}{dt} = -\eta_D\, p_i + \xi_i(t) \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t') \tag{2}$$

- η_D – momentum drag coefficient and 3κ is mean-squared momentum transfer per unit time, $\kappa = \frac{1}{3}\int_{-\infty}^{\infty} dt \sum_i \langle \xi_i(t)\xi_i(0)\rangle$.

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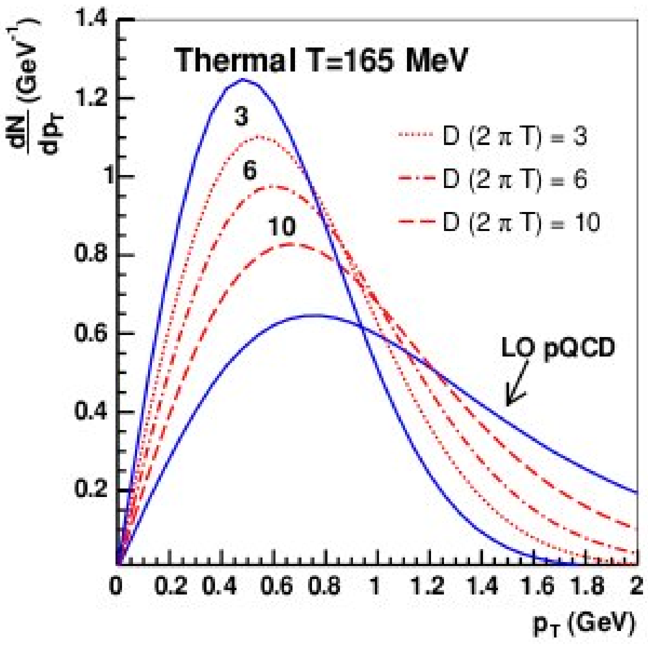
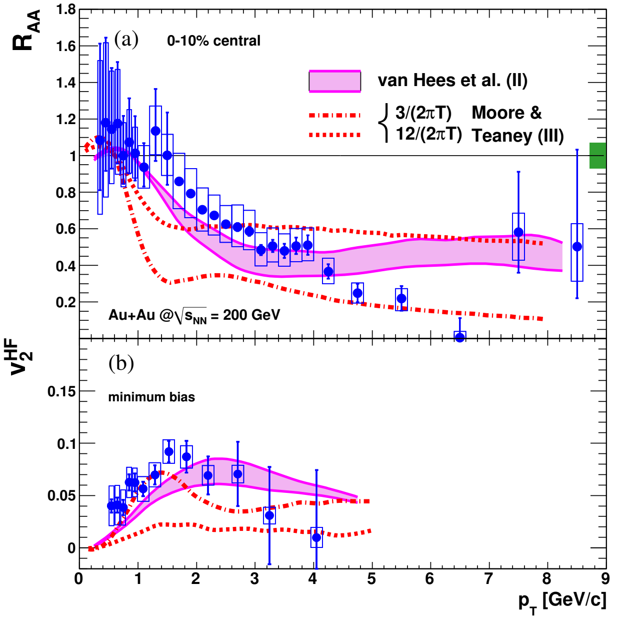
- Diffusion constant D can be shown to be $2T^2/\kappa$ with $\eta_D = \kappa/2MT$.

- Moore-Teaney assumed an initial power-law (LO pQCD) transverse momentum distribution of a heavy Q in an expanding QGP at $T_0 = 300$ MeV. Assuming an ideal Bjorken expansion of the plasma, they showed that by $T_f = 165$ MeV the charm distributition approximates a thermal one **provided** $D \leq 3/2\pi T$.
- Their comparison, including a more realistic hydro-simulation, which I showed earlier also, supports such a conclusion.





- Casalderrey-Solana & Teaney (PRD 74 (2006) 085012) suggested to obtain κ from a correlator of the (colour) force exerted on a heavy Q by the (deconfined & coloured) medium.

- Caron-Huot, Laine & Moore (JHEP 0904, 053) provided a suitable definition for κ for a lattice evaluation: The force acting on the heavy quark is given by $M\ dJ^i/dt$, where $J^\mu(\vec{x},t) = \bar{\psi}(\vec{x},t)\gamma^\mu\psi(\vec{x},t)$ is the conserved current for the heavy quark.

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- Using Heavy Quark Effective Theory, they narrowed it down to studying $G_E^{\rm Lat}(\tau) \ = \ -\frac{1}{3L}\sum_{i=1}^{3} \left\langle {\rm Re\ tr}\ \left[U(\beta,\tau)\ E_i(\tau,\vec{0})\ U(\tau,0)\ E_i(0,\vec{0})\right]\right\rangle$, where L is the Polyakov loop.

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- The spectral function, $\rho(\omega)$, is obtained from the $G_E(\tau)$, as usual, by

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi}\,\rho(\omega)\,\frac{\cosh\omega(\tau-\frac{1}{2T})}{\sinh\frac{\omega}{2T}}\,. \qquad (3)$$

- Then momentum diffusion coefficient $\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)$. where ρ is the spectral function obtained from G above.

- They also suggested a suitable discrete version for Lattice QCD : $E_i(\vec{x}, \tau) = U_i(\vec{x}, \tau)\, U_4(\vec{x} + \hat{i}, \tau) - U_4(\vec{x}, \tau)\, U_i(\vec{x} + \hat{4})$.

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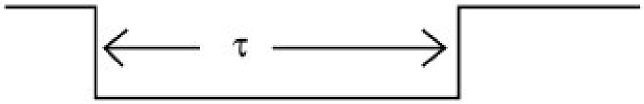
- Using this, the numerator can be written as a derivative of an extended (by spatial detour of a) Polyakov loop.
$G^i_{E,\mathrm{num}}(\tau) = C^i(\tau+1) + C^i(\tau-1) - 2C^i(\tau)$

$C^i(\tau) = \prod_{x_4=0}^{t-1} U_4(x_4) \cdot U_i(t) \cdot \prod_{x_4=t}^{t+\tau-1} U_4(x_4) \cdot U_i^\dagger(t+\tau) \cdot \prod_{x_4=t+\tau}^{\beta-1} U_4(x_4)$.



Graphical Representation of $C(\tau)$.

Our Lattice Results

- It is well-known that the Polyakov loop becomes exponentially small with N_τ. The extraction of κ, on the other hand, needs large N_τ.

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- We attempted $N_\tau =$ 12, 16, 20 and 24 for quenched QCD. Multilevel algorithm (Lüscher-Weisz, JHEP 0109 & 0207) was suitably adopted.

- For the same size error on $G(10)[G(3)]$ on $N_\tau = 20$ lattices, it was found to be $\sim 2500[200]$ times more efficient: Very crucial in getting κ.

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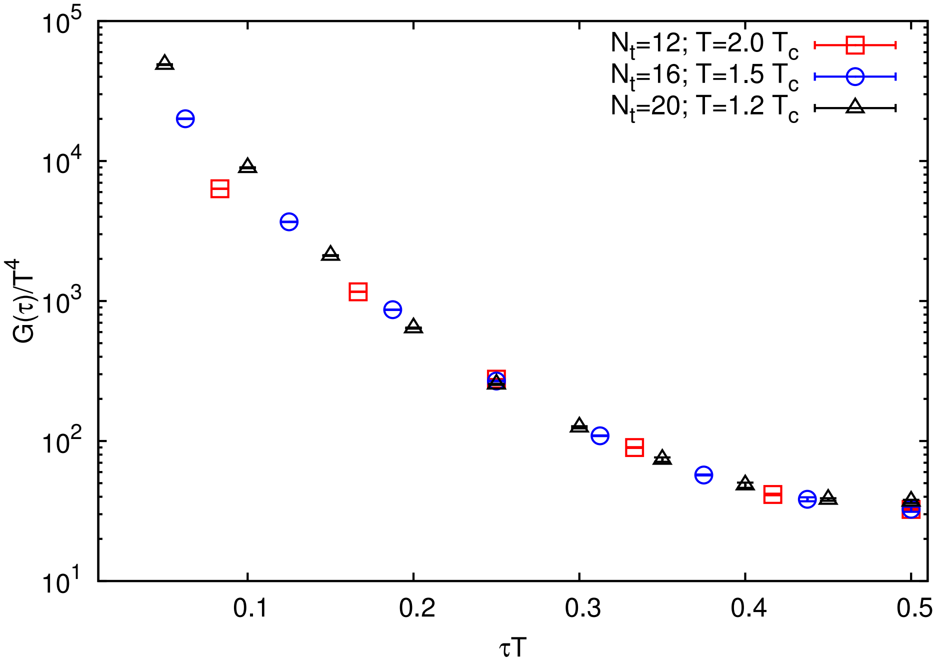
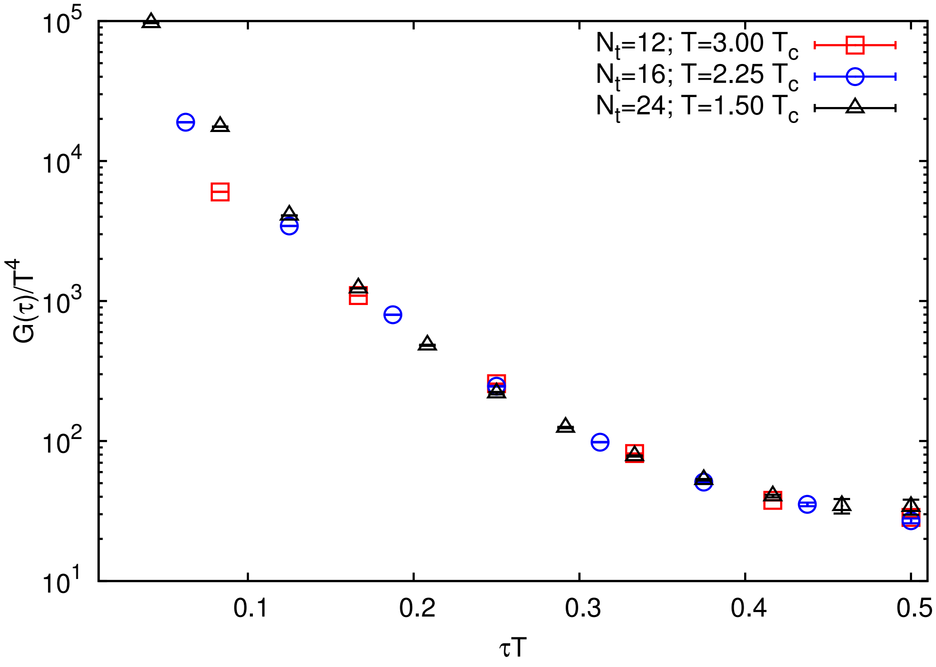
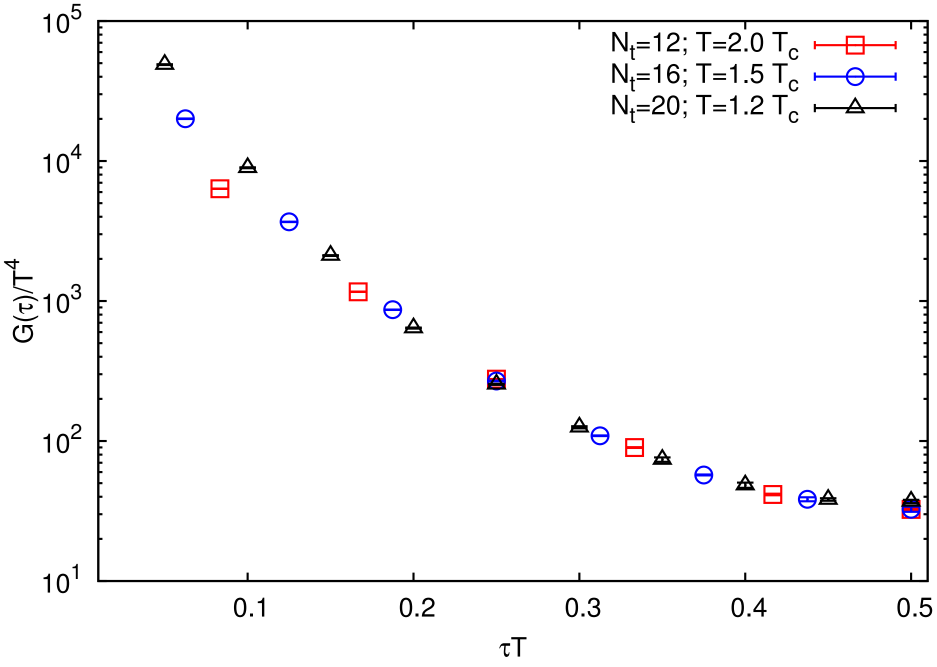
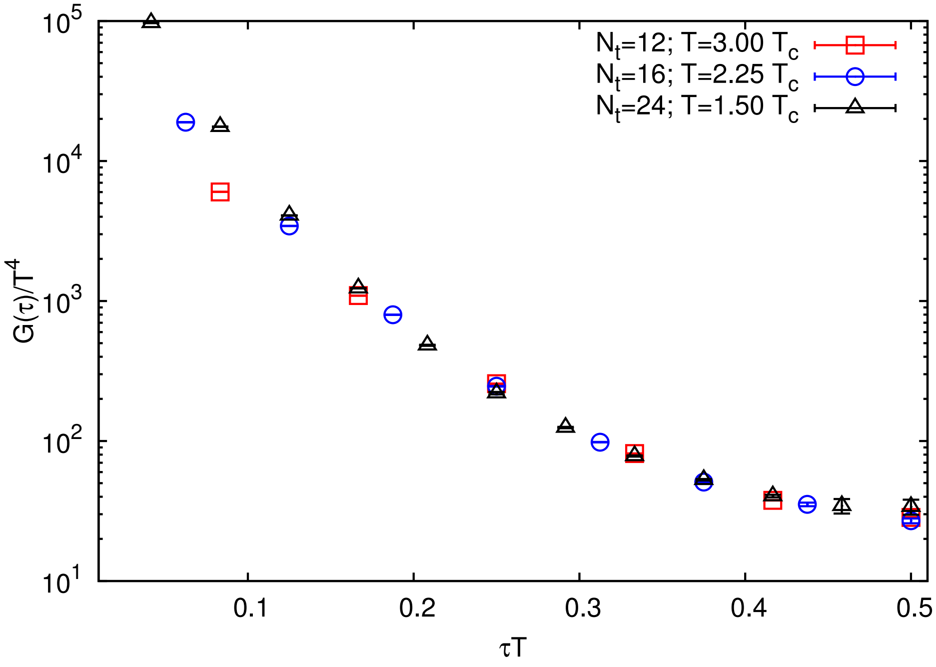
- For the same size error on $G(10)[G(3)]$ on $N_\tau =$ 20 lattices, it was found to be $\sim 2500[200]$ times more efficient: Very crucial in getting κ.
- Spatial volumes are such that $N_s \geq 2N_\tau$.
- Couplings were chosen suitably to make simulations at $T/T_c =$ 1.04, 1.09, 1.24, 1.5 and 1.96 for the two largest N_τ.
- Typical Statistics : Few hundred Independent Configurations, with a few thousand multilevel updates.

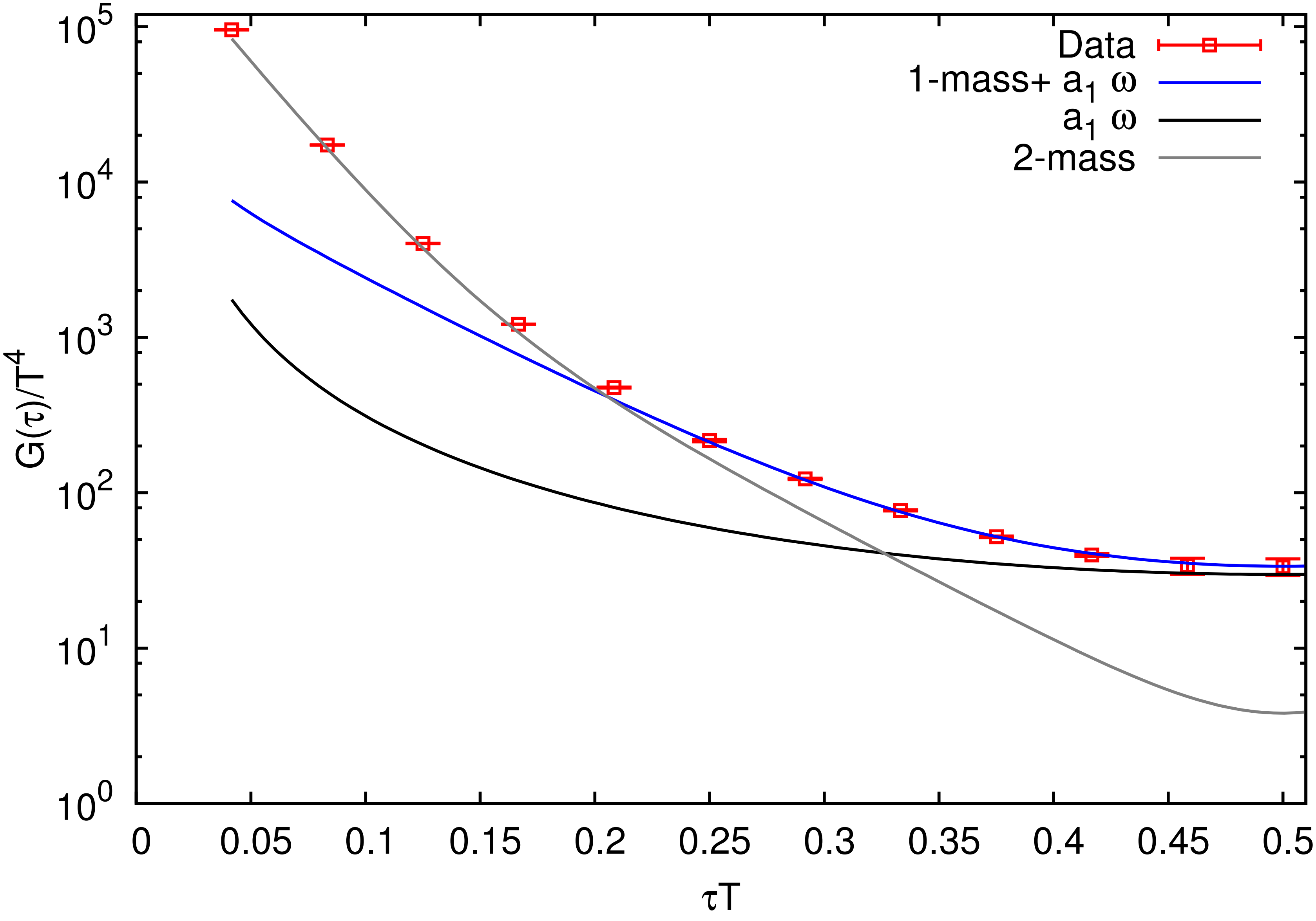

N_t=12; T=2.0 T_c
N_t=16; T=1.5 T_c
N_t=20; T=1.2 T_c
$G(\tau)/T^4$
10^1
10^2
10^3
10^4
10^5
0.1
0.2
0.3
0.4
0.5
τT


N_t=12; T=3.00 T_c
N_t=16; T=2.25 T_c
N_t=24; T=1.50 T_c
$G(\tau)/T^4$
10^1
10^2
10^3
10^4
10^5
0.1
0.2
0.3
0.4
0.5
τT





- Large τ region shows scaling.
- Low τ region, on the other hand, has only lattice artifacts.



10^5
10^4
10^3
10^2
10^1
10^0
$G(\tau)/T^4$
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
τT
Data
1-mass+ a_1 ω
a_1 ω
2-mass

Extracting D

- Getting to the spectral function ρ, an ill-posed problem, has attracted a lot of attention. Many methods can be tried.

- We use an $ansatz$ for ρ, obtain G from it, and then fit in the large τ range $[N_\tau/4, N_\tau/2]$

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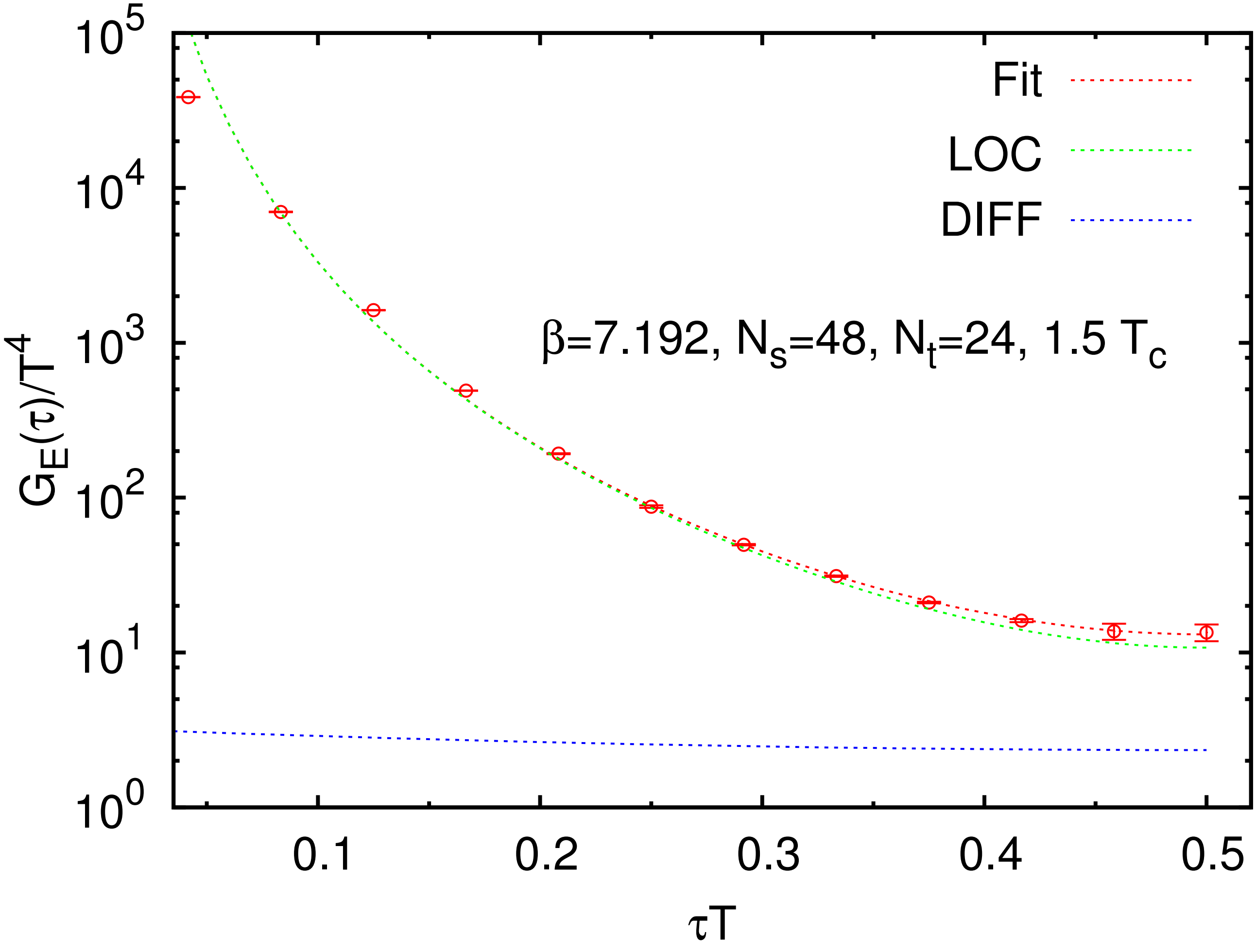
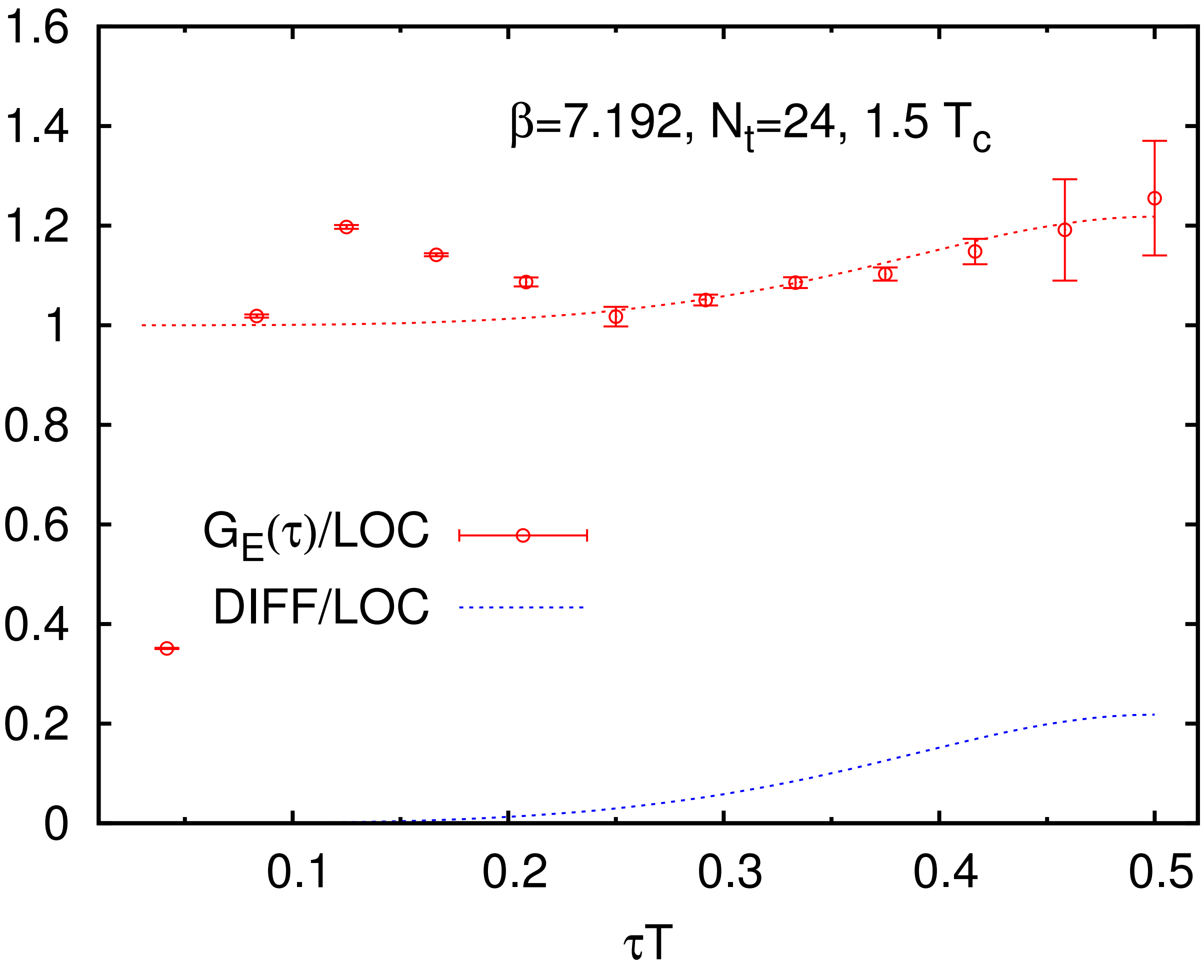
- We use an *ansatz* for ρ, obtain G from it, and then fit in the large τ range $[N_\tau/4,\, N_\tau/2]$

- $\rho(\omega) \;=\; a\omega\, \Theta(\omega - \Lambda) \;+\; b\omega^3$
  First term is the due to the expected DIFFusion constant, and the second is motivated by leading perturbation theory (LOC)

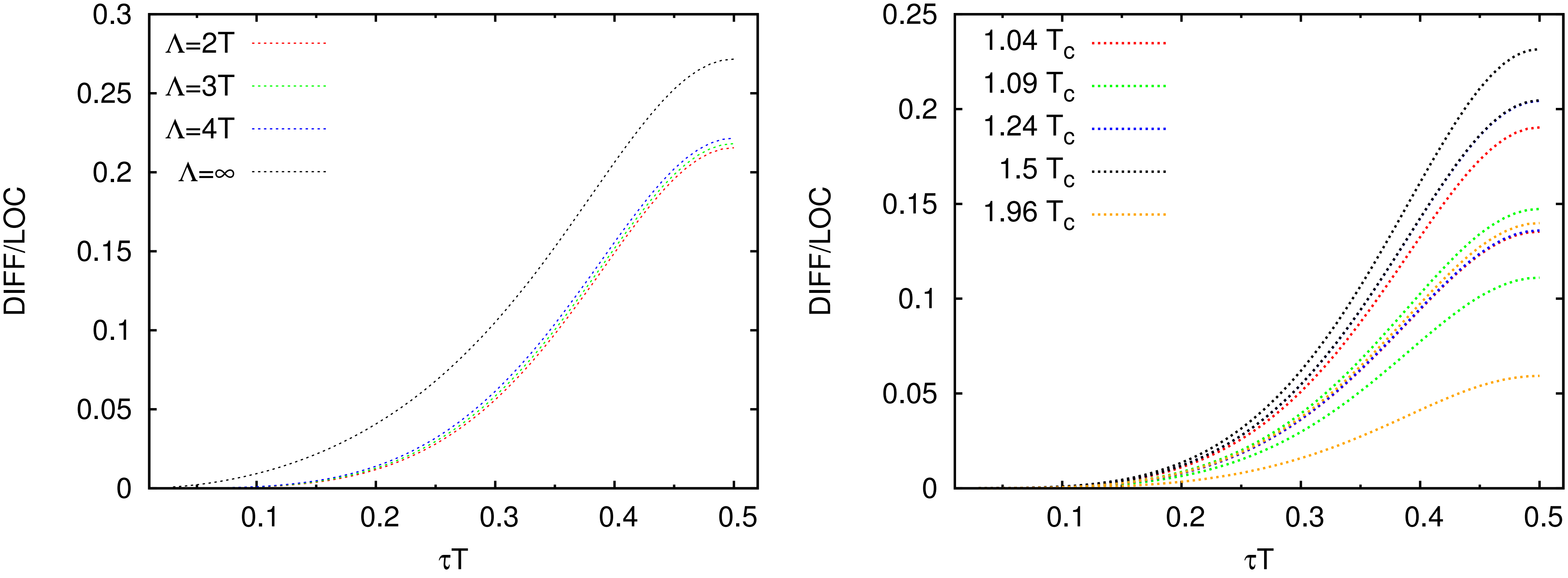
- $\Lambda = 3T$ used; varied from 2 to ∞ for systematic error.



- Contribution of the two terms shown as DIFF and LOC.



♠ Comparing the DIFF fit with the data after eliminating the LOC.



- Variation of a with the cut-off Λ and the temperature.

♠ Our fit parameter $a \rightsquigarrow \kappa$ modulo the renormalization factor for the electric fields.

♠ We use the tadpole factor. It is $\sim$ 1.2 as evaluated from our plaquette values.

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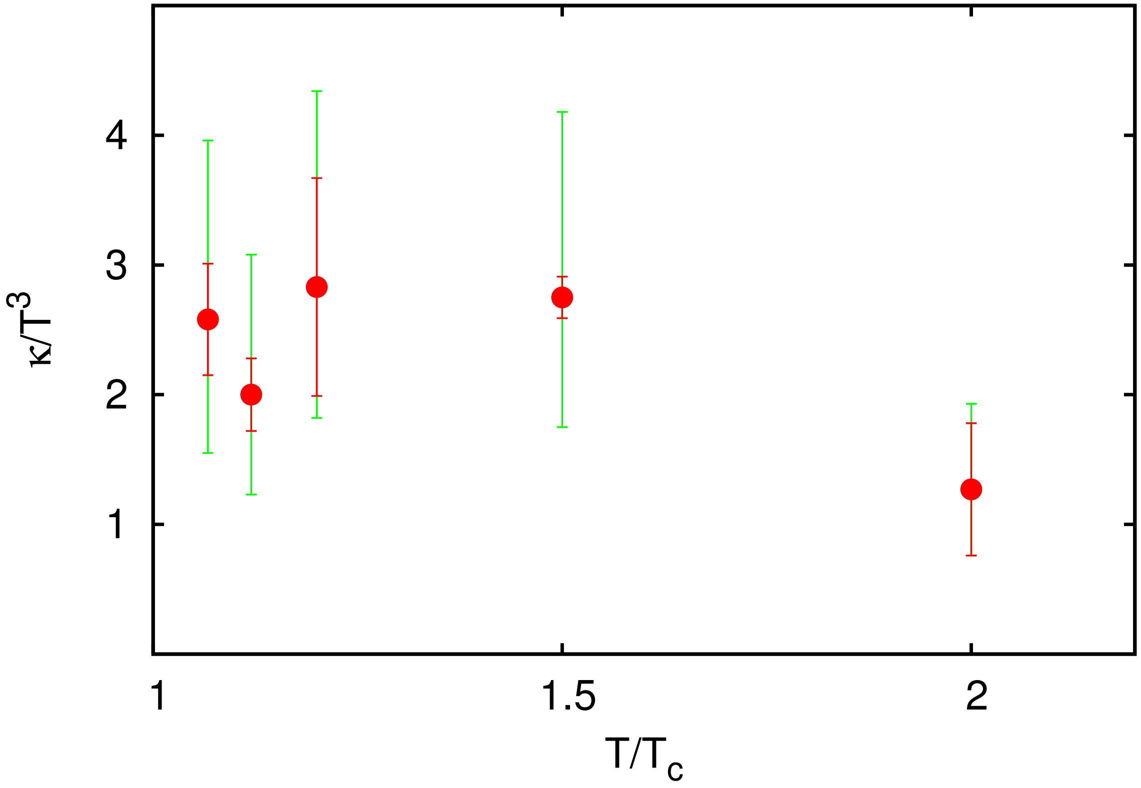
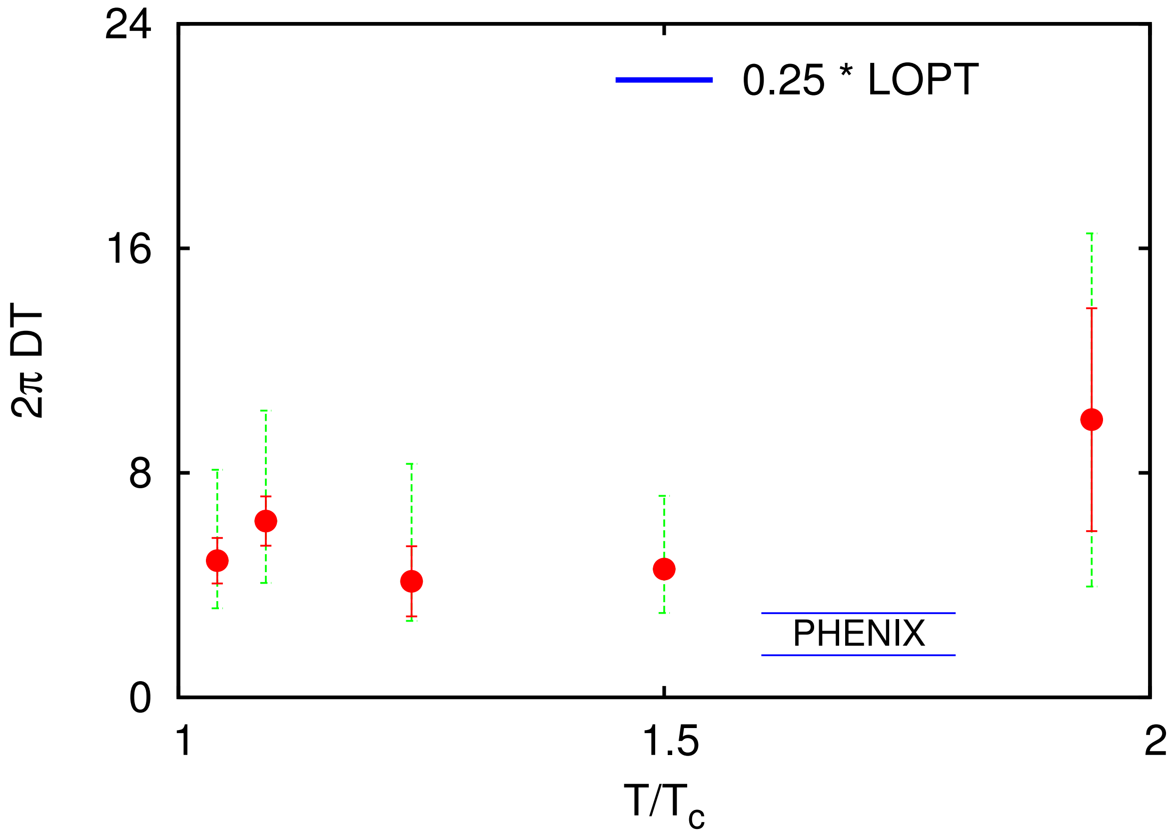
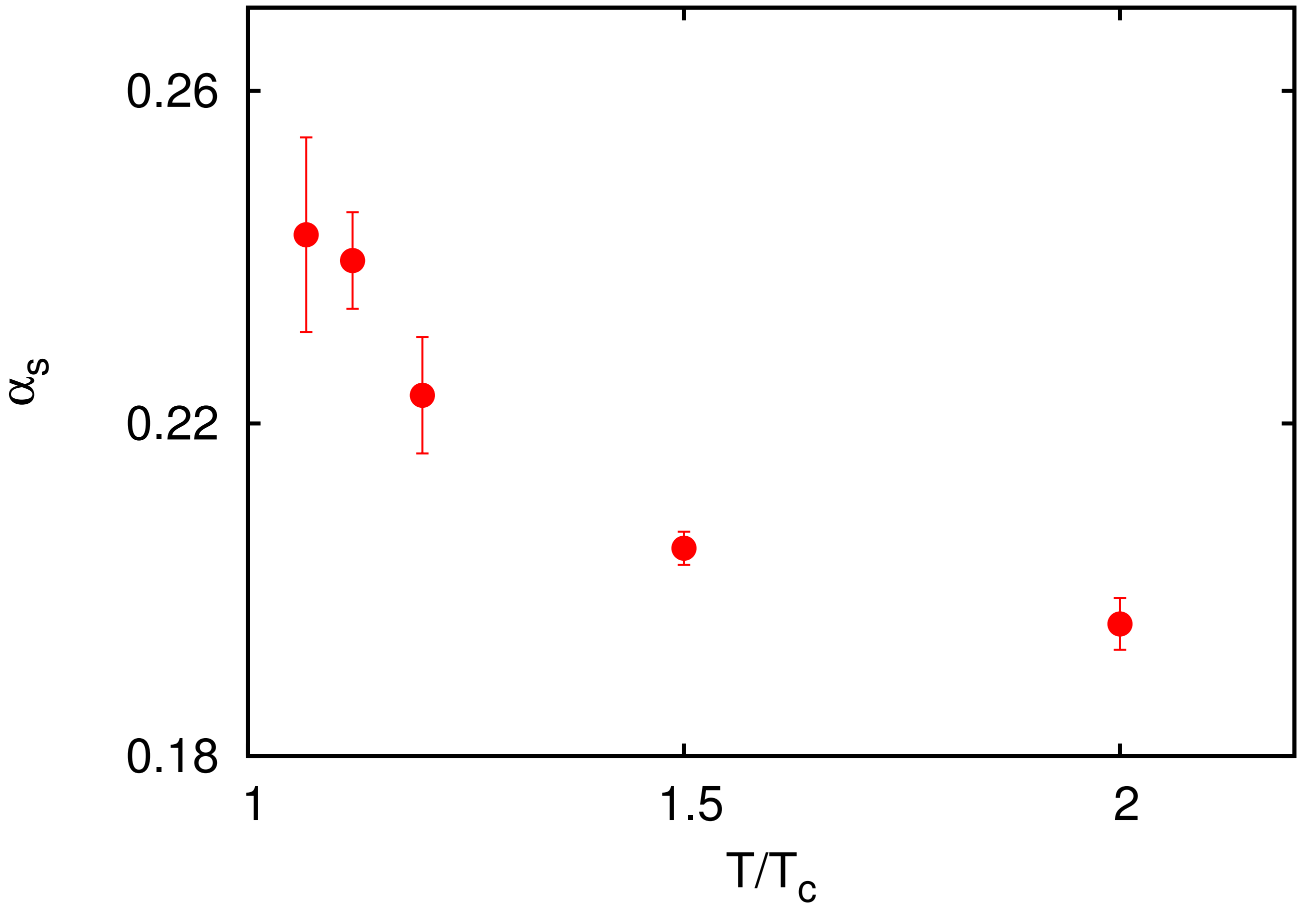


♠ Multiplying by T, obtain D, the quantity used by Moore-Teaney and PHENIX.



♡ In broad agreement with (preliminary) Bielefeld estimates (Ding et al. 1107.0311,1204.4945; Francis et al. 1109.3941): they get a factor ~ 2 smaller value with similar errorbars.

♠ The ω^3 term comes with g^2. Use as a scheme to define α_s non-perturbatively.



♡ In agreement with other similar estimates (Ding et al. PRD 83 (2011) 034504).

J/ψ: Flows or not ?

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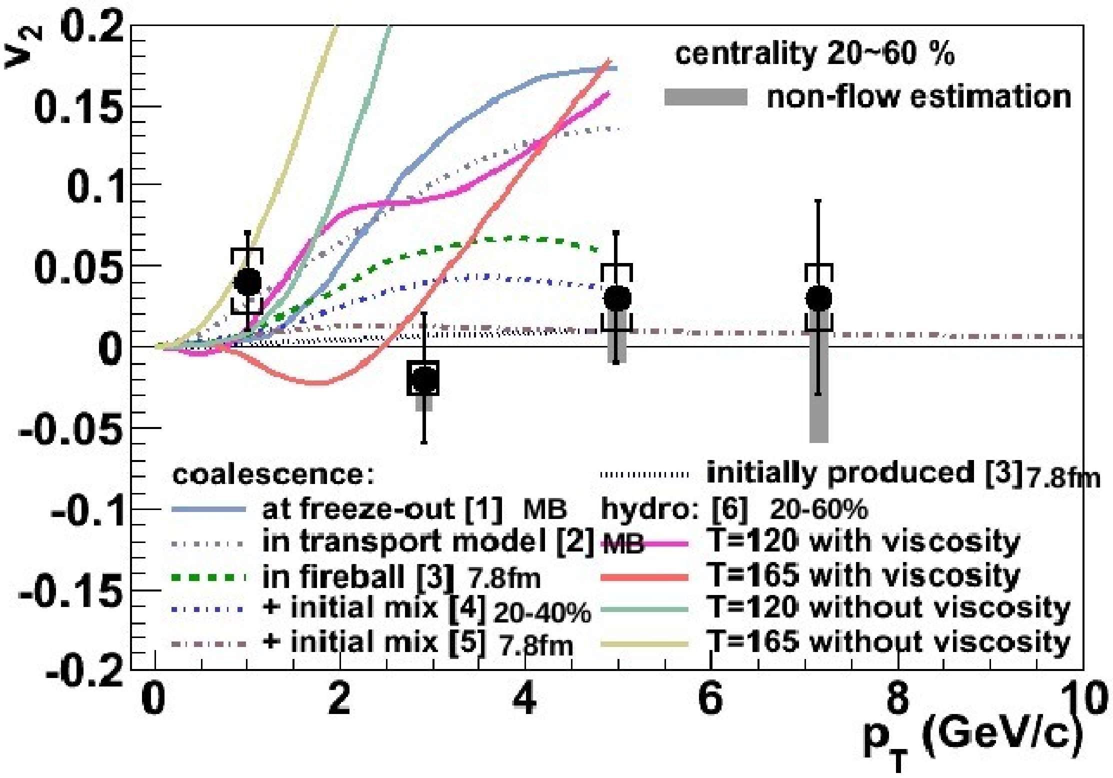
♢ But the thermal charm may be in abundance and may also obey the n_q-scaling.

♠ If thermal charm ‘recombines’ to produce many J/ψ, then one expects J/ψ to flow still.

♡ The STAR collaboration presented results for J/ψ flow in the recent Quark Matter 2011.

J/ψ elliptic flow v_2



:o, C.M. Ko, R. Rapp, PLB 595, 202.
ıgli, R. Rapp, PLB 655, 126.
. P. Zhuang, N. Xu, PRL 97, 232301.
ɔ, R. Rapp, 24th WWND, 2008.
[5] Y. Liu, N. Xu, P. Zhuang, Nucl. Phy. A, 834, 317
[6] U. Heinz, C. Shen, priviate communication.

Consistent with zero
Disfavor coalescence from thermalized charm quarks

Zebo Tang, USTC QM2011

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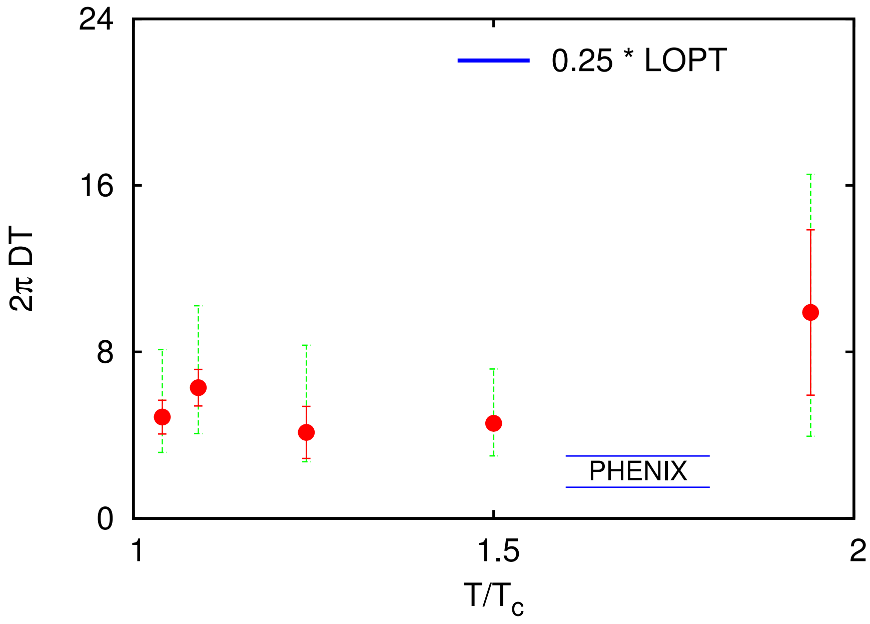

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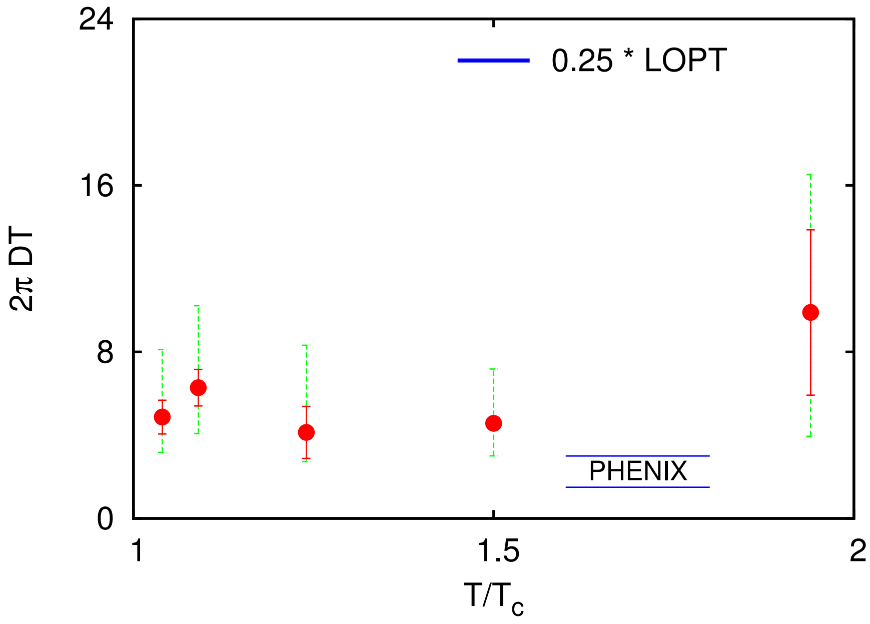
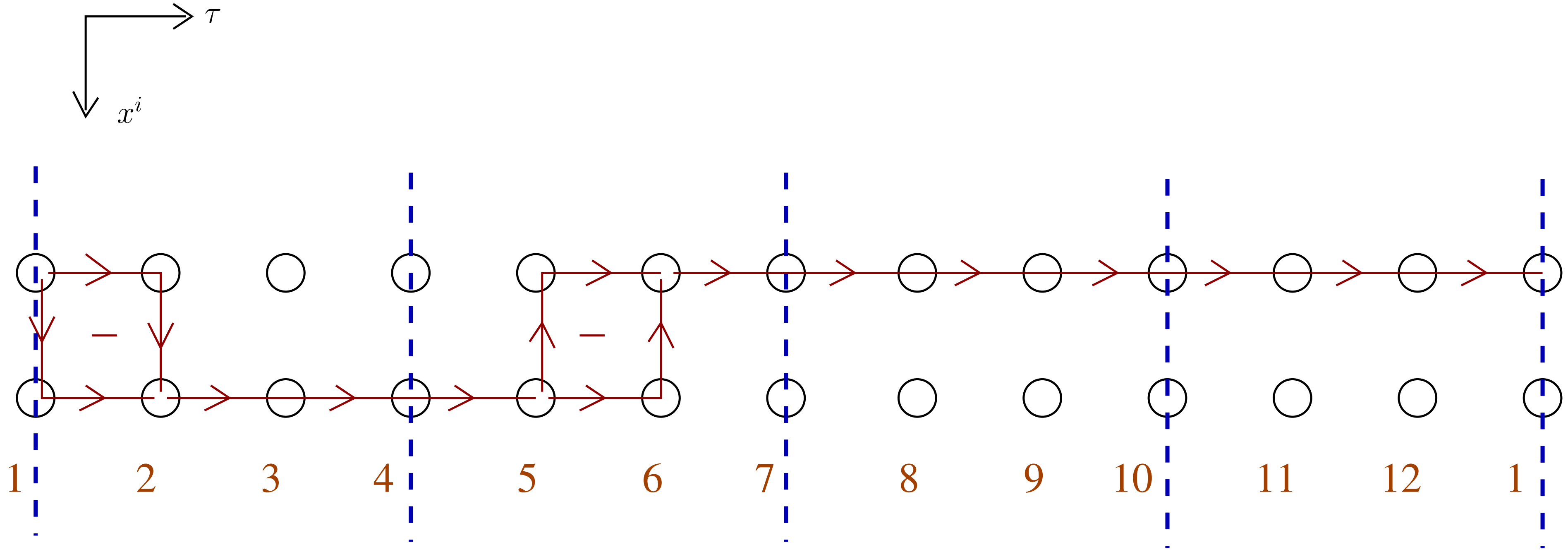


It would be interesting to see if DT vs. T/T_c exhibits similar flavour independence as the pressure.

τ
x^i
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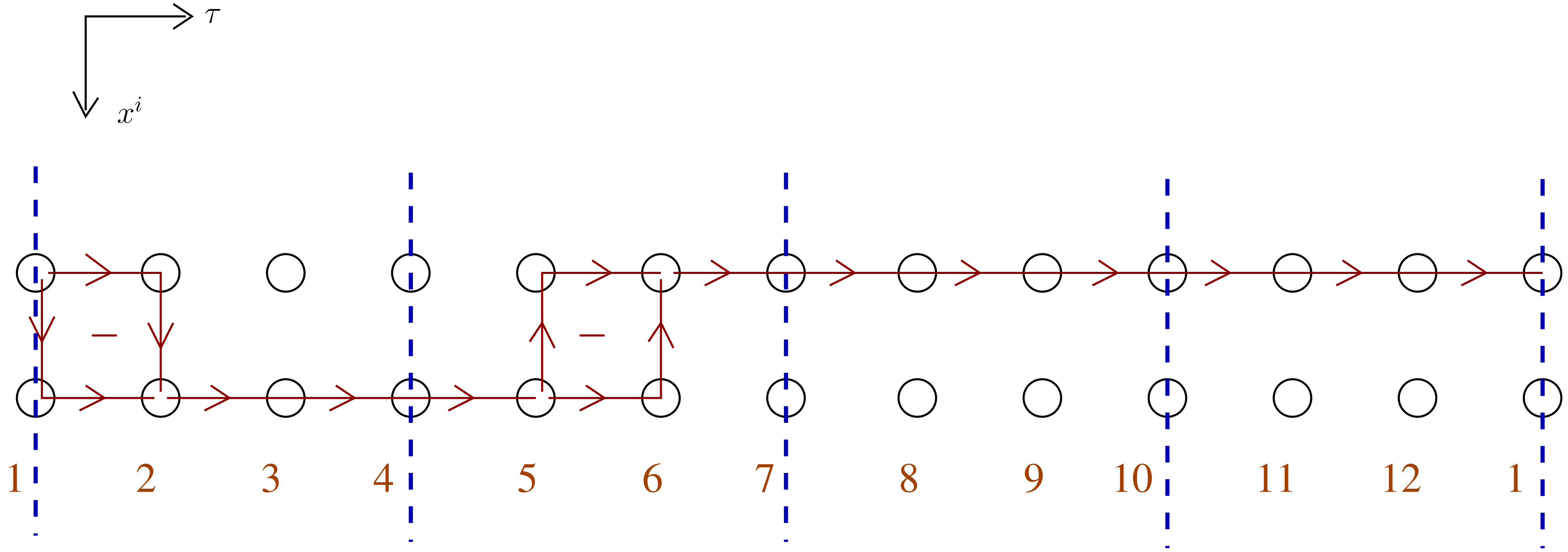
1



β	6.76	6.80	6.90	7.192	7.255
N_τ	20	20	20	24	20
T/T_c	1.04	1.09	1.24	1.5	1.96

Table 1: List of lattices on which diffusion coefficients were extracted, and the temperatures.