Hadronic Screening Lengths :
A window to Quark-Gluon Plasma

Rajiv V. Gavai *

T. I. F. R., Mumbai

* *In collaboration with Sourendu Gupta, TIFR, Mumbai and Robert Lacaze, SPhT, Saclay.*

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Introduction & Motivation

Hadronic Screening Lengths

Our Results

Summary

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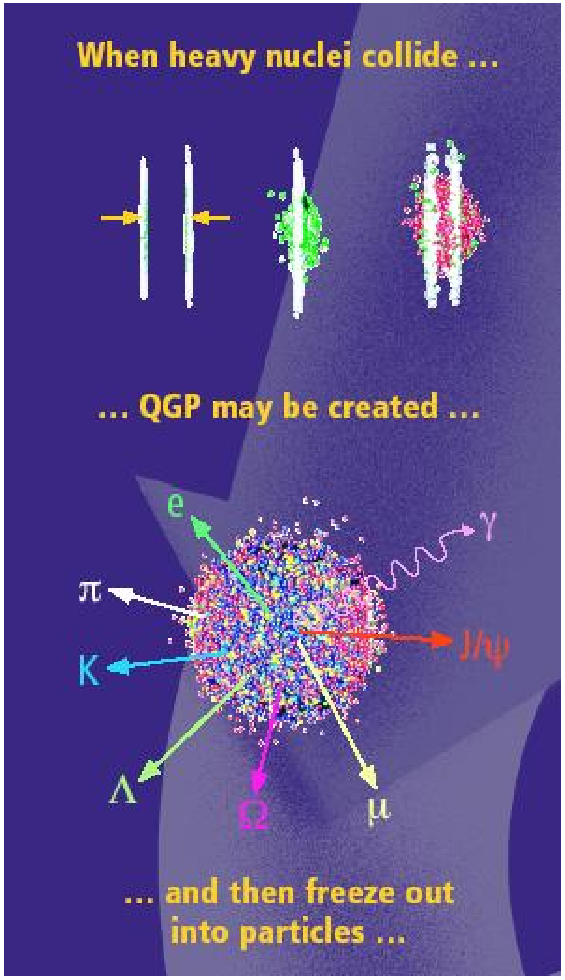
Introduction

♠ Quest for Quark-Gluon Plasma : Heavy Ion Collisions at SPS, RHIC and LHC.



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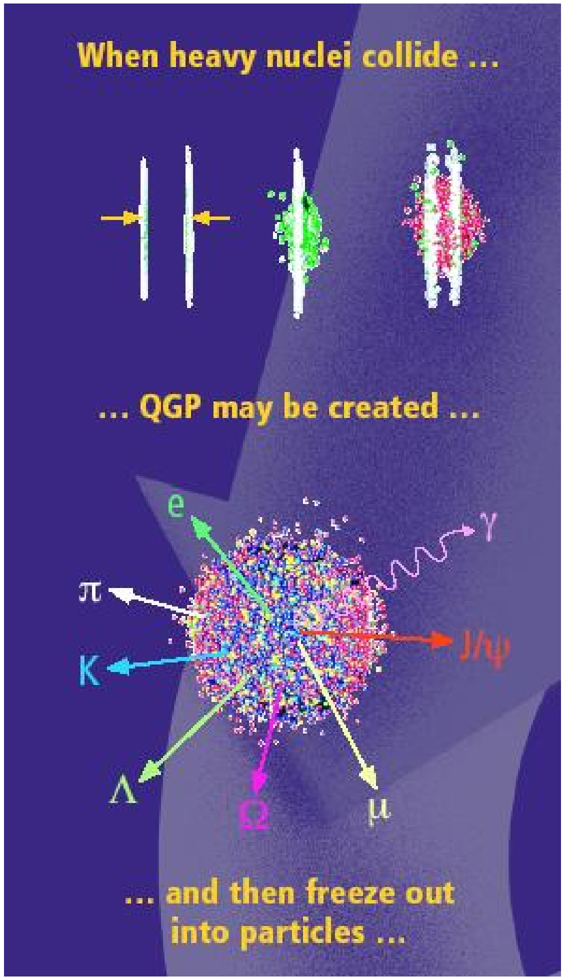
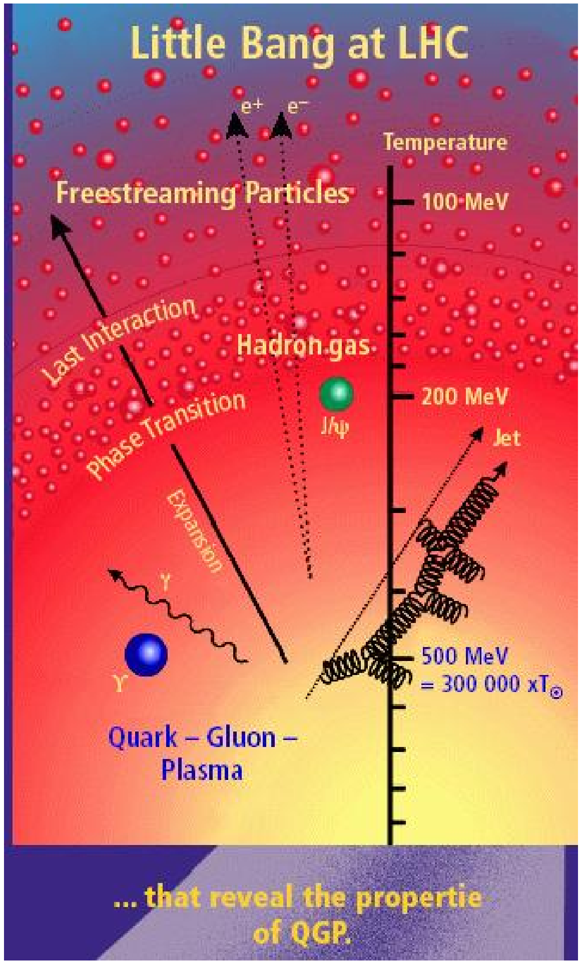
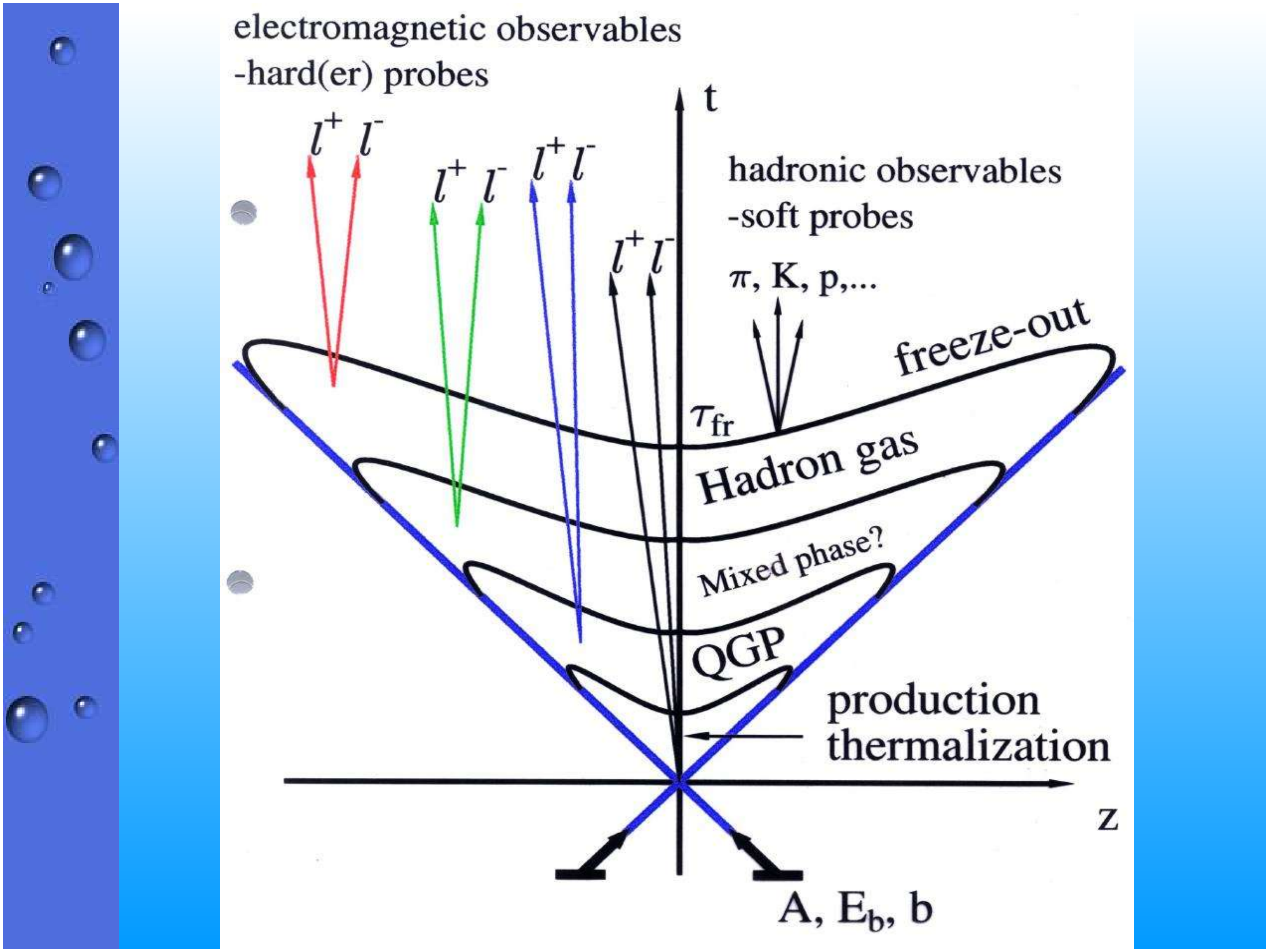
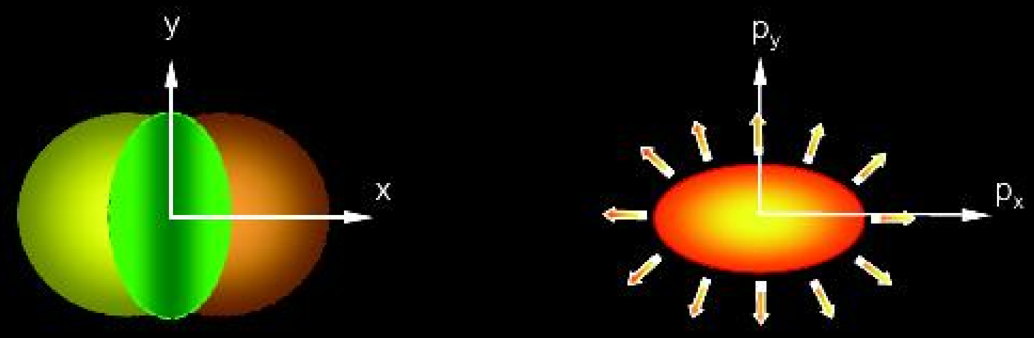





electromagnetic observables
-hard(er) probes
t
hadronic observables
-soft probes
π, K, p,...
l^+ l^-
l^+ l^-
l^+ l^-
l^+ l^-
freeze-out
τ_{fr}
Hadron gas
Mixed phase?
QGP
production
thermalization
z
A, E_b, b

Anisotropy Parameter v_2

coordinate-space-anisotropy ⇨ momentum-space-anisotropy

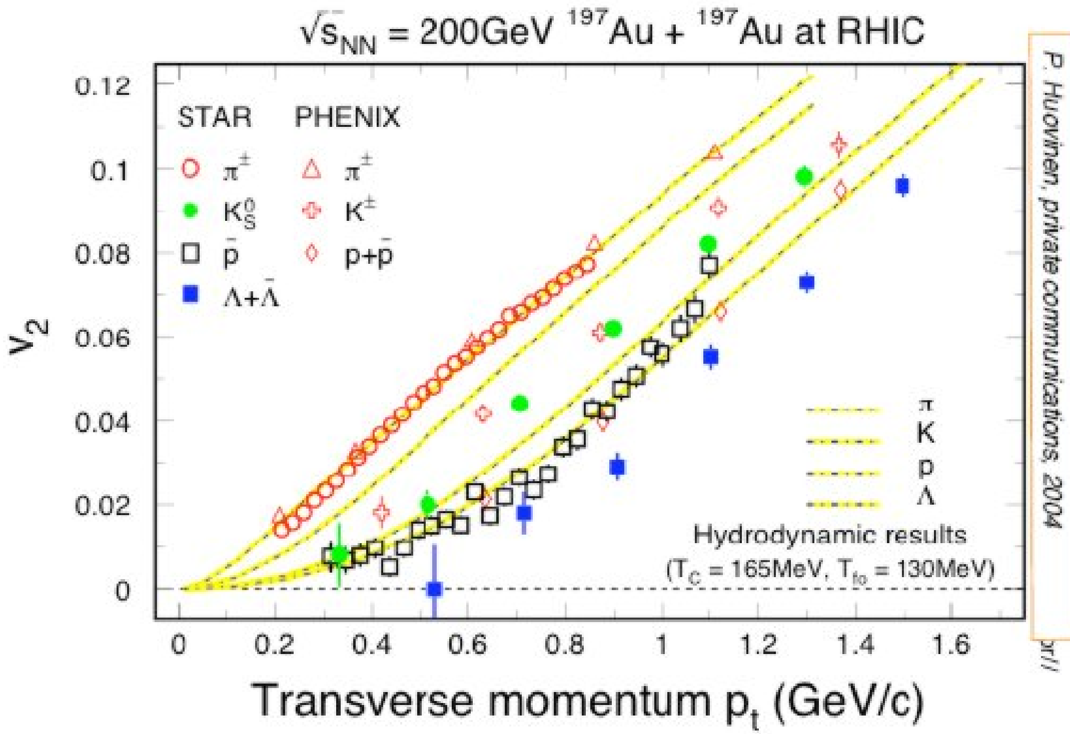


$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\varphi \rangle, \quad \varphi = \tan^{-1}\left(\frac{p_y}{p_x}\right)$$

Initial/final conditions, EoS, degrees of freedom

"ICHEP 2006" Moscow, Russia, July 26 - August 2, 2006

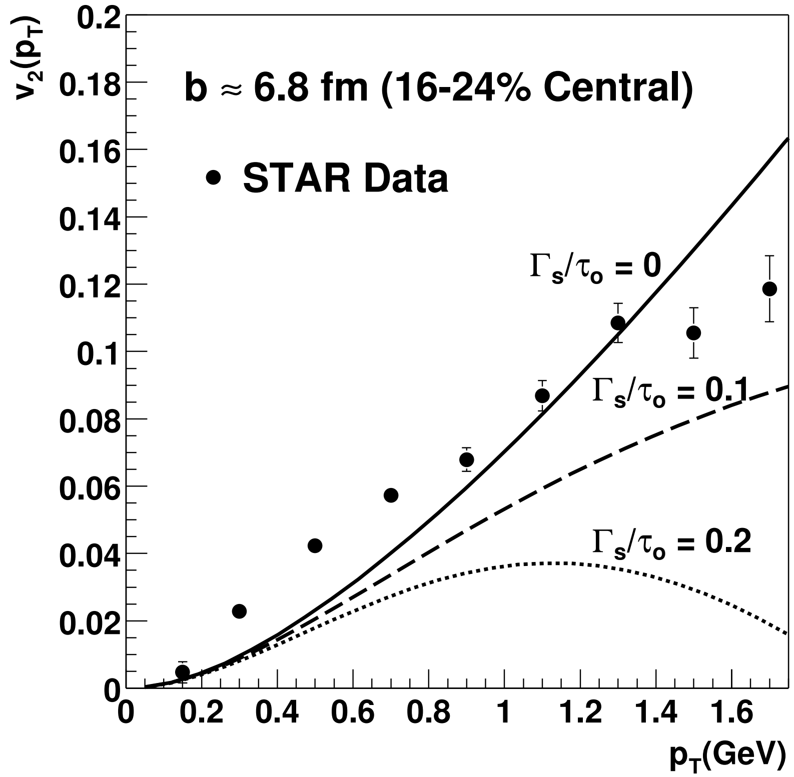
v_2 at Low p_T Region



- Minimum bias data! At low p_T, model result fits mass hierarchy well!
- Details do not work, need more flow in the model!

Nu Xu "ICHEP 2006" Moscow, Russia, July 26 - August 2, 2006 17/22

QGP - (Almost) Perfect Liquid



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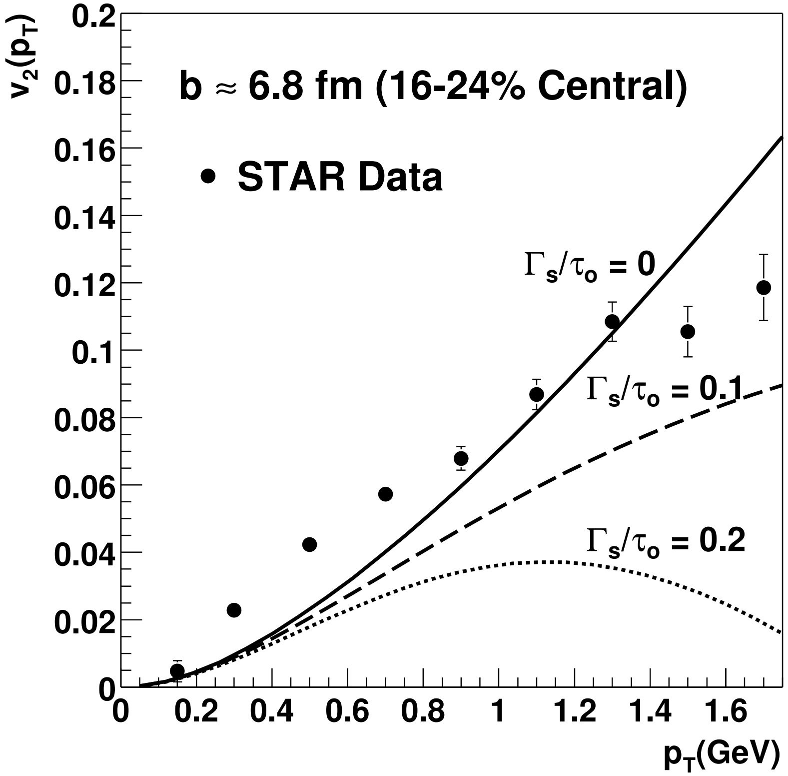

Elliptic flow consistent with Ideal Hydrodynamics $\rightsquigarrow$ bound on Shear viscosity. (D. Teaney, nucl-th/03010099; PRC 2003)

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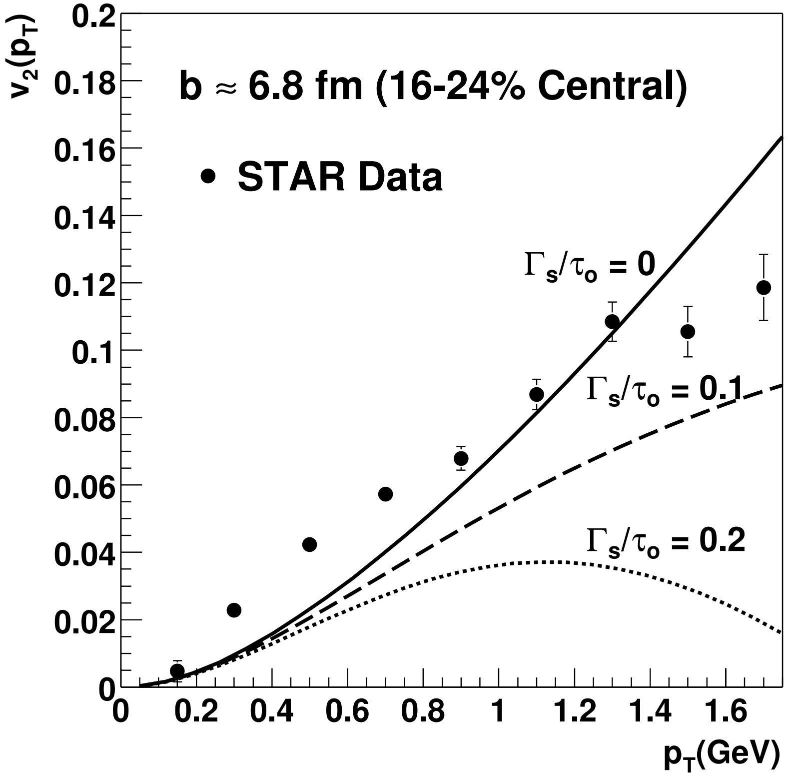

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$$\Gamma_s = \frac{\frac{4}{3}\eta}{sT} , \tag{1}$$

where η is Shear Viscosity and s is entropy density; $\tau = \sqrt{t^2 - z^2}$ is the time scale of expansion.

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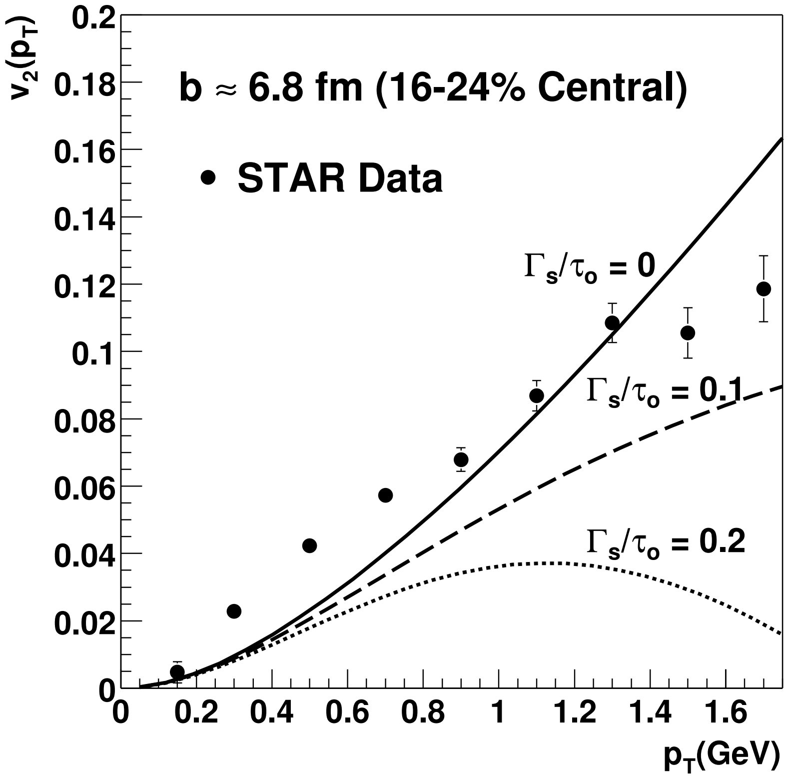

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Perturbation theory $\Rightarrow$ Large η/s
Small $\eta/s \longrightarrow$ Strongly Coupled Liquid.

Lattice QCD : What it can do

- Transition temperature, Critical energy density, Order of Phase Transition, Properties of QGP (EoS, Excitation types, Screening..)

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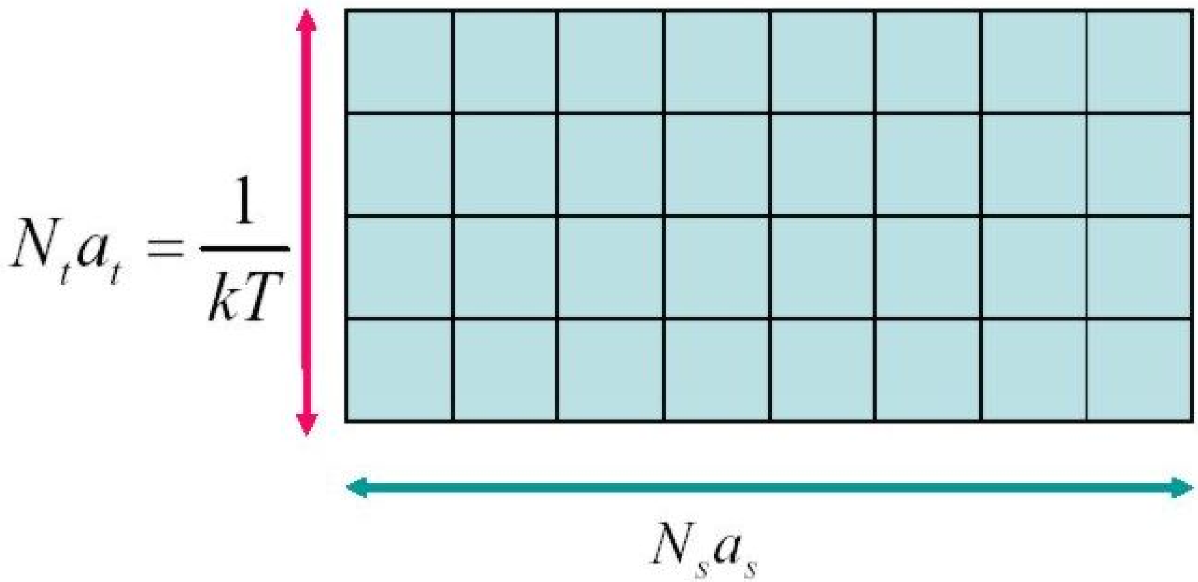
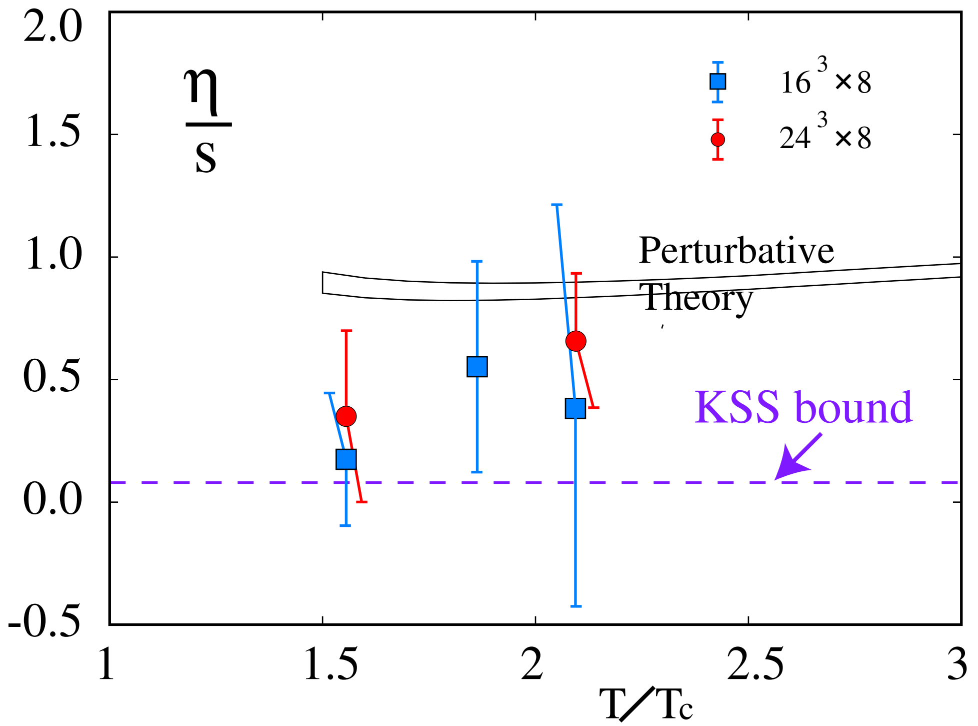
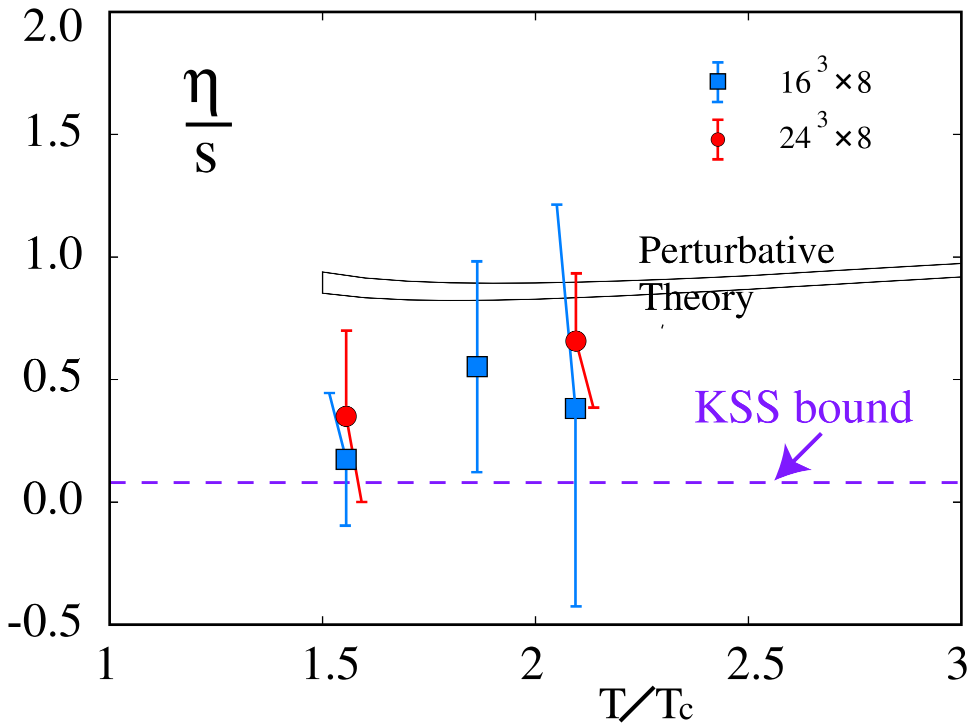


Need $N_s \gg N_t$ for thermodynamic limit and large N_t for continuum limit.



Nakamura and Sakai, PRL 94 (2005).



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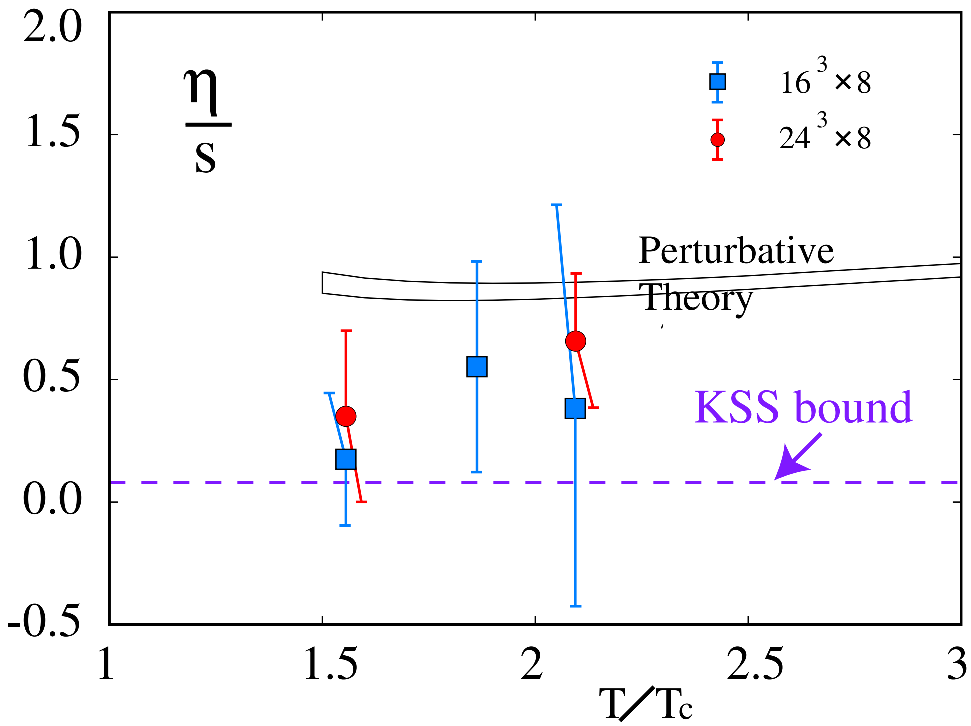
- Kubo's Linear Response Theory : Transport Coefficients in terms of equilibrium correlation functions.



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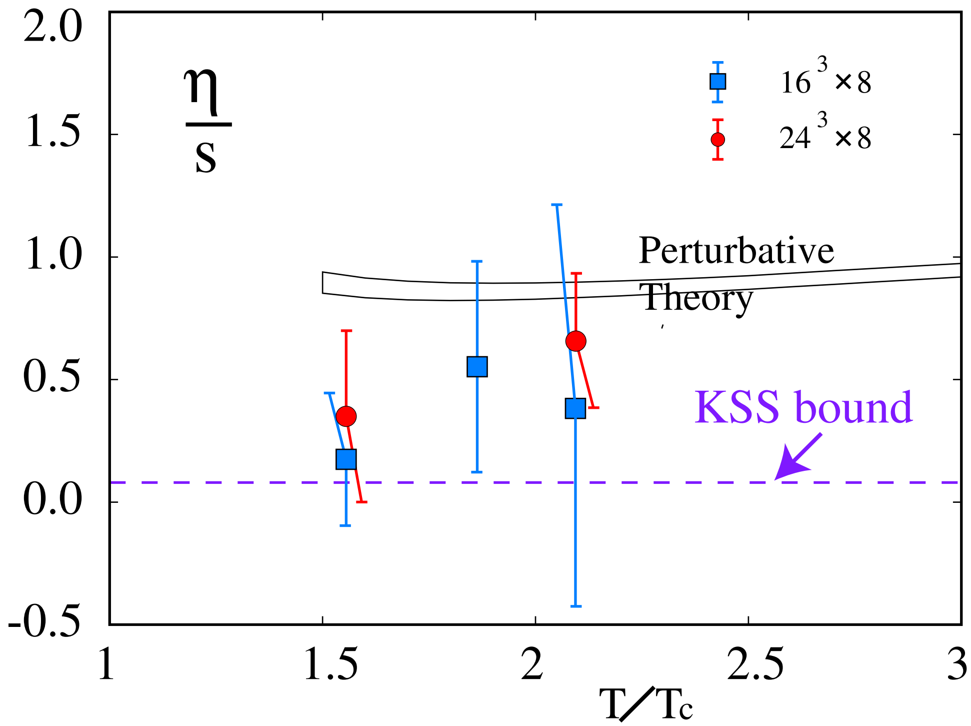

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- Obtain Energy-Momentum Correlation functions on Lattice (at discrete Matsubara frequencies).
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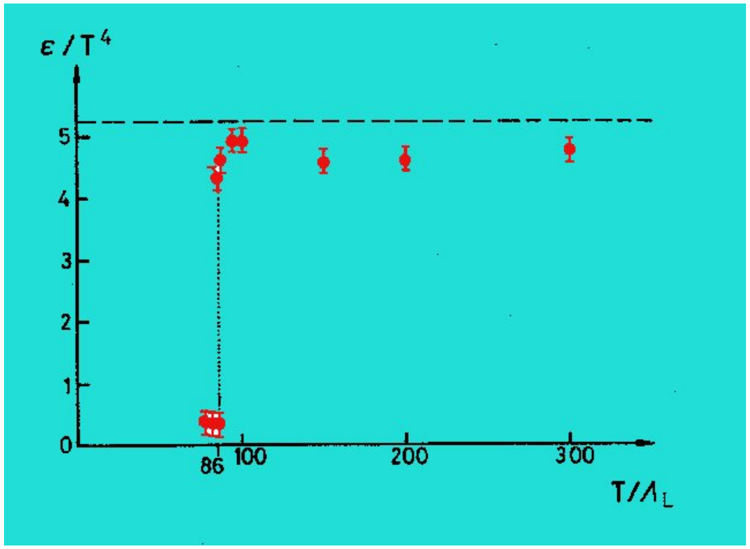
- Larger lattices and inclusion of dynamical quarks in future.

EoS of QGP

- First results from Bielefeld :



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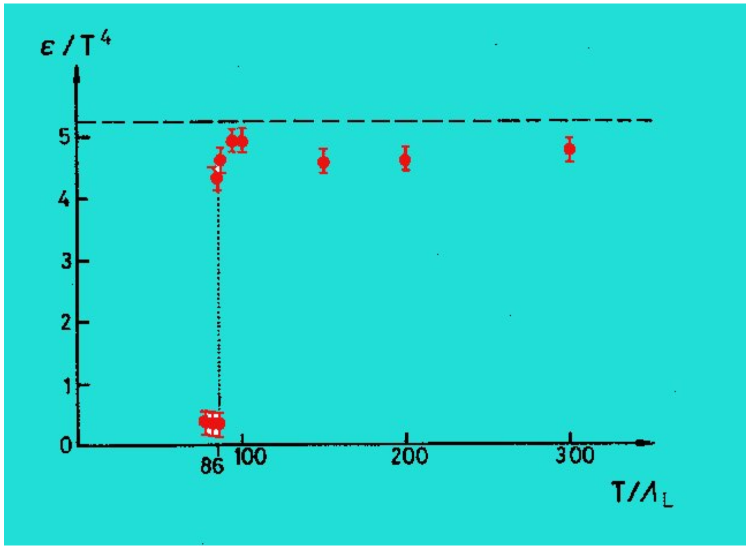

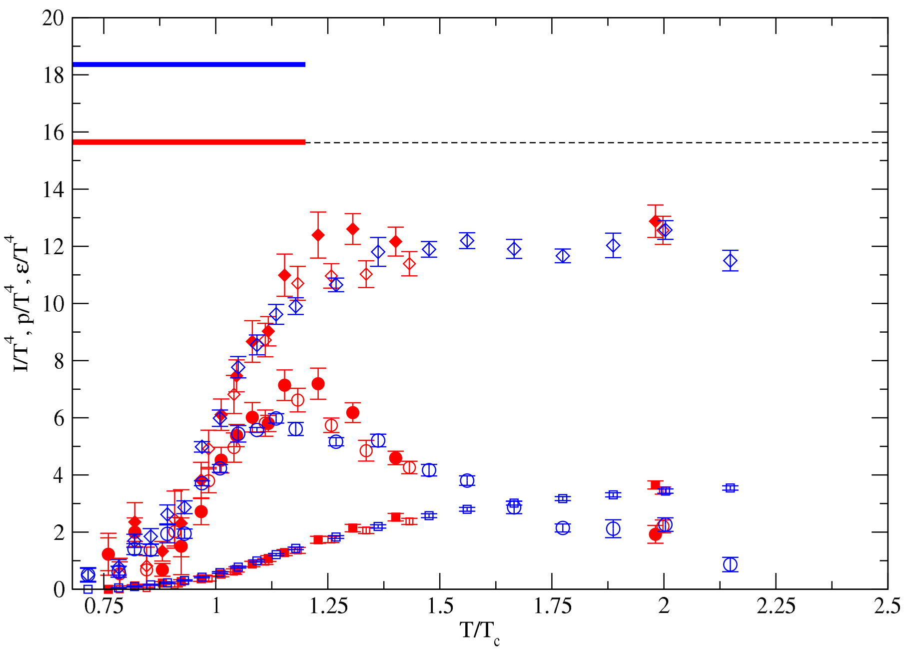

Celik, Engels & Satz, PLB129, 323 1983

Bernard et al., MILC hep-lat/0509053

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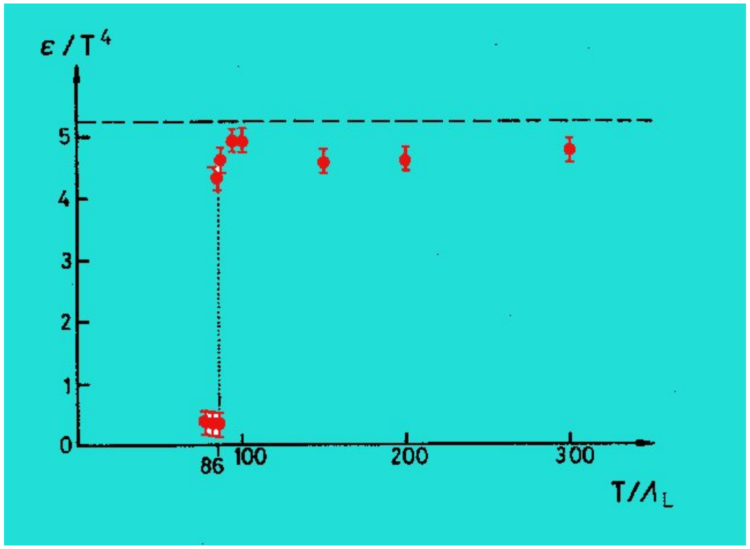

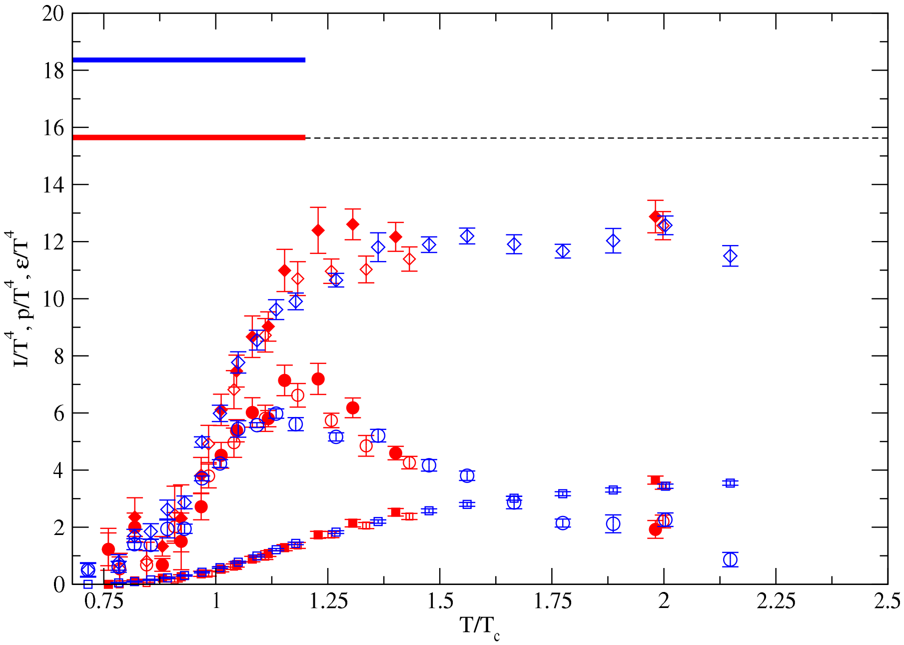

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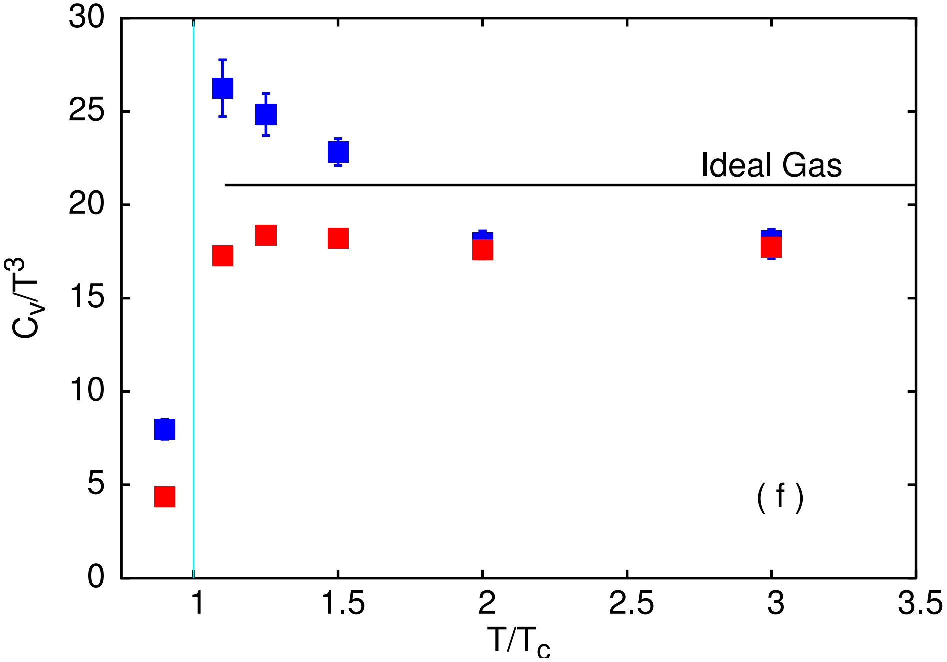
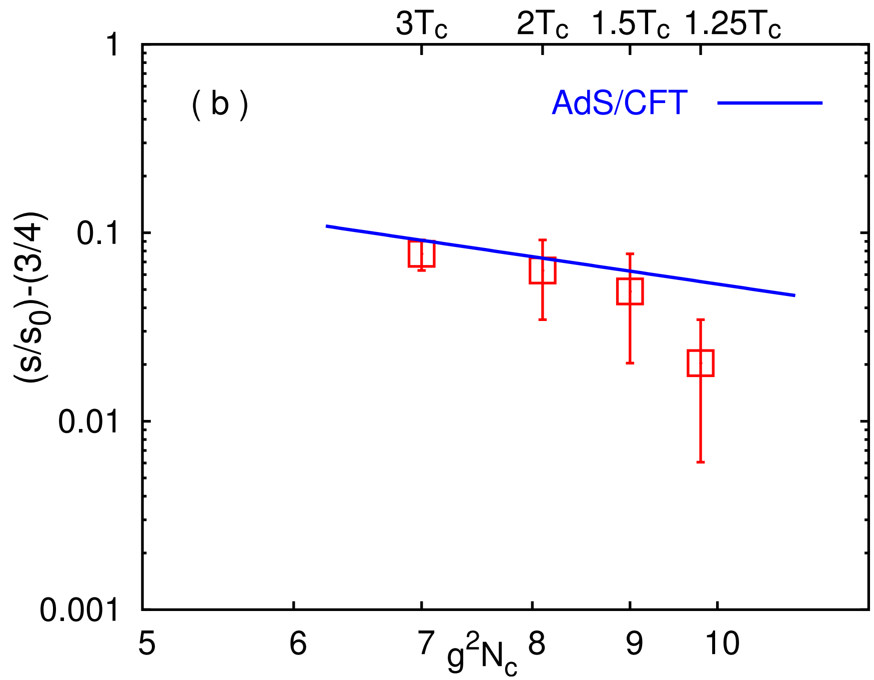
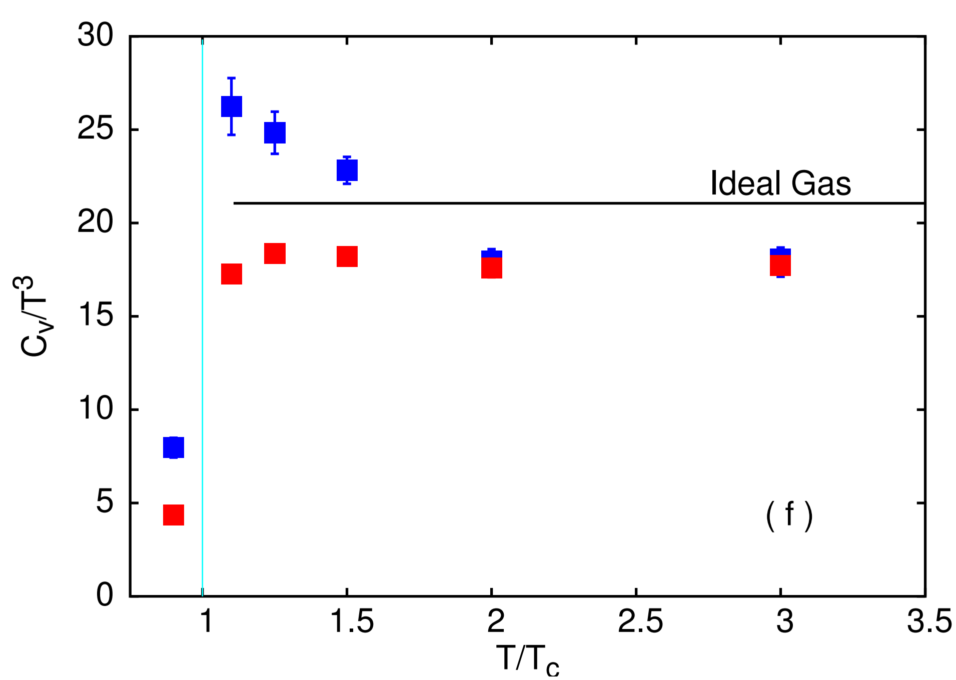
- Recent results for EoS : N_t=6, Smaller quark masses. Small differences for N_t = 4 & 6; $\epsilon(T_c) \sim 6T_c^4$ still. Too small volumes $\longrightarrow$ Thermodynamic Limit ?

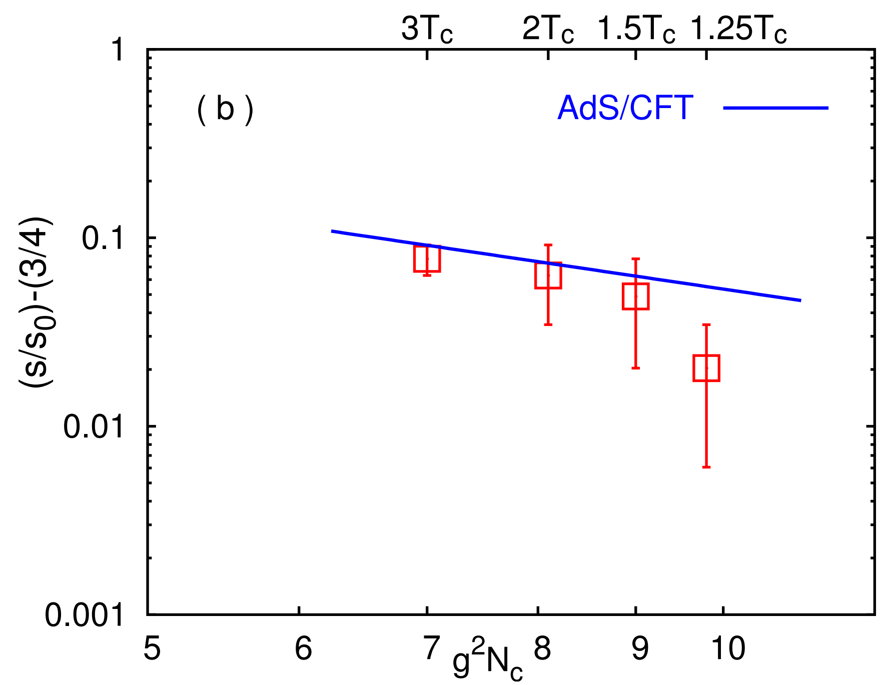




(RVG, S. Gupta and S. Mukherjee, hep-lat/0506015)

♠ $C_v \sim 4\epsilon$ for $2T_c$ but No Ideal Gas limit.





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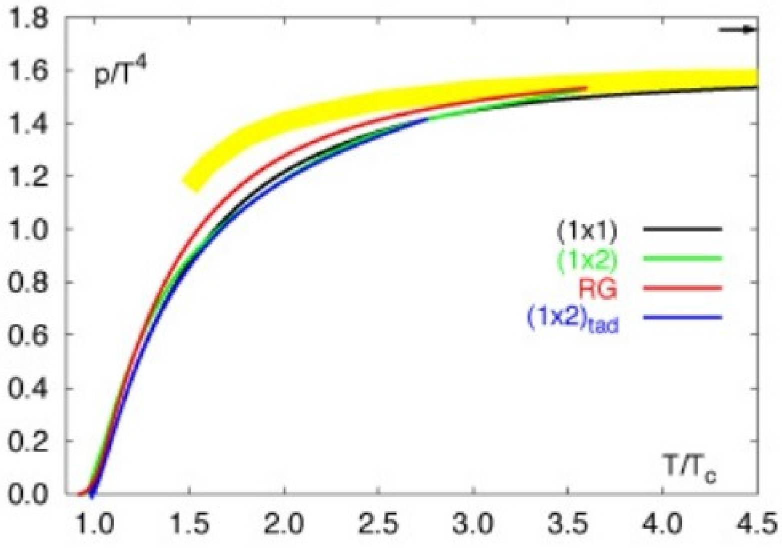
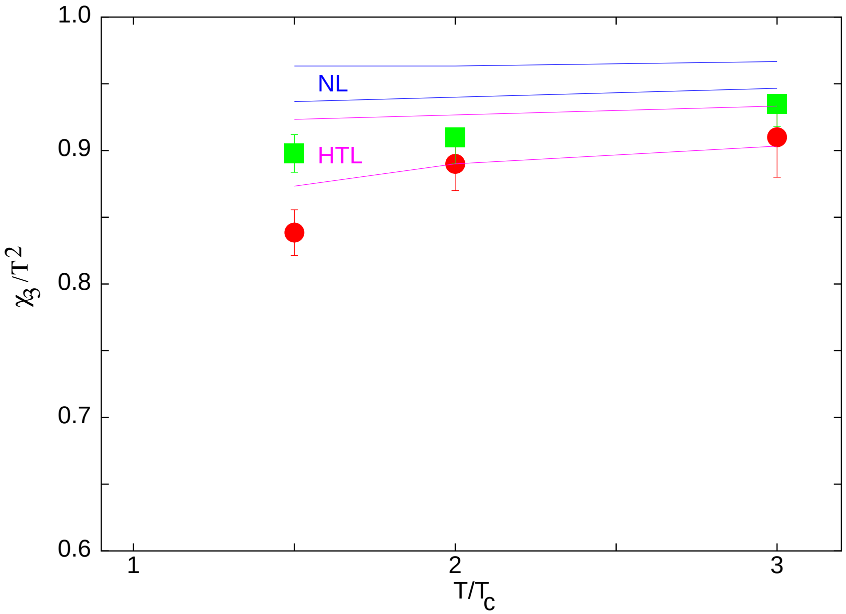
♠ Entropy agrees with strong coupling SYM prediction (Gubser, Klebanov & Tseytlin, NPB '98, 202) for $T = 1.5 - 3T_c$ but fails at lower T, as do various weak coupling schemes : $\frac{s}{s_0} = f(g^2 N_c)$, where $f(x) = \frac{3}{4} + \frac{45}{32}\zeta(3)x^{-3/2} + \cdots$ and $s_0 = \frac{2}{3}\pi^2 N_c^2 T^3$.

Weak Coupling



Blaizot, Iancu & Rebhan PRD 01, PLB 01



RVG & Gupta PRD 01, 03

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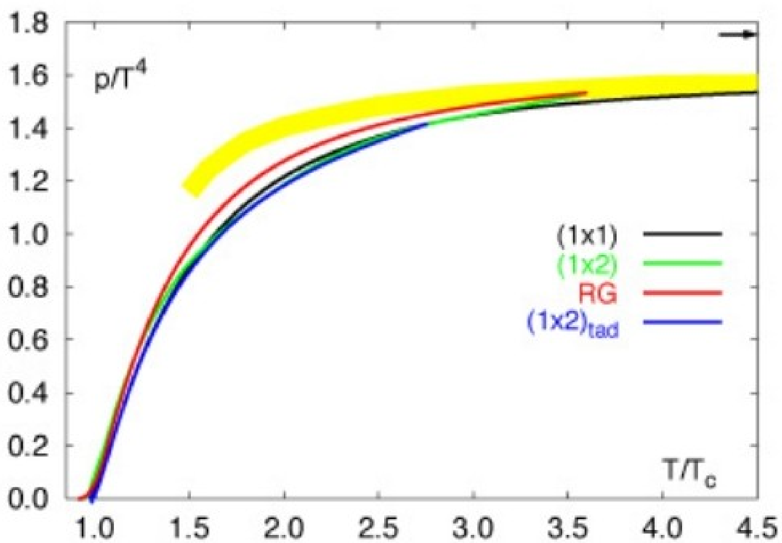

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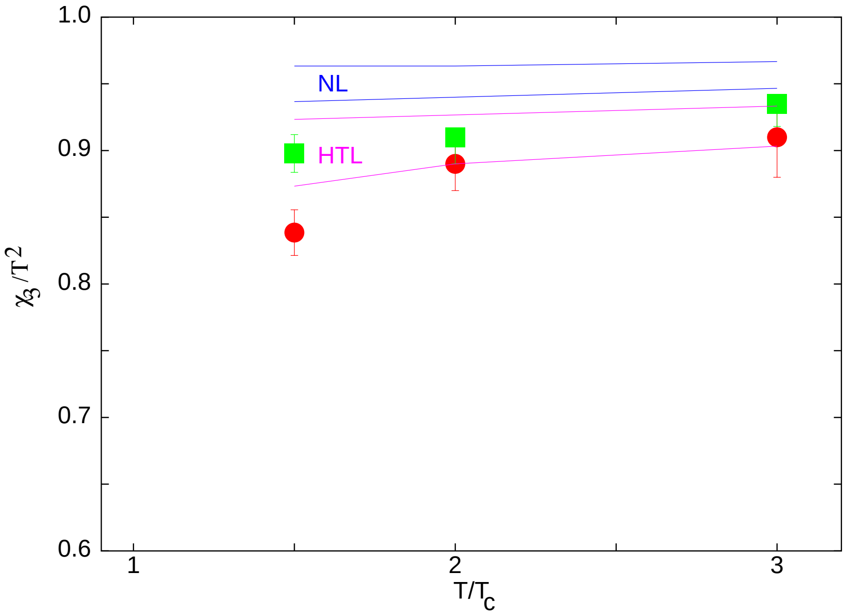

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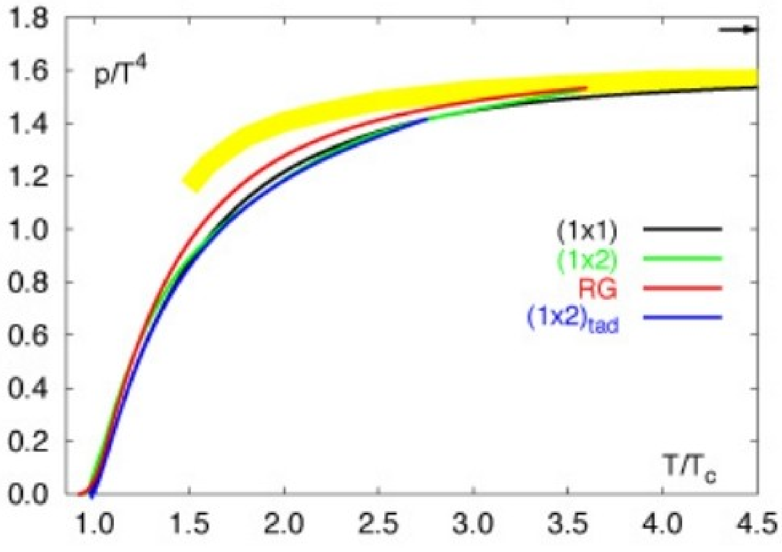

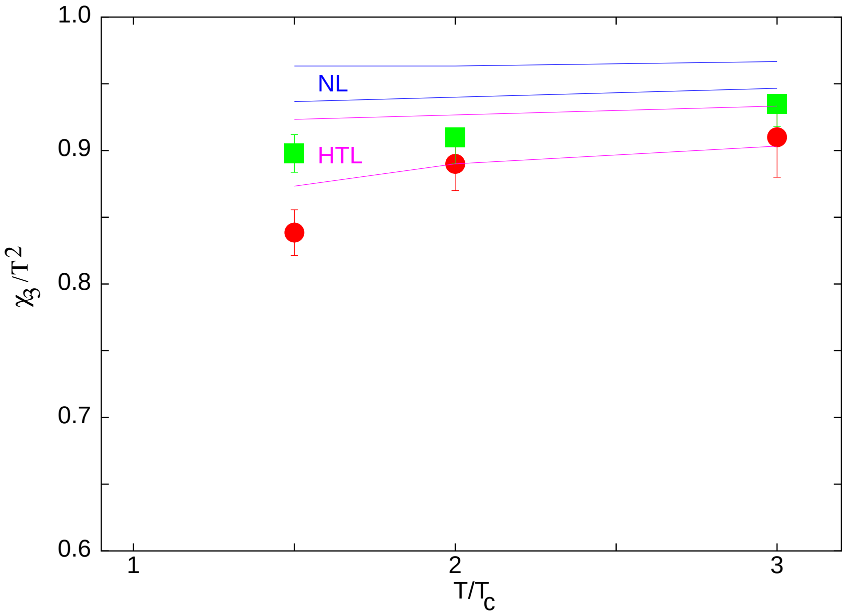

Blaizot, Iancu & Rebhan PRD 01, PLB 01 RVG & Gupta PRD 01, 03

♣ Re-summed weak coupling explains lattice results.

♣ So does dimensional reduction (Kajantie et al, Vourinen)

♣ Quasiparticle, PNJL models (Kampfer et al., Wiese et al.).

Baryon-Strangeness Correlation

♣Correlation between quantum numbers K and L can be studied through the ratio $C_{(KL)/L} = \frac{\langle KL\rangle - \langle K\rangle\langle L\rangle}{\langle L^2\rangle - \langle L\rangle^2}$.

♣ These are robust : theoretically & experimentally.

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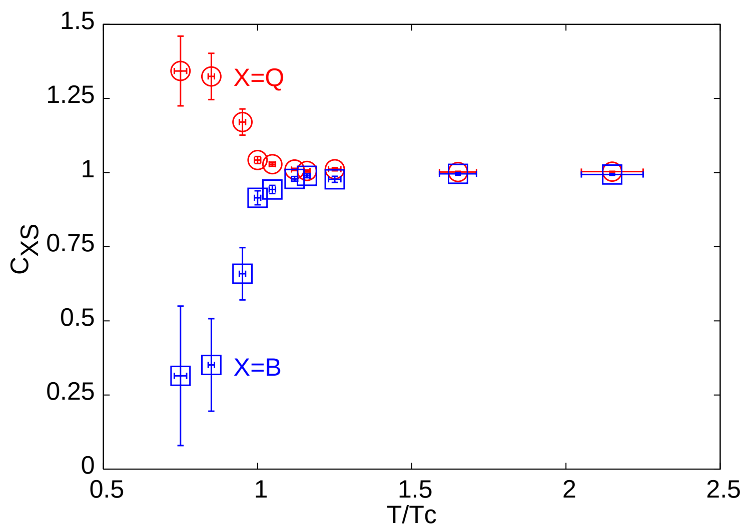

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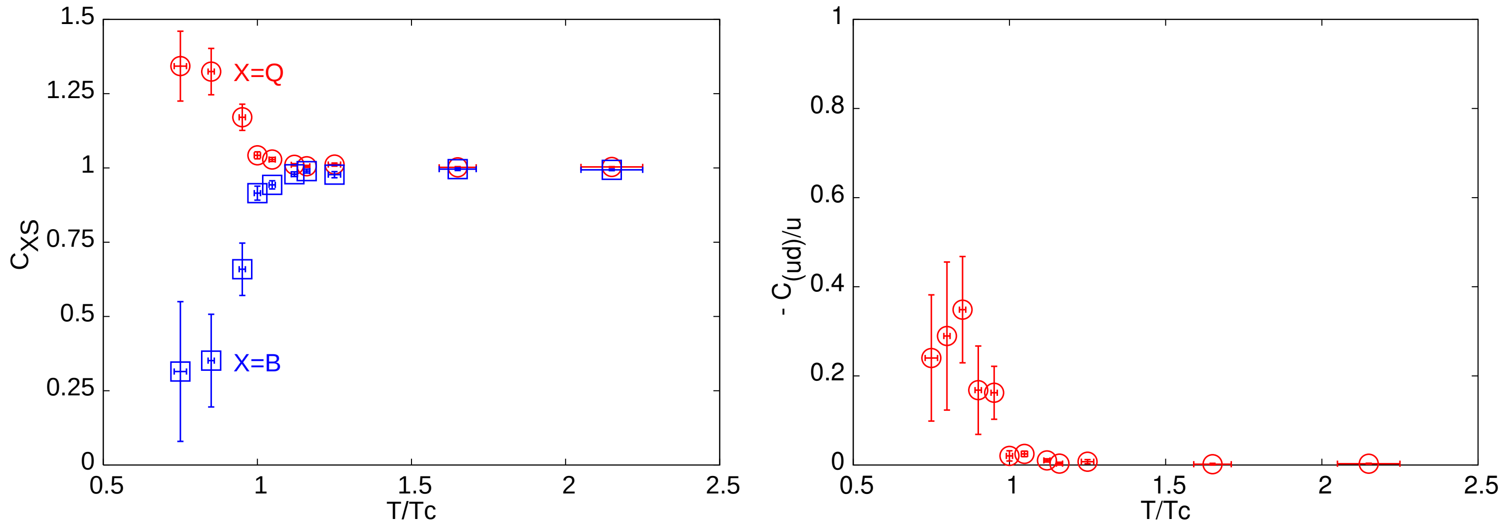

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- Here $\bar{A}(z) = \sum_{x,y,t} A(x,y,z,t)/N_s^2 N_t$ and is typically taken as a local meson or baryon operator. $\mu(T)^{-1}$ then is meson(baryon) screening length.

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- Their conclusion : Existence of hadronic modes in QGP, *unlike* expectations from naive pictures of deconfinement.

- MT_c-collaboration (Born et al. PRL '89) pointed out that lowest Matsubara frequency for small N_t is much larger than in continuum $\Longrightarrow$ can explain ρ (N)-screening mass as that for free $q\bar{q}$ (qqq)-pair. But μ_π was still very different.

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- Type of quarks ? Fermions on lattice have a well-known "No-Go" theorem due to Nielsen-Ninomiya : Popular choices
  - Wilson Fermions – Break *all* chiral symmetries.
  - Kogut-Susskind Fermions – Break some chiral symmetries *but* break also flavour symmetry.
  - Overlap Fermions – *both* correct chiral and flavour symmetry on lattice.

Overlap-Dirac Operator

♠ Neuberger (PLB 1998) proposed the overlap-Dirac operator :

$$aD = 1 + A(A^\dagger A)^{-1/2} \qquad \text{with} \qquad A = aD_w, \tag{2}$$

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$$aD_w = \frac{1}{2}\{\gamma_\mu(\partial^*_\mu + \partial_\mu) - a\partial^*_\mu\partial_\mu\} + M, \tag{3}$$

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♠ quark with a mass : $D(ma) = ma + (1 - ma/2)D$; Use $ma =$ 0.001 – 0.1

Computational Difficulties

- Quark Propagator, $Y = D^{-1}X$, needs inversion of D. Usually done iteratively (Conjugate Gradient).
- At each iteration for overlap, need $M^{-1/2}X$: Iterations within each iteration.

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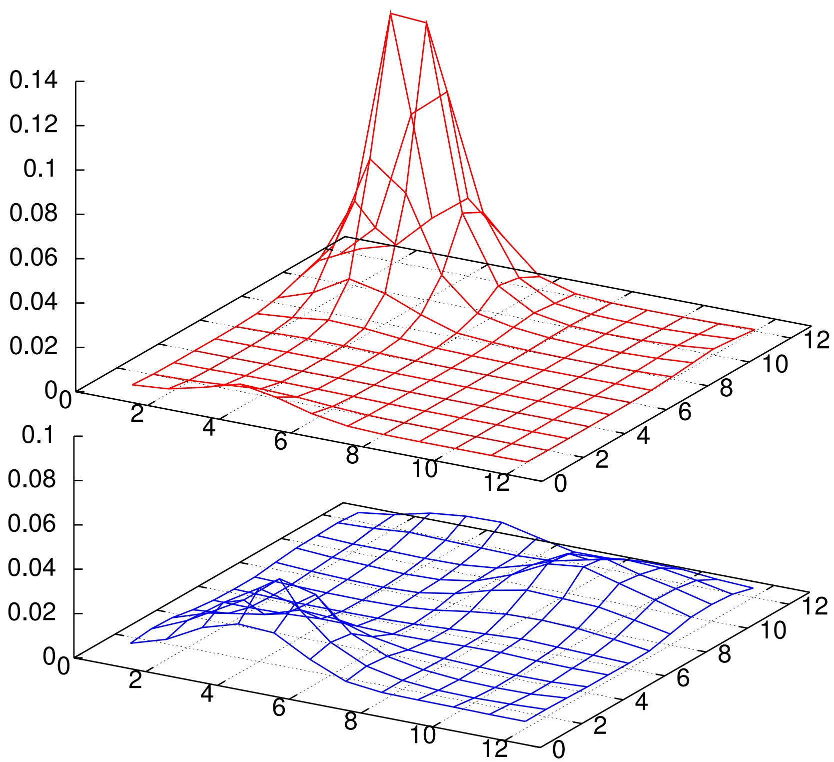
- Several methods for computing $M^{-1/2}X$, including one by us (PRD 2002, CPC 2003).

- We use two algorithms : Conjugate Gradient based CGA, and Zolotarev Approximation.

Our Results

Gavai, Gupta, Lacaze PRD 2002



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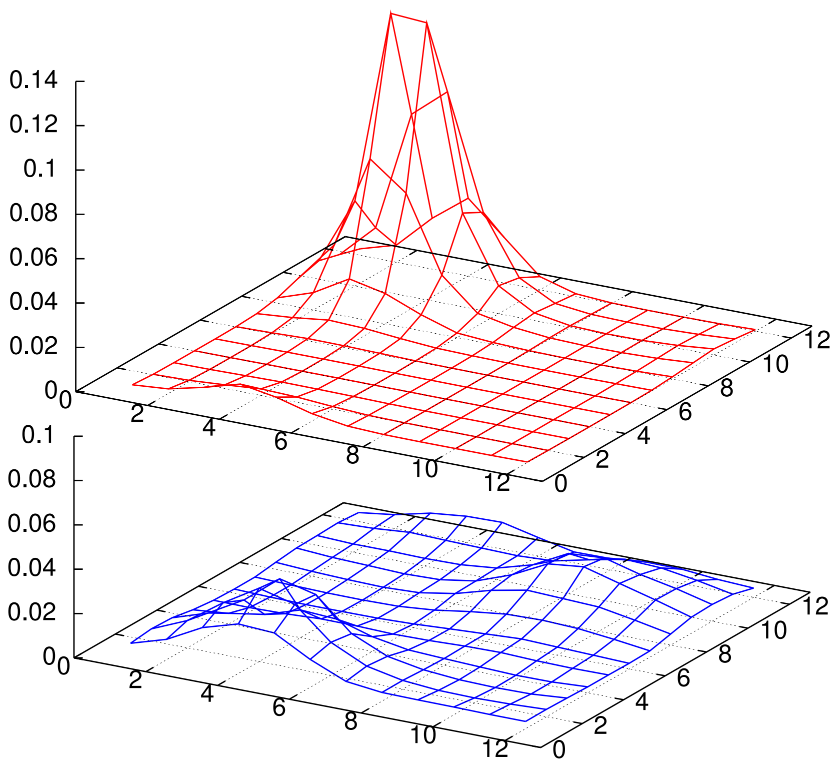

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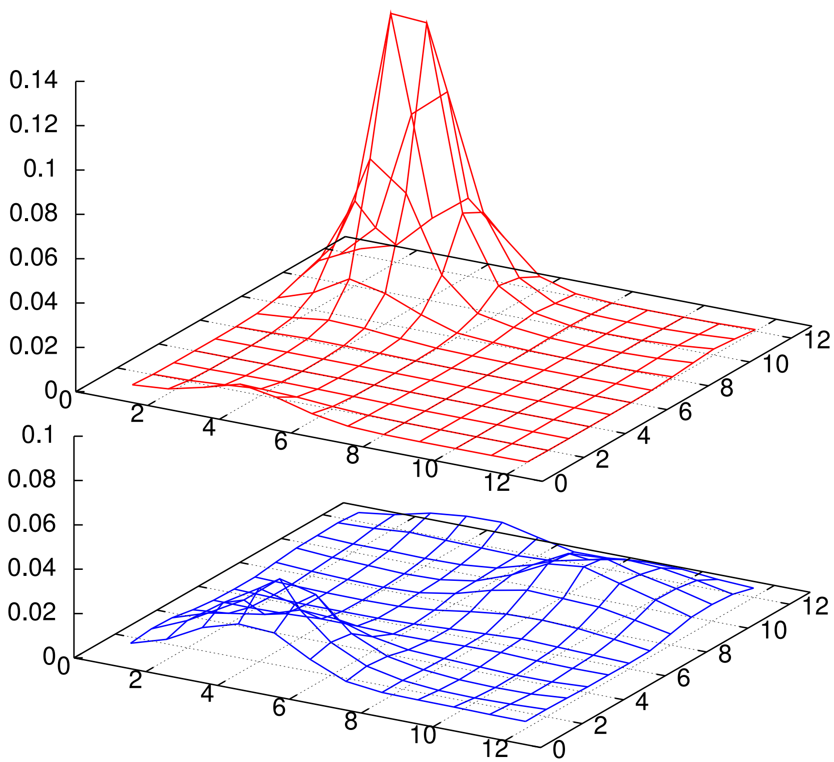

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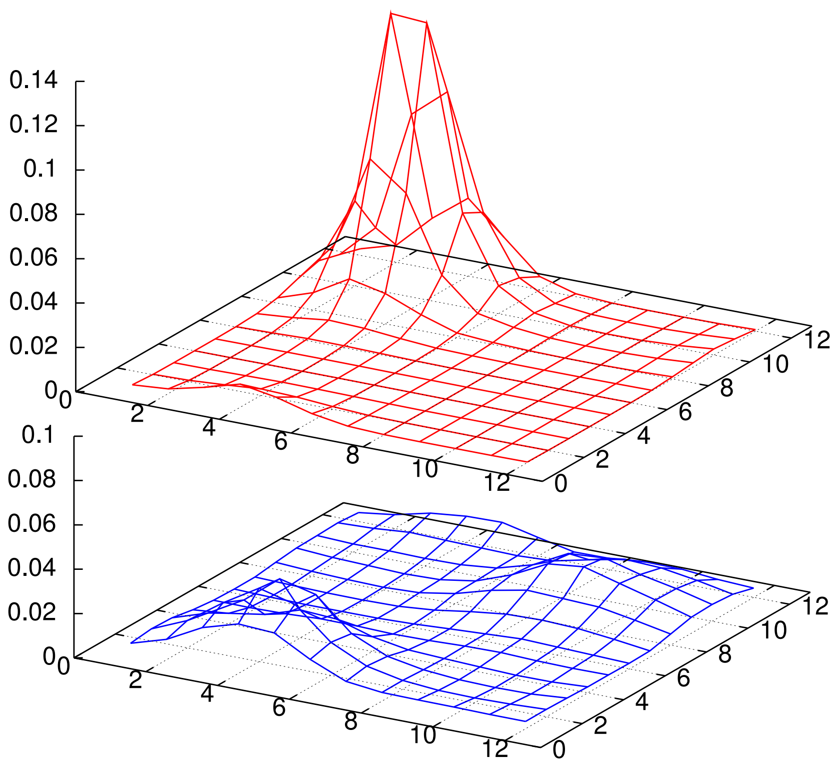

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$C_V = C_A$ & $C_{PS} = -C_S$ after subtraction of zero modes.

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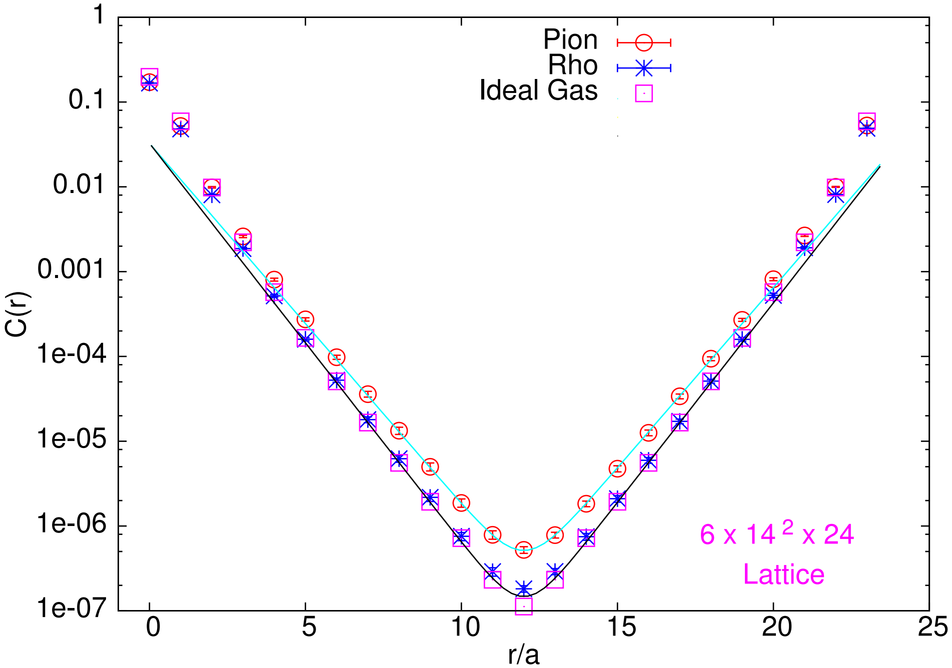

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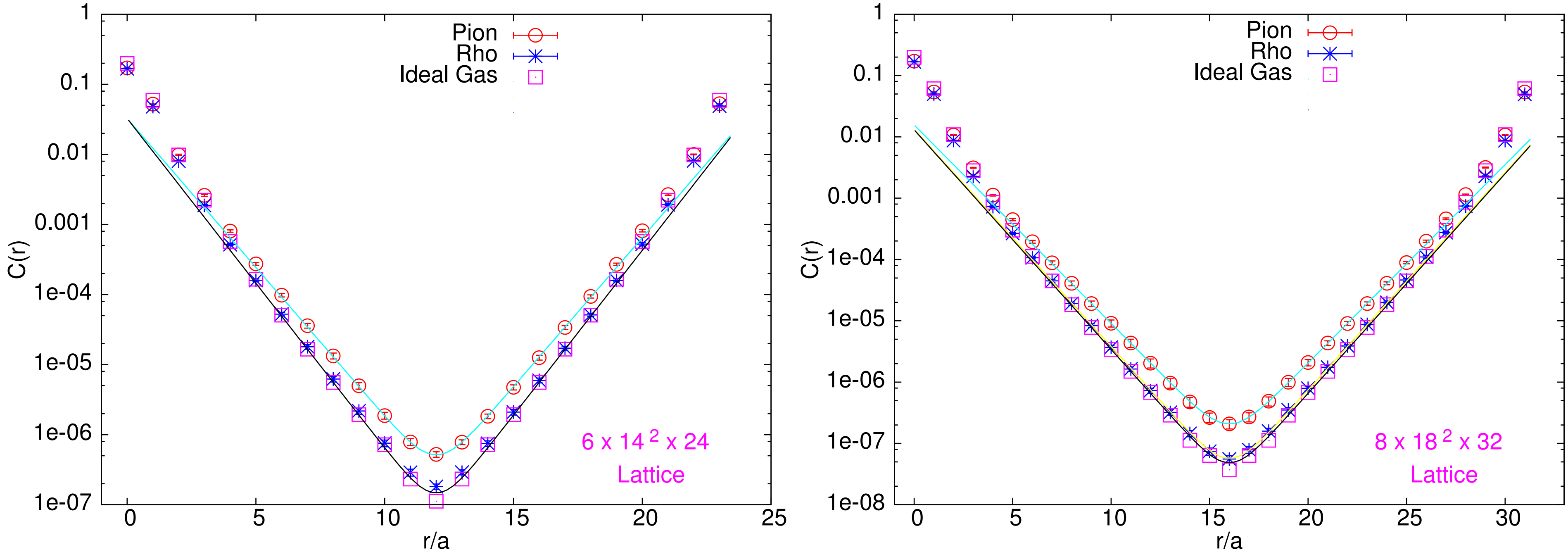

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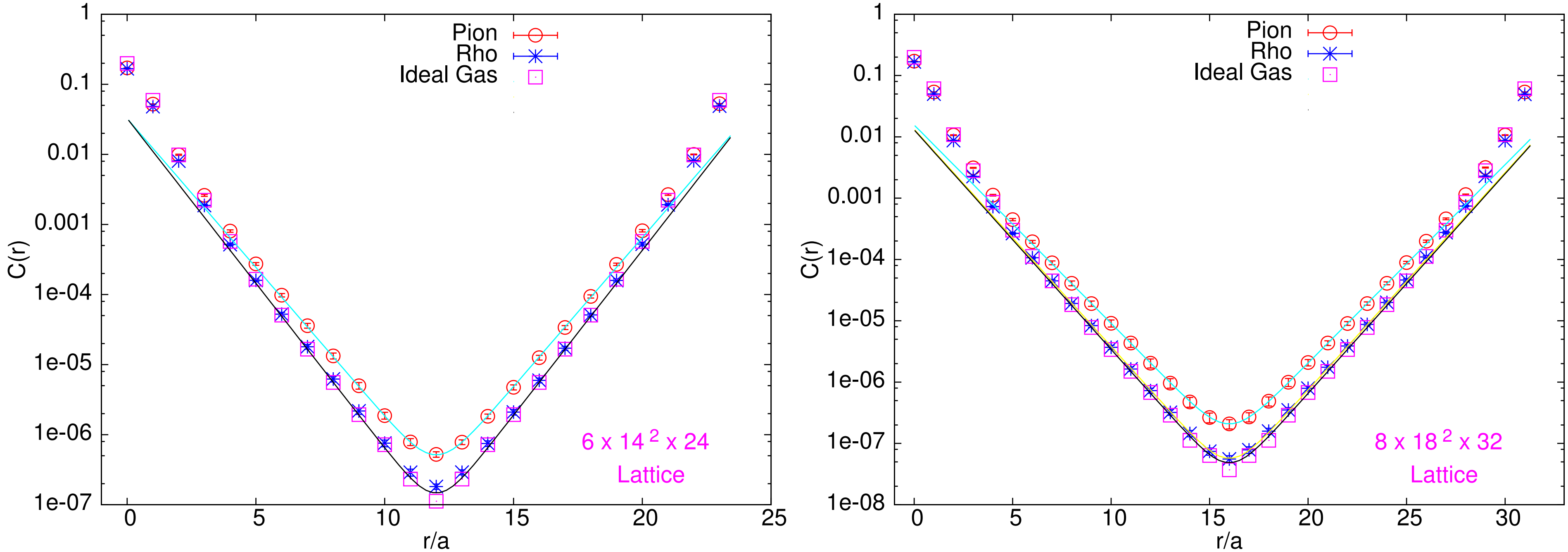

♣ On both N_t =6 and 8, cosh-like behaviour is seen.

♣ Ideal gas correlator very close in each case.

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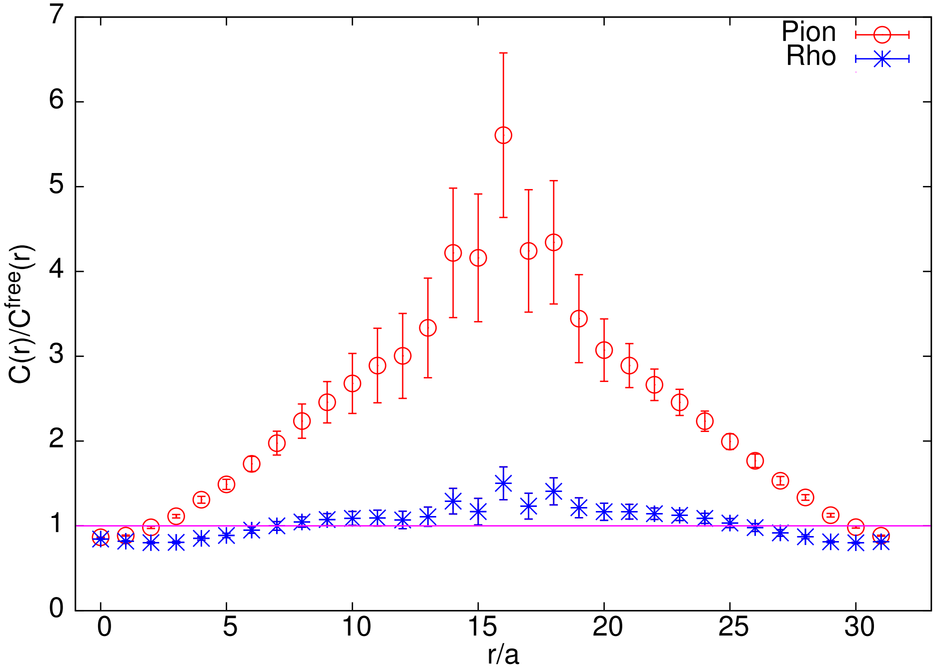

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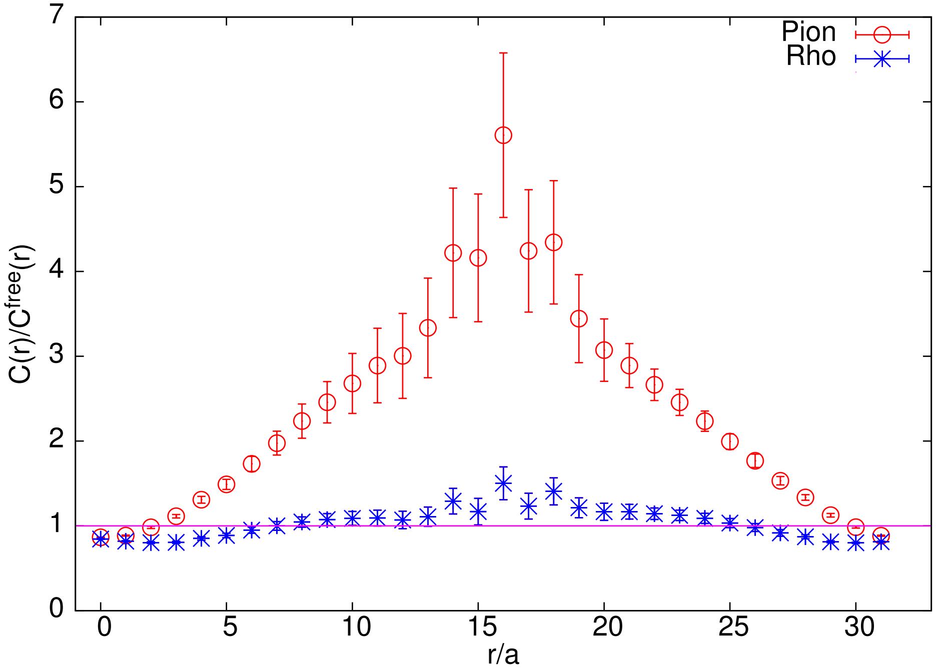

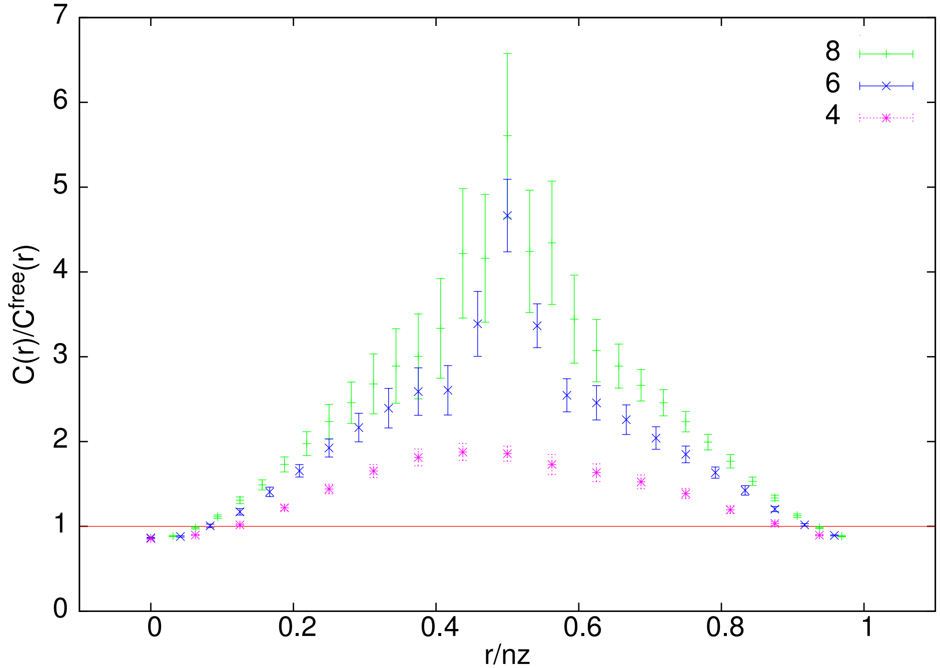

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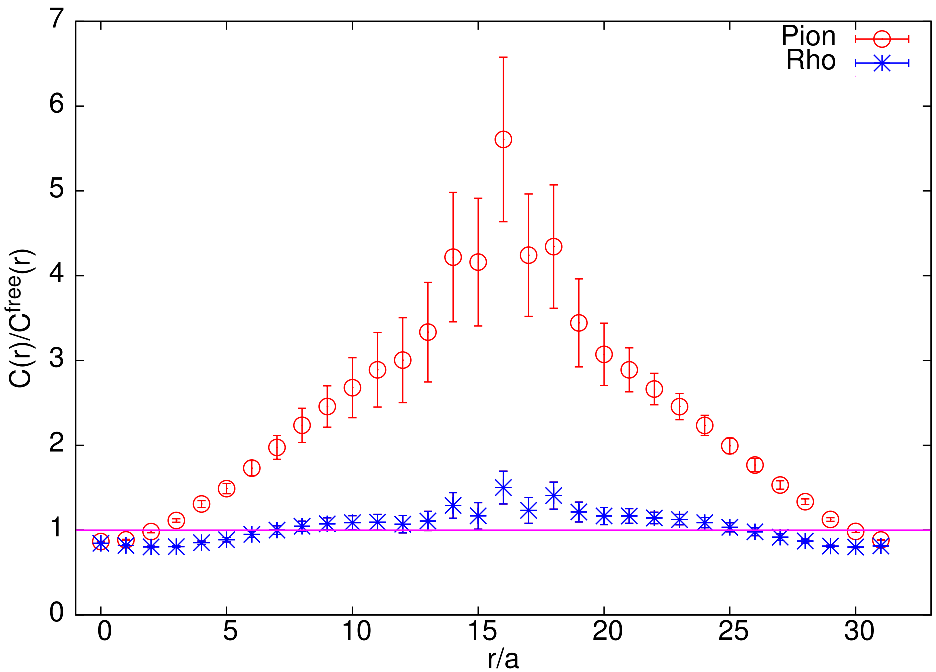

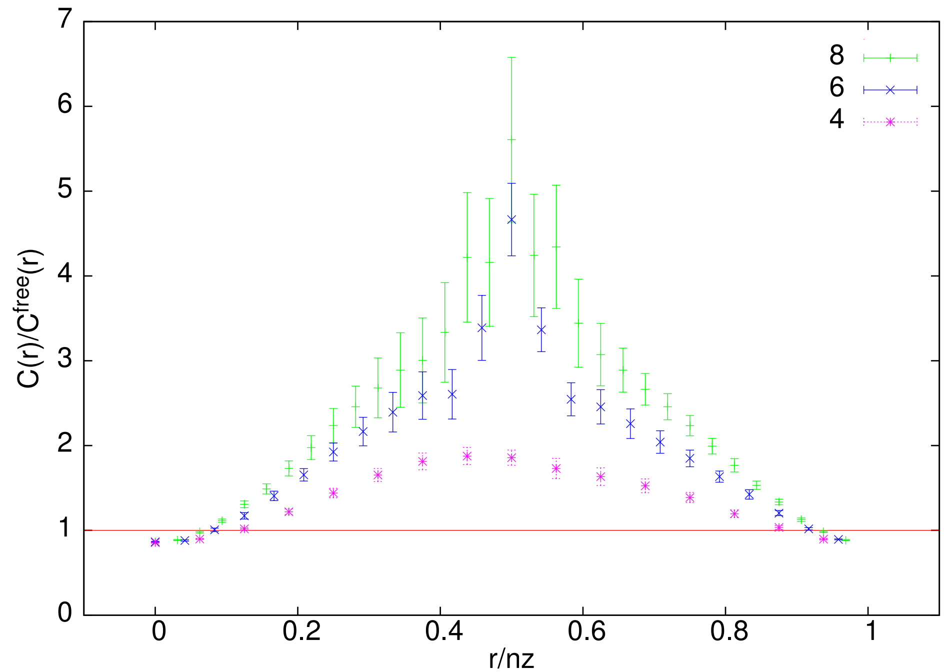

♣ As a gets smaller, the pion deviations increase.

♣ Local masses [$\sim \ln(C(r)/C(r+1)$] show nice plateau behaviour for pi & rho.

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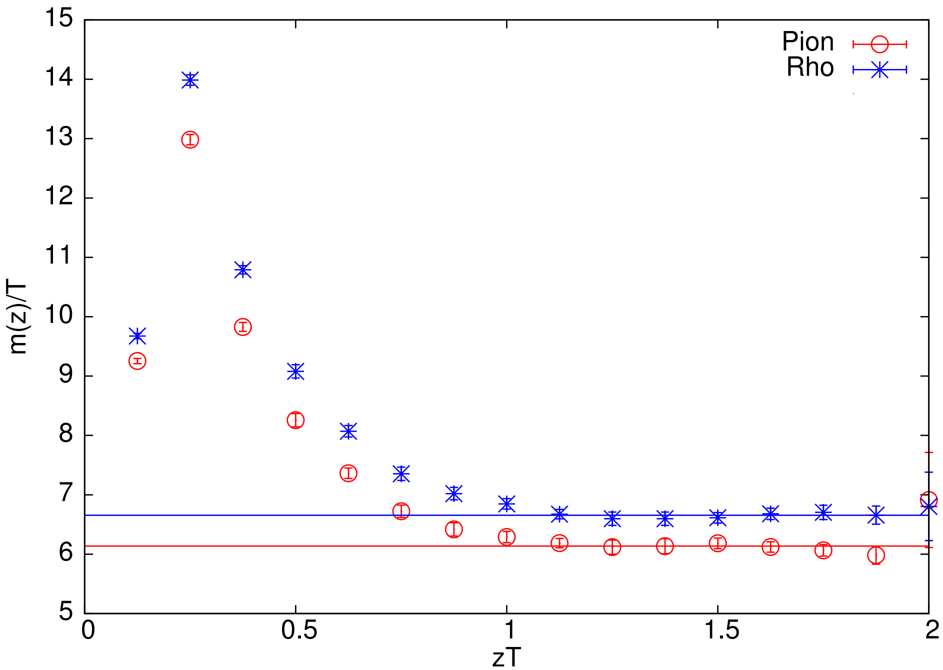

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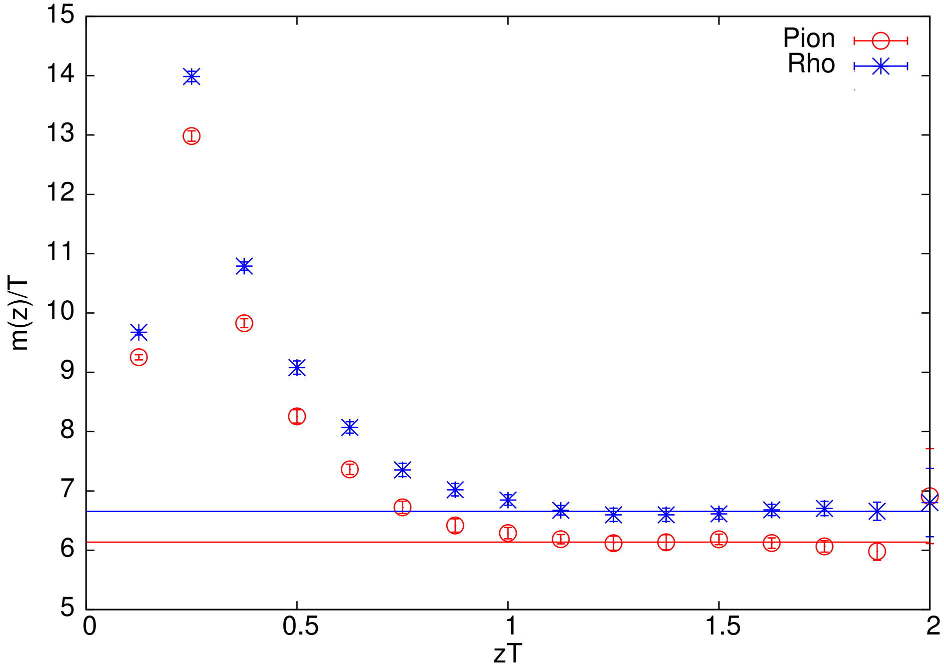

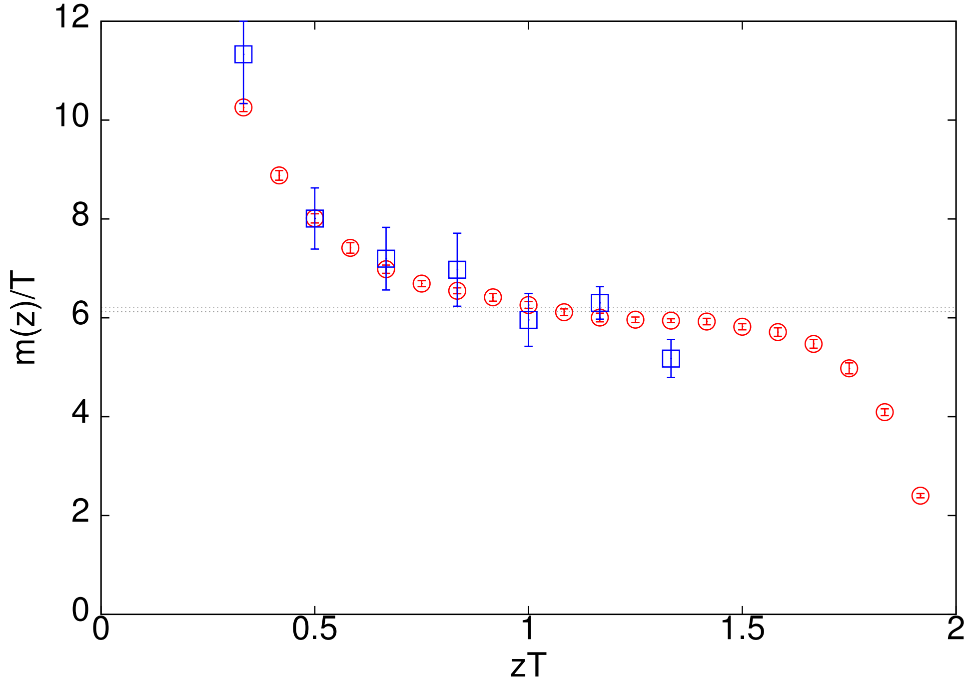

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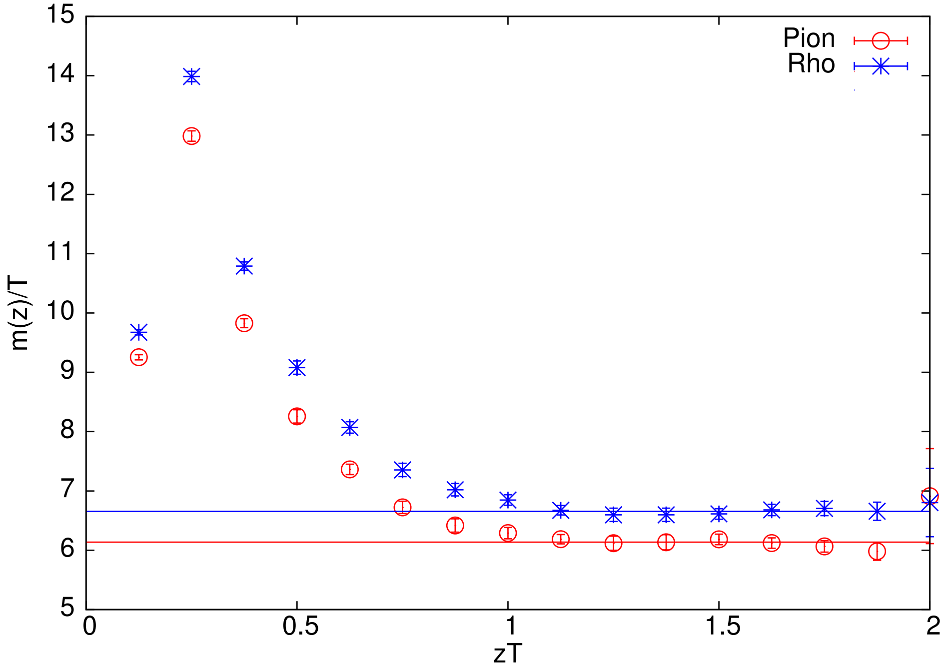

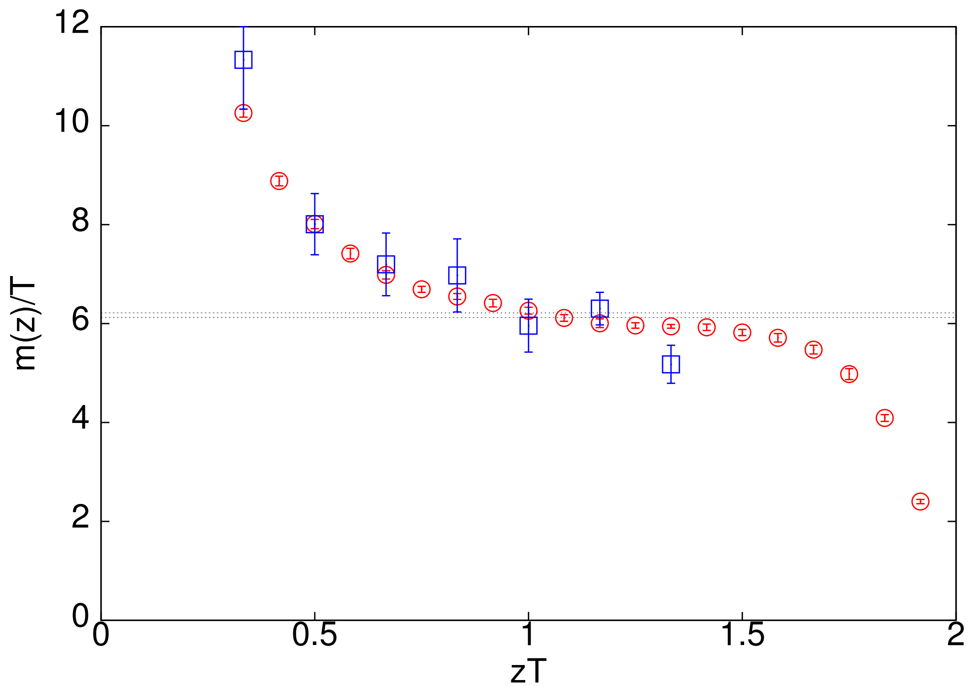

♣ Contrast this with the staggered effective mass (Gavai & Gupta PRD 2002).

Comparison with Wilson Fermions

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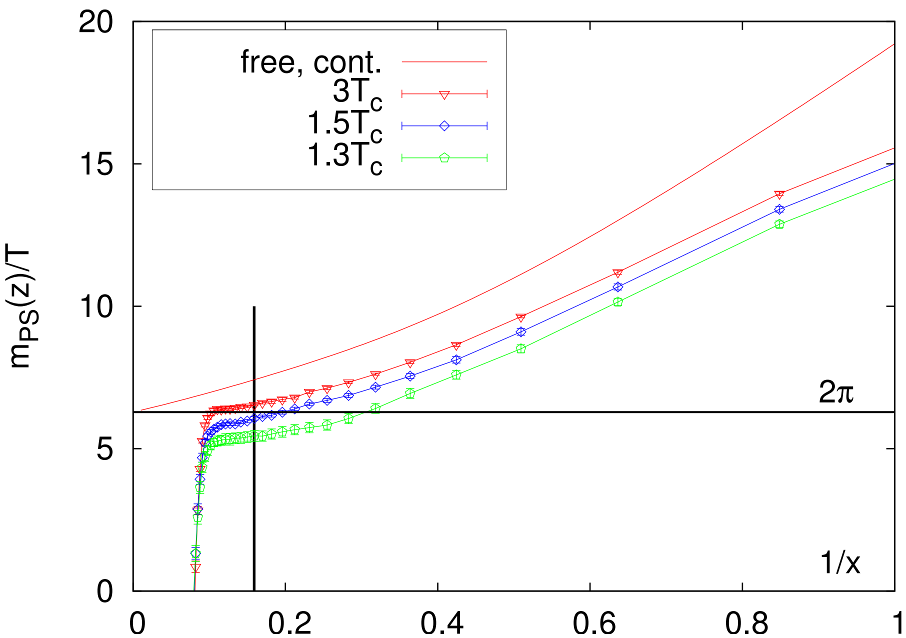

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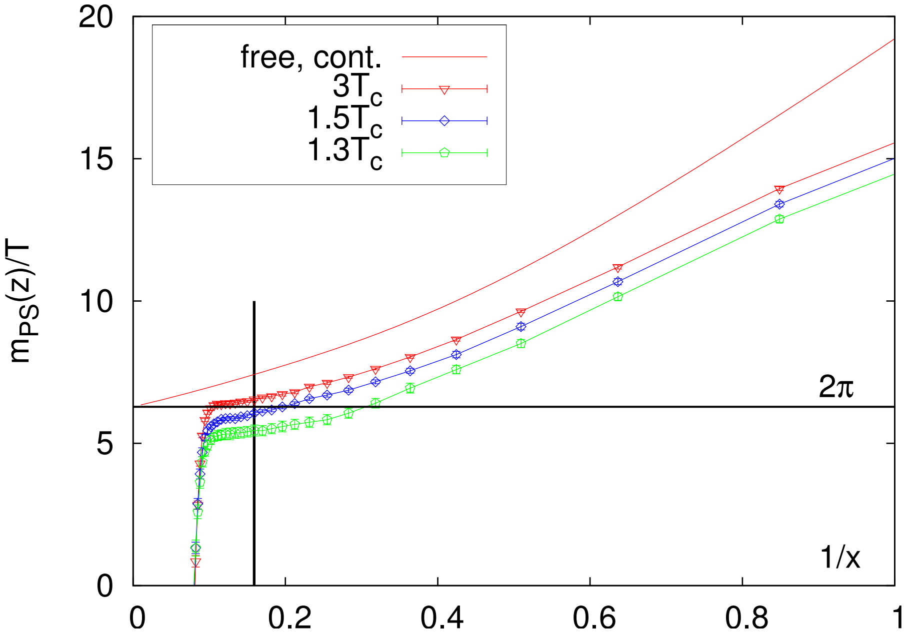

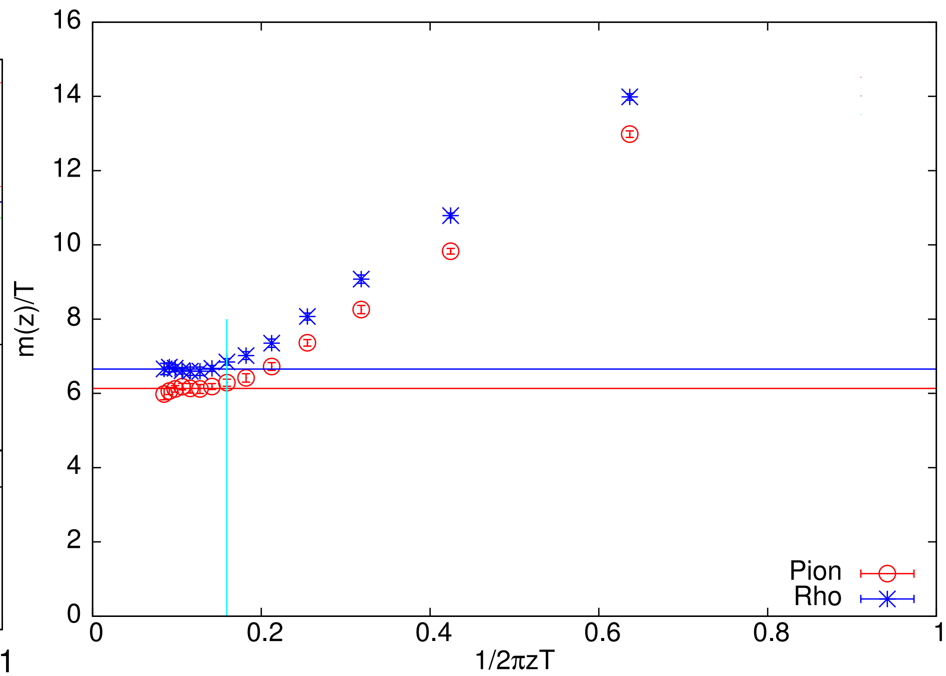

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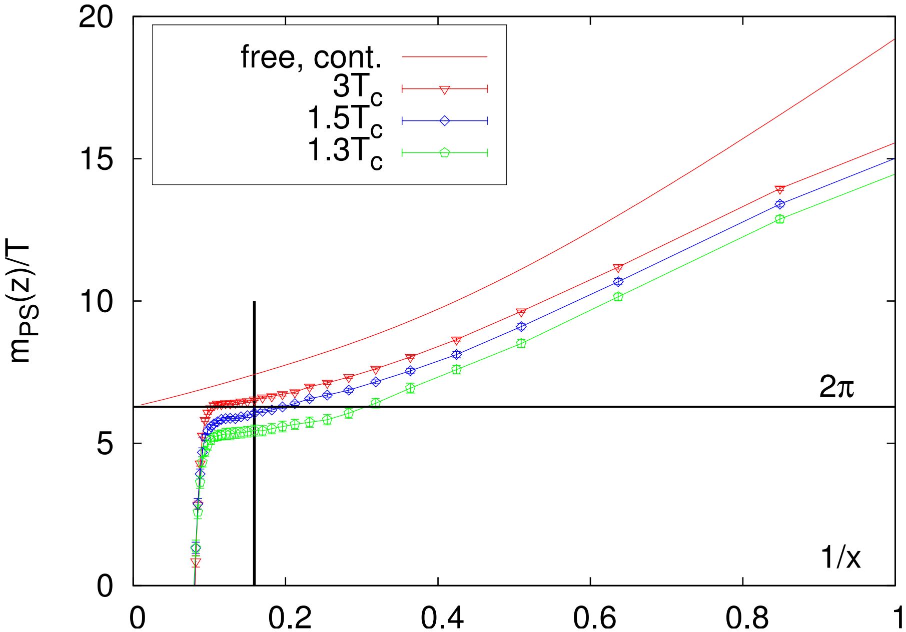
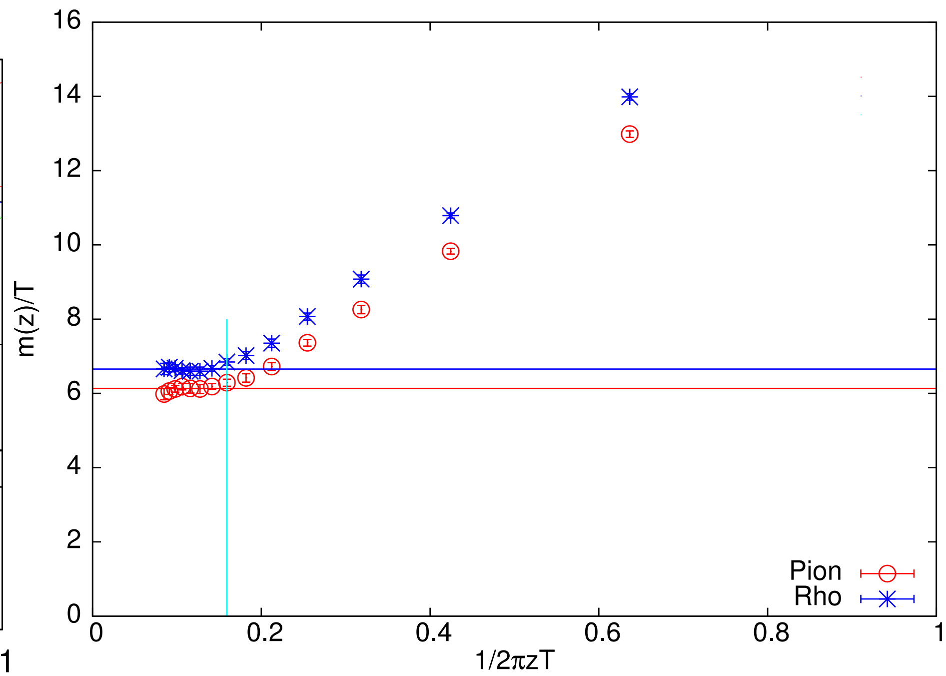
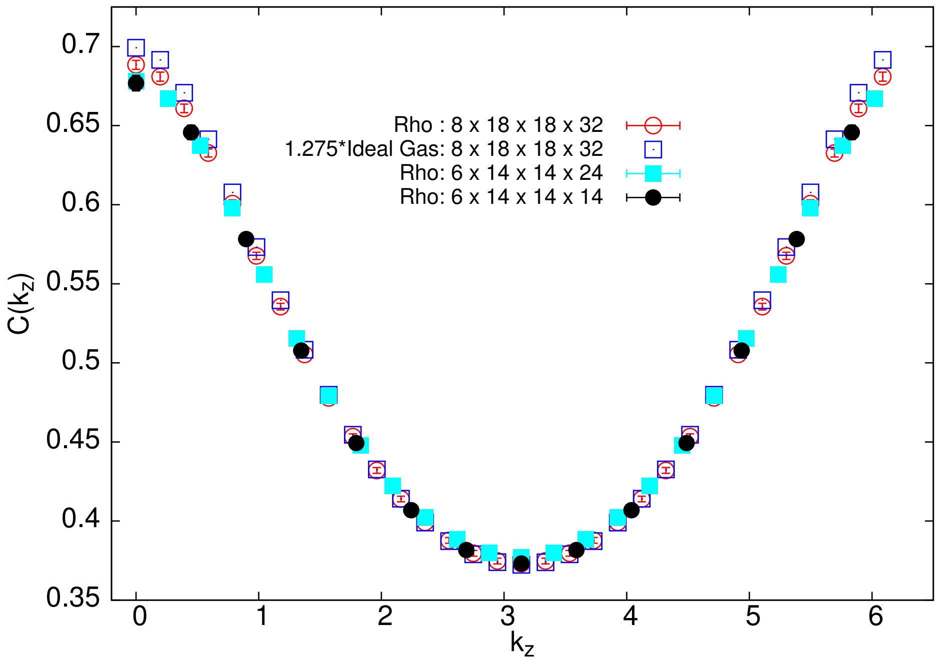




♣ Nice plateau behaviour for Overlap fermions.

Momentum Space Correlators



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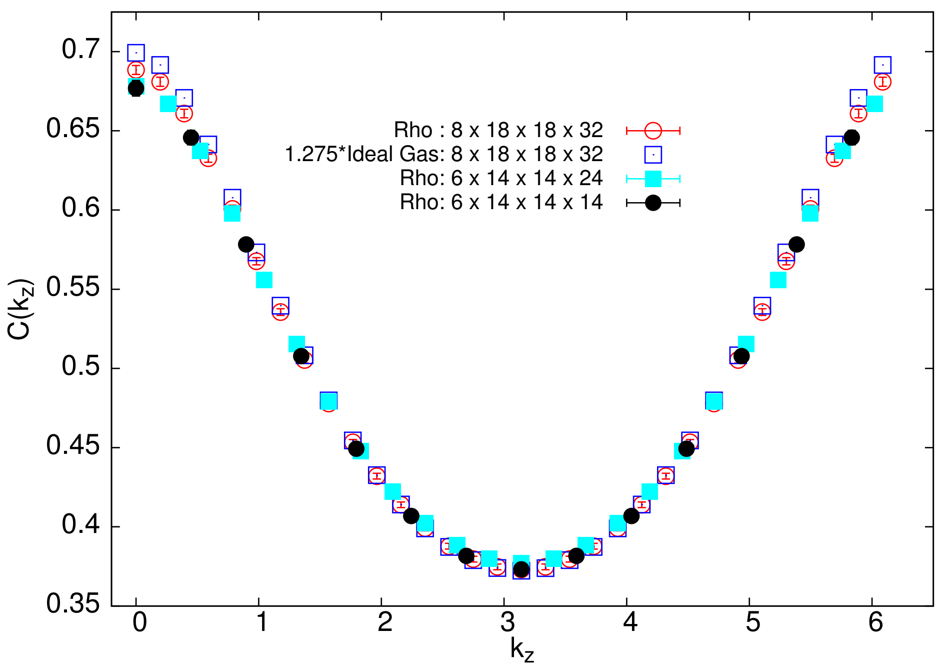
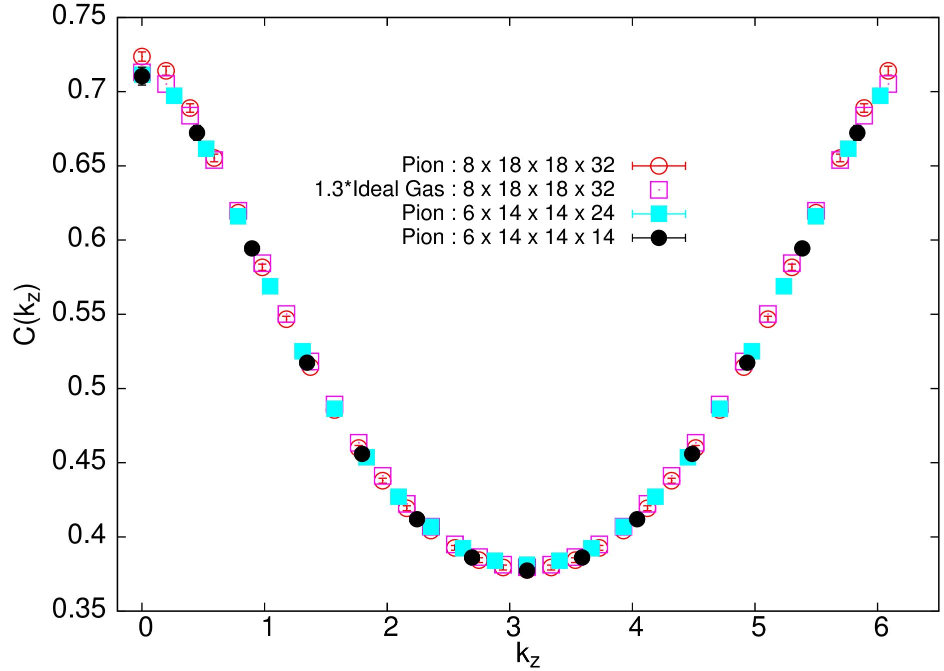
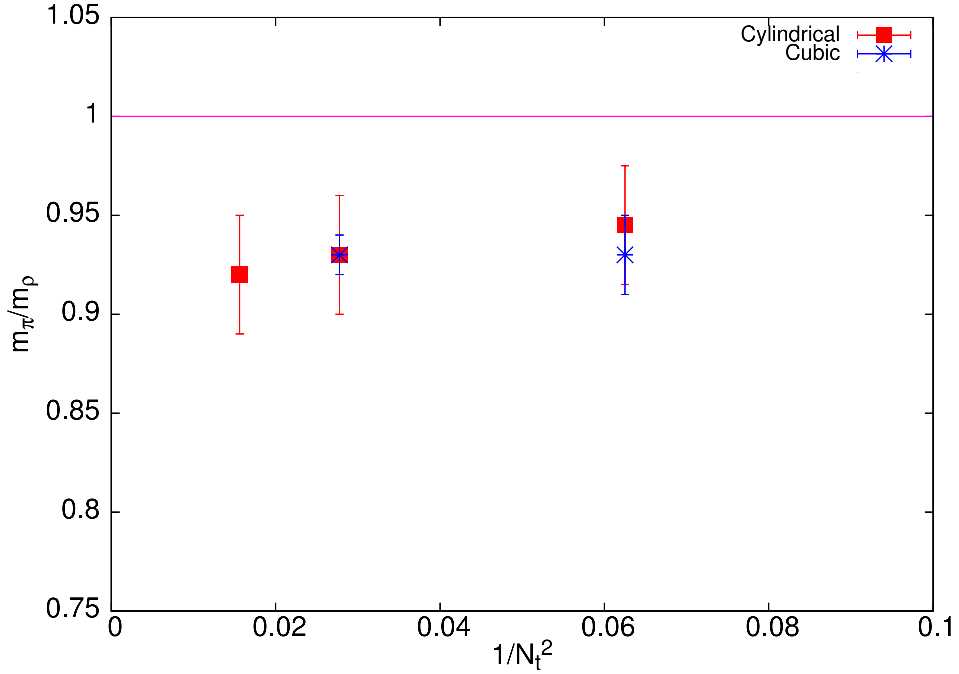




Screening masses vs. a



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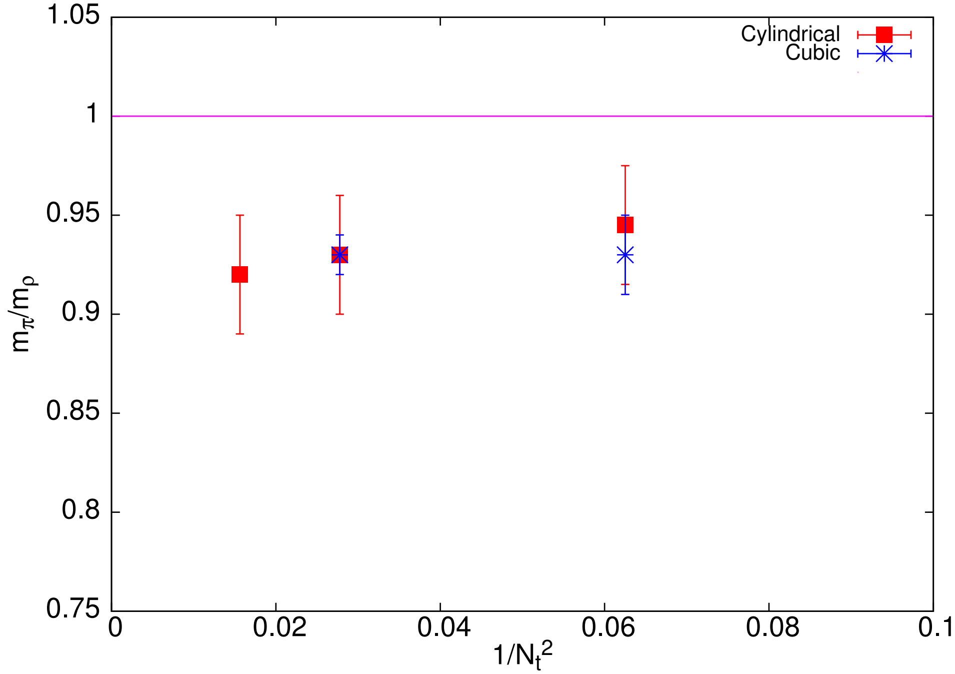

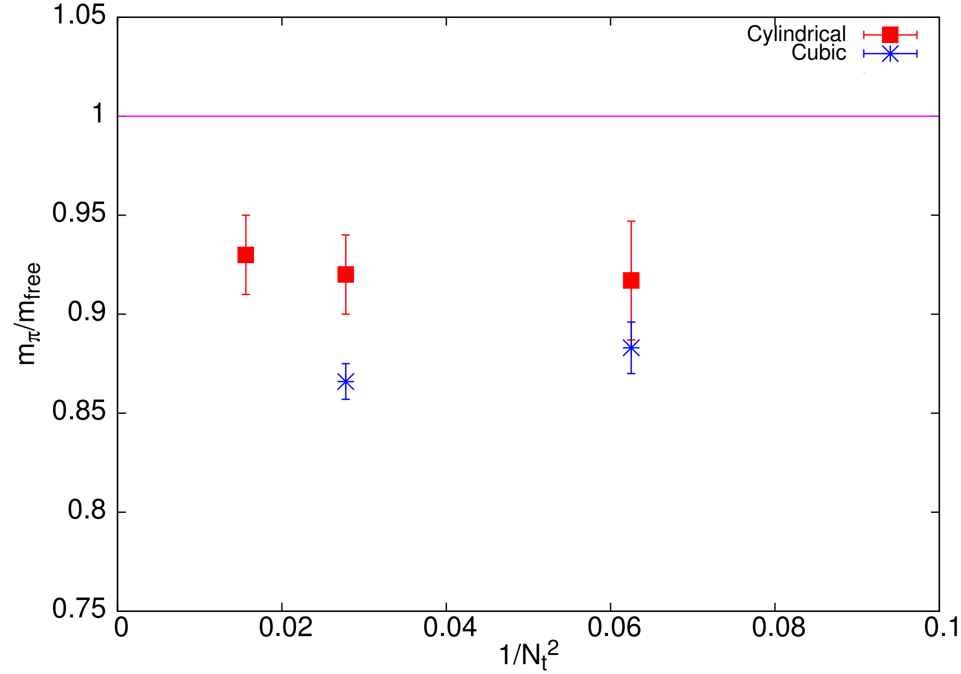

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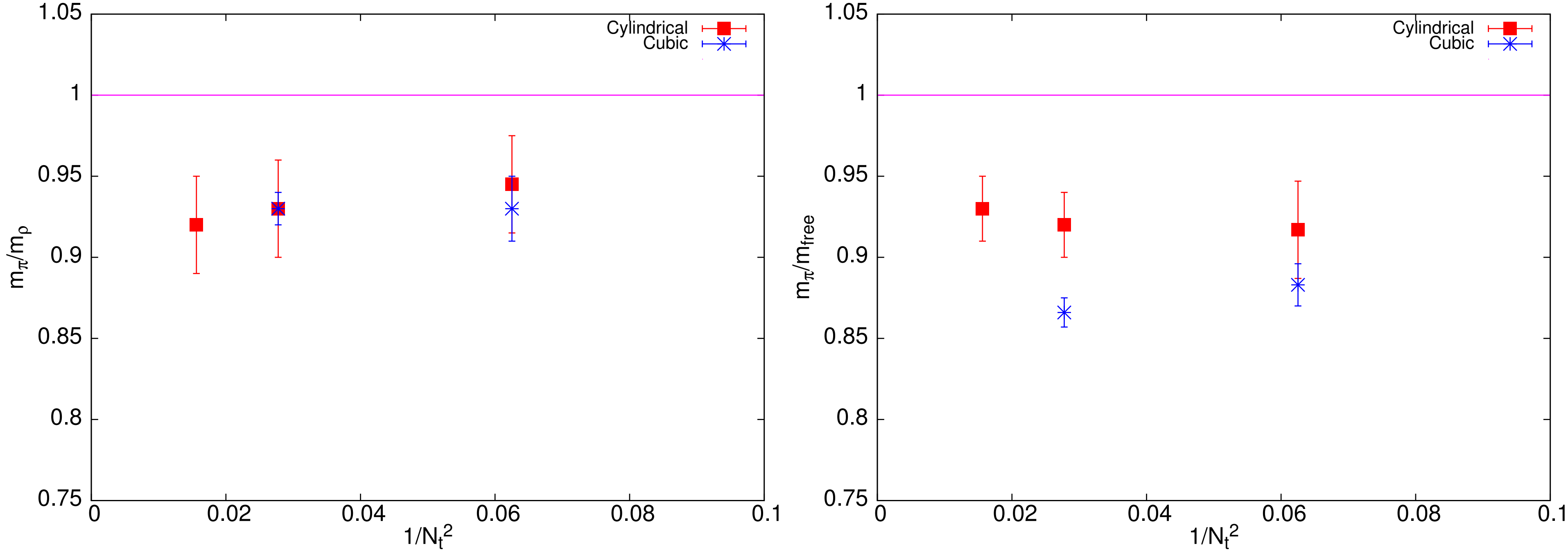

♣ Very small a dependence.

♣ m_ρ consistent with Ideal Gas but m_π smaller by about 10 %.

Summary

- Single *cosh* behaviour, leading to nice plateau in local masses, seen on *ALL* N_t= 4, 6 and 8.
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- Rho correlator in very good agreement with ideal gas one, but pion differs on all N_t; Deviations increase in continuum limit.
- Pion screening mass remained different from the ideal gas at $\sim$ 10 % or 3σ level, while rho mass was in agreement.
- Very little, if any, a dependence $\Longrightarrow$ difference to persist on very large N_t.

Summary II

- Lattice QCD **predicts** transition to Quark-Gluon Plasma and several of its properties, T_c, EoS, μ_π, λ_s, η ...

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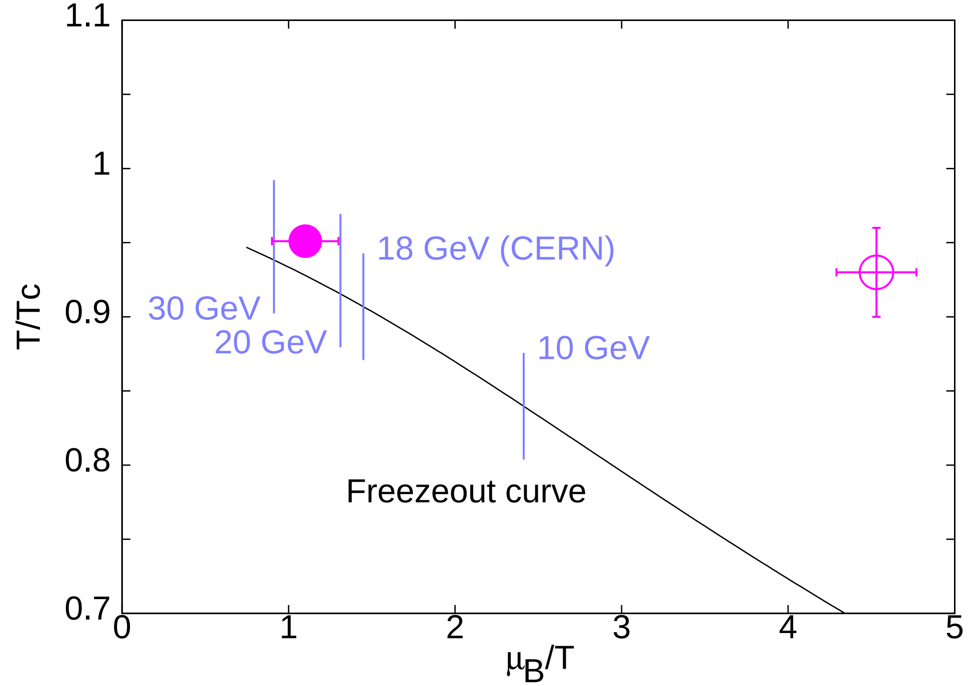

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- Finally, make a relaxation time approximation $(\omega\tau \gg 1)$ $\rightsquigarrow$ ratio of real parts is the same as the ratio of imaginary parts.

We use $m/T_c = 0.03$ for u, d and $m/T_c = 1$ for s quark;
At each T, ratio of χ_s and χ_{ud} $\quad \rightarrow \quad \lambda_s(T)$.

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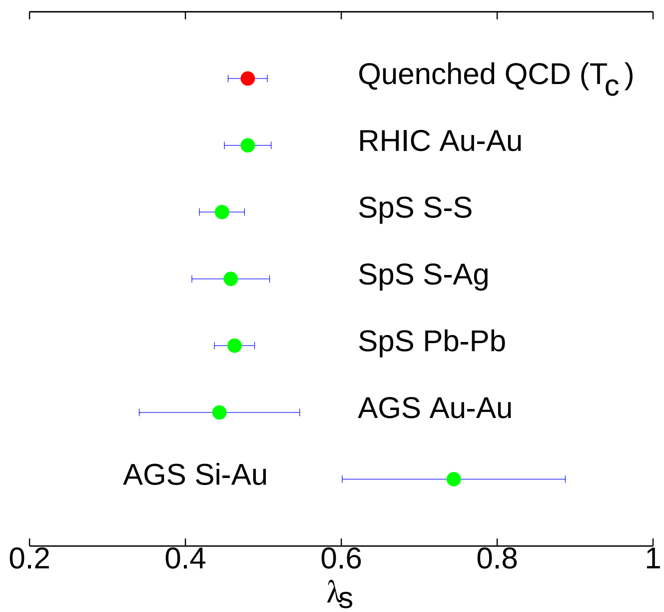

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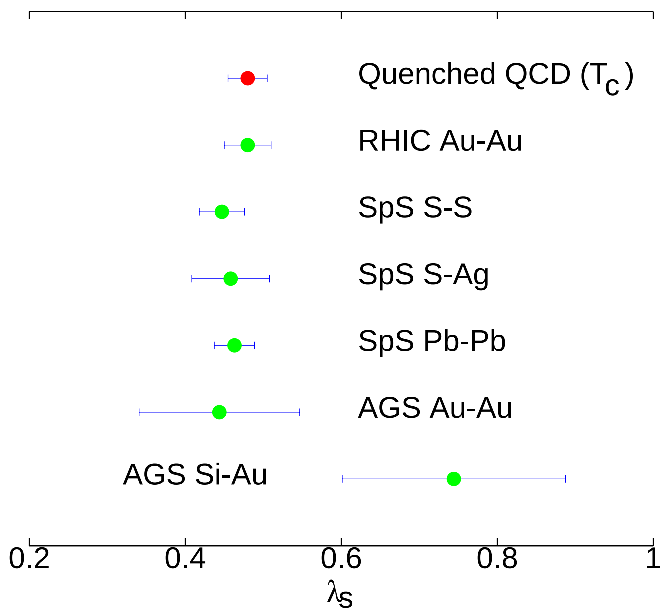

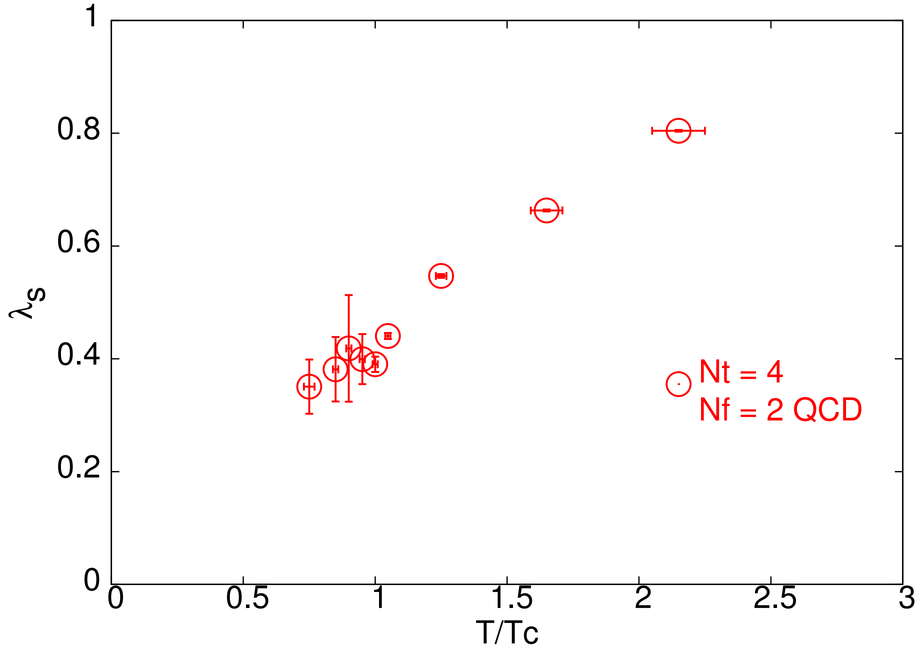

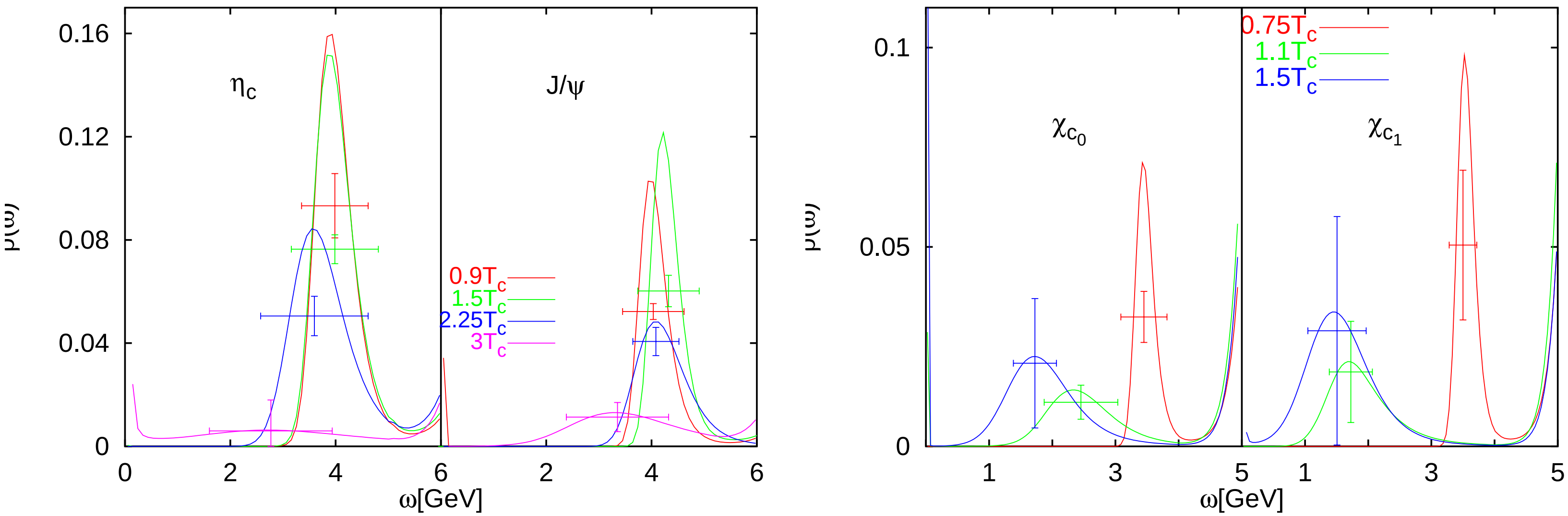

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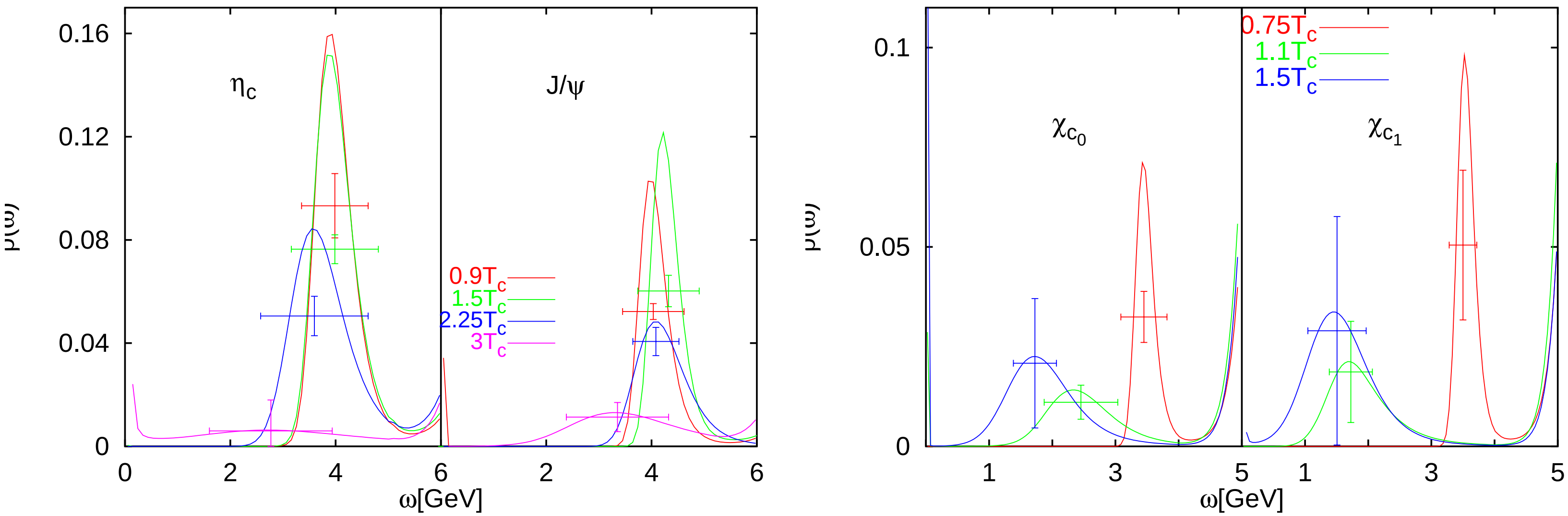

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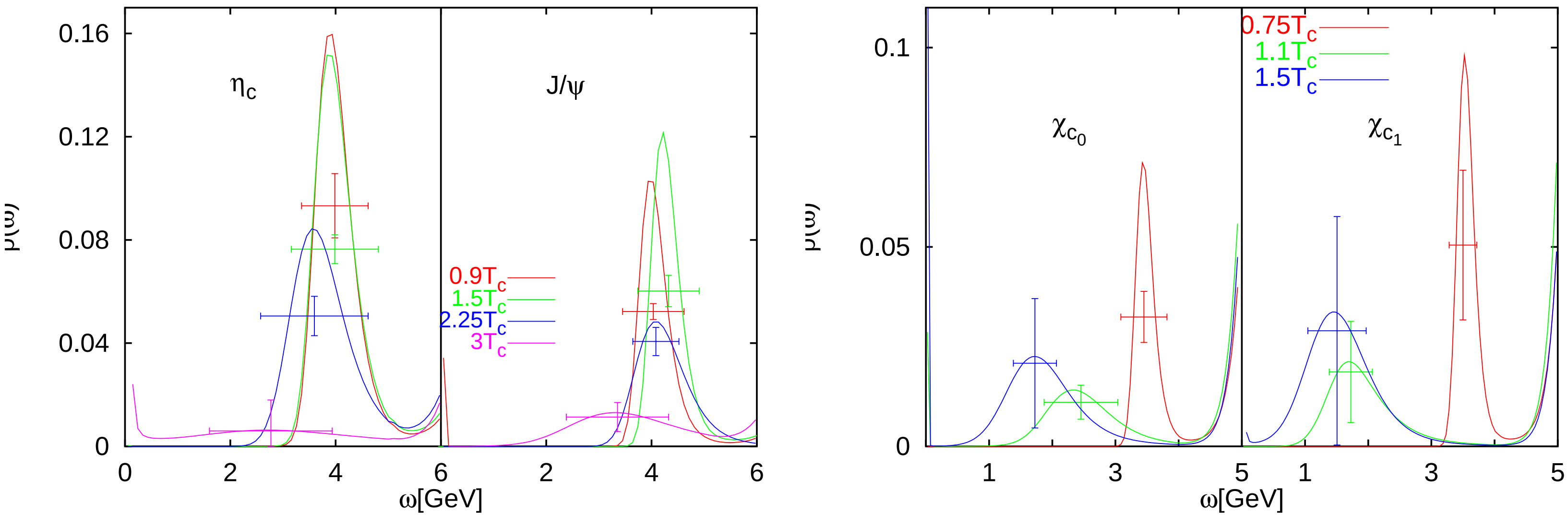

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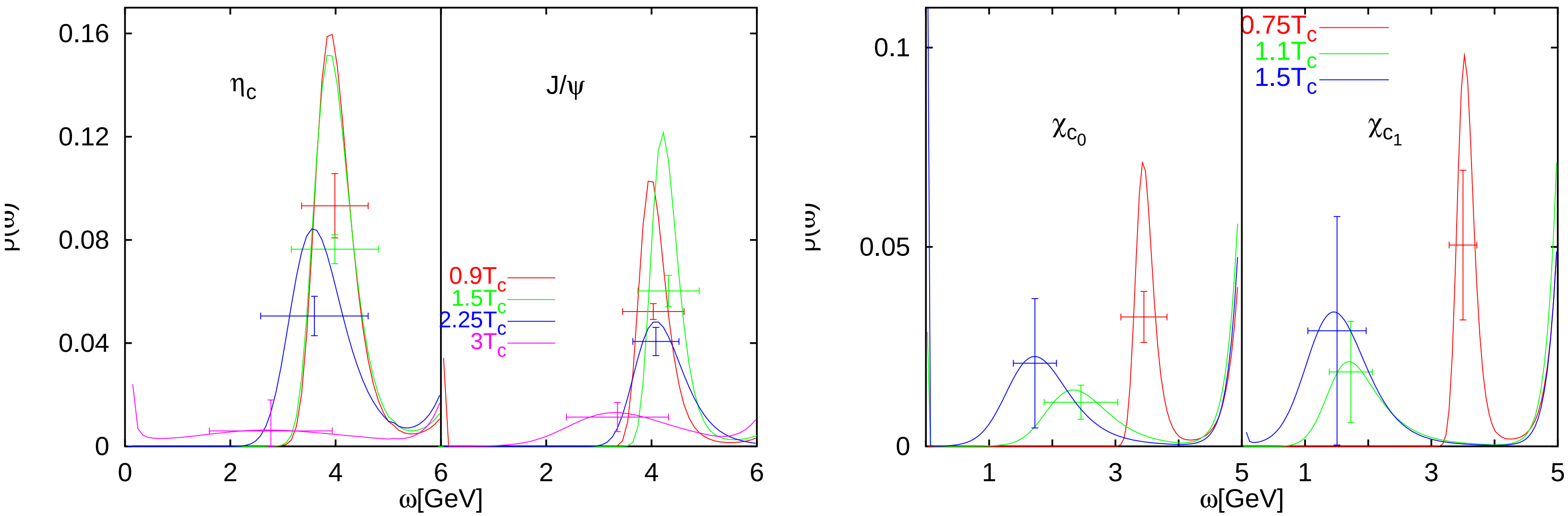

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♠ No Significant Effect of inclusion of dynamical fermions ?

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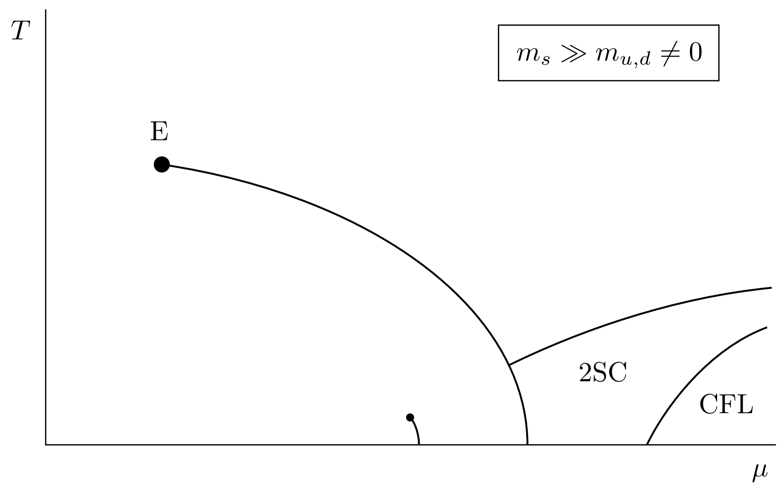

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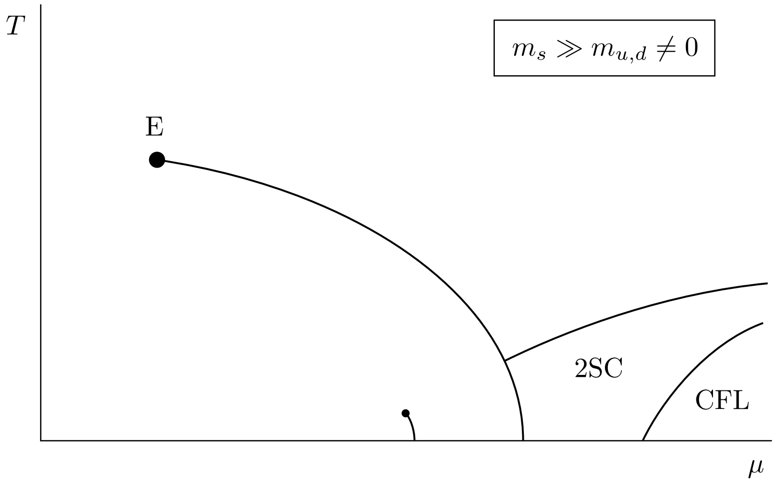

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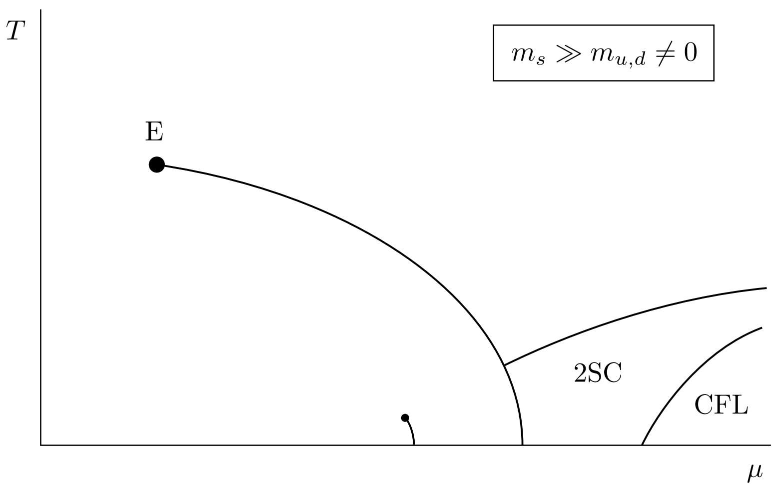

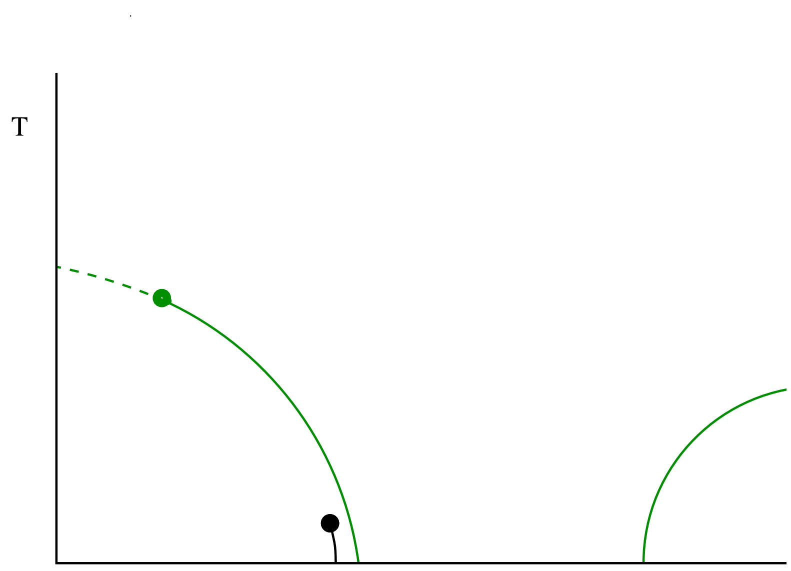

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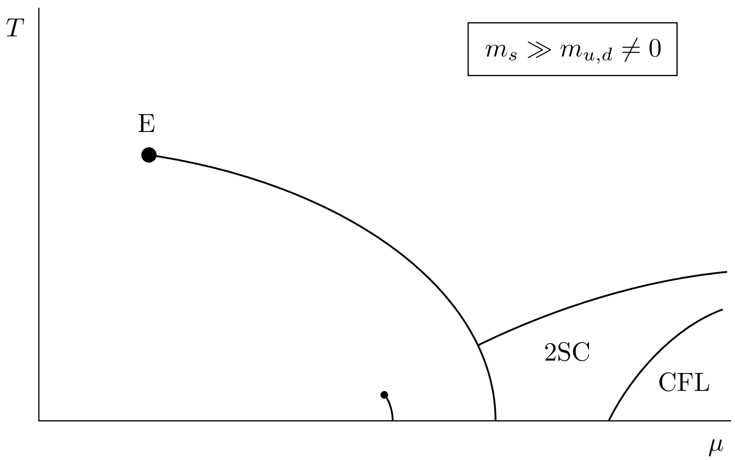

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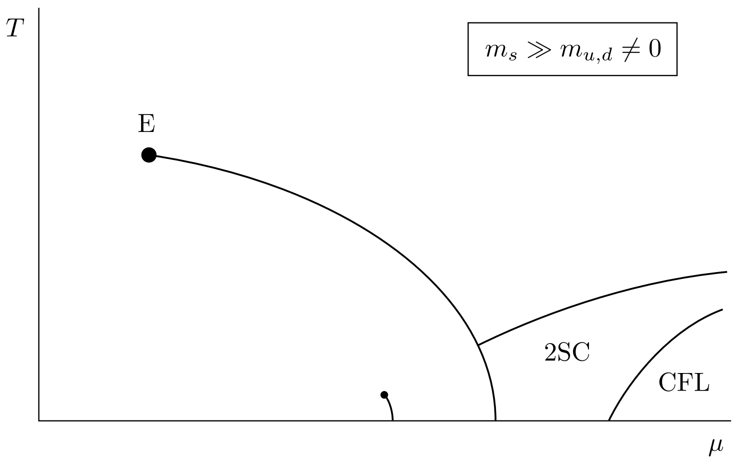

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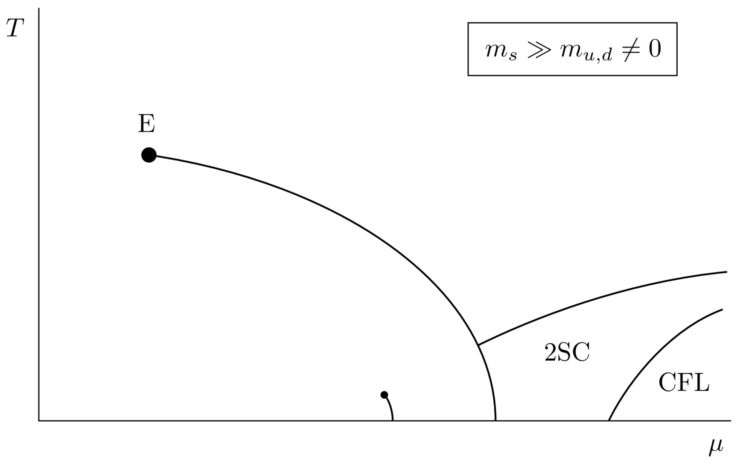

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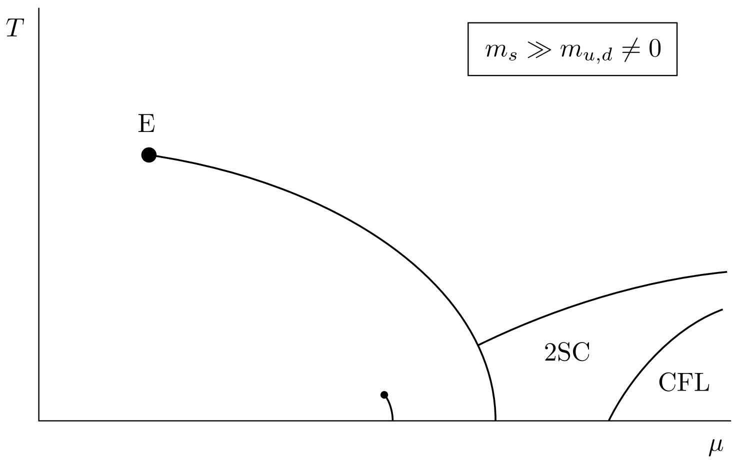

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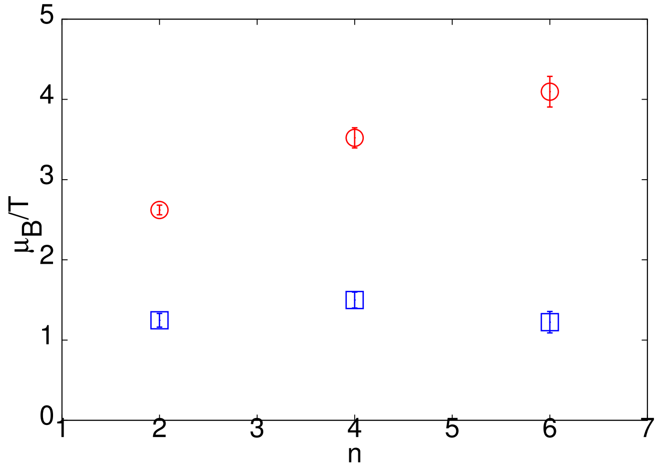
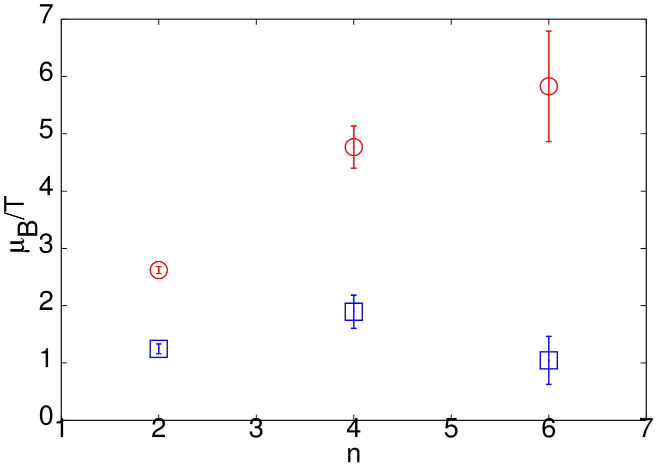
Critical Point Estimate

RVG & S. Gupta, PR D 71 2005.





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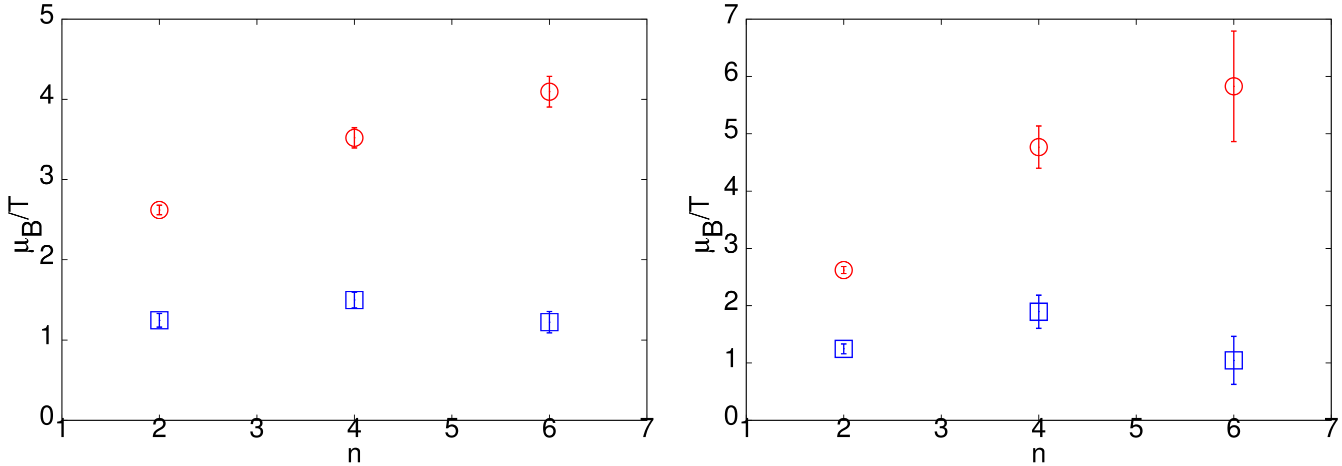
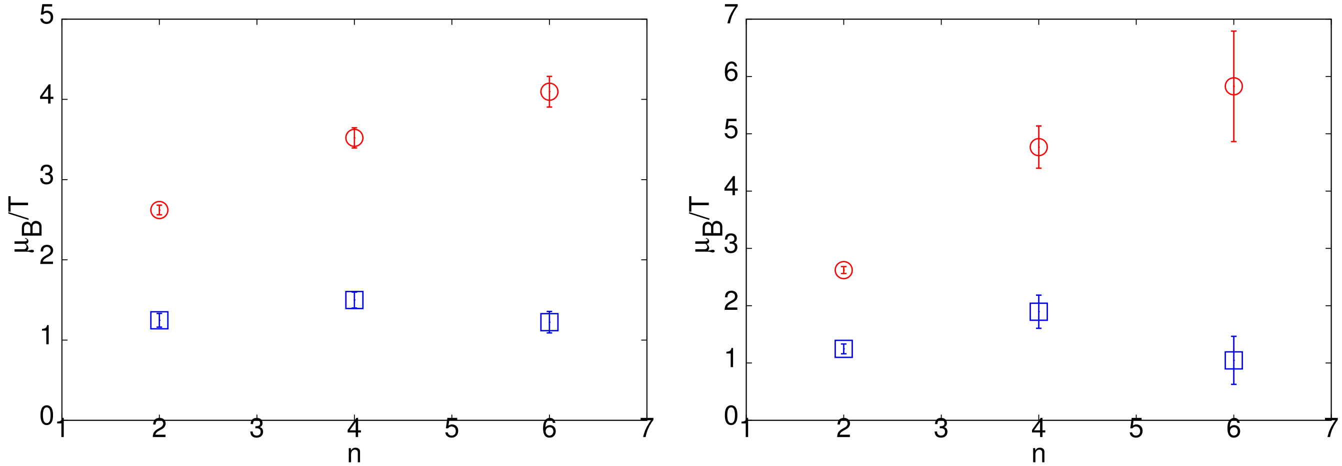


♠ Radii of convergence as a function of the order of expansion at $T = 0.95T_c$ on $N_s = 8$ (circles) and 24 (boxes). Left panel for ρ_n and right one for r_n.

Critical Point Estimate

RVG & S. Gupta, PR D 71 2005.



♠ Radii of convergence as a function of the order of expansion at $T = 0.95T_c$ on $N_s = 8$ (circles) and 24 (boxes). Left panel for ρ_n and right one for r_n.

♠ Extrapolation in $n \rightsquigarrow \mu^E/T^E = 1.1 \pm 0.2$ at $T^E = 0.95T_c$. Finite volume shift consistent with Ising Universality class.

m_ρ/T_c	m_π/m_ρ	m_N/m_ρ	$N_s m_\pi$	flavours	T^E/T_c	μ_B^E/T^E
5.372 (5)	0.185 (2)	—	1.9–3.0	2+1	0.99 (2)	2.2 (2)
5.12 (8)	0.307 (6)	—	3.1–3.9	2+1	0.93 (3)	4.5 (2)
5.4 (2)	0.31 (1)	1.8 (2)	3.3–10.0	2	0.95 (2)	1.1 (2)
5.4 (2)	0.31 (1)	1.8 (2)	3.3	2	—	—
5.5 (1)	0.70 (1)	—	15.4	2	—	—

Table 1: Summary of critical end point estimates— the lattice spacing is $a = 1/4T$. N_s is the spatial size of the lattice and $N_s m_\pi$ is the size in units of the pion Compton wavelength, evaluated for $T = \mu = 0$. The ratio m_π/m_K sets the scale of the strange quark mass.

Results are sequentially from Fodor-Katz '04, Fodor-Katz '02, Gavai-Gupta, de Forcrand- Philipsen and Bielefeld-Swansea.