Lattice Perspective on Strangeness and Quasi-Quarks

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Introduction

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- Quark Number Susceptibilities (QNS) measure these fluctuations. Lattice approach yields these directly from QCD. (Gottlieb et al, '86,'87, .. , Gavai et al. '89...)

- Ratios of the susceptibilities, $C_{K/L} \equiv \frac{\chi_K}{\chi_L} = \frac{\sigma_K^2}{\sigma_L^2}$ are robust variables in high T Phase: both theoretically and experimentally.

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Nature of QGP Excitations

- Outstanding question in spite of long history of several investigations:
  - Equation of State : $T \geq 3 - 5T_c$ agrees with weak coupling schemes.
  - Quark Number Susceptibilities : Successful check on them.

- We address this directly using $C_{(KL)/L} = \frac{\langle KL \rangle - \langle K \rangle \langle L \rangle}{\langle L^2 \rangle - \langle L \rangle^2}$. Physically, create an excitation of quantum number K and ask what else (like L) does it carry.

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Quark Number Susceptibility

Assuming three flavours, u, d, and s quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \prod_{f=u,d,s} \mathrm{Det}\ M(m_f, \mu_f) \quad . \tag{1}$$

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Defining $\mu_B = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, baryon and isospin density/susceptibilities can be obtained as :
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$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}, \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}, \qquad i, j = 0, 3, u, d, s$$

Similarly, Charge (Q), Hypercharge (Y), Strangeness (S) susceptibilities can be defined. Higher order susceptibilities are defined by

$$\chi_{fg\cdots} = \frac{T}{V} \frac{\partial^n \log Z}{\partial\mu_f \partial\mu_g \cdots} = \frac{\partial^n P}{\partial\mu_f \partial\mu_g \cdots} \ . \tag{2}$$

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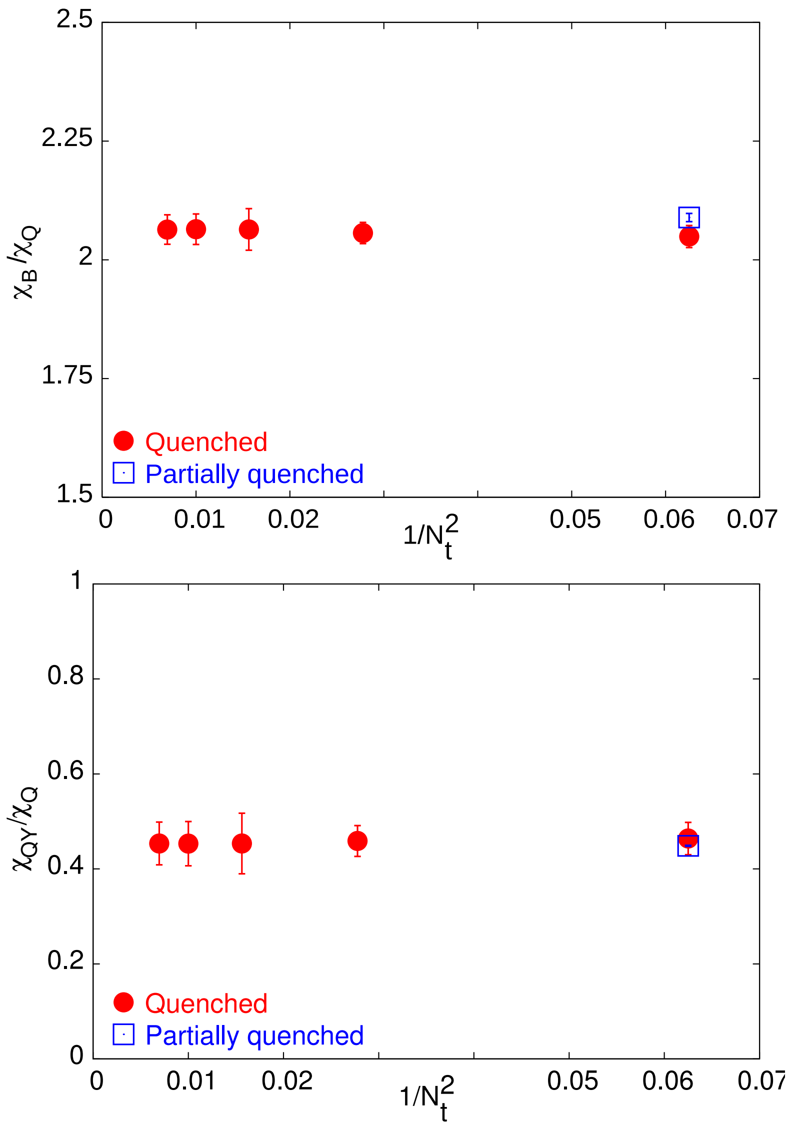
♠ Theoretical Checks : Resummed Perturbation expansions, Dimensional Reduction, Models of (s)QGP, ..

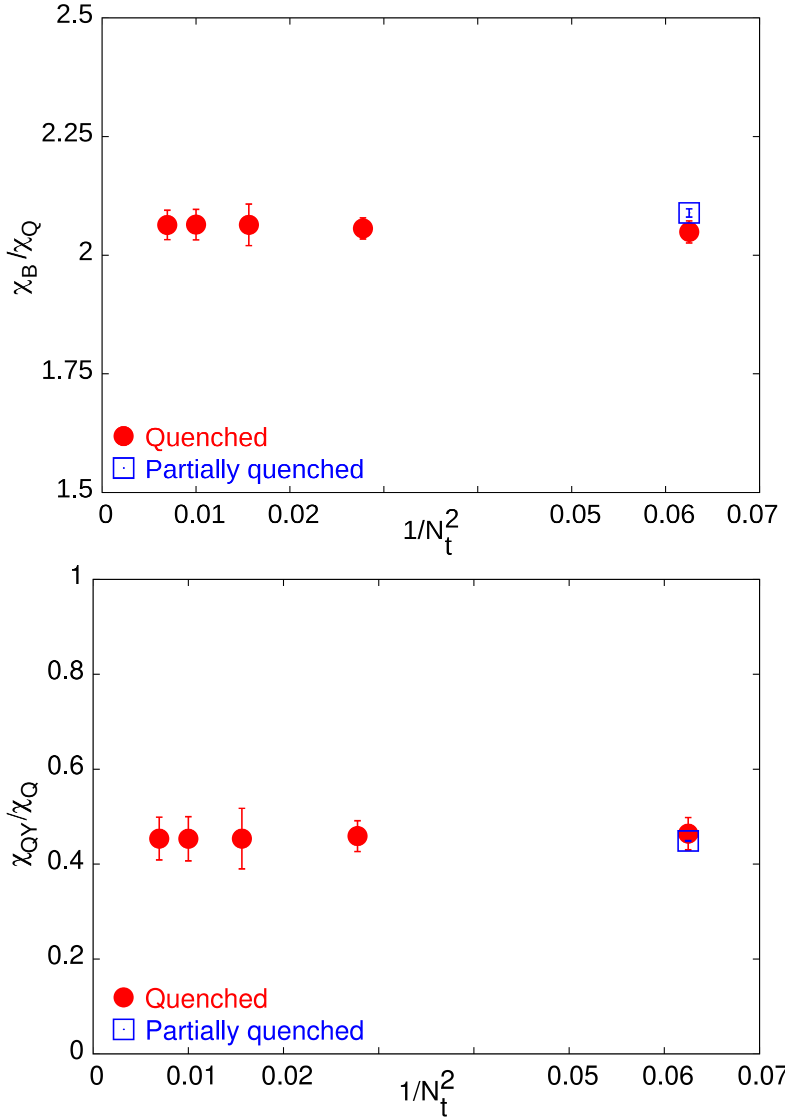
♠ We (Gavai & Gupta, PR D '02) have argued that

$$\lambda_s = \frac{2\chi_s}{\chi_u + \chi_d} . \tag{3}$$

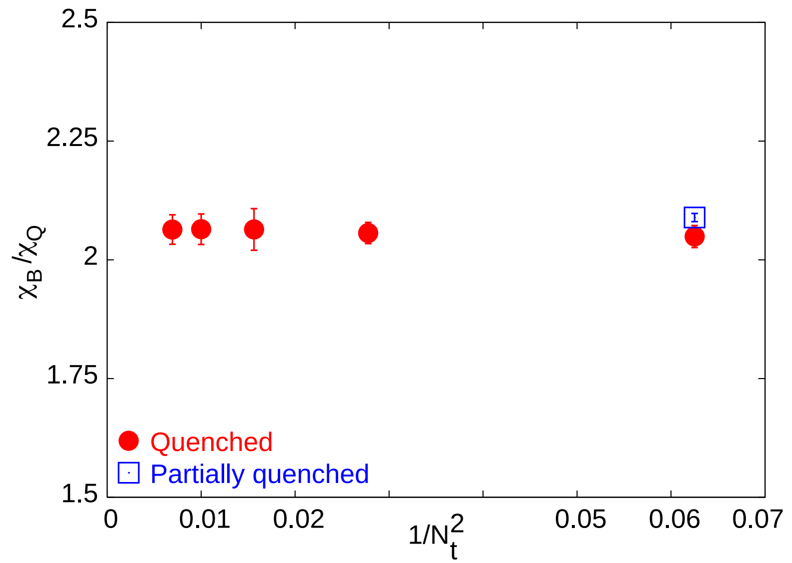
Robustness of Ratios C

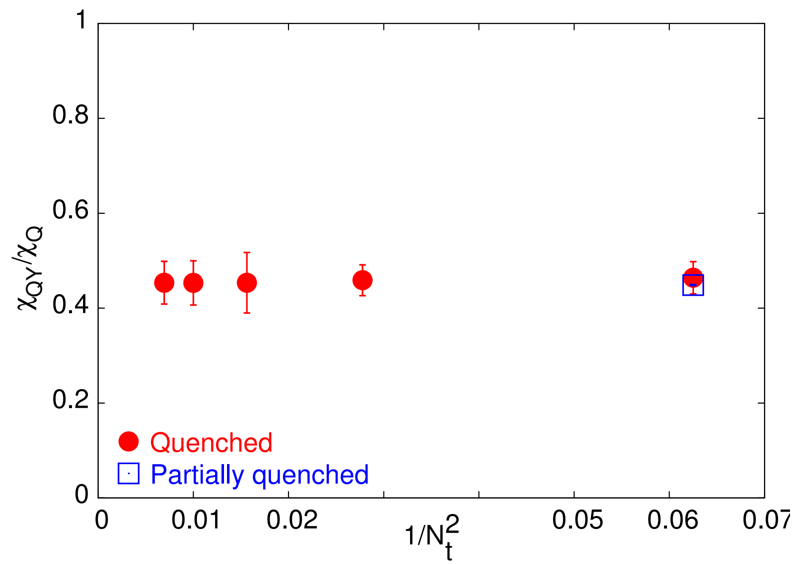


2.5
2.25
2
1.75
1.5
χ_B/χ_Q
Quenched
Partially quenched
0
0.01
0.02
0.05
0.06
0.07
$1/N_t^2$
1
0.8
0.6
0.4
0.2
0
χ_{QY}/χ_Q
Quenched
Partially quenched
0
0.01
0.02
0.05
0.06
0.07
$1/N_t^2$



Here
1) $C_{B/Q}$ and $C_{(QY)/Q}$ at $T = 2T_c$ exhibited as a function of lattice spacing.





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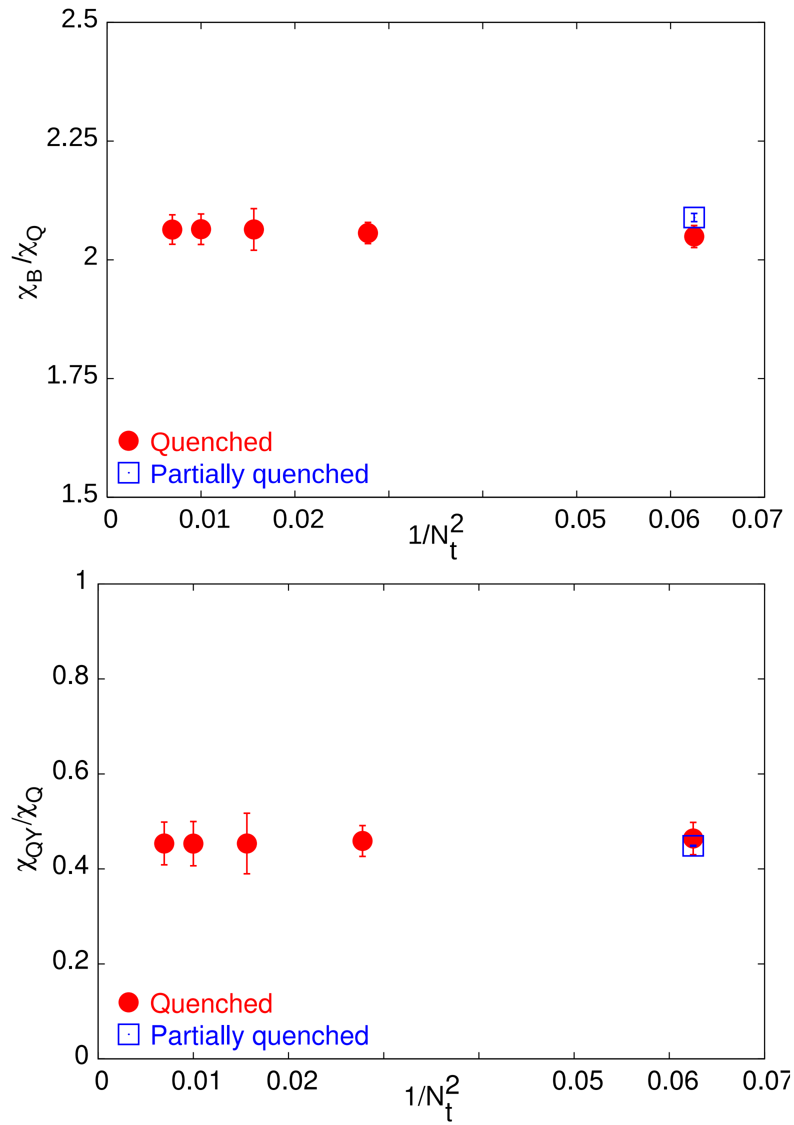
2) Partially Quenched $\Leftrightarrow$ Dynamical quarks of mass $0.1T_c$ on $16^3 \times 4$, corresponding to $m_\rho/T_c = 5.4$ and $m_\pi/m_\rho = 0.3$



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3) Valence light and strange quark masses : $m_v^{up}/T_c = 0.03$ and $m_v^{strange}/T_c \simeq$ 0.75-1.0.

Some Robust Predictions for various fluctuations thus are :

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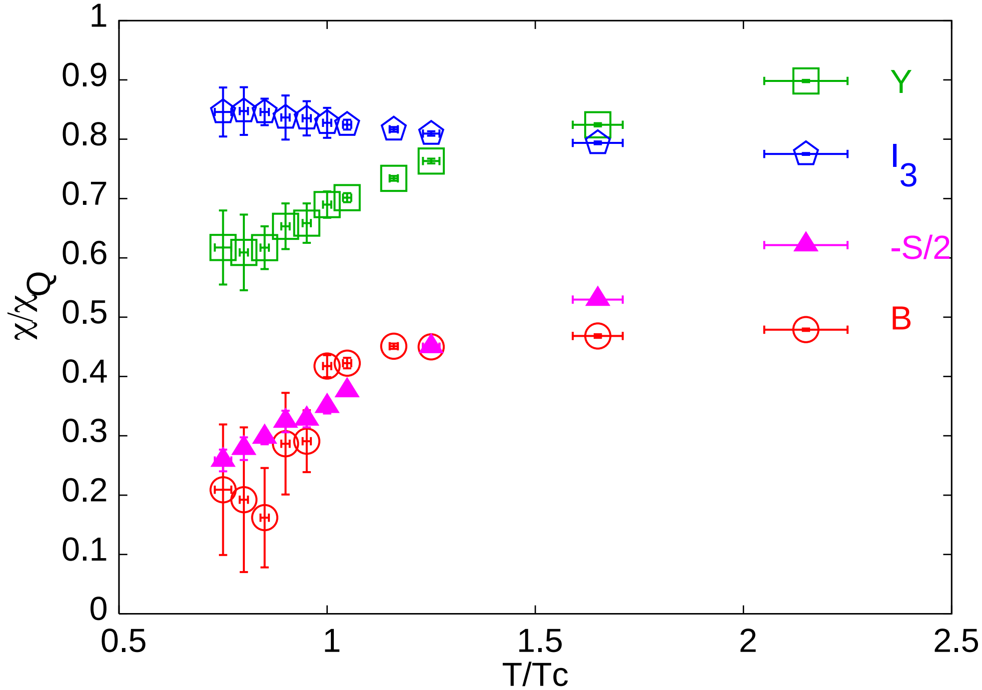

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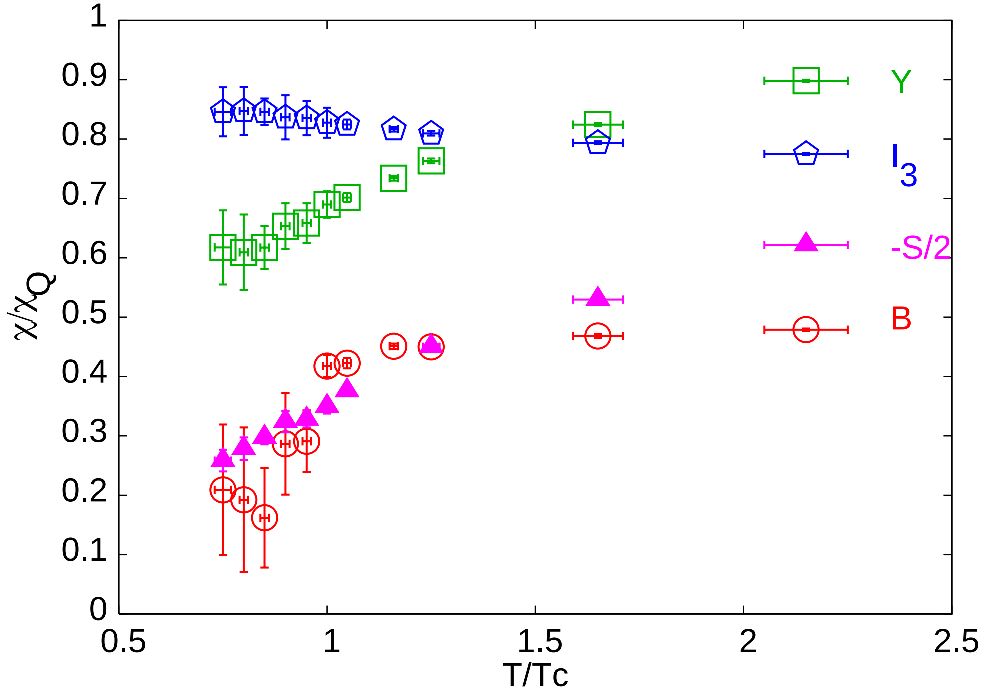

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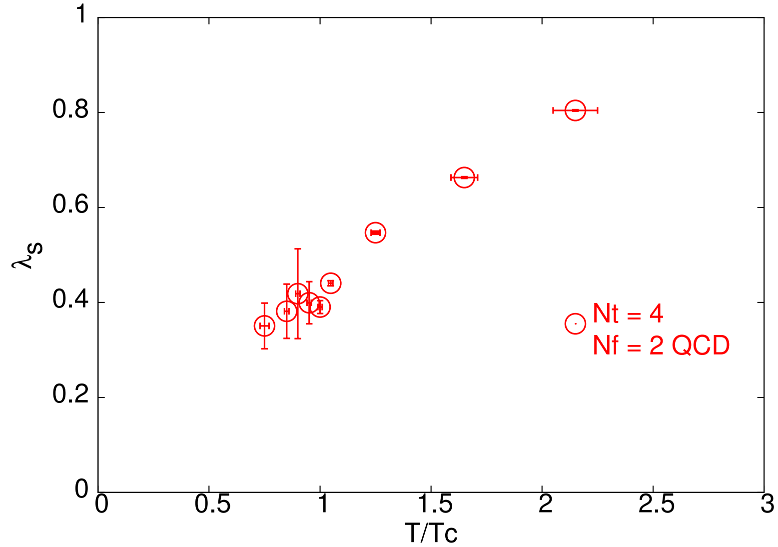
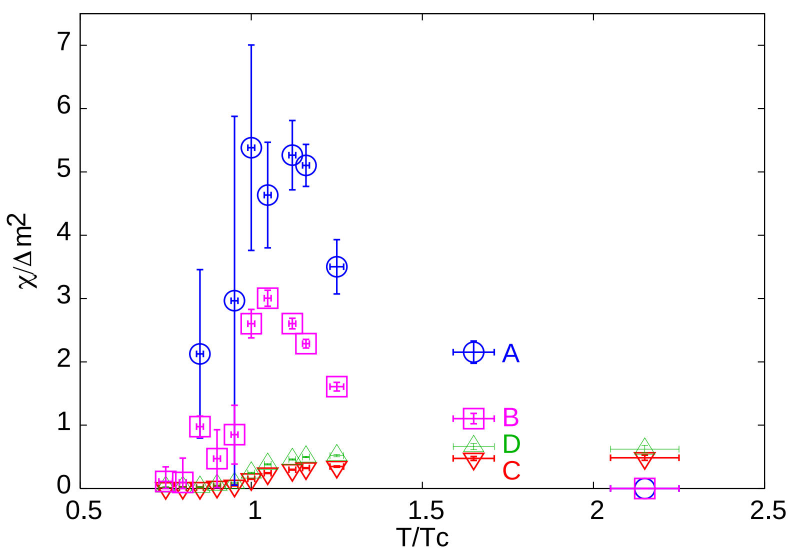
♡ $C_{S/Q}$ and $C_{B/Q}$ exhibit a large change in going from Hadronic phase to QGP.

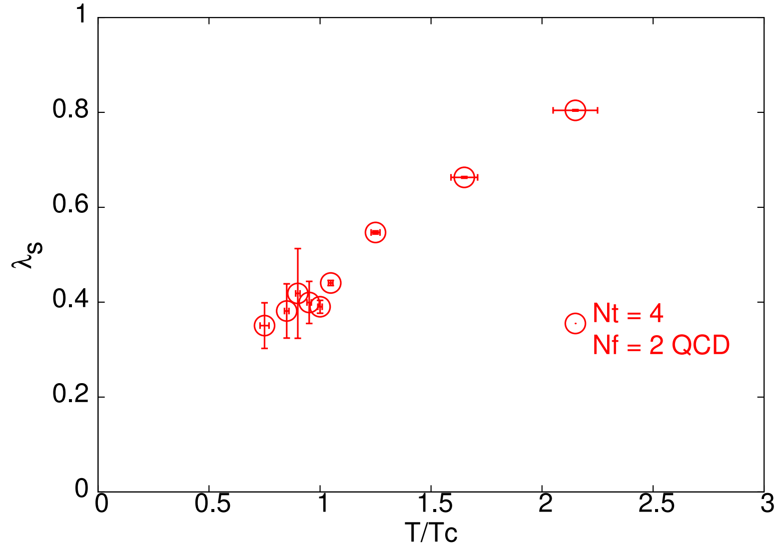
Wróblewski Parameter

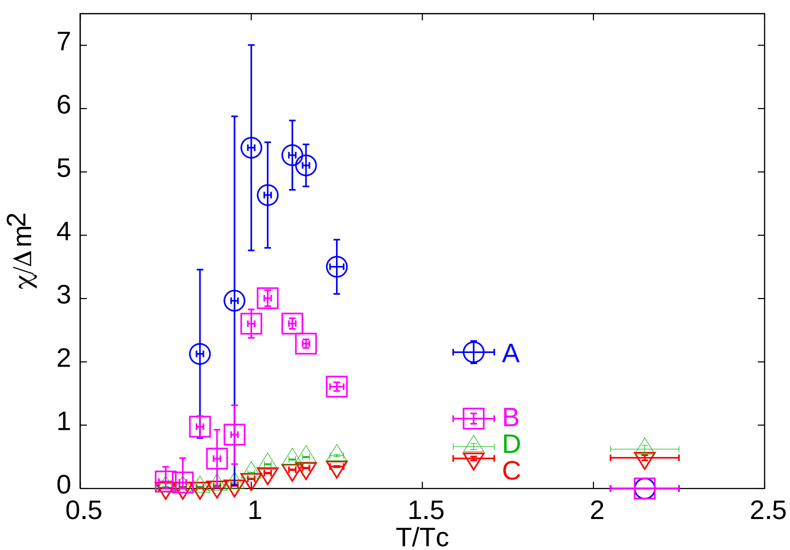


λs
T/Tc
Nt = 4
Nf = 2 QCD

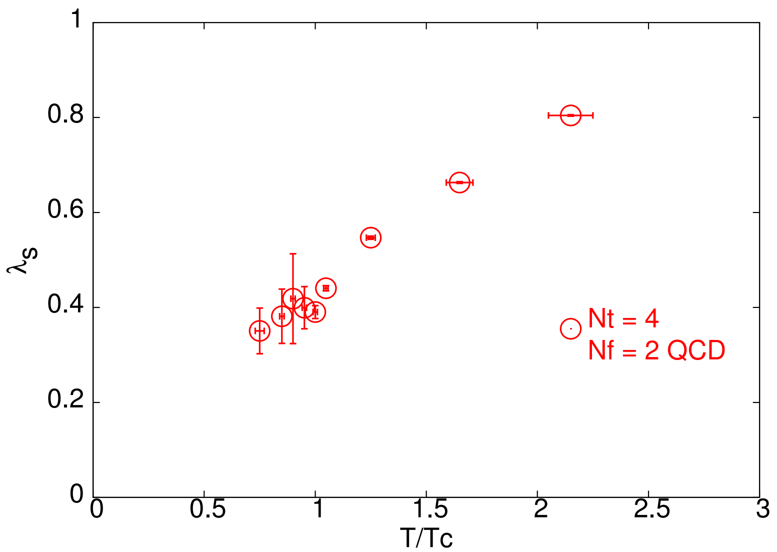


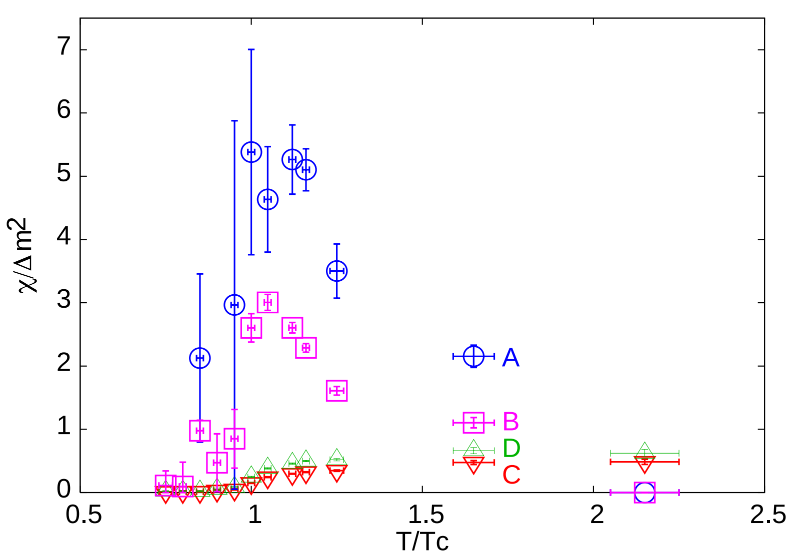
χ/Δ m2
T/Tc
A
B
D
C





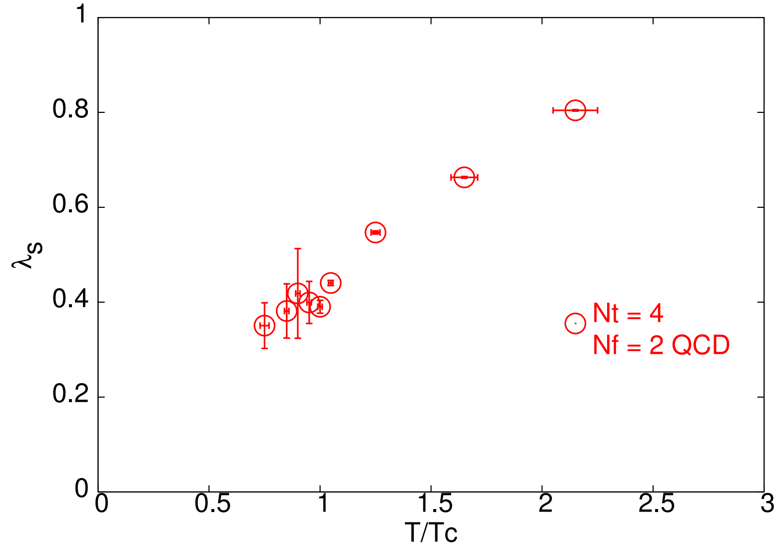
- Fluctuation-Dissipation Theorem, Kramers - Krönig relation & a relaxation time approximation $\Longrightarrow$ robust observable $C_{s/u} \equiv \lambda_s$.

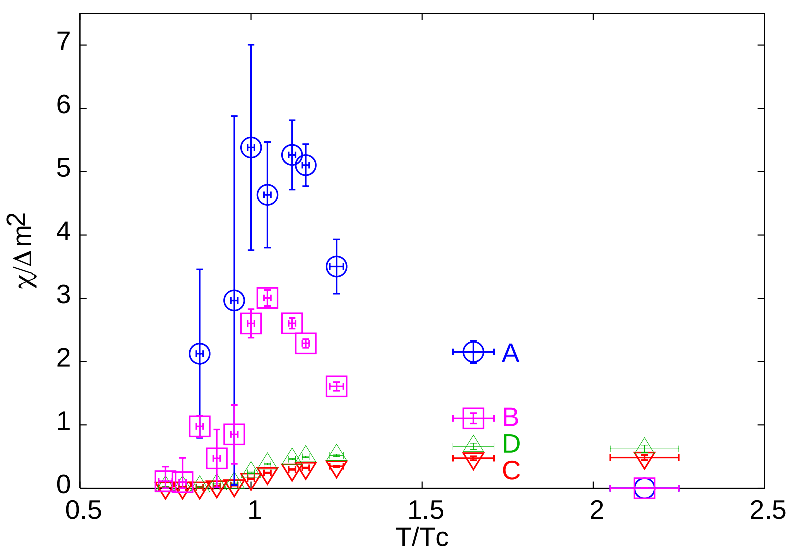




- Fluctuation-Dissipation Theorem, Kramers - Krönig relation & a relaxation time approximation $\Longrightarrow$ robust observable $C_{s/u} \equiv \lambda_s$.

- $\lambda_s \simeq 0.4$ close to T_c, in agreement with extractions from experiment (See, e.g., Cleymans, JPG 28 (2002) 1575.) and our own earlier result in Quenched QCD. Goes down at lower temperature.





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- Strongly dependent on m_s for $T \leq T_c$. χ_{BY}/Δ^2_{us}, curves A, D and C with $m_s/T_c = 0.1$, 0.75 and 1, hint at kinematic effects in the shape of λ_s.

Flavour Carriers : Quasi-quarks ?

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♠ Koch, Majumder and Randrup (PRL 95,182301(2005)) introduced

$$C_{BS} = -3C_{(BS)/S} = 1 + \frac{\chi_{us}+\chi_{ds}}{\chi_s} \ ,$$

to distinguish models of QGP excitations : $C_{BS} \approx 2/3$ for sQGP and unity for (ideal) quarks.

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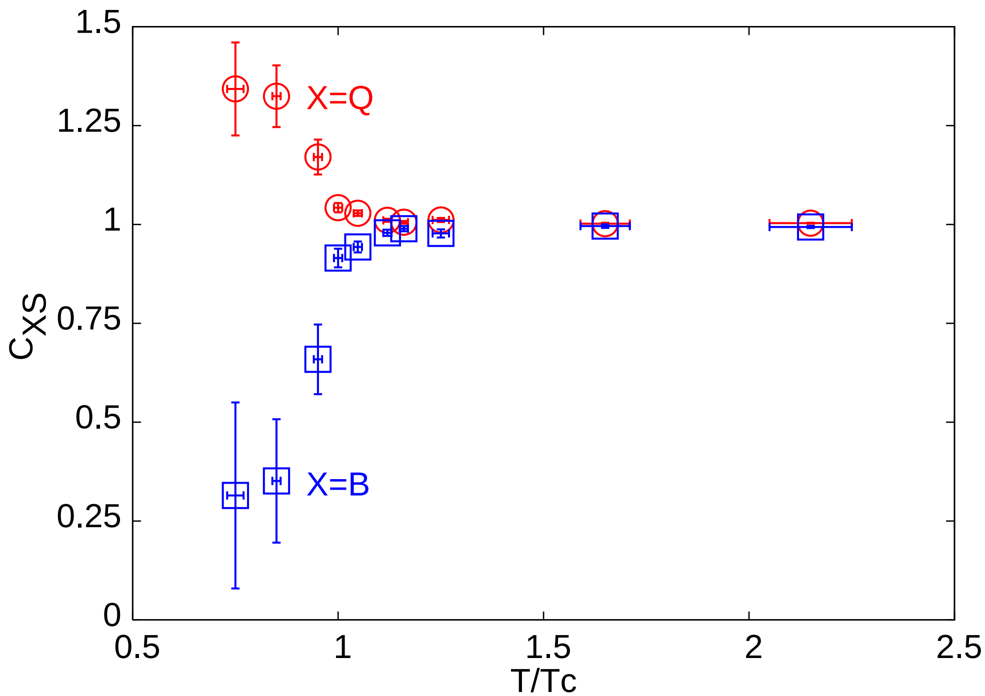
♣ Charge and Strangeness Correlation offers another similar possibility of being unity, if strangeness is carried by quarks :

$$C_{QS} = 3C_{(QS)/S} = 1 - \frac{2\chi_{us}-\chi_{ds}}{\chi_s} .$$

- First Results on C_{BS} and C_{QS} :



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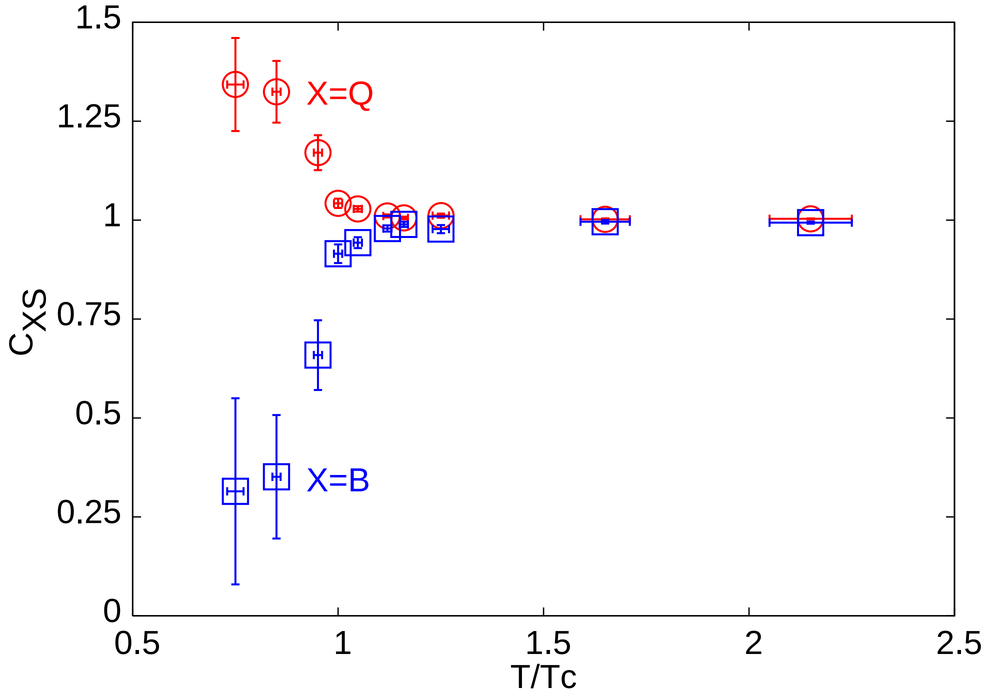

• Note that while both are different from unity below T_c, they become close to unity immediately above T_c:
$\Longrightarrow$ Unit strangeness is carried by objects with baryon number -1/3 and charge 1/3 near T_c.

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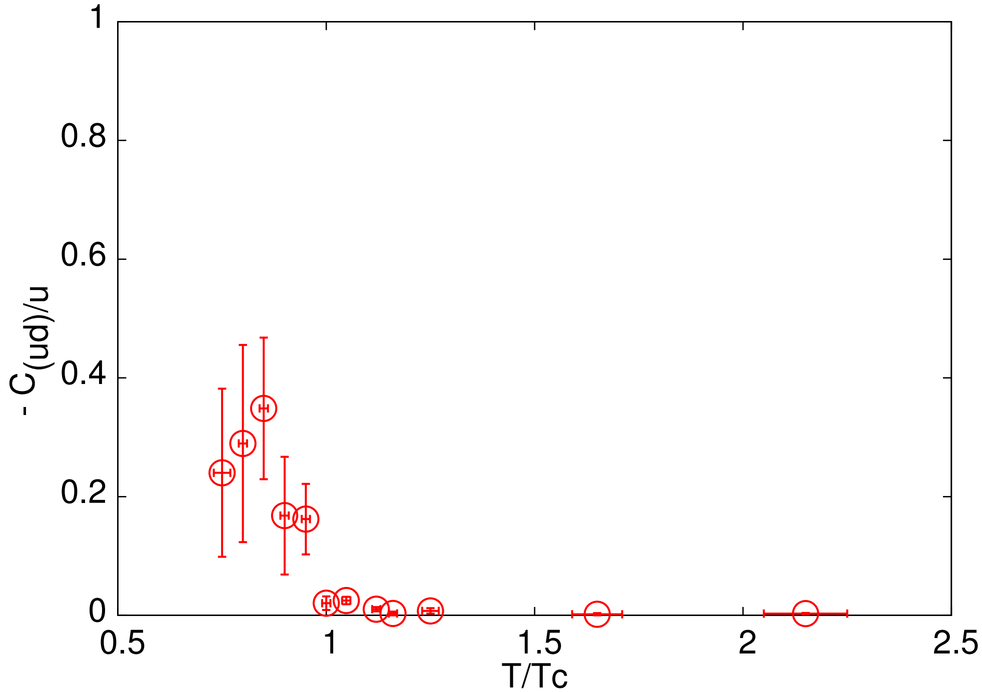
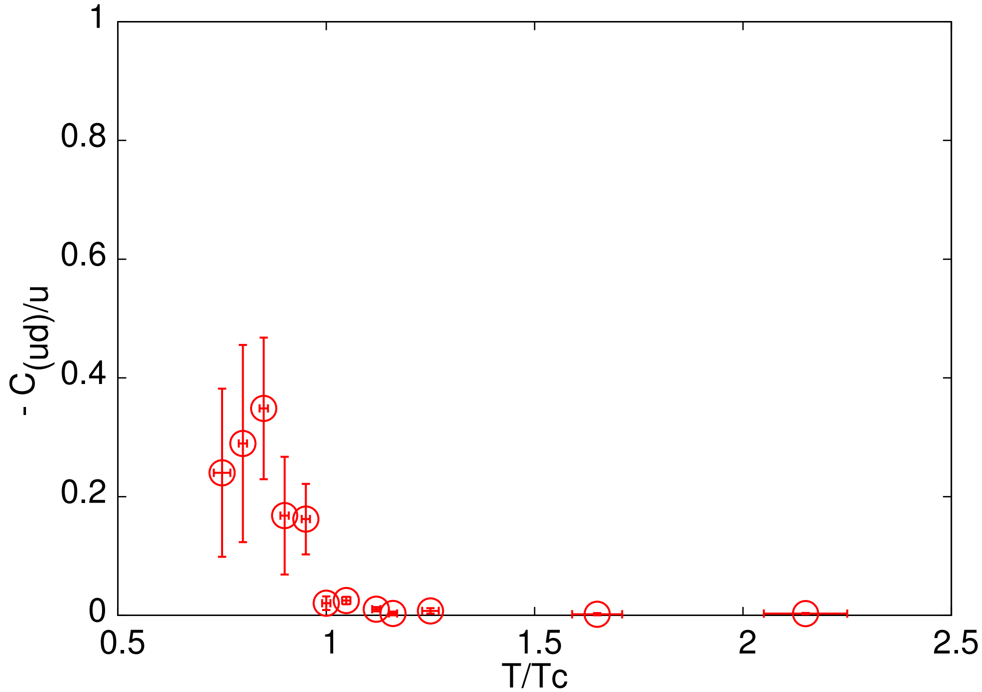
- Similar results in the light quark sector:
  From e.g., $C_{(BU)/U}$ and $C_{(QU)/U}$, or $C_{(BD)/D}$ and $C_{(QD)/D}$,
  $\Rightarrow u$ (d)-flavour is carried by $B = 1/3$ and $Q = 2/3$ (-1/3) objects.


1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
- C(ud)/u
T/Tc



- Interactions dress up quarks. Close to T_c the coupling is presumably not weak, but these flavour linkages seem to persist $\Rightarrow$ quasi-quarks.

Screening Lengths

- Using overlap quarks, we obtained screening lengths for $T \geq 1.25 T_c$ earlier for $N_t = 4$ & found better agreement for even π.
- Extend to larger N_t to check whether continuum limit improves it further and closer to ideal gas of quarks.

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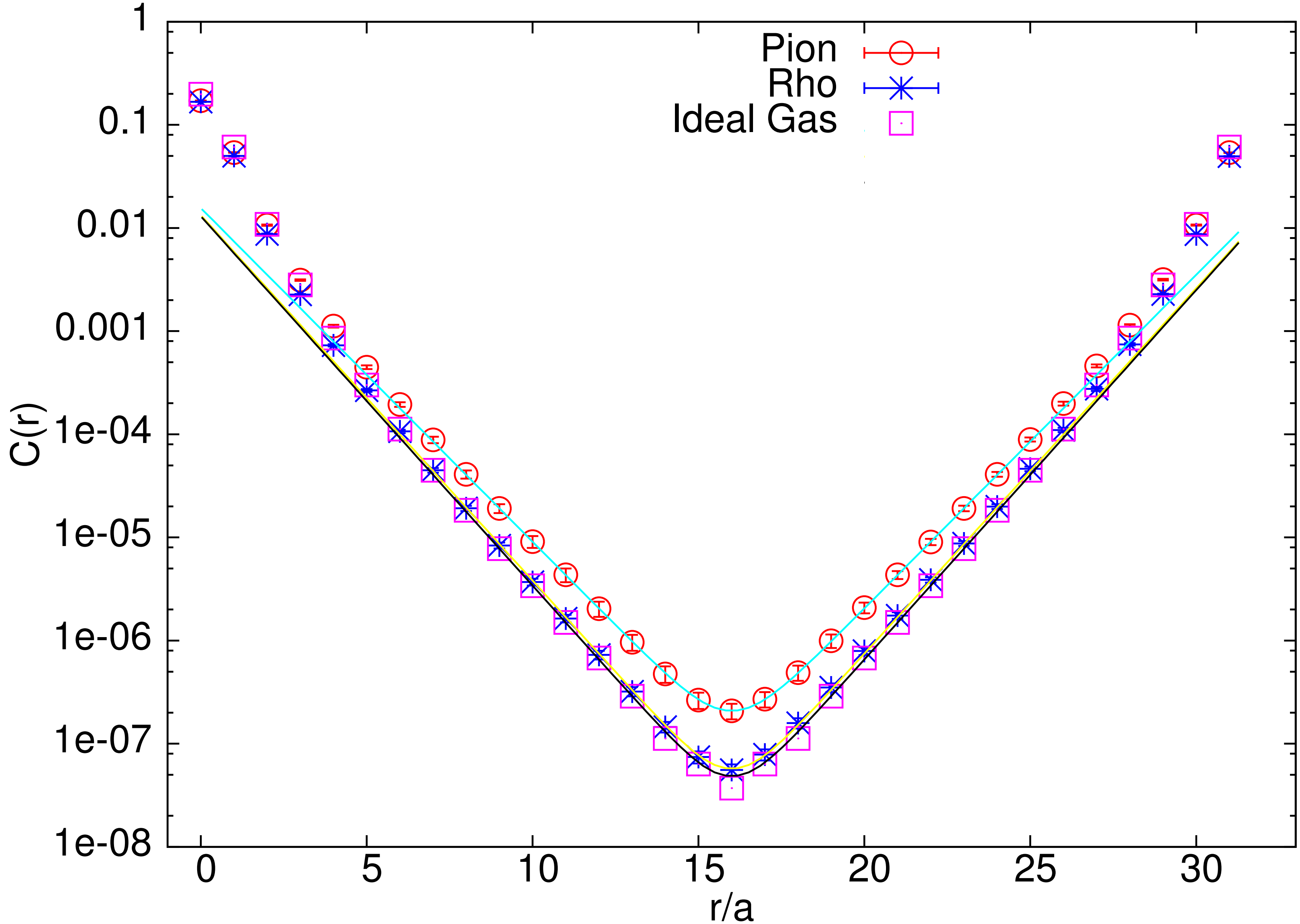
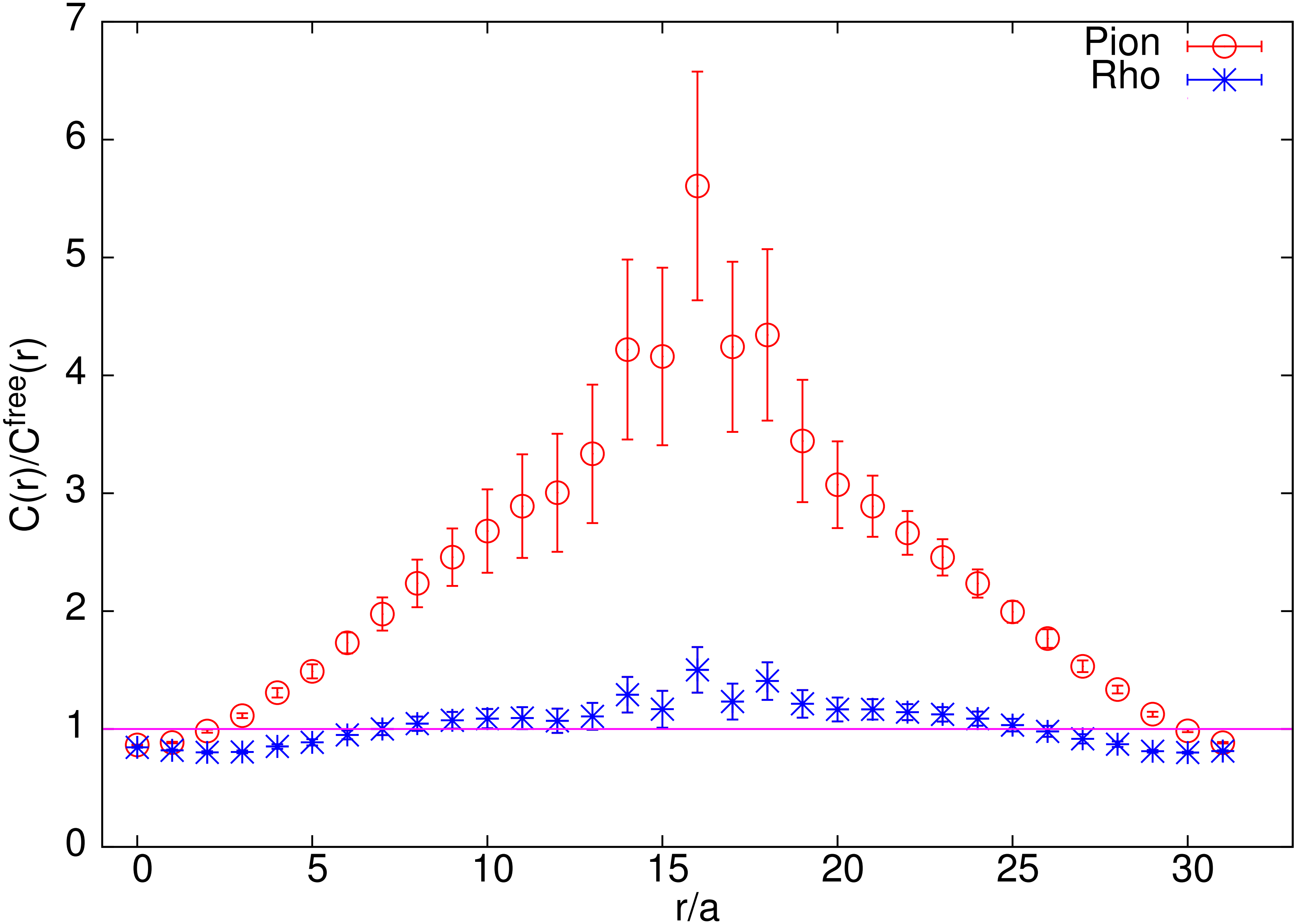
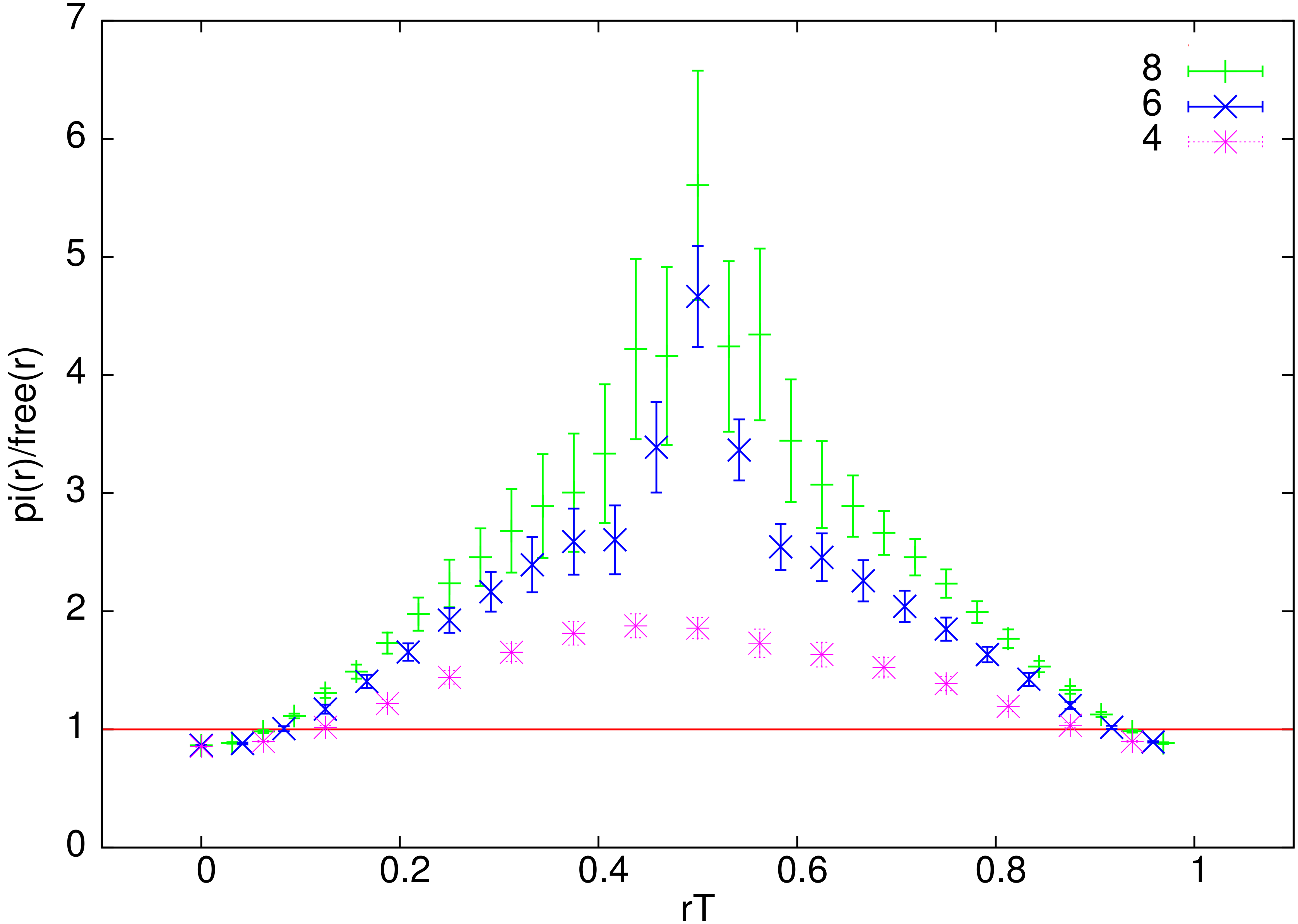
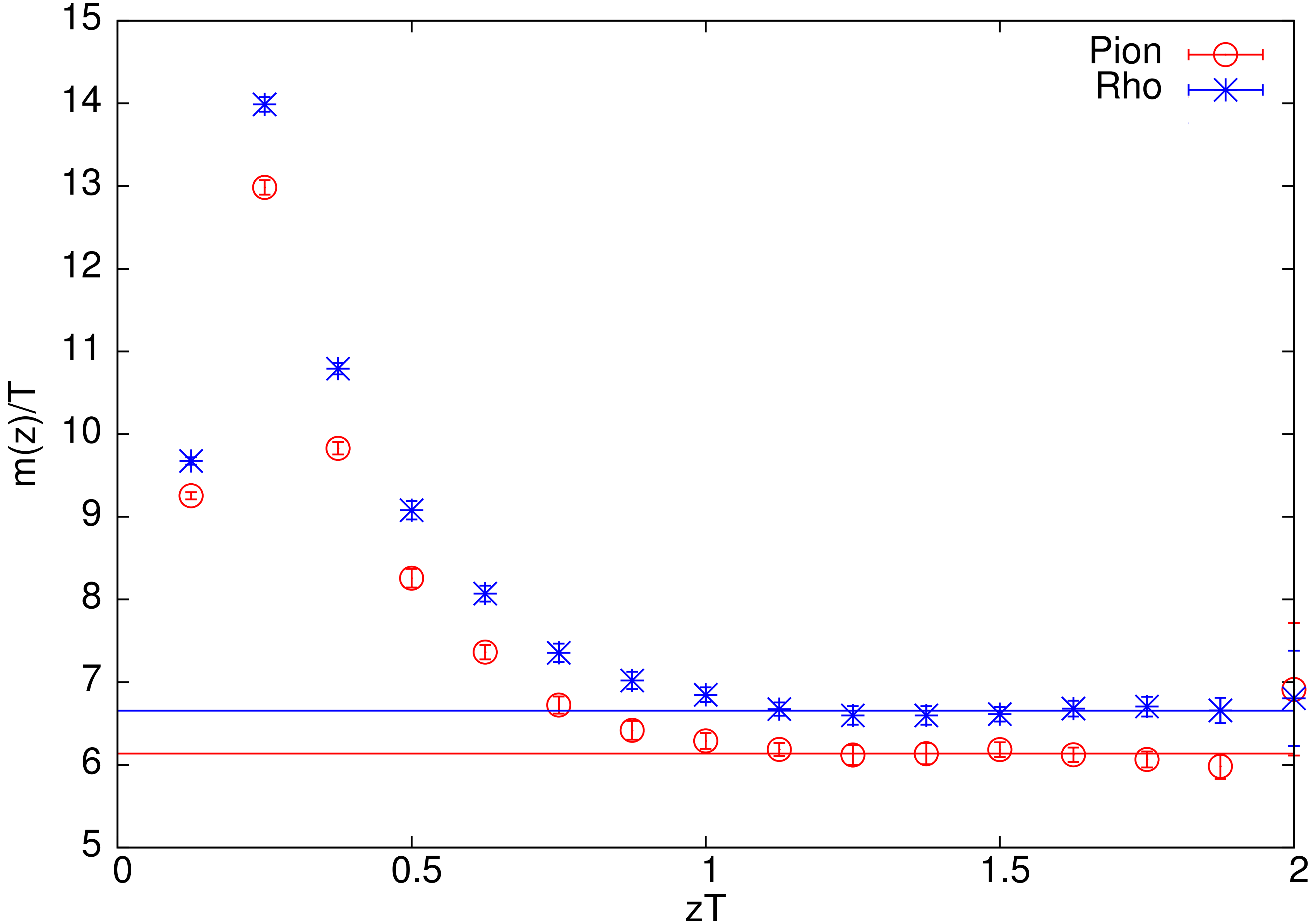
- Extend to larger N_t to check whether continuum limit improves it further and closer to ideal gas of quarks.
- Lattices used : $4 \times 10^2 \times 16$, $6 \times 14^2 \times 24$, $8 \times 18^2 \times 32$, 4×12^3, and 6×14^3.
- β values : 6.0625, 6.3384 and 6.55, β_c for $N_t = 8$, 12 and 16 respectively.
- Zolotarev Algorithm and Multi-Shift CG inversion used.


Pion
Rho
Ideal Gas
C(r)
r/a
1
0.1
0.01
0.001
1e-04
1e-05
1e-06
1e-07
1e-08
0
5
10
15
20
25
30

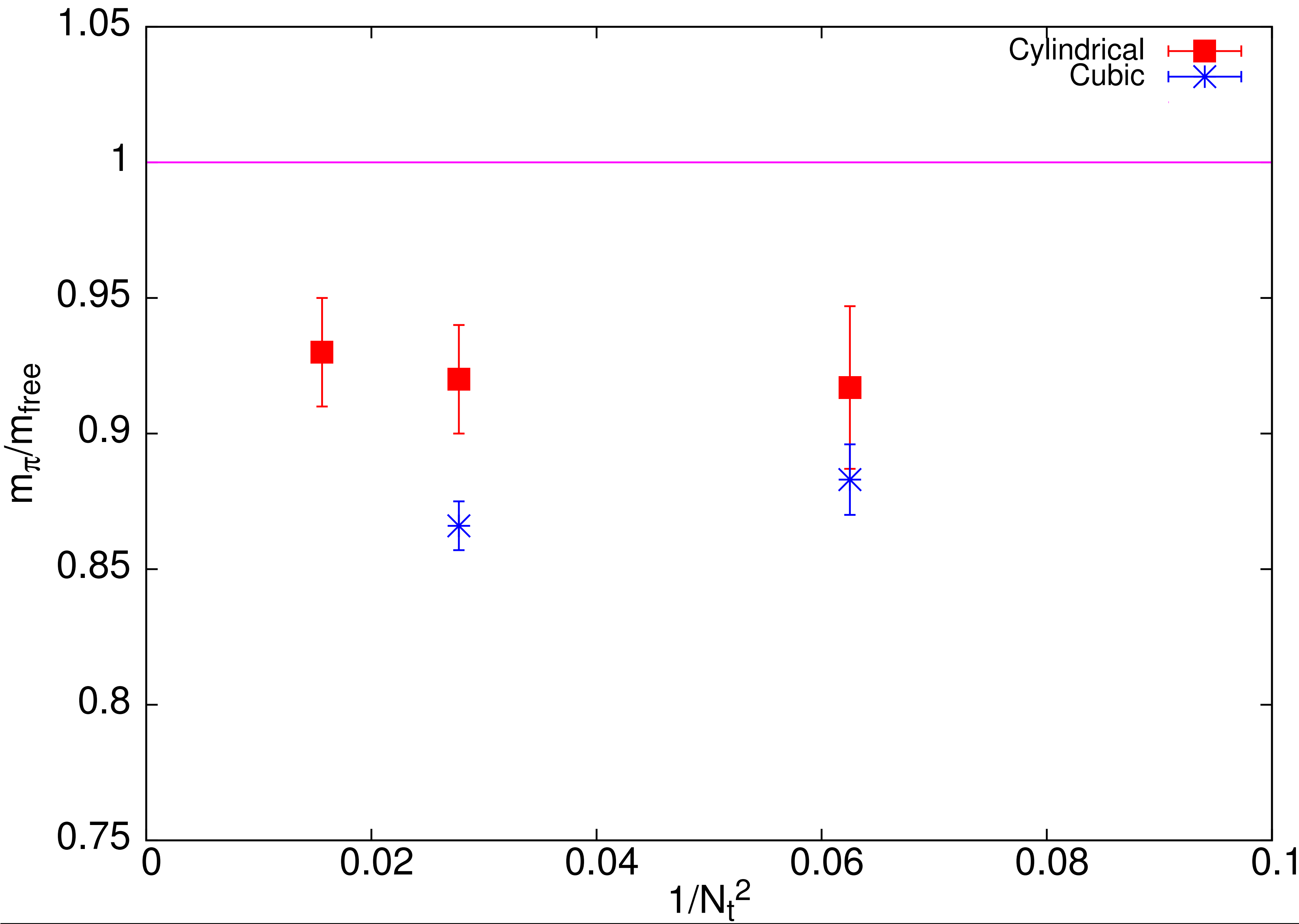


Pion
Rho
C(r)/C^{free}(r)
r/a
0
1
2
3
4
5
6
7
0
5
10
15
20
25
30


pi(r)/free(r)
rT
8
6
4
0
1
2
3
4
5
6
7
0
0.2
0.4
0.6
0.8
1



Pion
Rho
m(z)/T
zT
15
14
13
12
11
10
9
8
7
6
5
0
0.5
1
1.5
2



Cylindrical
Cubic
1.05
1
0.95
0.9
0.85
0.8
0.75
0
0.02
0.04
0.06
0.08
0.1
m_π/m_{free}
$1/N_t^2$

Summary

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- Flavour linkages of excitations demonstrate that High Temperature phase of QCD essentially consists of quasi-quarks.

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- Screening lengths exhibit excellent *single cosh* behaviour & very little a-dependence : π continues to be ~ 10 % below ideal gas value.