Excursions in QCD Phase Diagram

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Motivation

Quark Number Susceptibility

EoS for nonzero baryon density

Screening Lengths

Summary

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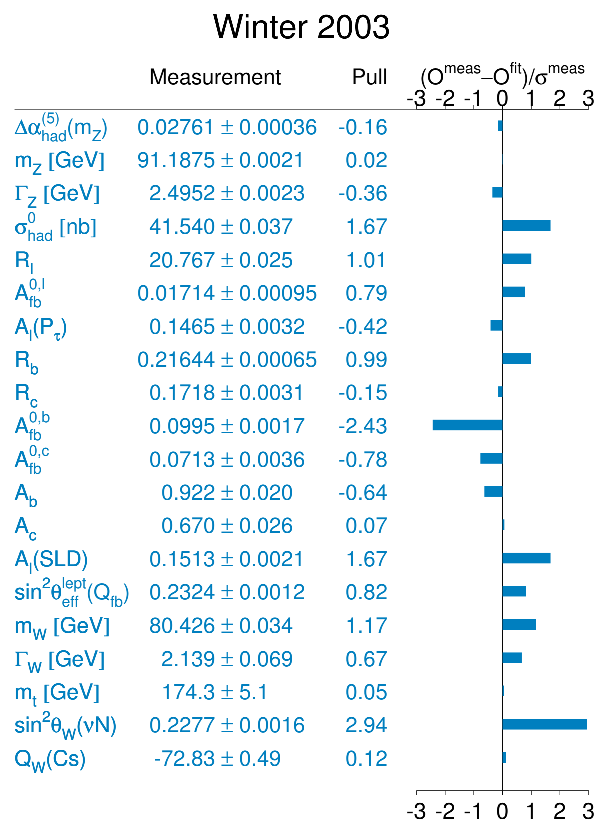
- Standard Model – Very Successful !
- Precision tests from LEP
- All tests based on perturbation theory
- Need to understand non-perturbative QCD to explain baryonic matter in our Universe, i.e., us.
- Lattice QCD – only well-understood, viable tool for this.



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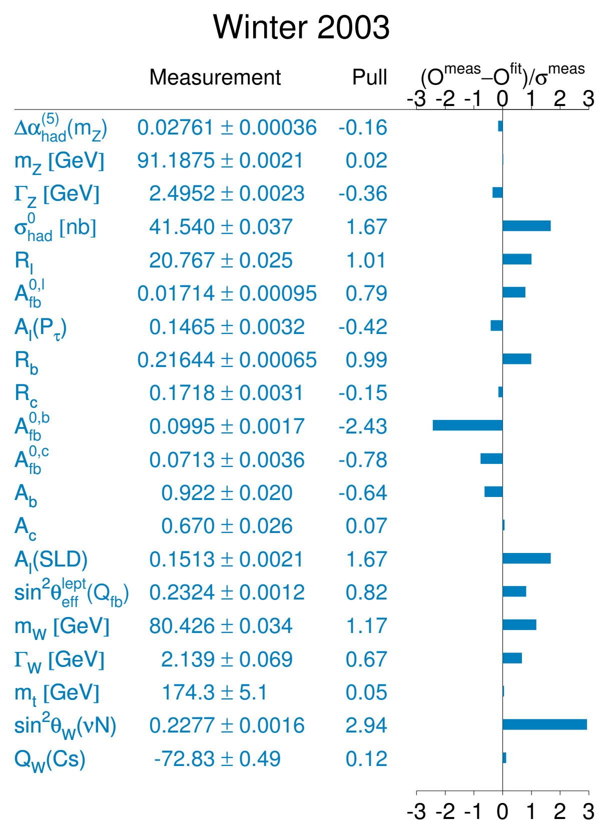

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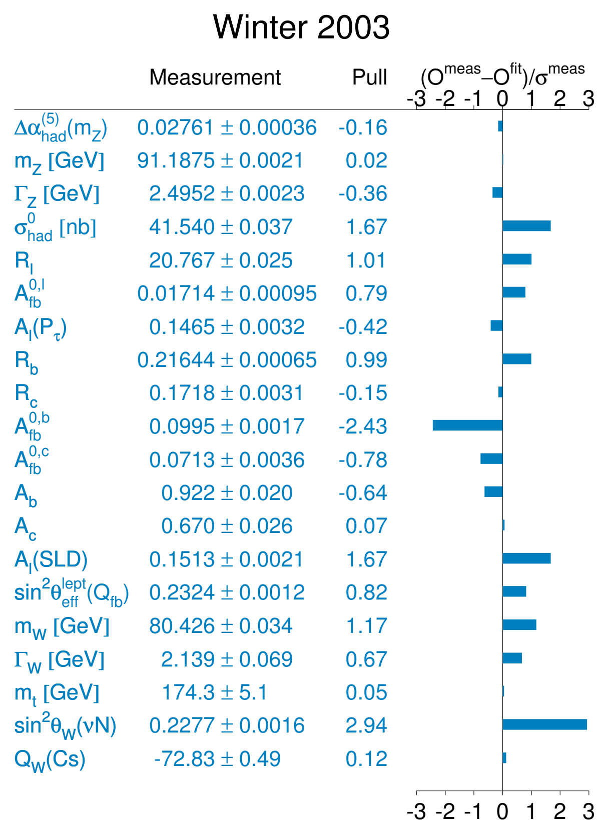

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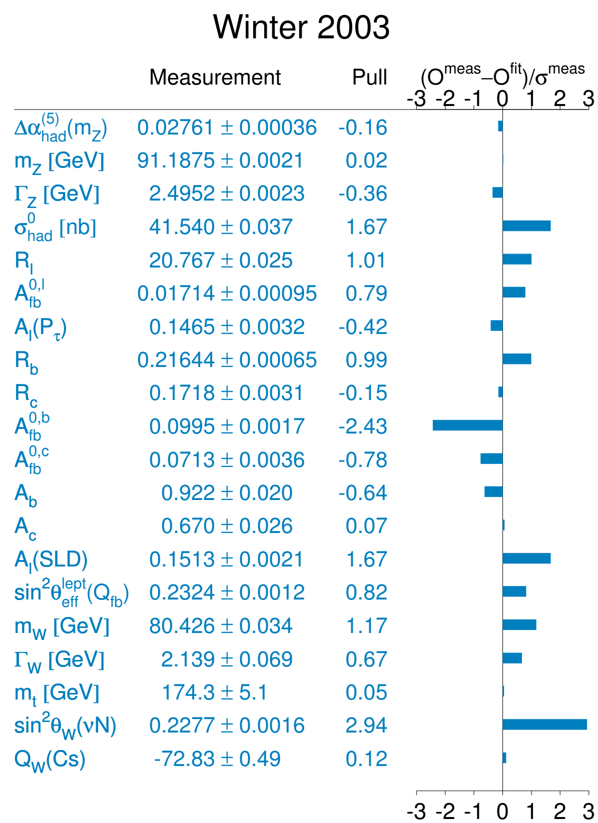

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- Gauge invariance $\longrightarrow$ Actions from Closed Wilson loops, e.g., plaquette.
- Fermion Doubling Problem $\longrightarrow$
  - Staggered Fermions (partial chiral and flavour symmetry),
  - Wilson fermions (only flavour symmetry),

– Recent Overlap fermions (exact chiral and flavour symmetry).

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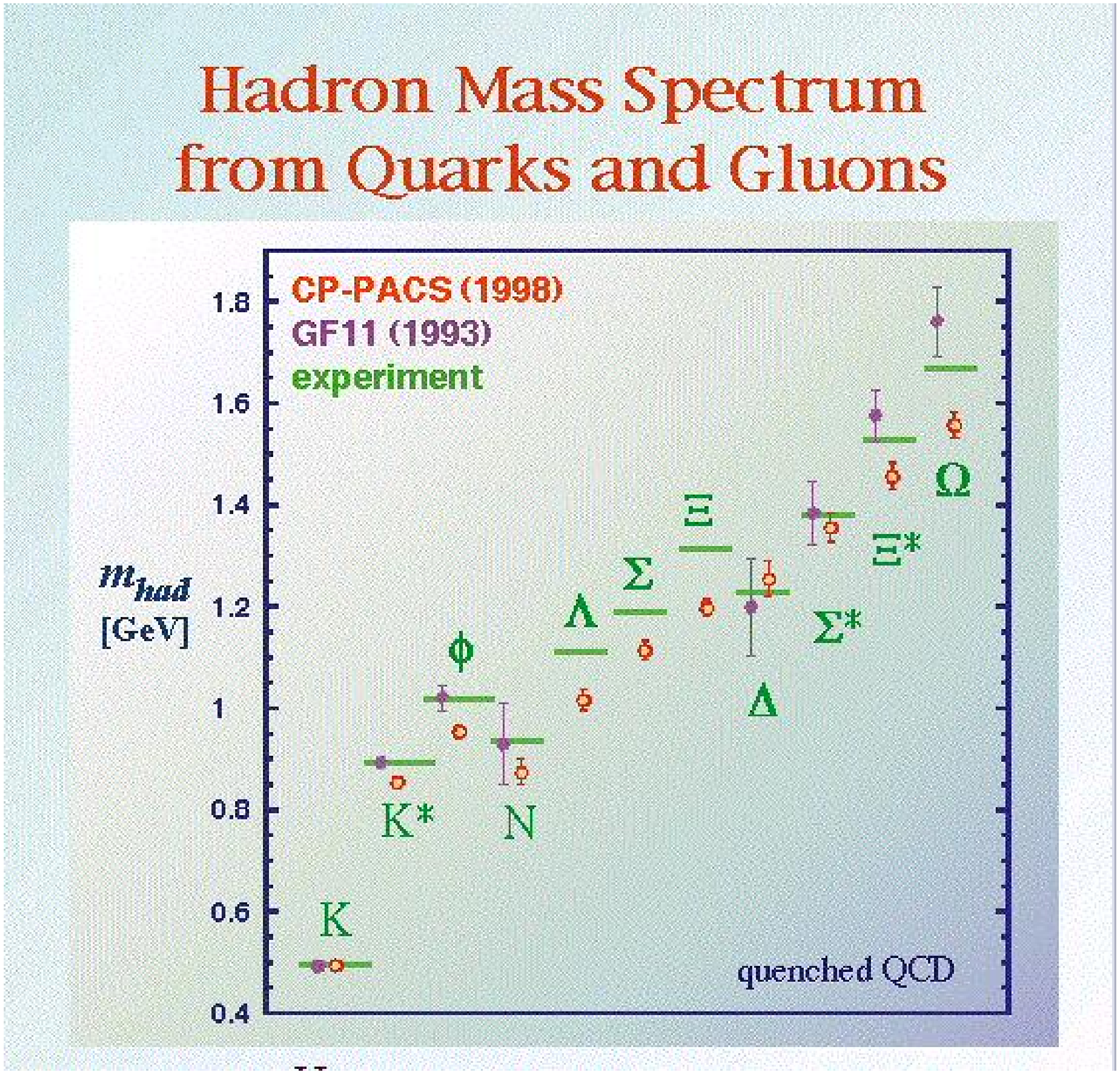
Typically, we need to evaluate

$$\langle \Theta(m_v) \rangle = \frac{\int DU \exp(-S_G)\Theta(m_v) \ \mathrm{Det} \ M(m_s)}{\int DU \exp(-S_G) \ \mathrm{Det} \ M(m_s)} \ , \tag{1}$$

where M is the Dirac matrix in x, colour, spin, flavour space for fermions of mass m_s, S_G is the gluonic action, and the observable Θ may contain fermion propagators of mass m_v.

Since $\langle \Theta \rangle$ is computed by averaging over a set of configurations $\{U_\mu(x)\}$ which occur with probability $\propto \exp(-S_G) \cdot \mathrm{Det}\ M$, the complexity of evaluation of Det $M \Longrightarrow$ approximations : Quenched ($m_s = \infty$ limit), Partially Quenched (low $m_s = m_u = m_d$), and Full (including a heavier s quark).

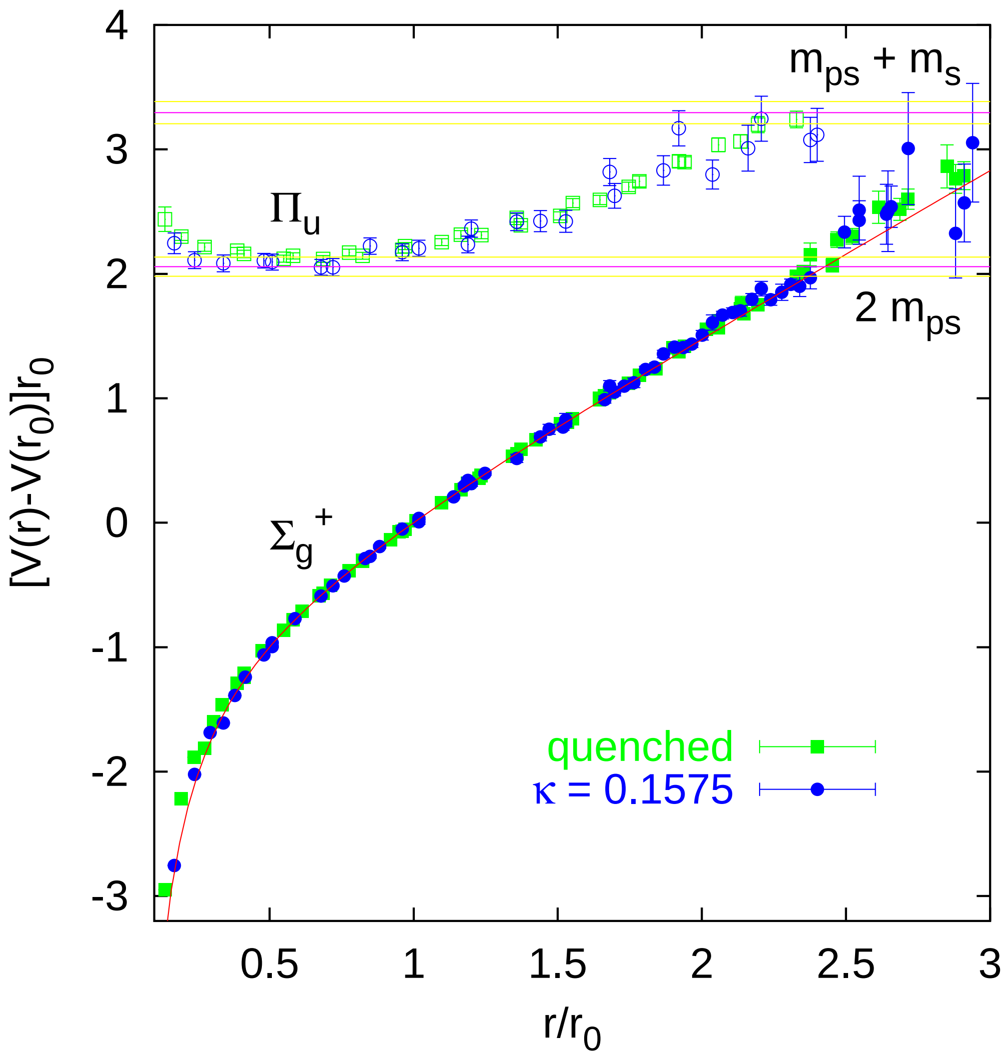
Q $\rightarrow$ PQ $\rightarrow$ Full $\rightsquigarrow$ Computer time $\uparrow$ and Precision $\downarrow$.



Baryon mass comes out (almost) right.

At least in Quenched Approximation

(From CP-PACS Collaboration, Japan)



As does the heavy quark potential $V_{Q\bar{Q}}$.

Here r_0 is roughly 0.5 fm.

(Bali, Phys. Rep. 343 (2001) 1.)

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- Relevant for physics of Heavy Ion collisions, Early Universe and perhaps quark stars.
- Theoretically profound : A new critical point ?
- Lattice details :
  - $N_s^3 \times N_t$ Lattice, $N_s \gg N_t$ for $T \neq 0$,
  - Spatial Volume $V = N_s^3 a^3$,
  - Temperature $T = 1/N_t a(\beta)$,
  - Chemical potential: Multiply each $U_4(x)$ by $f(a\mu)$ and $U_4^\dagger(x)$ by $1/f(a\mu)$, where $f(a\mu) = 1 + a\mu + \mathcal{O}(a^2)$. (Gavai, PRD '85)

- Known choices : $f_{HK}(x) = \exp(x)$ and $f_{BG} = (1+x)/\sqrt{1-x^2}$.

  (Hasenfratz-Karsch '83, Bilić-Gavai, '84)

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- Order Parameters : Chiral condensate $\langle \bar{\psi}\psi \rangle$,
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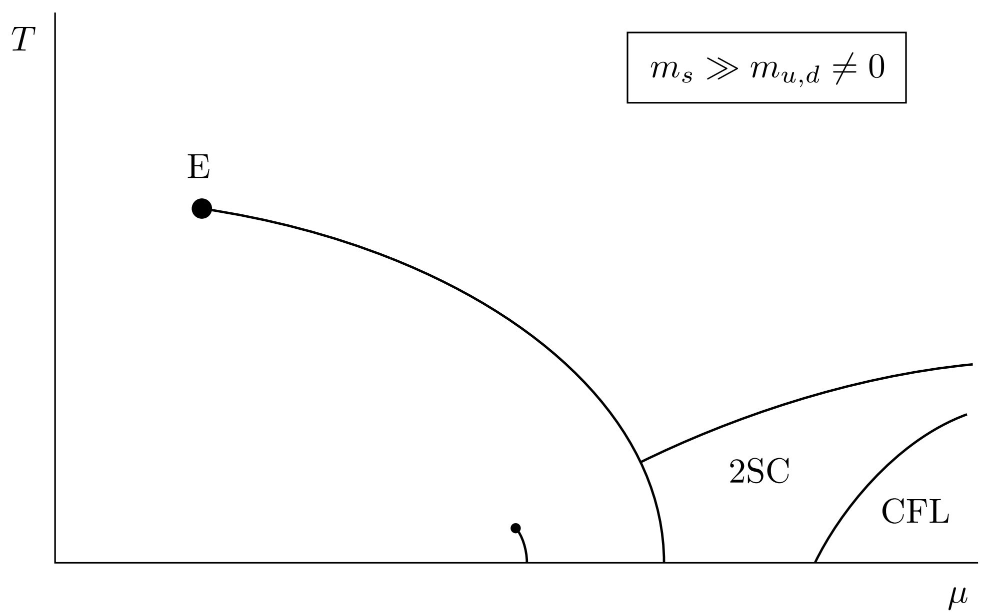

$\mu_B \neq 0$

- Phase Problem : Det $M(\mu)$ is complex for $\mu \neq 0$.
- Early results in quenched approximation and $T = 0$:- $\langle \bar{\psi}\psi \rangle = 0$ at $\mu_B \sim m_\pi$!
- Exciting results in recent past for small μ, starting in the $T_c(\mu = 0)$ neighbourhood.
  - Re-weighting Method (Fodor & Katz, JHEP '02)
  - Imaginary μ (de Forcrand & Philipsen, NPB '02, D'Elia & Lombardo, PRD '03)
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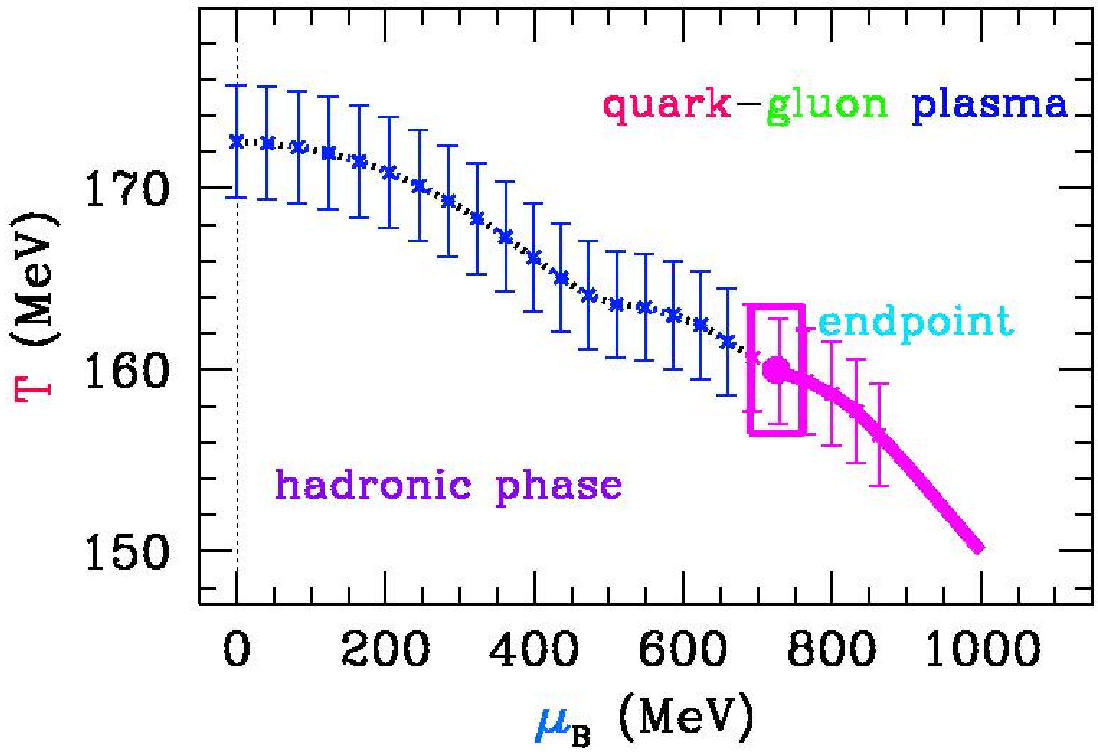
Fodor-Katz Results



$N_s^3 \times 4$ Lattices,
$N_s =$ 4,6,8;
Bit heavy u,d quarks.
Critical End-point :
$T =$ 160(4) MeV,
$\mu =$ 725(35) MeV

How reliable are these results ? Methods, Prescription dependence...

We address some of these issues via Quark Number Susceptibilities.

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Definitions: For u, d, and s quarks, the partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \prod_{f=u,d,s} \mathrm{Det}\ M(m_f, \mu_f) \quad , \tag{2}$$

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$$n_i = \frac{T}{V}\frac{\partial \ln \mathcal{Z}}{\partial \mu_i}, \qquad \chi_{ij} = \frac{T}{V}\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}$$

Setting $\mu_i = 0$, n_i =0 but χ_{ij} are nontrivial. Diagonal χ's are

$$\chi_0 = \frac{T}{2V}[\langle \mathcal{O}_2(m_u) + \frac{1}{2}\mathcal{O}_{11}(m_u)\rangle] \quad (3)$$

$$\chi_3 = \frac{T}{2V} \ \langle \mathcal{O}_2(m_u)\rangle \quad (4)$$

$$\chi_s = \frac{T}{4V}[\langle \mathcal{O}_2(m_s) + \frac{1}{4}\mathcal{O}_{11}(m_s)\rangle] \quad (5)$$

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$\mathrm{Tr}\ A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\mathrm{Tr}\ A)^2 = 2\sum_{i>j=1}^{L} (\mathrm{Tr}\ A)_i (\mathrm{Tr}\ A)_j / L(L-1)$,
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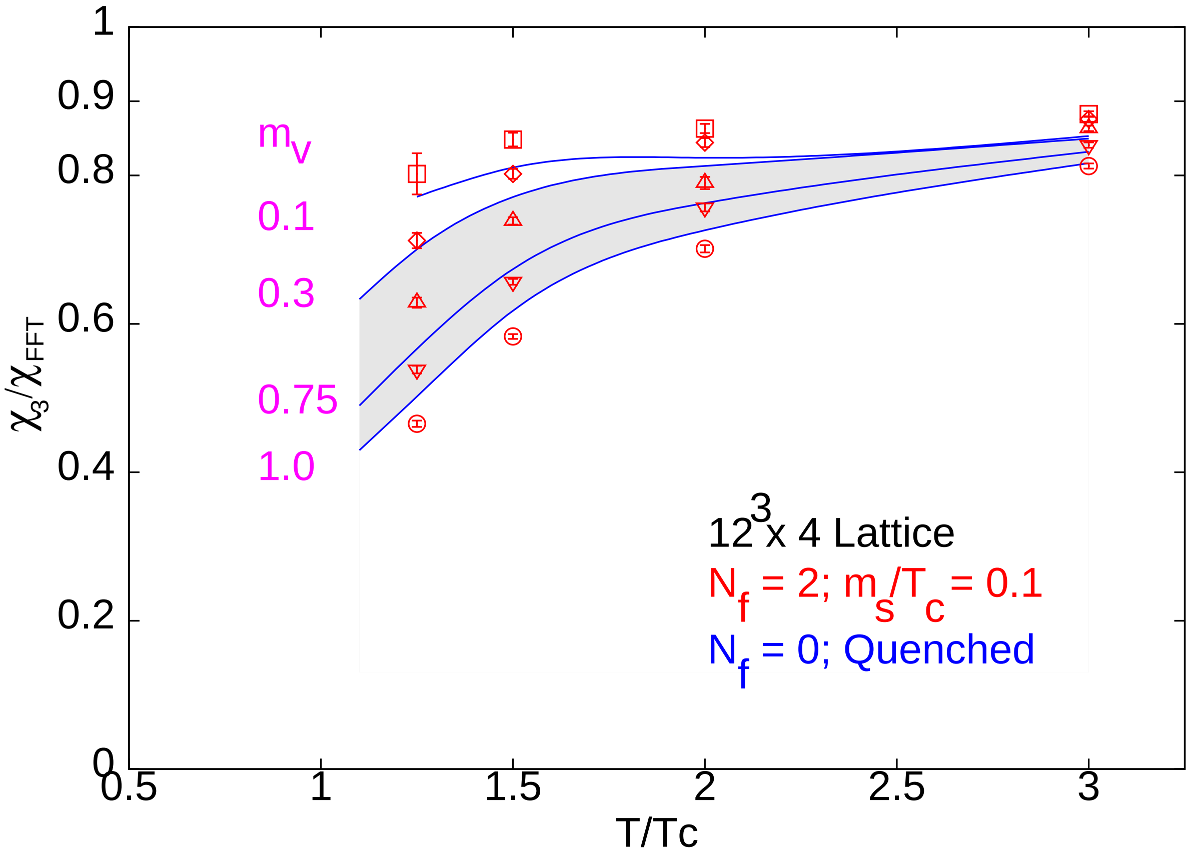
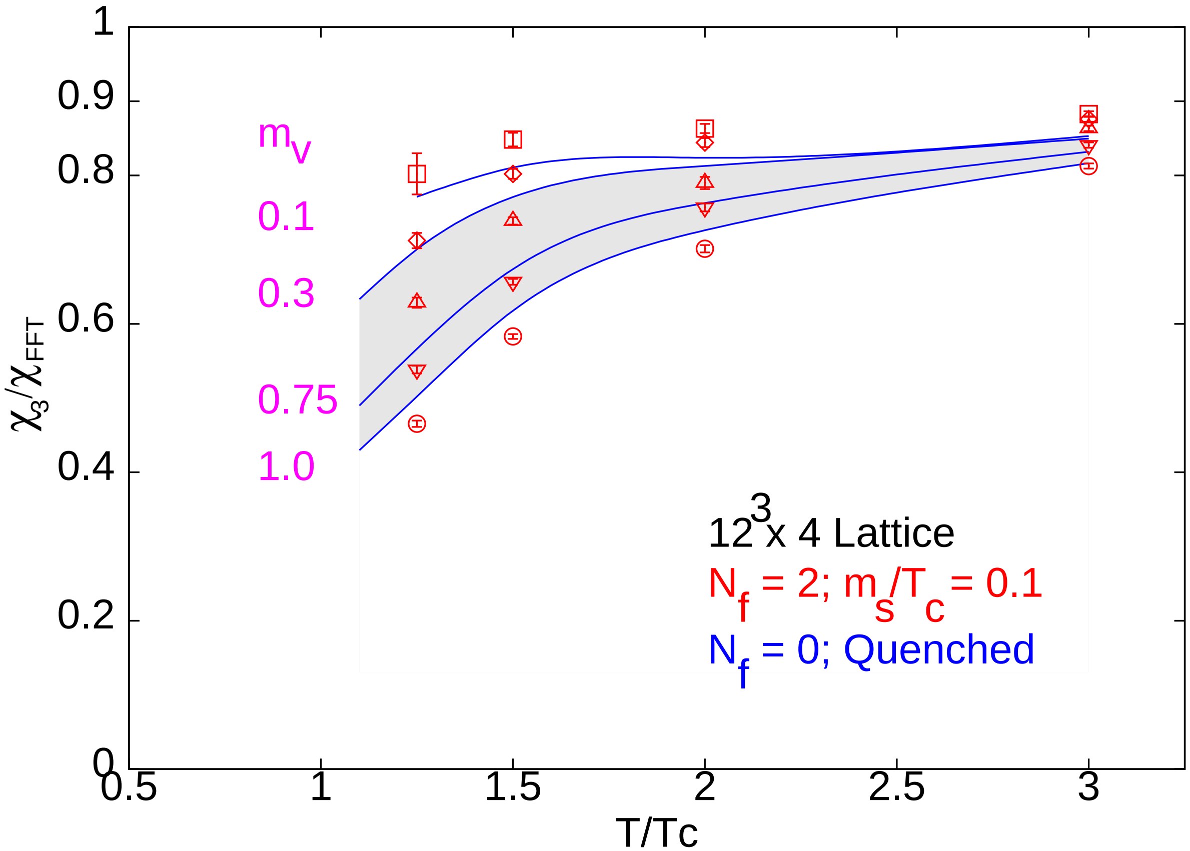
Gavai & Gupta PR D '01; Gavai, Gupta & Majumdar, PR D 2002

χ_{FFT} — Ideal gas results for same Lattice.



m_V
0.1
0.3
0.75
1.0
12^3 x 4 Lattice
N_f = 2; m_s/T_c = 0.1
N_f = 0; Quenched
χ_3/χ_{FFT}
T/Tc



Note that PDG values for strange quark mass $\Longrightarrow$

$m_v^{strange}/T_c \simeq$ 0.3-0.7 (N_f=0); 0.45-1.0(N_f=2).

Perturbation Theory

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Weak coupling expansion gives:

$$\frac{\chi}{\chi_{FFT}} = 1 - 2(\frac{\alpha_s}{\pi}) + 8\sqrt{(1 + 0.167N_f)}(\frac{\alpha_s}{\pi})^{\frac{3}{2}}$$

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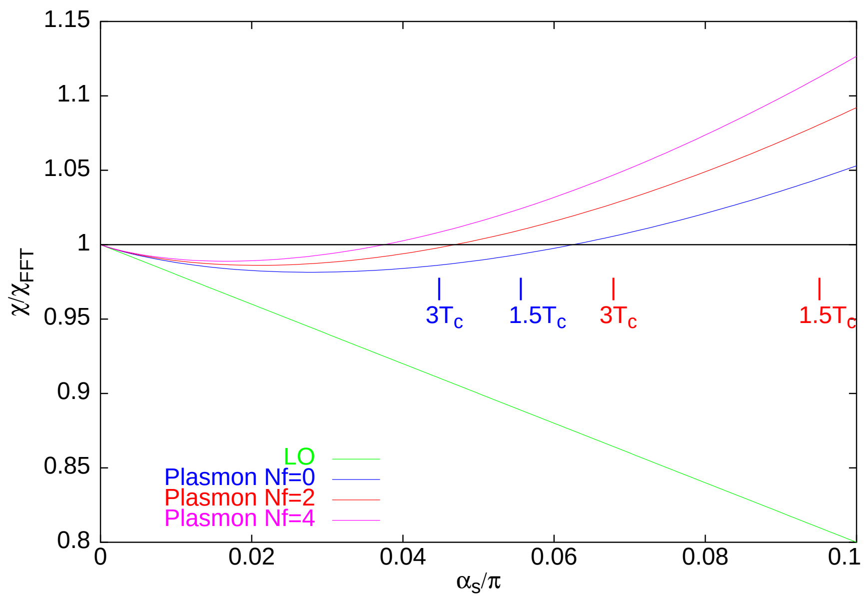

♣ Minm 0.981 (0.986) at 0.03 (0.02) for $N_f = 0$ (2).

♣ For $1.5 \leq T/T_c \leq 3$ pert. theory $\longrightarrow$ 0.99-0.98 (1.08=1.03) for $N_f = 0$ (2).

Resumed Perturbation Theory

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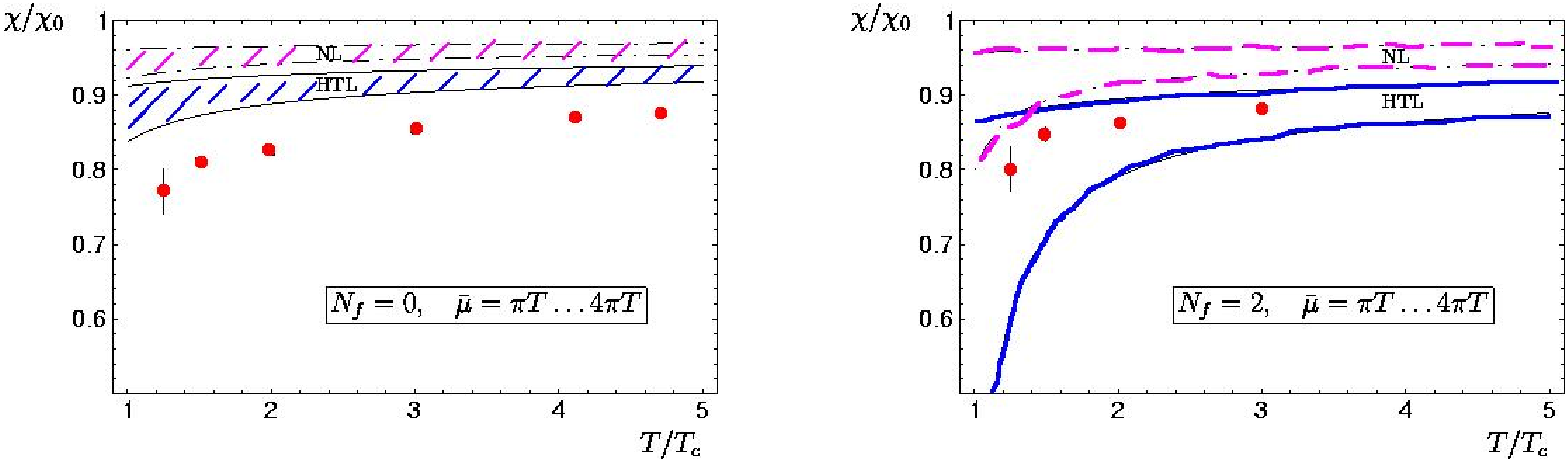

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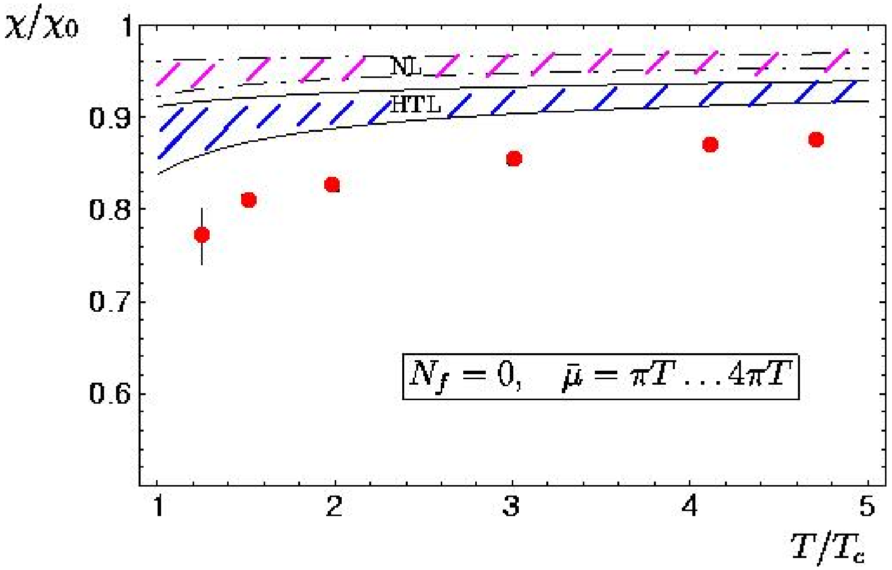

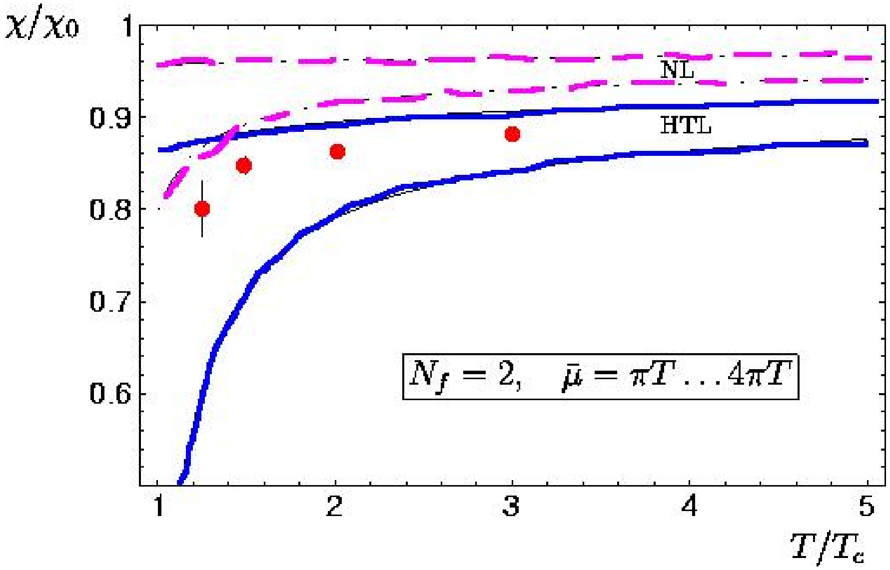

Our results for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Check for larger N_t and improved actions.

χ_{ud}

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Off-diagonal Susceptibility : $\chi_{ud} = \langle \frac{T}{V} \text{ Tr } M_u^{-1} M_u' \text{Tr } M_d^{-1} M_d' \rangle$

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Using the same scale and α_s as for $\chi_3 \longrightarrow \chi_{ud} \sim O(10^{-4})$!!

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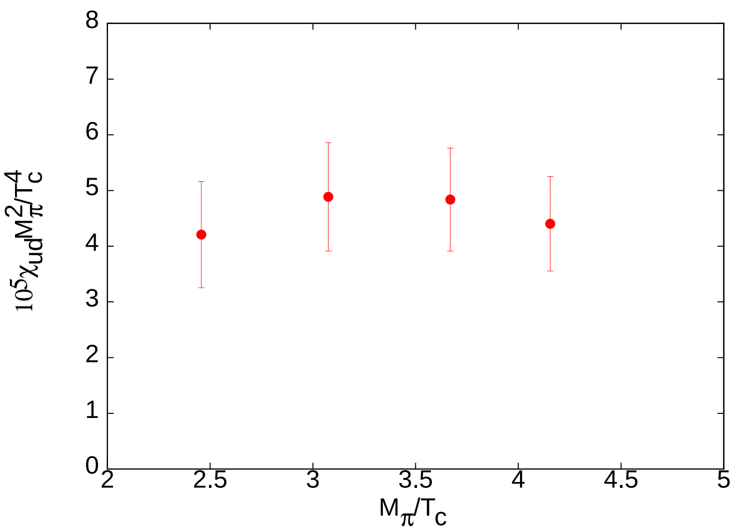

♣ $12^3 \times 4$ Lattice; Quenched.
♣ $T = 0.75T_c$
♣ Gavai, Gupta & Majumdar, PR D 2002

Taking Continuum Limit

(Gavai & Gupta, PR D '02 and hep-lat/0211015)

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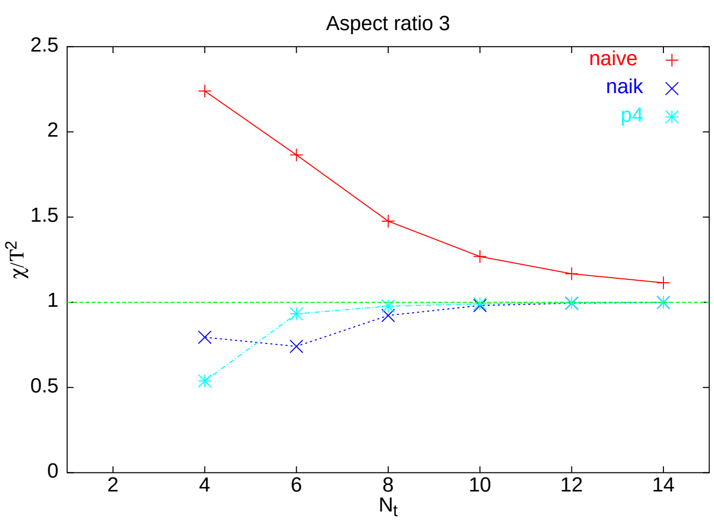
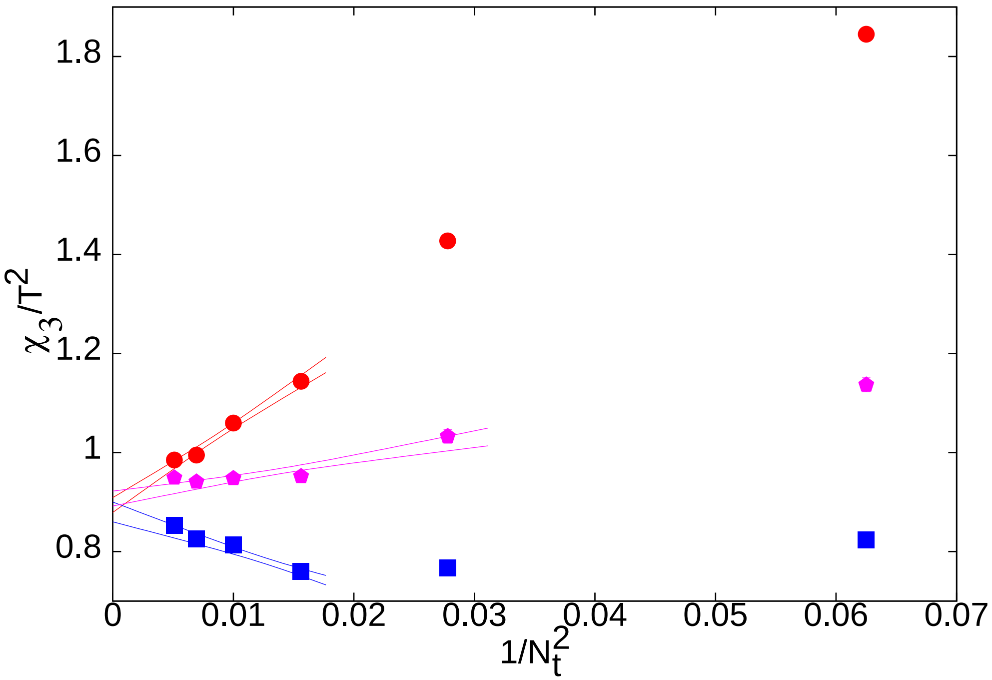


♠ Does improve the N_t-dependence of the free fermions.

Results at $2T_c$:



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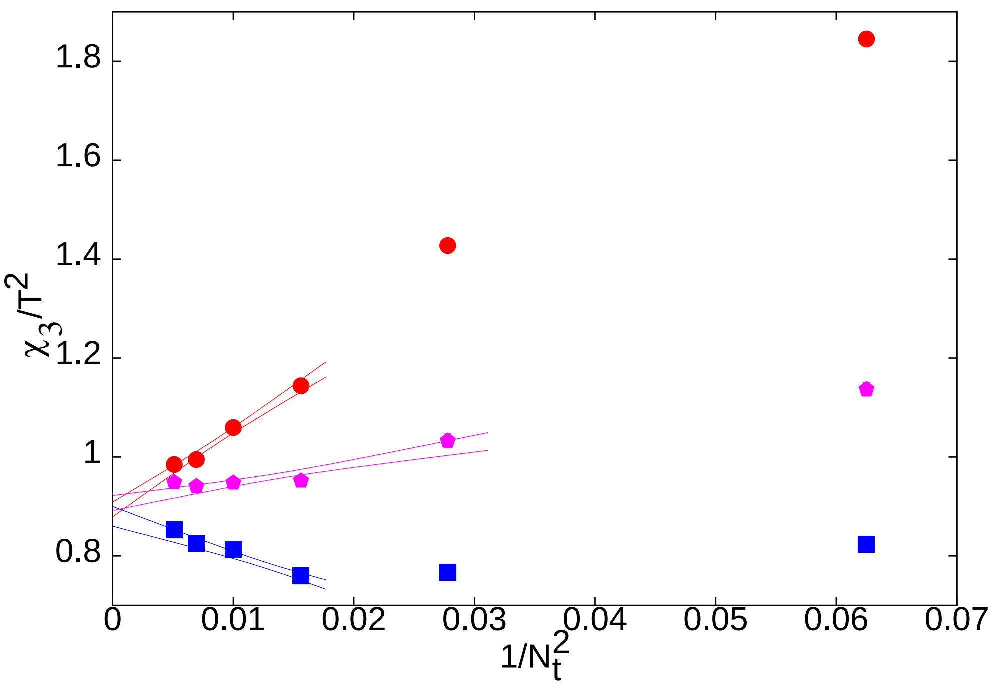

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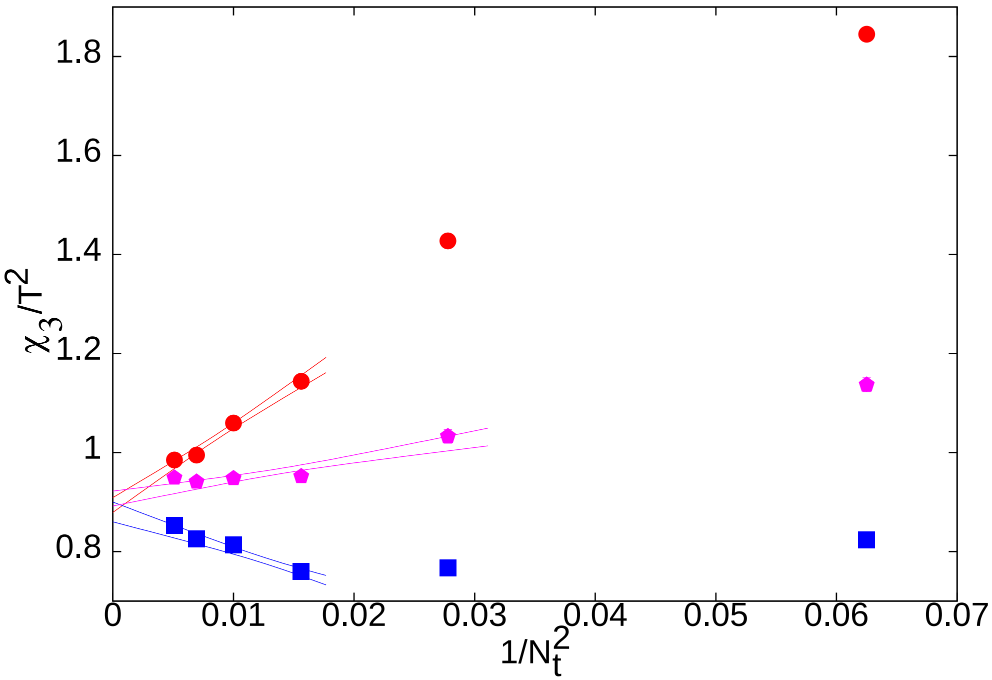

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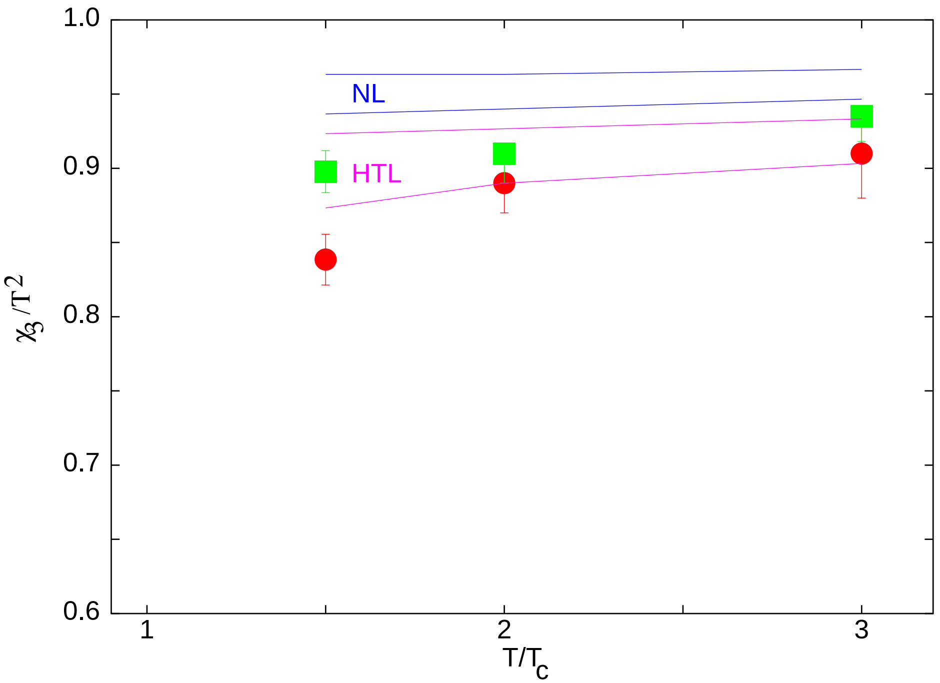

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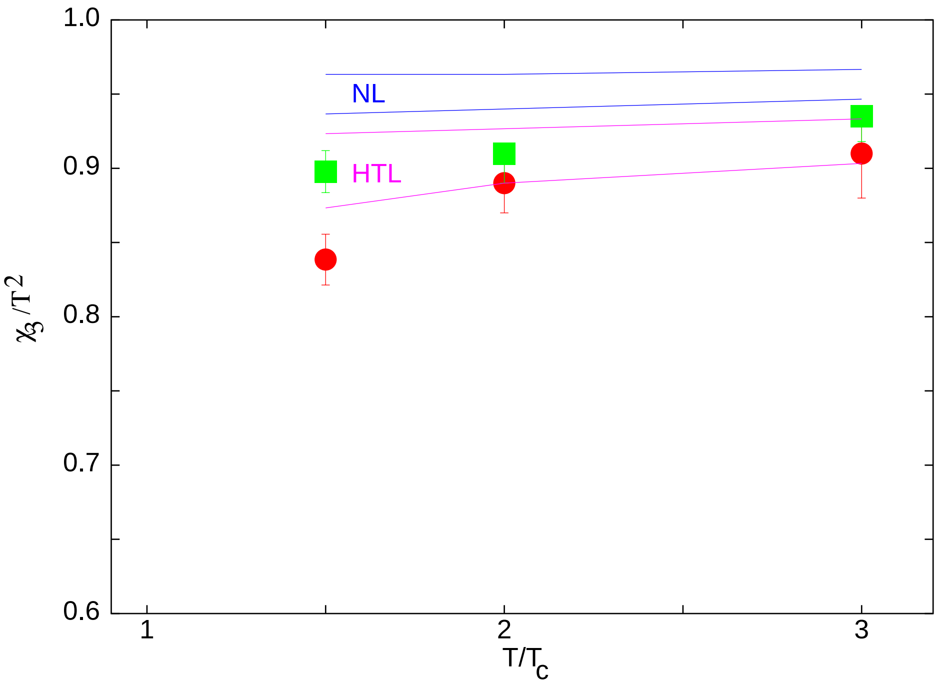

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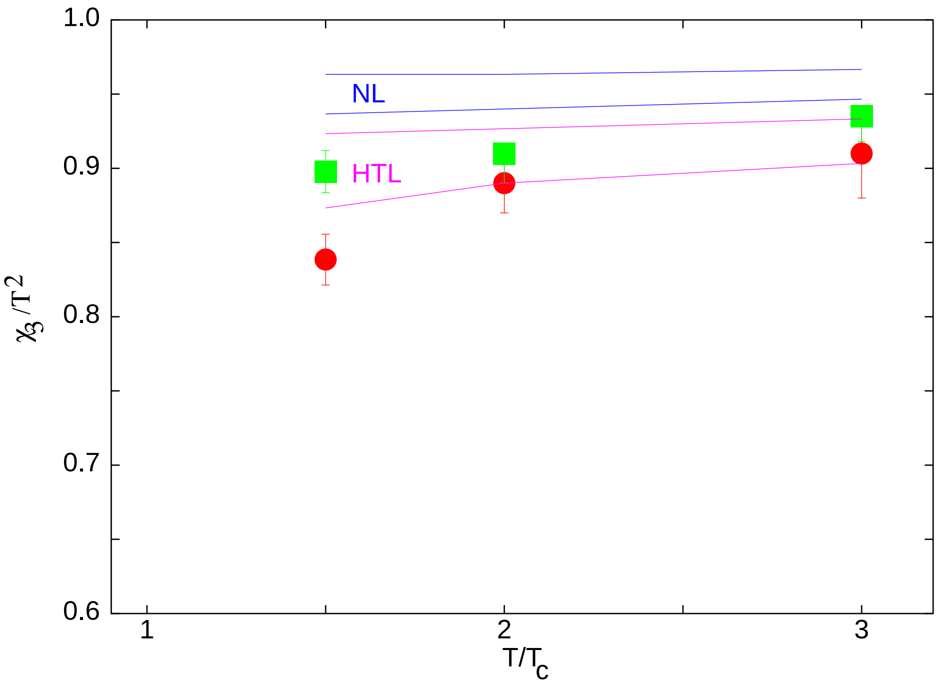

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♡ Note that χ_{ud} behaves the same way for ALL N_t and both fermions, leading to the same $O(10^{-6})$ values in continuum too.

Wroblewski Parameter

Enhancement of strangeness production – A signal of Quark-Gluon Plasma.

Wroblewski Parameter – ratio of newly created strange quarks to light quarks.

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle}. \tag{6}$$

Using our continuum QNS, it is a ratio χ_s/χ_u.

$m/T_c = 0.03$ for u, d and $m/T_c = 1$ for s quark $\longrightarrow \quad \lambda_s(T)$. Extrapolate to T_c.

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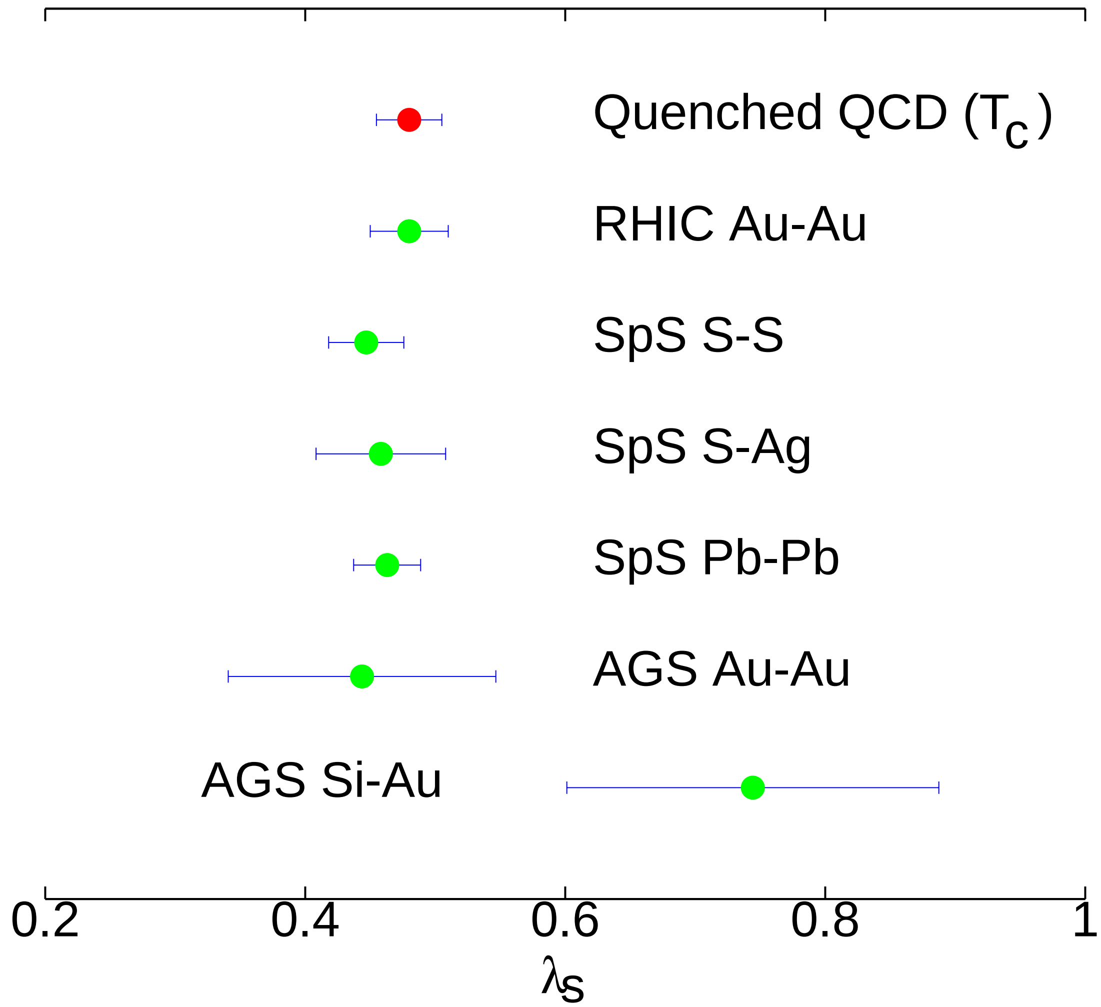
Quenched QCD (T_c)
RHIC Au-Au
SpS S-S
SpS S-Ag
SpS Pb-Pb
AGS Au-Au
AGS Si-Au
0.2
0.4
0.6
0.8
1
λ_s

Caveats

- Quenched approximation – Expect a shift of 5-10 % in full QCD.
- Extrapolation to T_c – Straightforward but better to do it for full QCD .
- At SPS and RHIC, $\mu_{\rm B} \neq 0$; But observed λ_s is insensitive to it. .
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EoS for nonzero baryon density

Higher order susceptibilities are defined by

$$\chi_{fg\cdots} = \frac{T}{V}\frac{\partial^n \log Z}{\partial\mu_f \partial\mu_g \cdots} = \frac{\partial^n P}{\partial\mu_f \partial\mu_g \cdots} \ . \tag{7}$$

These are Taylor coefficients of the pressure P in its expansion in μ.

Can be written as traces of products of M^{-1} and various derivatives of M. *E.g.* , χ_{uuuu} involves terms having fourth derivative w. r. to μ while χ_{uudd} only second derivatives.

In continuum, $f(a\mu) = 1 + a\mu \rightarrow f''(0) = 0$.
On lattice, in general, all derivatives exist and depend on the nature of function : prescription dependence !

Fodor-Katz used f_{HK} and got $\mu_E = 725$ MeV for $N_t = 4$. If they were to use f_{BG}, then $\mu_E = 692$ MeV.

Easy to show that $f''(0) = 1$ always but all higher derivatives depend on choice of f. Thus, one can write

$$\chi_{uuuu} = \chi^{HK}_{uuuu} + \Delta f^{(3)} \left(\frac{\chi_{uu}}{T^2}\right) \left(\frac{4}{N_t^2}\right) , \tag{8}$$

where $\Delta f^{(3)} = f^{(3)} - 1$ is 2 for f_{BG}.

Prescription dependence must go away for small a or large enough N_t.
How large an N_t needed ? $N_t \geq 10$, see below.

Defining

$$\frac{\mu_*}{T} = \sqrt{\frac{12\chi_{uu}/T^2}{|\chi_{uuuu}|}} , \tag{9}$$

and $\Delta P = P(\mu) - P(\mu = 0)$, the Taylor series expansion for Pressure P for 2 flavours can be re-organized as,

$$\frac{\Delta P}{T^4} = \left(\frac{\chi_{uu}}{T^2}\right) \left(\frac{\mu}{T}\right)^2 \left[1 + \left(\frac{\mu/T}{\mu_*/T}\right)^2 + \mathcal{O}\left(\frac{\mu^4}{\mu_*^4}\right)\right] . \tag{10}$$

Note that

- Each term in ΔP is prescription dependent, except the 1st. Physical ΔP may be best obtained by evaluating each in continuum limit, as we do below. More important for larger μ.

- The above is true for all physical quantities.

- $\mu \ll \mu_*$ for prescription independence, provided still higher susceptibilities $\leq \chi_{uuuu}$.

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Our Results

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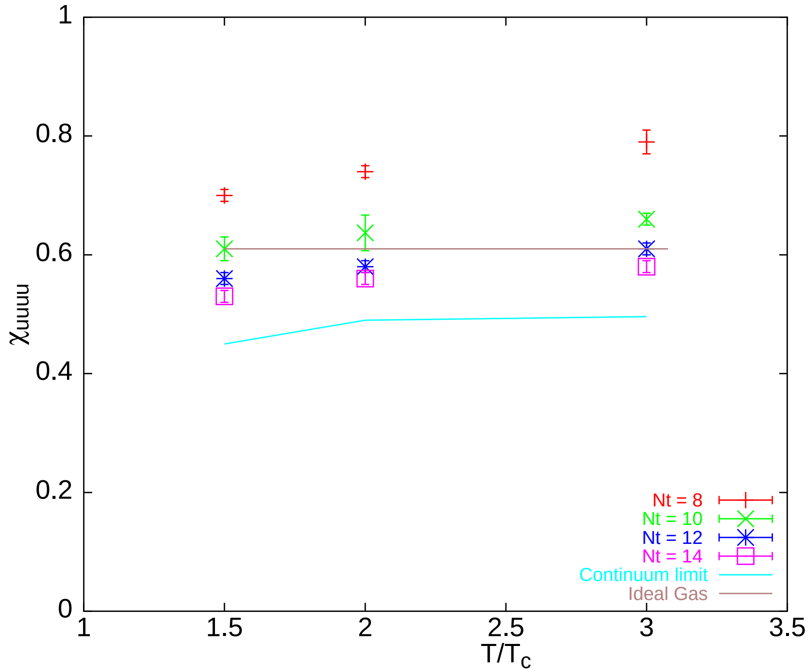

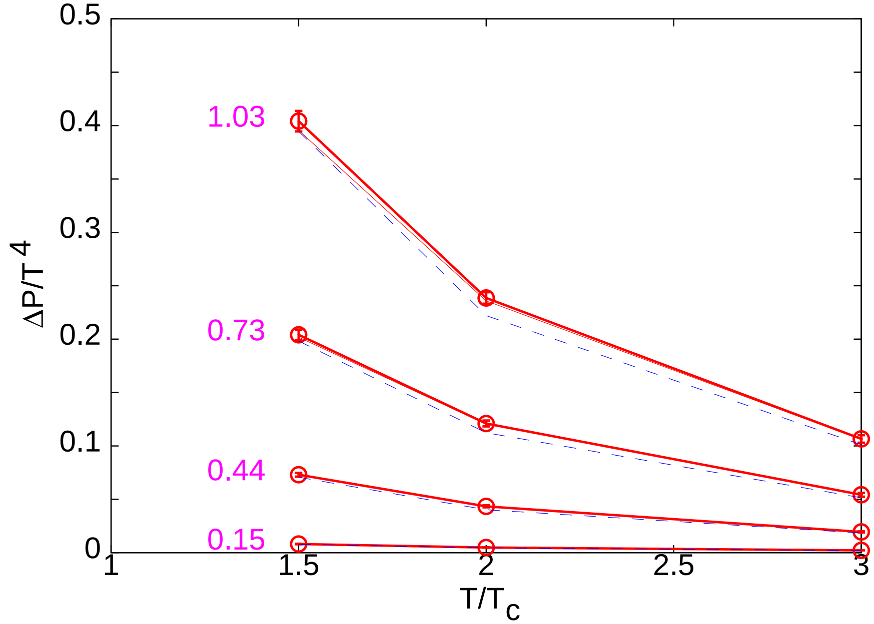

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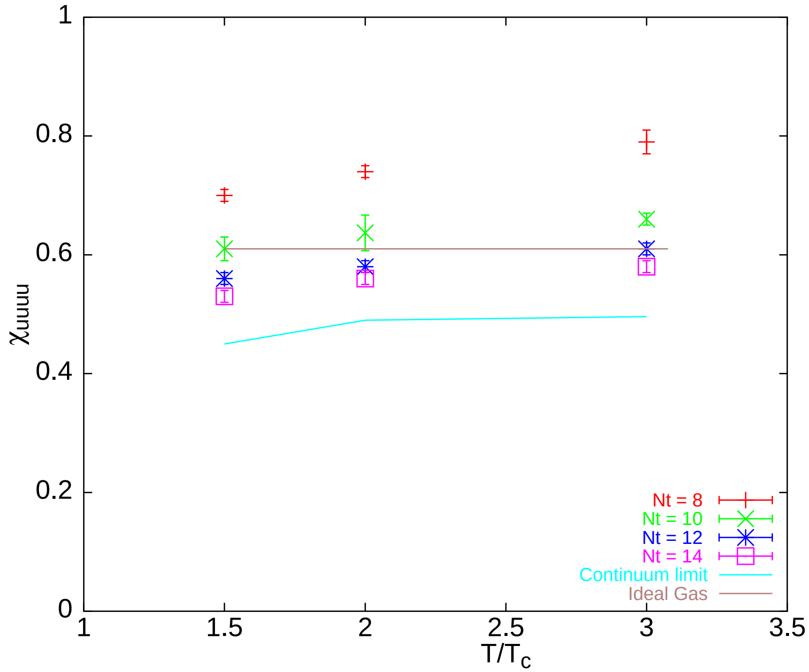

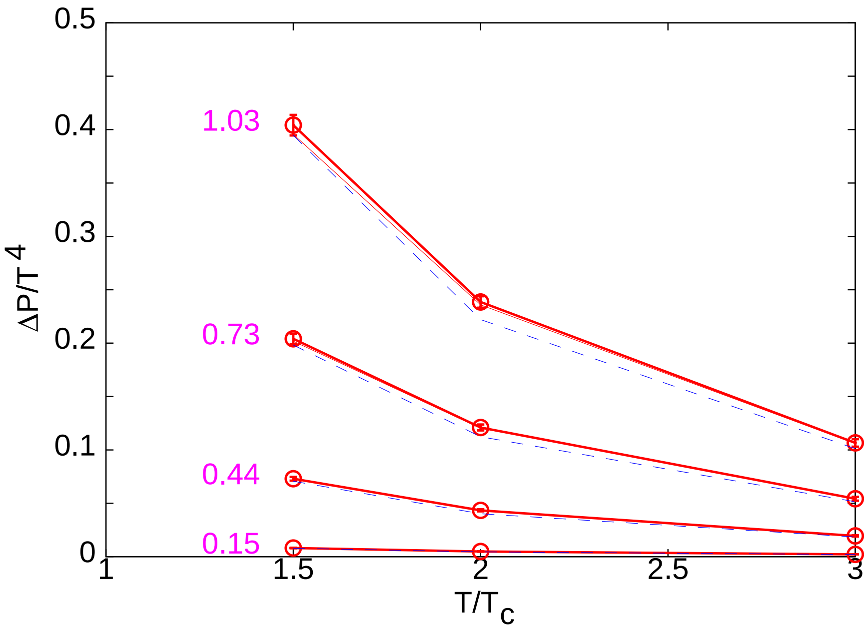

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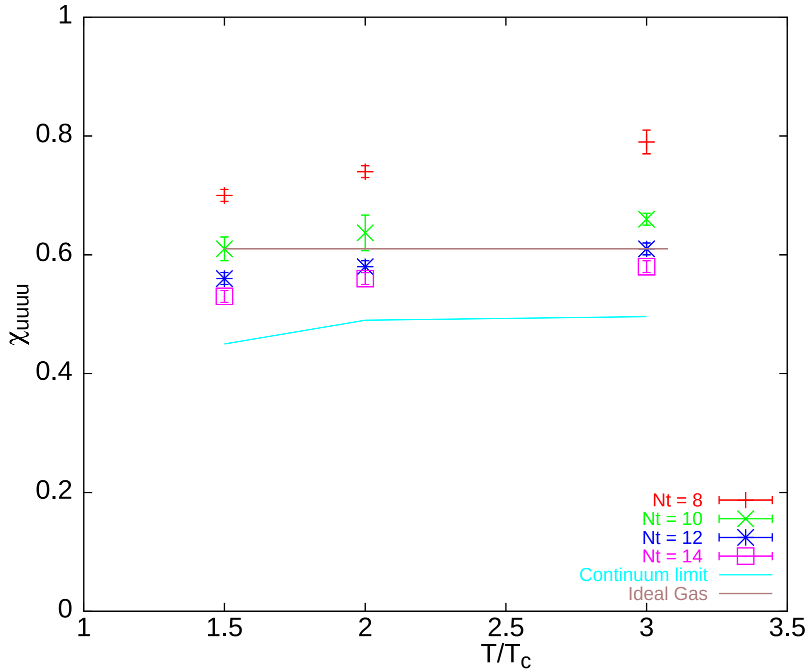

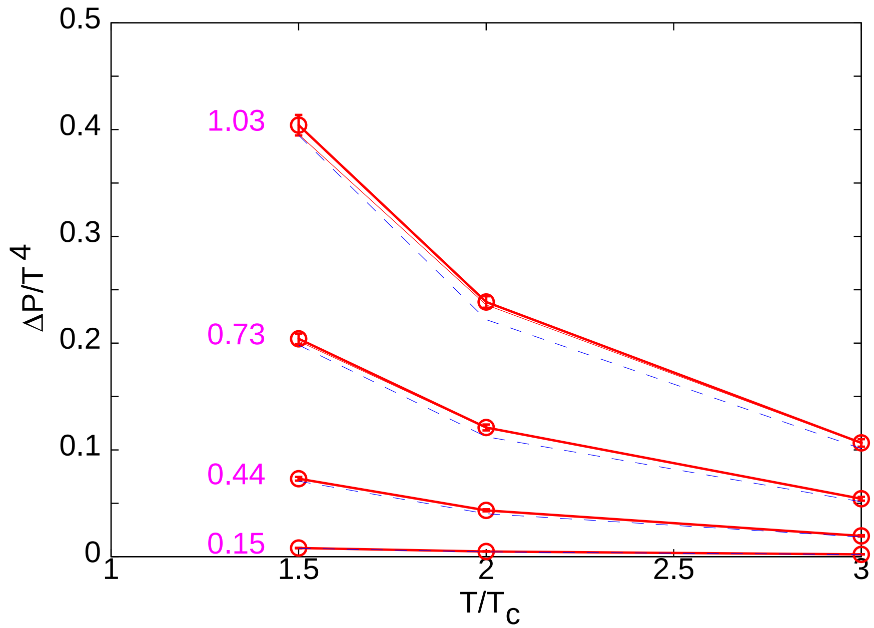

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- Continuum limit of χ_{uuuu} in Quenched QCD obtained. $\sim$to dimensional reduction.
- Pressure for nonzero μ obtained. At both SPS and RHIC, χ_{uu} is the major contribution.

- Many questions still for full 2+1 QCD : Order, Large N_t, $\cdots$.

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- Obtained from the exponential decay of

$$C_{\Gamma}(z) = \sum_{x,y,t} \langle M^{-1}_{\alpha\beta}(x,y,z,t)\Gamma M^{\dagger -1}_{\beta\alpha}(x,y,z,t)\Gamma\rangle \qquad (11)$$

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- Summing up the C_Γ for pion $\rightarrow$ Pion susceptibility.

$N_t = 4$ Lattices with $N_z = 16$.
$4\chi_3/T^2$ (open symbols) and $\chi_\pi/10T^2$ (filled)
at $2T_c$ (lower set) and $3T_c$.
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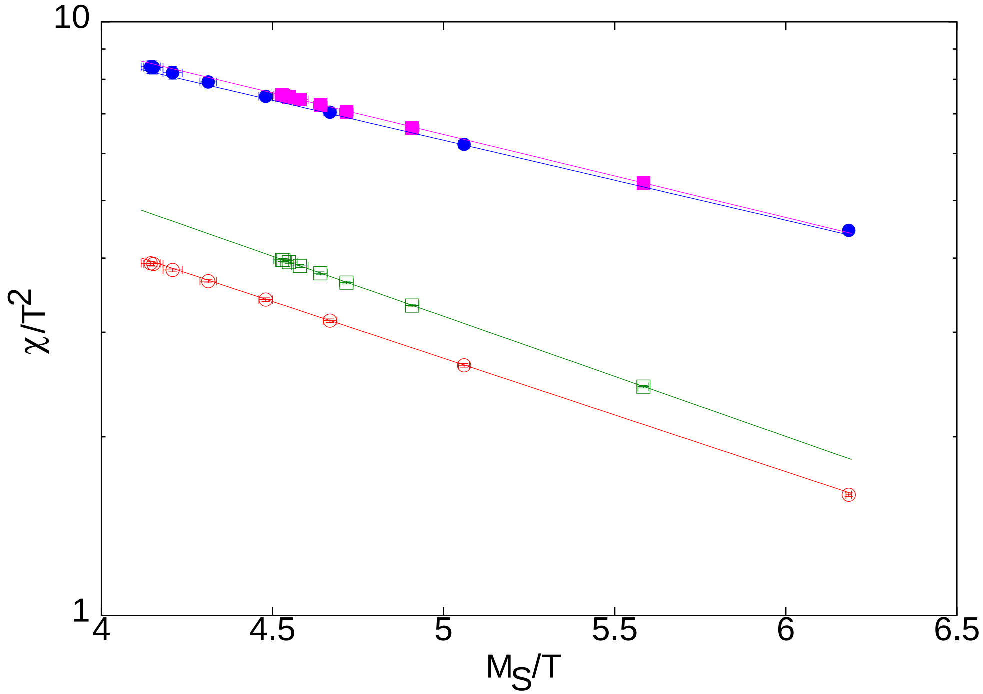

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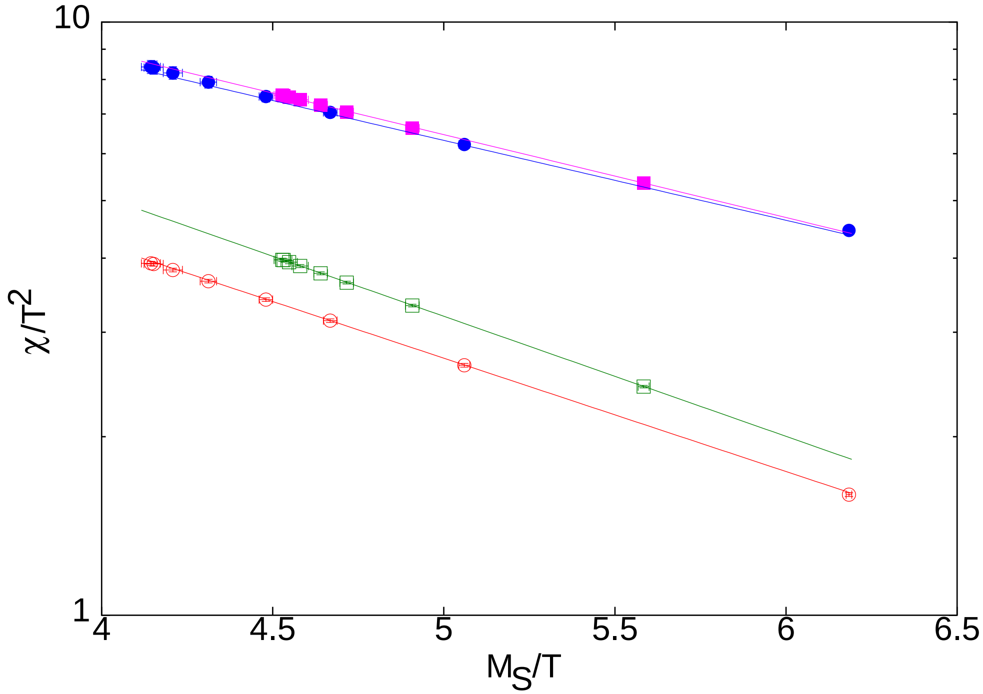

Why ? $\chi_3 \sim \sum$ propagator of nonlocal vector meson.

Taking Continuum Limit

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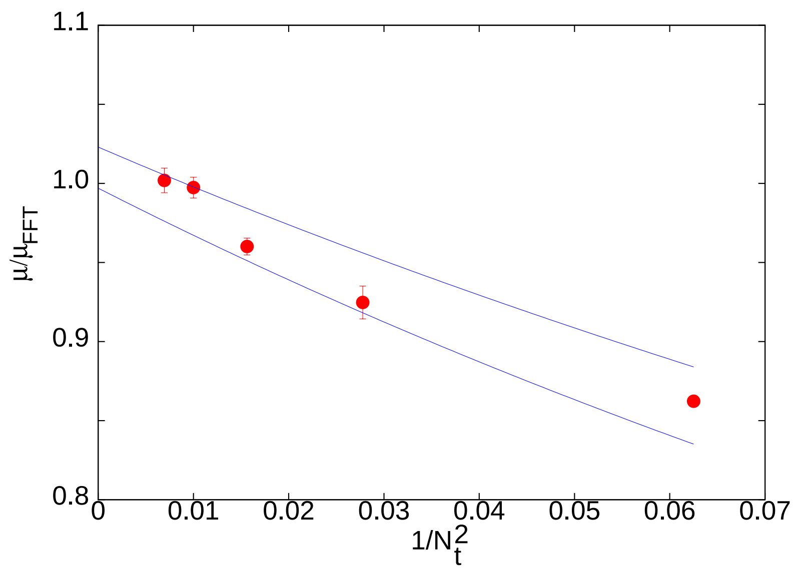

- $m_v/T_c = 0.03$,
- Lattices up to 48×26^2.

Overlap Fermions agree:

On coarse lattices, a $= 1/4T$, Pion screening lengths become degenerate with those of ρ , i.e, also close to FFT!! (Gavai, Gupta & Lacaze, PR D '02)

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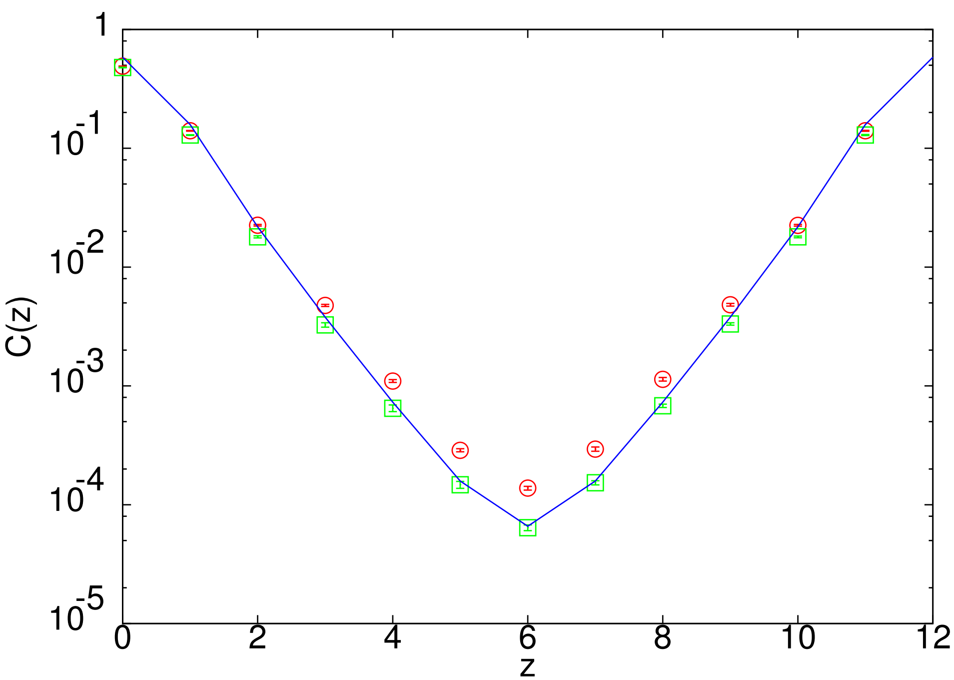

Configurations with zero modes excluded. $12^3 \times 4$ lattice at $T = 1.5T_c$. Quenched Approximation. $m/T_c = 0.006$

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Note that both PS and V have SAME fit with changed normalization.