Dark Energy
a cosmic mystery
Quintessence

C. Wetterich

A. Hebecker, M. Doran, M. Lilley, J. Schwindt,
C. Müller, G. Schäfer, E. Thommes,
R. Caldwell, M. Bartelmann,
K. Karwan, G. Robbers
What is our universe made of?

quintessence!
fire, air,
water, soil!
Dark Energy dominates the Universe

Energy - density in the Universe

= 

Matter + Dark Energy

25 % + 75 %
critical density

\[ \rho_c = 3 H^2 M^2 \]

critical energy density of the universe

( \( M \) : reduced Planck-mass, \( H \) : Hubble parameter)

\[ \Omega_b = \frac{\rho_b}{\rho_c} \]

fraction in baryons

energy density in baryons over critical energy density
What is Dark Energy?
Matter: Everything that clumps
Dark Matter

- \( \Omega_m = 0.25 \) total “matter”
- Most matter is dark!
- So far tested only through gravity
- Every local mass concentration \( \Rightarrow \) gravitational potential
- Orbits and velocities of stars and galaxies \( \Rightarrow \) measurement of gravitational potential and therefore of local matter distribution
\( \Omega_m = 0.25 \)
spatially flat universe

\[ \Omega_{\text{tot}} = 1 \]

- theory (inflationary universe)
  \[ \Omega_{\text{tot}} = 1.0000 \ldots \times \]

- observation (WMAP)
  \[ \Omega_{\text{tot}} = 1.02 (0.02) \]
picture of the big bang
\( \Omega_{\text{tot}} = 1 \)

Mean values:

- \( \Omega_{\text{tot}} = 1.02 \)
- \( \Omega_m = 0.27 \)
- \( \Omega_b = 0.045 \)
- \( \Omega_{\text{dm}} = 0.225 \)
WMAP 2006

Polarization
Dark Energy

\( \Omega_m + X = 1 \)

\( \Omega_m : 25\% \)

\( \Omega_h : 75\% \quad \text{Dark Energy} \)

h : homogenous, often \( \Omega_\Lambda \) instead of \( \Omega_h \)
Space between clumps is not empty: Dark Energy!
Dark Energy density is the same at every point of space

“homogeneous“

No force in absence of matter –

“In what direction should it draw?“
Predictions for dark energy cosmologies

The expansion of the Universe accelerates today!
Power spectrum

Baryon - Peak

galaxy –
correlation –
function

Structure formation :
One primordial
fluctuation- spectrum

SDSS
consistent cosmological model!
Composition of the Universe

\[ \Omega_b = 0.045 \quad \text{visible clumping} \]

\[ \Omega_{dm} = 0.2 \quad \text{invisible clumping} \]

\[ \Omega_h = 0.75 \quad \text{invisible homogeneous} \]
Dark Energy- a cosmic mystery
What is Dark Energy?

Cosmological Constant or Quintessence?
Cosmological Constant
- Einstein -

- Constant $\lambda$ compatible with all symmetries
- No time variation in contribution to energy density

- Why so small? $\lambda/M^4 = 10^{-120}$
- Why important just today?
Einstein equation

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi}{M_p^2} T_{\mu\nu}
\]

\[M_p = 1.22 \cdot 10^{19} \text{ GeV} = G_N^{-\frac{1}{2}}\]

Energy-momentum-tensor

\[T_{\mu\nu} = T_{\mu\nu}^{(\text{baryon})} + T_{\mu\nu}^{(\text{radiation})} + T_{\mu\nu}^{(\text{dark matter})} + T_{\mu\nu}^{(\text{homogeneous})}\]
\[ T_{\mu \nu} \] (homogenous) \quad \pm \quad \lambda \ g_{\mu \nu} : \text{cosmological const.} \\
\quad T_{\mu \nu}^{(q)} : \text{quintessence} \\
\quad \text{scalar field} \quad ? \\
\quad \text{nonlocal gravity} \quad ?
Gravitational action

\[ S = + \int d^4x \, g^{-\frac{1}{2}} \left( -\frac{\mathcal{H}_p^2}{16\pi} \, R + \lambda \right) \]

\( \lambda \): cosmological constant

Field equations

\[ R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = \frac{8\pi}{\mathcal{H}_p^2} \left( T_{\mu\nu}^{\text{eff}} = \lambda \, g_{\mu\nu} \right) \]

\( \mathcal{H}_p \approx 10^{19} \text{ GeV} \): Planck mass

\( T_{\mu\nu}^{\text{eff}} \): matter energy momentum tensor

accounts for nonvanishing entropy in the universe

without matter:

\[ R = + \frac{32\pi}{\mathcal{H}_p^2} \lambda \]
Cosmological constant

\[ T_{\mu \nu} = T^\Lambda_{\mu \nu} - \lambda g_{\mu \nu} \]

\[ \rho \rightarrow \rho + \lambda = \rho + \rho_\Lambda \]

\[ p \rightarrow p - \lambda = p + p_\Lambda \]

\[ \rho + p \rightarrow \rho + p \]

"Equation of state"

\[ \frac{p_\Lambda}{\rho_\Lambda} = -1 \]
Friedman universe \(( \lambda = 0 )\)

Einstein equations \(\Rightarrow\)

(i) \( H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi G}{3 H^2} \rho - \frac{k}{a^2} \)

(evolution equation)

(ii) \( \dot{\rho} + 3 H (\rho + p) = 0 \)

(energy-momentum conservation)

\[ \Rightarrow \frac{d}{dt} \left[ a^3 (\rho + p) \right] = a^3 \frac{d}{dt} \rho \]

radiation \((p = \frac{1}{3} \rho)\) \(\rho \sim a^{-4}\)

matter \((p = 0)\) \(\rho \sim a^{-3}\)

\(k = 0\) (always applicable for early universe)

Field equations involve only the Hubble parameter \(H = \dot{a}/a\)
Einstein equations \( \rightarrow \)

(i) \( H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3H^2} (\rho + \lambda) - \frac{k}{a^2} \)

\( \text{(evolution equation)} \)

(ii) \( \dot{\rho} + 3H(\rho + \pi) = 0 \)

\( \text{(energy-momentum conservation)} \)

\( \Leftrightarrow \frac{d}{dt} \left[ a^3 (\rho + \pi) \right] = a^3 \frac{d}{dt} \rho \)

radiation (\( \rho = \frac{1}{3} \pi \)) \( \rho \sim a^{-4} \)

matter (\( \rho = 0 \)) \( \rho \sim a^{-3} \)

\( k = 0 \) \( \text{(always applicable for early universe)} \)

Field equations involve only the Hubble parameter \( H = \dot{a}/a \)
Only minor modification for

\[ \lambda \ll \rho \]

For \( t \to \infty \): \( \lambda \neq 0 \) has always important effects!
asymptotic solution for cosmological constant ($k=0$)

$\lambda > 0$

\[ H^2 \rightarrow \frac{8\pi}{3H^2} \lambda \]

$\lambda = 0$

\[ H \rightarrow \eta t^{-1} \quad a \sim t^{\eta} \]

$\lambda < 0$

\[ H \rightarrow \left( \frac{8\pi}{3H^2} |\lambda| \right)^{\frac{1}{3}} \sqrt{\sinh (c_1 - c_2 t)} \]

e.g. a expands to maximal $a_0$

and shrinks subsequently
For

\[ |\lambda| \ll \frac{3M_p^2H^2}{8\pi} : \]

only small corrections to
standard cosmology

\[ \lambda \approx (0.6 - 0.7) \rho_c \]

good candidate for dark energy!

compatible with observation!
problems with small $\lambda$

- no symmetry explanation for $\lambda/M^4 = 10^{-120}$
- quantum fluctuations contribute

Zero point energies for normal modes of field with mass $m$,
for wave numbers $|k| < \Lambda$ ($m^2 \ll \Lambda^2$)

$$\langle \mathcal{O} \rangle_{\text{vac}} = \sum \frac{a_{k}^* a_{k}}{(2\pi)^3} \cdot \frac{1}{\sqrt{k^2 + m^2}} \propto \frac{\Delta^4}{16\pi^2}$$

In QED, zero point energies are measured by Casimir effect —

detection of cosmological constant:

link between quantum fluctuations and gravity!
For

\[ \lambda \leq -\frac{1}{2} \rho_c \]

or

\[ \lambda \geq (10^{-100}) \rho_c \]

we simply would not exist!
Cosmological Constant
- Einstein -

- Constant $\lambda$ compatible with all symmetries
- No time variation in contribution to energy density

- Why so small? $\lambda/M^4 = 10^{-120}$
- Why important just today?
Cosm. Const. | Quintessence
static | dynamical

\[ \lambda = 10^{-124} \]

\[ \Phi \sim t^{-2} \]
Cosmological mass scales

- Energy density
  \[ \rho \sim (2.4 \times 10^{-3} \text{ eV})^{-4} \]

- Reduced Planck mass
  \[ M = 2.44 \times 10^{18} \text{ GeV} \]

- Newton’s constant
  \[ G_N = (8\pi M^2) \]

Only ratios of mass scales are observable!

- Homogeneous dark energy:
  \[ \rho_h/M^4 = 6.5 \times 10^{-121} \]

- Matter:
  \[ \rho_m/M^4 = 3.5 \times 10^{-121} \]
Time evolution

\[ \rho_{m}/M^4 \sim a^{-3} \sim t^{-2} \] matter dominated universe

\[ \rho_{r}/M^4 \sim a^{-4} \sim t^{-3/2} \] radiation dominated universe

Huge age \( \Rightarrow \) small ratio

Same explanation for small dark energy?
Quintessence

Dynamical dark energy, generated by scalar field

(cosmon)

Prediction:

homogeneous dark energy influences recent cosmology

- of same order as dark matter -

Original models do not fit the present observations
.... modifications
Quintessence

Cosmon – Field $\varphi(x,y,z,t)$

similar to electric field, but no direction (scalar field)

Homogeneous und isotropic Universe: $\varphi(x,y,z,t)=\varphi(t)$

Potential und kinetic energy of the cosmon-field contribute to a dynamical energy density of the Universe!
Cosmon

- Scalar field changes its value even in the \textit{present} cosmological epoch
- Potential und kinetic energy of cosmon contribute to the energy density of the Universe
- Time - variable dark energy : \(\varrho_b(t)\) decreases with time!
Cosmon

- Tiny mass

- $m_c \sim H$

- New long-range interaction
“Fundamental” Interactions

Strong, electromagnetic, weak interactions

On astronomical length scales:

graviton + cosmon
Evolution of cosmon field

Field equations

$$\ddot{\phi} + 3H \dot{\phi} = -dV/d\phi$$

$$3M^2 H^2 = V + \frac{1}{2} \dot{\phi}^2 + \rho$$

Potential $V(\varphi)$ determines details of the model

e.g. $V(\varphi) = M^4 \exp( - \varphi/M )$

for increasing $\varphi$ the potential decreases towards zero!
Cosmological equations

\[ \ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi} \]

\[ 3M^2H^2 = V + \frac{1}{2} \dot{\phi}^2 + \rho \]
Cosmic Attractors

Solutions independent of initial conditions

typically $V \sim t^{-2}$

$\varphi \sim \ln(t)$

$\Omega_h \sim \text{const.}$

details depend on $V(\varphi)$ or kinetic term

early cosmology
Cosmological equations

\[ \mathcal{L} = \sqrt{g} \left\{ \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + V(\phi) \right\} \]

\[ \ddot{\varphi} + 3H \dot{\varphi} + \frac{\partial V}{\partial \varphi} = 0 \]

\[ 3M^2 H^2 = V + \frac{1}{2} \dot{\varphi}^2 + \rho \]

\[ \dot{\rho}_M + 3H(\rho_M + p_M) = 0 \]

\[ p_M = \frac{n - 3}{3} \rho_M \]
asymptotic solution for large time

Cosmological solutions with scalar field (cosmon)

for exponential potential $V \sim \exp(-a\frac{\varphi}{M})$

\[ \Rightarrow \]

asymptotic solution for $t \to \infty$:

$V \sim t^{-2}$, $\varphi^2 \sim t^{-2}$

$\varphi = \frac{2M}{a} \ln t$

stable attractor!

independent of initial conditions
“tracker solution”

$\Omega_{\text{om}} = \frac{3}{2a^2}$

fixed fraction in dark energy!

$M \to \hat{M}$

\[ H^2 = \frac{1}{6M^2}(\rho_M + V(\varphi) + \frac{1}{2}\dot{\varphi}^2) \]

$\rho_M + 3H(\rho_M + p_M) = 0$

$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} = 0$

$p_M = \frac{n-3}{3}\rho_M$

$V = V_0 \exp\{-a\frac{\varphi}{M}\}$

$\phi = \frac{2M}{\alpha} \ln(t/\tilde{t})$, $\frac{1}{2}\dot{\phi}^2 = \frac{2M^2}{\alpha^2} t^{-2}$, $V = \frac{2M^2(6-n)}{\alpha^2} n^{-1} t^{-2}$

$H = \frac{2}{n} \frac{1}{t}$, $\rho \sim t^{-2}$

$\Omega_d = (V + \frac{1}{2}\dot{\phi}^2)/\rho_c = \rho_0/\rho_c = \frac{n}{2a^2}$
exponential potential
constant fraction in dark energy

\[ \Omega_M = 1 - \frac{n}{2a^2} \]
\[ \Omega_V = \frac{V}{\rho_c} = \frac{n(6 - n)}{12a^2} \]
\[ \Omega_{kin} = \frac{\dot{\varphi}^2}{2\rho_c} = \frac{n^2}{12a^2} \]
\[ \Omega_h = \Omega_V + \Omega_{kin} = \frac{n}{2a^2} \]

asymptotic solution

for \( t \to \infty \)

“attractor” for \( a^2 > \frac{n}{2} \)

\[ \Omega_x = \frac{8\pi}{3} \frac{\rho_x}{M_p^2 H^2} \]
realistic quintessence

fraction in dark energy has to increase in “recent time”!
Quintessence becomes important “today”
Equation of state

\[ p = T - V \]  \hspace{1cm} \text{pressure} \hspace{1cm} \rho = T + V \]  \hspace{1cm} \text{energy density}

\[ T = \frac{1}{2} \phi^2 \]

\[ w = \frac{p}{\rho} = \frac{T - V}{T + V} \]

Depends on specific evolution of the scalar field
Negative pressure

- $w < 0$  \quad $\Omega_h$ increases  \quad (with decreasing $z$)

Late universe with small radiation component:

\[ w_h = \frac{1}{3\Omega_h(1-\Omega_h)} \frac{\partial \Omega_h}{\partial \ln(1+z)} \]

- $w < -1/3$  \quad expansion of the Universe is accelerating

- $w = -1$  \quad cosmological constant
Quintessence becomes important “today”

No reason why $w$ should be constant in time!
How can quintessence be distinguished from a cosmological constant?
Time dependence of dark energy

\[ \Omega_h \sim t^2 \sim (1+z)^{-3} \]

M.Doran,…
small early and large present
dark energy

fraction in dark energy has substantially
increased since end of structure formation

expansion of universe accelerates in present
epoch

\[ \omega_h = \frac{1}{3 \Omega_h (1 - \Omega_h)} \frac{\partial \Omega_h}{\partial \ln (1 + z)} \]
Early Dark Energy

A few percent in the early Universe

Not possible for a cosmological constant

1σ and 2σ limits

Doran, Karwan, ..
Little Early Dark Energy can make large effect!

More clusters at high redshift

Cluster number relative to $\Lambda$CDM

Two models with 4% Dark Energy during structure formation

Fixed $\sigma_8$ (normalization dependence!)

Early Quintessence slows downs the growth of structure

Dark Energy during structure formation
How to distinguish \( Q \) from \( \Lambda \)?

A) Measurement \( \Omega_h(z) \leftrightarrow H(z) \)
   i) \( \Omega_h(z) \) at the time of structure formation, CMB - emission or nucleosynthesis
   ii) equation of state \( w_h(\text{today}) > -1 \)

B) Time variation of fundamental “constants”

C) Apparent violation of equivalence principle

D) Possible coupling between Dark Energy and Dark Matter
Cosmodynamics

Cosmon mediates new long-range interaction

Range: size of the Universe – horizon
Strength: weaker than gravity

- photon         electrodynamics
- graviton       gravity
- cosmon         cosmodynamics

Small correction to Newton’s law
“Fifth Force”

- **Mediated by scalar field**

- **Coupling strength: weaker than gravity**
  - (nonrenormalizable interactions $\sim M^{-2}$)

- **Composition dependence**
  - $\rightarrow$ violation of equivalence principle

- **Quintessence: connected to time variation of fundamental couplings**
Violation of equivalence principle

Different couplings of cosmon to proton and neutron

Differential acceleration

“Violation of equivalence principle”

only apparent: new “fifth force”!
(1) \( \alpha_x(\varphi) \rightarrow \Lambda_{QCD}(\varphi) \rightarrow m_n(\varphi) \)

nucleon mass depends on value of the cosmon field
(and therefore on time)

(2) expand around cosmological value \( \varphi_0(t) \):

\[
\varphi(\vec{x},t) = \varphi_0(t) + \delta\varphi(\vec{x},t)
\]

\[
m_n = m_n(\varphi_0) + \frac{\partial m_n}{\partial \varphi} \varphi_0 \delta\varphi
\]

\[\Rightarrow \text{cosmon-nucleon vertex} \sim \overline{m}m \delta\varphi\]

\[\Rightarrow \text{earth is source for surrounding local cosmon field} \delta\varphi(\vec{x})\]
(3) Test body carries effective "cosmon charge"

\[ Q_c = \hbar^{-1} \frac{\partial m_t}{\partial \phi} \]

to be compared with "gravitational charge"

\[ Q_g = \frac{m_t}{\frac{\alpha^2}{2} F_p} \]

\[ \Rightarrow \text{Correction to Newtonian potential} \]

\[ V_N = -\frac{G_N M m_t}{r} (1 + \alpha_t) \]

\[ \alpha_t = \frac{2F_p^2}{\hbar^2} \frac{\partial \ln M}{\partial \phi} \frac{\partial \ln m_t}{\partial \phi} \]

(4) Protons and neutrons have different cosmon charges, \( \frac{\partial m_p}{\partial \phi} \neq \frac{\partial m_n}{\partial \phi} \)
Differential acceleration

Two bodies with equal mass experience a different acceleration!

\[ \eta = \frac{a_1 - a_2}{a_1 + a_2} \]

bound: \( \eta < 3 \times 10^{-14} \)
Cosmon coupling to atoms

- Tiny !!!
- Substantially weaker than gravity.
- Non-universal couplings bounded by tests of equivalence principle.
- Universal coupling bounded by tests of Brans-Dicke parameter $\omega$ in solar system.
- Only very small influence on cosmology.
Cosmon coupling to Dark Matter

- Only bounded by cosmology
- Substantial coupling possible
- Can modify scaling solution and late cosmology
- Role in clustering of extended objects?

L. Amendola
Quintessence and time variation of fundamental constants

Generic prediction

Strength unknown

C. Wetterich,

Strong, electromagnetic, weak interactions

gravitation

cosmodynamics
Time varying constants

- It is not difficult to obtain quintessence potentials from higher dimensional or string theories.
- Exponential form rather generic (after Weyl scaling).
- But most models show too strong time dependence of constants!
Are fundamental “constants” time dependent?

- Fine structure constant $\alpha$ (electric charge)
- Ratio electron mass to proton mass
- Ratio nucleon mass to Planck mass
Quintessence and Time dependence of “fundamental constants”

- Fine structure constant depends on value of cosmon field: $\alpha(\varphi)$

  (similar in standard model: couplings depend on value of Higgs scalar field)

- Time evolution of $\varphi$ 
  Time evolution of $\alpha$

  Jordan,…
Standard – Model of electroweak interactions: Higgs - mechanism

- The masses of all fermions and gauge bosons are proportional to the (vacuum expectation) value of a scalar field $\phi_H$ (Higgs scalar).
- For electron, quarks, W- and Z- bosons:

$$m_{\text{electron}} = h_{\text{electron}} \times \phi_H$$

etc.
Restoration of symmetry at high temperature in the early Universe

Low T
SSB
$\langle \phi_H \rangle = \phi_0 \neq 0$

High T
SYM
$\langle \phi_H \rangle = 0$

high T:
less order
more symmetry

example:
magnets
In the hot plasma of the early Universe:

No difference in mass for electron and muon!
symmetrisches Vakuum

unser Vakuum
Quintessence:
Couplings are still varying now!

Strong bounds on the variation of couplings - interesting perspectives for observation!
baryons:

the matter of stars and humans

\[ \Omega_b = 0.045 \]
Abundancies of primordial light elements from nucleosynthesis
Allowed values for variation of fine structure constant:

$$\frac{\Delta \alpha}{\alpha} (z=10^{10}) = -1.0 \times 10^{-3} \quad \text{GUT 1}$$
$$\frac{\Delta \alpha}{\alpha} (z=10^{10}) = -2.7 \times 10^{-4} \quad \text{GUT 2}$$

C. Mueller, G. Schaefer, …
variation of Li- abundance

$S = 160, R = 0, 36, 60, \Delta \alpha/\alpha = 2 \Delta h/h$

Coc, Nunes, Olive, Uzan, Vangioni
10/06
Variation of fine structure constant as function of redshift

Three independent data sets from Keck/HIRES

\[ \Delta \alpha / \alpha = -0.54 (12) \times 10^{-5} \]

Murphy, Webb, Flambaum, June 2003

VLT

\[ \Delta \alpha / \alpha = -0.06 (6) \times 10^{-5} \]

Srianand, Chand, Petitjean, Aracil, Feb. 2004

\[ z \approx 2 \]
Atomic clocks and OKLO

* Atomic clocks:

\[
\frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} = -5.4 \cdot 10^{-10} \quad \frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} (z=0.13) \text{ yr}^{-1}
\]

Observation: \( \frac{\Delta \alpha_{\text{em}}}{\alpha_{\text{em}}} = (4.2 \pm 6.3) \cdot 10^{-15} \text{ yr}^{-1} \)

Sortais et al.

assumes that both effects are dominated by change of fine structure constant
Time variation of coupling constants must be tiny —

would be of very high significance!

Possible signal for Quintessence
Παντα θεί

Everything is flowing
Apparent violation of equivalence principle

and

time variation of fundamental couplings

measure both the

cosmon – coupling to ordinary matter
Differential acceleration $\eta$

For unified theories (GUT):

$$\eta = -1.75 \, 10^{-2} \, \Delta R_z \left( \frac{\partial \ln \alpha}{\partial z} \right)^2 \frac{1 + \tilde{Q}}{\Omega_h (1 + w_h)}$$

$$\Delta R_z = \frac{\Delta Z}{Z + N} \approx 0.1$$

$\eta = \Delta a / 2a$

Q: time dependence of other parameters
Link between time variation of $\alpha$

and violation of equivalence principle

typically: $\eta = 10^{-14}$

if time variation of $\alpha$

near Oklo upper bound

to be tested (MICROSCOPE, …)
small change of couplings in space

- Fine structure constant depends on location in space
- Experiments with satellites?

\[
\delta \frac{\alpha_{em}}{\alpha_{em}} = 3 \times 10^{-19} / k^2
\]

for \( r = 2 R_E \)
Summary

- $\Omega_h = 0.7$

- $Q/\Lambda$: dynamical and static dark energy will be distinguishable

- $Q$: time varying fundamental coupling “constants”

violation of equivalence principle
Why becomes Quintessence dominant in the present cosmological epoch?
Are dark energy and dark matter related?
Can Quintessence be explained in a fundamental unified theory?
Quintessence and solution of cosmological constant problem should be related!
A few references


P.Viana, A.Liddle, Phys.Rev.D57,674(1998)


Dynamics of quintessence

- **Cosmon** $\phi$ : scalar singlet field

- Lagrange density $L = V + \frac{1}{2} k(\phi) \partial \phi \partial \phi$
  (units: reduced Planck mass $M=1$)

- Potential : $V = \exp[-\phi]\]

- "Natural initial value" in Planck era $\phi=0$

- Today: $\phi=276$
cosmon mass changes with time!

for standard kinetic term

\[ m_c^2 = V'' \]

for standard exponential potential, \( k = \text{const.} \)

\[ m_c^2 = \frac{V''}{k^2} = \frac{V}{(k^2 M^2)} = 3 \Omega_h (1 - w_h) \frac{H^2}{(2 k^2)} \]
Quintessence models

- Kinetic function $k(\varphi)$: parameterizes the details of the model - “kinetial”
  - $k(\varphi) = k=\text{const.}$: Exponential Q.
  - $k(\varphi) = \exp \left( \left( \varphi - \varphi_1 \right) / \alpha \right)$: Inverse power law Q.
  - $k^2(\varphi) = \left( 1 / \left( 2E(\varphi_c - \varphi) \right) \right)$: Crossover Q.

- possible naturalness criterion:

  $k(\varphi=0) / k(\varphi_{\text{today}})$: not tiny or huge!

  - else: explanation needed -
More models ...

- **Phantom energy** (Caldwell)
  - negative kinetic term \( w < -1 \)
  - consistent quantum theory?

- **K – essence** (Amendariz-Picon, Mukhanov, Steinhardt)
  - higher derivative kinetic terms
  - why derivative expansion not valid?

- **Coupling cosmon / (dark) matter** (C.W., Amendola)
  - why substantial coupling to dark matter and not to ordinary matter?

- **Non-minimal coupling to curvature scalar** – \( f(\varphi) R \)
  - can be brought to standard form by Weyl scaling!
Small almost constant $k$:

- Small almost constant $\Omega_h$

Large $k$:

- Cosmon dominated universe (like inflation)