The Stability Issue in Models of Neutrino Dark Energy

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Outline

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2. Neutrino Dark Energy–The Mass Varying Neutrino (MaVaN) Scenario
3. A No-Go theorem for Neutrino Dark Energy?
4. Summary

Neutrino Dark Energy (Mass Varying Neutrinos)

[Fardon,Nelson,Weiner ’03]

Idea of varying neutrino masses in other contexts

[Kawasaki,Murayama,Yanagida ’92,Stephenson et al ’97]

- Attractive scalar force between Big Bang relic neutrinos (the analog of the Cosmic Microwave Background (CMB) photons)$\rightarrow$ smooth background, can form a negative pressure fluid
- $\rightarrow$ acts as a form of Dark Energy $\rightarrow$ accelerated expansion
- $\rightarrow$ neutrino mass m_ν becomes a function of neutrino energy density $\rho_\nu(z)$, which evolves on comological time scales (here parametrized in terms of cosmic redshift z)

$\rightarrow$ Neutrino mass not constant but promoted to a dynamical quantity $m_\nu(z)$!

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Mass Varying Neutrino (MaVaN) Scenario

The non-SM neutrino interaction mediated by a scalar field

- Introduce light scalar field ϕ with mass
  $H_0 \sim 10^{-33}\mathrm{eV} \ll m_\phi \lesssim 10^{-4}\mathrm{eV}$
- Introduce a coupling between neutrinos ν and ϕ
- $\rightarrow$ Consider class of models with

$$\mathcal{L} \supset \mathcal{L}_\phi + \mathcal{L}_{\nu_{\mathrm{kin}}} + \mathcal{L}_{\nu_{\mathrm{mass}}}, \text{ where} \tag{1}$$

$$\mathcal{L}_\phi = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V_\phi(\phi) \tag{2}$$

$$\mathcal{L}_{\nu_{\mathrm{mass}}} = -m_\nu(\phi)\bar{\nu}\nu \tag{3}$$

$\rightarrow$ neutrino mass generated from the VEV of ϕ, $m_\nu(\phi)$, and becomes linked to its dynamics
$\rightarrow$ neutrinos interact through a new non-SM force

Dynamics of the MaVaN Scenario

Complex interplay between the neutrinos and the scalar field

- Neutrino energy density ρ_ν and pressure p_ν are functions of neutrino mass $m_\nu(\phi) \longrightarrow \rho_\nu(m_\nu(\phi)), p_\nu(m_\nu(\phi))$
- $\longrightarrow$ Neutrinos can stabilize ϕ by contributing to its effective potential $V_{\rm eff}(\phi) = [\rho_\nu(m_\nu(\phi)) - 3p_\nu(m_\nu(\phi))] + V_\phi(\phi)$
- Evolution of ϕ governed by modified Klein-Gordon equation

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V'_\phi = -a^2 \underbrace{\frac{d \log m_\nu}{d\phi}}_{\text{coupling } \beta} (\rho_\nu - 3p_\nu), \text{ with } ('= d/d\phi) \quad (4)$$

- Extra source term on RHS accounts for energy exchange between ϕ and neutrinos
- As long as neutrinos relativistic, coupling term suppressed $(\rho_\nu - 3p_\nu \sim 0)$

Dynamics of the MaVaN Scenario

Adiabatic evolution in the non-relativistic neutrino regime

- Consider late-time dynamics of MaVaNs in the non-relativistic limit
  $m_\nu \gg T_\nu \to p_\nu \sim 0,\ \rho_\nu = m_\nu n_\nu$ ($n_\nu \equiv$ neutrino number density)
  $\longrightarrow V_{\rm eff}(\phi) = \rho_\nu(m_\nu(\phi)) + V_\phi(\phi)$
- In the limit $H^2 \ll V''_{\rm eff}(\mathcal{A}) = m_\phi^2$ adiabatic solution to EOM of ϕ apply (can safely neglect effects of kinetic energy terms)
- $\longrightarrow \phi$ instantaneously tracks the minimum of its effective potential $V_{\rm eff} \longrightarrow$

$$V'_{\rm eff}(\phi, z) = V'_\phi(\phi) + \underbrace{\rho'_\nu(m_\nu(\phi), z)}_{m'_\nu(\phi) n_\nu(m_\nu(\phi), z)} = 0 \text{ with } (' = \partial/\partial\phi) \quad (5)$$

Crucial effect: $n_\nu(m_\nu(\phi), z)$ is diluted by expansion $\to \phi$ varies on comological time scales (slowly)

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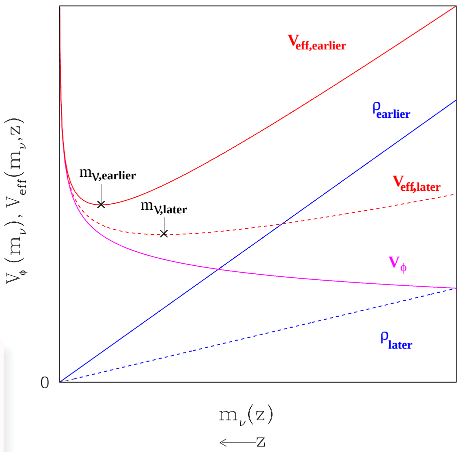
Neutrino mass varies!

- $m_\nu(\phi) = m_\nu(\phi, z)$, $\rightarrow$ $V_{\rm eff}(\phi, z) = V_{\rm eff}(m_\nu(\phi), z)$
- $\rightarrow \frac{\partial V_{\rm eff}(\phi)}{\partial \phi} = \frac{\partial m_\nu}{\partial \phi} \frac{\partial V_{\rm eff}(m_\nu)}{\partial m_\nu}|_{m_\nu = m_\nu(\phi)} = 0$
- Neutrino mass variation determined from $\frac{\partial V_{\rm eff}(m_\nu, z)}{\partial m} =$ $0 = n_\nu(m_\nu, z) + \frac{\partial V_\phi(m_\nu)}{\partial m_\nu}$

$\rightarrow$ Combined scalar-neutrino fluid has dynamical Eq. of State $\omega(z) = \frac{p_{\rm DE}(z)}{\rho_{\rm DE}(z)}$

$\omega(z) + 1 = -\frac{m_\nu(z) V'_\phi(m_\nu(z))}{m'_\nu(z) V_{\rm eff}(m_\nu(z))}$



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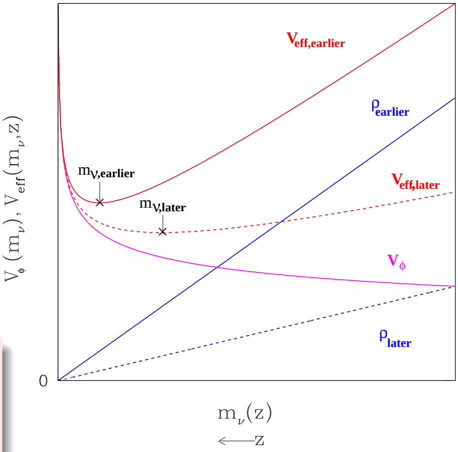

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A No-Go theorem for Neutrino Dark Energy?

Instabilities? Formation of neutrino bound states?

'In the non-relativistic neutrino regime any realistic MaVaN scenario with $m_\phi^2 > 0$ is characterized by an imaginary sound speed c_s and thus becomes unstable to hydrodynamic perturbations...with the likely outcome of the formation of non-linear neutrino structure ('neutrino nuggets')'

[Afshordi,Zaldarriaga,Kohri '05]

Note: Outcome of neutrino instability inherently non-linear process ...but if 'nuggets' really form, neutrino fluids redshifts similar to cold dark matter with $\omega \sim 0 \not\approx -1 \rightarrow$ no acceleration

Crucial claims to check

As soon as MaVaNs turn non-relativistic

- $c_s^2 < 0$ in *any* MaVaN scenario with $m_\phi^2 > 0$
- $c_s^2 < 0$ *sufficient* criterion for MaVaN instabilities

A No-Go theorem for Neutrino Dark Energy?

Instabilities

- Neutrino instabilities driven by attractive force mediated by ϕ
- Phenomenon similar to gravitational instabilities of CDM
- Good observational evidence, at early times universe homogeneouse and isotropic on all scales
- Apart from small primeval peturbations $\delta\rho_i$ in densities ρ_i of each individual particle i

$$\rho_i(\mathbf{x},\tau) = \underbrace{\rho_i(\tau)}_{\text{mean background density}} + \underbrace{\delta\rho_i(\mathbf{x},\tau)}_{\text{small perturbation}} \;,\; \underbrace{\delta_i(\mathbf{x},\tau) \equiv \frac{\delta\rho_i(\mathbf{x},\tau)}{\rho_i(\tau)}}_{\text{density constrast}}$$

- $\rightarrow$ grew by gravity into observable structure on scales of galaxies and clusters of galaxies
- Small amplitudes $|\delta\rho_i(\mathbf{x},\tau)| \ll \rho_i(\tau) \leftrightarrow |\delta_i(\mathbf{x},\tau)| \ll 1 \rightarrow$ growth of fluctuations can be solved from linear perturbation theory

A No-Go theorem for Neutrino Dark Energy?

Gravitational instability in Newtonian theory

- Assume static (non-expanding) universe, consider perfect fluid, density ρ, pressure p, velocity $\mathbf{v}$ (Continuity eq. + Euler eq. + Newtonian gravity)
- Add small perturbations δp, $\delta\rho$, $\delta\mathbf{v}$ and linearise $\rightarrow$ for k^{th} Fourier component

$$\ddot{\delta}_k + (c_s^2 k^2 - 4\pi G\rho)\delta_k = 0, \text{ where } \underbrace{\omega = \sqrt{c_s^2 k^2 - 4\pi G\rho}}_{\text{dispersion relation}} \quad (6)$$

- Perturbations adiabatic ($c_s^2 = \frac{\dot{p}}{\dot{\rho}}$ adiabatic sound speed squared)
- $\rightarrow$ sign of ω^2 determines perturbation evolution
- ω becomes imaginary for k below critical value $k_{\text{Jeans}} = \left(\frac{4\pi G\rho}{c_s^2}\right)^{1/2}$
- $\rightarrow \delta_k \propto e^{\pm\omega t}$, one exponentially growing for $k < k_{\text{Jeans}}$ $(\lambda > \lambda_{\text{Jeans}})$ (gravity overcomes pressure)
- $\rightarrow$ no growth but acoustic oscillations $\delta_k \propto e^{\pm i\omega t}$ for $k > k_{\text{Jeans}}$ $(\lambda < \lambda_{\text{Jeans}})$

$\rightarrow$ sound speed squared c_s^2 governs evolution of density contrast δ_k

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A No-Go theorem for Neutrino Dark Energy?

Make contact with MaVaN instabilities

- MaVaNs interact through gravity and the force mediated by ϕ (both attractive), $4\pi G \to 4\pi G_{\rm eff}(\beta(\phi))$
- Sound speed squared? For a general fluid i (with 'c_g' general, 'c_s' adiabatic, 'Γ_i' intrinsic entropy perturbation of i)

$$w_i \Gamma_i = (c_{gi}^2 - c_{ai}^2)\,\delta_i, \; c_g^2 = \frac{\delta p_i}{\delta \rho_i}, \; c_s^2 = \frac{\dot{p}_i}{\dot{\rho}_i}, \; \delta_i(\mathbf{x},\tau) \equiv \frac{\delta\rho_i(\mathbf{x},\tau)}{\rho_i(\tau)} \qquad (7)$$

- Dissipative processes invoke entropy perturbations ($\Gamma_i \neq 0$)
- For MaVaNs? Depends on scales/regimes one considers!
- Relativistic neutrinos: neutrino pressure support overcomes attractive force $\to$ no growth (on all scales)
- Non-relativistic neutrinos: $p_\nu \sim 0$
- m_ϕ^{-1} sets physical length scales a/k as of which gradient terms become unimportant ($\Gamma_\phi \sim 0$) (for small deviations away from its minimum, ϕ re-adjusts to new minimum on a time scale $m_\phi^{-1} \ll H^{-1}$

[Afshordi, Zaldarriaga, Kohri'05]

On scales $m_\phi^{-1} < a/k < H^{-1}$ MaVaN perturbations adiabatic $\to \nu - \phi$ system can be treated as unified fluid with $\Gamma_{DE} = 0$ and $c_s^2 = \frac{\dot{p}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)}$

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Evolution of scalar field perturbations

- Perturbed Klein-Gordon equation in the non-relativistic neutrino regime ($\rightarrow$ neglect terms $\propto p_\nu, \omega_\nu, c_\nu^2$ and $\dot{\phi}$)

$$\ddot{\delta\phi} + 2H\dot{\delta\phi} + \left[k^2 + a^2 \underbrace{\left(V''_\phi + \beta' \rho_\nu\right)}_{m_\phi^2 - \beta^2 \rho_\nu} \right] \delta\phi = -a^2 \beta \delta_\nu \rho_\nu \tag{8}$$

- Solution of homogenous equation is oscillating with decaying amplitude
- Particular solution given by forcing term on RHS

$$\delta\phi = -\frac{a^2 \beta \rho_\nu \delta_\nu}{a^2 (V''_\phi + \rho_\nu \beta') + k^2} \tag{9}$$

A No-Go theorem for Neutrino Dark Energy?

Equation of motion of the neutrino density contrast $\delta_\nu = \frac{\delta\rho_\nu}{\rho_\nu}$

In the non-relativistic limit $p_\nu \sim \omega_\nu$ on length scales $m_\phi^{-1} < a/k < H^{-1}$

$$0 = \ddot{\delta}_\nu + H\dot{\delta}_\nu - 4\pi a^2 G\left(\rho_{\rm CDM}\delta_{\rm CDM} + \rho_b\delta_b\right) + \underbrace{c_\nu^2 k^2 - 4\pi a^2 G\left(1 + \frac{2\beta^2 M_{\rm pl}^2}{1 + a^2(V_\phi'' + \rho_\nu\beta')/k^2}\right)\rho_\nu\delta_\nu}_{\simeq \frac{c_s^2}{(c_s^2+1)}k^2 - 4\pi G\rho_\nu\delta_\nu}$$

where $G_{\rm eff} = G\left(1 + \frac{2\beta^2 M_{\rm pl}^2}{1+a^2(V_\phi''+\rho_\nu\beta')/k^2}\right)$

For $G_{\rm eff}\rho_\nu\delta_\nu < G\rho_{\rm CDM}\delta_{\rm CDM}$ dynamics of δ_ν governed by CDM $\rightarrow$ growth like ordinary gravitational instabilities, ('neutrinos follow CDM') $\delta_\nu = \delta_{\rm CDM}(a)(1 - \frac{b}{a})$ (a scale factor, b some constant)

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A No-Go theorem for Neutrino Dark Energy?

Any realistic MaVaN scenario $c_s^2 < 0$?

- Require $c_s^2 = \frac{\dot{p}_{\rm DE}}{\dot{\rho}_{\rm DE}} = \omega - \frac{\dot{\omega}}{3H(1+\omega)} > 0$ for $m_\nu(z) \gg T_\nu(z)$ (take into account finite temperature effects)

$$\rightarrow \sum_{i=1}^{3} \frac{\partial m_{\nu_i}(z)}{\partial z} \left(1 - \frac{5\alpha T_{\nu,0}^2 (1+z)^2}{3 m_{\nu_i}^2(z)}\right) + \sum_{i=1}^{3} \frac{25\alpha T_{\nu,0}^2 (1+z)}{3 m_{\nu_i}(z)} > 0, \text{ with } \alpha \equiv \frac{\int_0^\infty \frac{dy\, y^4}{e^y+1}}{2\int_0^\infty \frac{dy\, y^2}{e^y+1}}$$

[Takahashi,Tanimoto '06]

- Assume degenerate mass spectrum with $m_{\nu_i}(0) \sim m_\nu(0) = 0.312$ eV, $i = 1, 2, 3$ $\rightarrow$ determine maximally allowed neutrino mass variation

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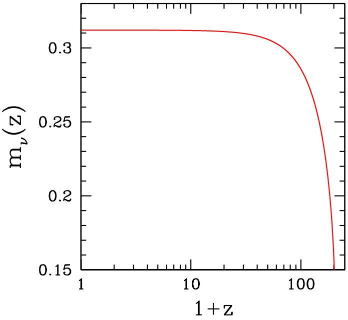

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A concrete model

Consider model proposed in the context of 'Chameleon cosmologies'

[Khoury, Weltman '03, Brax, van de Bruck, Davis, Khoury, Weltman '04, ...]

- Inverse power-law potential (at late times $\phi \gtrsim M \sim \rho_{\rm DE}^{(0)}$ to accomodate $\Omega_{\rm DE} \sim 0.7$)

$$V_\phi(\phi) = M^4 e^{\frac{M^n}{\phi^n}} \sim M^4 + M^{4+n}\phi^{-n} \text{ for } \phi \gtrsim M \tag{10}$$

- Recall: evolution of ϕ determined from $V'_{\rm eff} = 0 = V'_\phi + \rho'_\nu$
- Mass dependence on ϕ

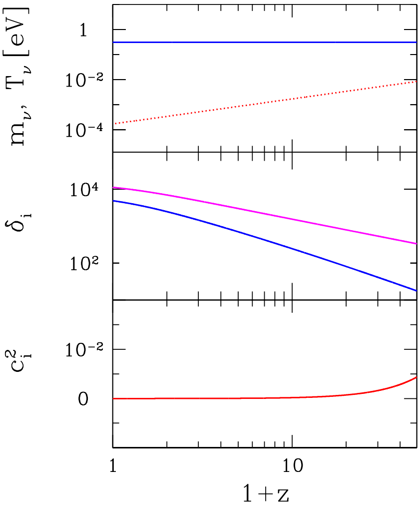
$$m_\nu(\phi) = m_0 e^{\beta\phi}, \text{ where } \overbrace{\beta}^{\rm coupling} = \frac{d\log m_\nu}{d\phi} = \text{const.} \tag{11}$$

- For β of $O(1/M_{\rm pl})$ $\phi \ll 1 M_{\rm pl} \to m_\nu$ very weakly depends on changes in the neutrino energy density $\to$ hardly evolves with time
- $\to$ attractive force between neutrinos essentially time independent

A concrete model

- Normalization?
- $\rightarrow$ For $k = 0.11h\,\mathrm{Mpc}^{-1}$
  $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2} \propto \delta^2_{\mathrm{CDM}} < 1$
  $\rightarrow$linear

  [Percival et al.'06]
- Since $\delta^2_\nu < \delta^2_{\mathrm{CDM}} < 1 \rightarrow$ neutrino density contrast linear!
- $\rightarrow$ Adiabatic model of Neutrino Dark Energy stable also in the highly non-relativistic regime$\rightarrow$ viable dark energy candidate



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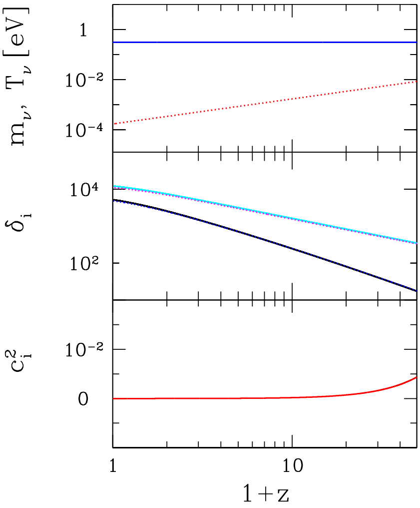

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Summary

- Possible growth of MaVaN perturbations depends on relative influence of CDM
- Viable model of neutrino dark energy found with $c_s^2 > 0$? $\rightarrow$ growth of perturbations as in General Relativity, but allowed mass variation strongly restricted at late times
- $c_s^2 < 0$? If relative influence of CDM negligible wrt to coupling term $\rightarrow$ perturbations tend to go non-linear?? Needs to be checked numerically...
- Note: 'Hybrid' models involving two light scalar fields can be stable until the present time even in the presence of unstable neutrino component

[Fardon, Nelson, Weiner '06, Spitzer '06]