Neutrino mass matrices from type II seesaw and an extension of S_3

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Type II seesaw mechanism

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Scalar triplets $\Delta_j = \frac{1}{\sqrt{2}} \vec{H} \cdot \vec{\tau} = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix}$

Higgs doublets ϕ_α

$$V_{\text{trilinear}} = \sum_{i,\alpha,\beta} t_{i\alpha\beta}\, \tilde{\phi}_\alpha^\dagger \Delta_i^\dagger \phi_\beta + \text{H.c.}$$

VEVs: $\langle H_i^0 \rangle = \delta_i$, $\phi_\alpha^0 = v_\alpha$

$$\begin{aligned} V_0 &= \left(\mu_\phi^2\right)_{\alpha\beta} v_\alpha^* v_\beta + \left(\mu_\Delta^2\right)_{ij} \delta_i^* \delta_j + \left(t_{i\alpha\beta}\delta_i^* v_\alpha v_\beta + \text{c.c.}\right) \\ &\quad + \lambda_{\alpha\beta\gamma\delta} v_\alpha^* v_\beta v_\gamma^* v_\delta + \lambda_{ijkl}\delta_i^*\delta_j\delta_k^*\delta_l + \lambda_{\alpha\beta ij} v_\alpha^* v_\beta \delta_i^* \delta_j \end{aligned}$$

Assumptions:

i) μ_Δ^2 positive definite, μ_ϕ^2 *not* positive definite $\Rightarrow$ spontaneous symmetry breaking
Note: $t_{i\alpha\beta} = 0 \Rightarrow \delta_i = 0$

ii) Eigenvalues of μ_Δ^2, $t_{i\alpha\beta}$ of order M $\longrightarrow$ seesaw scale
v_α of order of electroweak scale

$$\boxed{\delta_i \approx -\left(\mu_\Delta^2\right)^{-1}_{ij} t_{j\alpha\beta} v_\alpha v_\beta}$$

$$\delta_i \sim v^2/M \text{ with } v \approx 174 \text{ GeV}$$

The model

The multiplets: $\alpha = e, \mu, \tau$

Three left-handed lepton doublets $D_{L\alpha}$

Three right-handed charged-lepton singlets α_R

Three Higgs doublets ϕ_α

Four Higgs triplets Δ_α and Δ_4

The symmetries:

S_3: permutation of indices e, μ, τ

Two sets of three $\mathbb{Z}_2$ symmetries:

$$\mathrm{z}_\alpha^{(1)}: \quad \phi_\alpha \to -\phi_\alpha, \; \alpha_R \to -\alpha_R$$

$$\mathrm{z}_\alpha^{(2)}: \quad D_{L\alpha} \to -D_{L\alpha}, \; \alpha_R \to -\alpha_R, \Delta_\beta \to -\Delta_\beta \text{ iff } \beta \neq \alpha.$$

Comments:

Higgs “glue” numbers $\mathrm{z}_\alpha^{(1)}$

Discrete lepton numbers $\mathrm{z}_\alpha^{(2)}$

Δ_4 invariant under all these symmetries

The Yukawa Lagrangian:

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -y_0 \bar{D}_{L\alpha} \phi_\alpha \alpha_R + \frac{1}{2} y_1 D_{L\alpha}^T C^{-1} \varepsilon \Delta_4 D_{L\alpha} \\ & +y_2 \Big(D_{Le}^T C^{-1} \varepsilon \Delta_\mu D_{L\tau} + D_{L\mu}^T C^{-1} \varepsilon \Delta_\tau D_{Le} + \\ & D_{L\tau}^T C^{-1} \varepsilon \Delta_e D_{L\mu} \Big) + \text{H.c.}\end{aligned}$$

Charged-lepton masses: $m_\alpha = |y_0 v_\alpha|$

Neutrino mass matrix:

$$\mathcal{M}_\nu = \begin{pmatrix} y_1\delta_4 & y_2\delta_\tau & y_2\delta_\mu \\ y_2\delta_\tau & y_1\delta_4 & y_2\delta_e \\ y_2\delta_\mu & y_2\delta_e & y_1\delta_4 \end{pmatrix}$$

Type II seesaw and symmetry breaking:

$$V_{\text{trilinear}} = \sum_\alpha t\, \tilde{\phi}_\alpha^\dagger \Delta_4^\dagger \phi_\alpha + \text{H.c.} \;\Rightarrow\; t_{i\alpha\beta} = t\delta_{i4}\delta_{\alpha\beta}$$

$$\delta_i = -t v_\alpha v_\alpha \left(\mu_\Delta^2\right)^{-1}_{i4} \quad (i = e, \mu, \tau, 4)$$

However, this does not work: $\mathrm{z}_\alpha^{(2)} \Rightarrow \mu_\Delta^2$ **diagonal** $\Rightarrow$
$\delta_e = \delta_\mu = \delta_\tau = 0$, **symmetry too strong!**
Remedy: Soft symmetry breaking by dimension-2 terms
Symmetry-breaking options:

1. **Flavour symmetries:**
   (a) ***All*** **softly broken** $\Rightarrow$ **all** δ_i **different**
   (b) $\mu \leftrightarrow \tau$ **exchange symmetry left unbroken** $\Rightarrow$
   $\underline{\delta_e = \delta_\mu \neq \delta_\tau}$
2. **CP violation:**
   (a) **Hard breaking** $\Rightarrow$ **no phase relation for** $\delta_i \neq \delta_j$
   (b) **Spontaneous breaking** $\Rightarrow$ **phases of all** δ_i **the same**

A 3-parameter neutrino mass matrix

❶ $\mu \leftrightarrow \tau$ exchange symmetry intact in Lagrangian

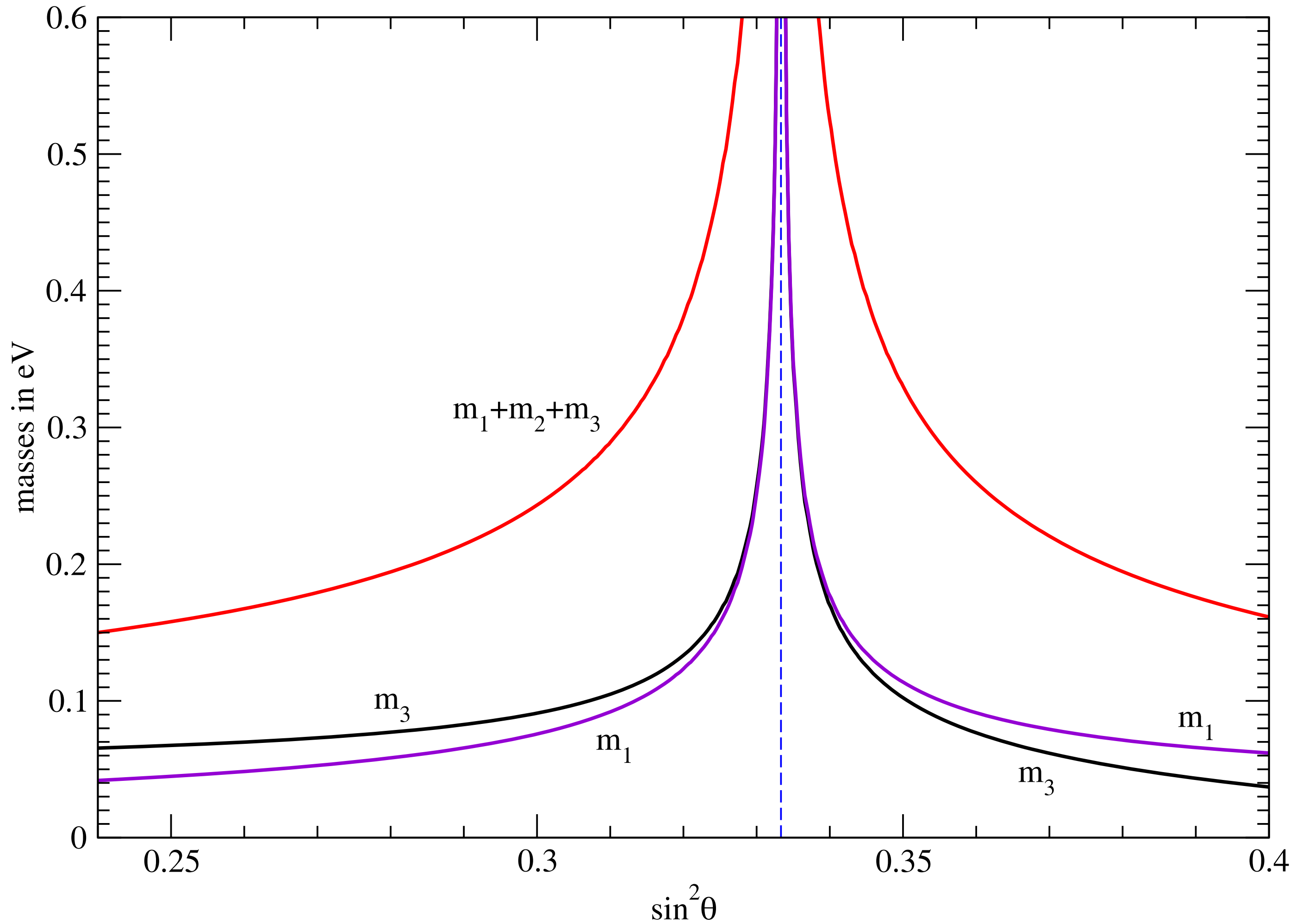
❷ CP spontaneously broken

Note: $\mu \leftrightarrow \tau$ exchange symmetry spontaneously broken by $v_\mu \neq v_\tau$ or $m_\mu \neq m_\tau$

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & x & w \\ y & w & x \end{pmatrix} \quad \text{with} \quad x,\, y,\, w \in \mathbb{R}$$

Predictions:

- $\theta_{13} = 0^\circ$, $\theta_{23} = 45^\circ$
- Absolute mass scale function of $\theta_{12} \equiv \theta$:
  $s^2_{12} < 1/3 \Rightarrow$ normal spectrum
  $s^2_{12} > 1/3 \Rightarrow$ inverted spectrum
- $m_{\beta\beta} \approx m_3/3$



Dashed vertical line: singularity at $\sin^2\theta = 1/3$
(Harrison - Perkins Scott value of solar mixing angle, $\theta \approx 35.26^\circ$)

The group structure

S_3 commutes neither with the $\mathrm{z}_\alpha^{(1)}$ nor with the $\mathrm{z}_\alpha^{(2)}$
Define $N \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \to 3 \times 3$ diagonal sign matrices

$$G: \begin{pmatrix} ms & 0 \\ 0 & ns \end{pmatrix}, \quad m, n \in N, \ s \in \hat{S}_3 \equiv 3{\times}3 \text{ permutation matrices}$$

Multiplication law: $(m, n, s) \in G$ with n, $m \in N$, $s \in \hat{S}_3$

$$(m_1, n_1, s_1)(m_2, n_2, s_2) = (m_1 s_1 m_2 s_1^{-1}, n_1 s_1 n_2 s_1^{-1}, s_1 s_2)$$

$$G = \{(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2) \times (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2)\} \rtimes S_3$$

Number of elements of G: $2^3 \times 2^3 \times 3! = 384$

Irreducible representations:

$\underline{\mathbf{1}}^{(p,q,r)}: \ (m,n,s) \to (\det m)^p (\det n)^q (\det s)^r$ **with** $p,q,r \in \{0,1\}$

$$\begin{aligned}
\underline{\mathbf{3}}_1^{(p,q,r)}: &\quad (m,n,s) \to (\det m)^p (\det n)^q (\det s)^r \, ms \\
\underline{\mathbf{3}}_2^{(p,q,r)}: &\quad (m,n,s) \to (\det m)^p (\det n)^q (\det s)^r \, ns \\
\underline{\mathbf{3}}_3^{(p,q,r)}: &\quad (m,n,s) \to (\det m)^p (\det n)^q (\det s)^r \, mns
\end{aligned}$$

3-dimensional irreps used in the model:

$$\begin{array}{rll}
ms & \text{for} & (\phi_e, \phi_\mu, \phi_\tau) \\
mns & \text{for} & (e_R, \mu_R, \tau_R) \\
ns & \text{for} & (D_{Le}, D_{L\mu}, D_{L\tau}) \\
(\det n)\, ns & \text{for} & (\Delta_e, \Delta_\mu, \Delta_\tau)
\end{array}$$

Further irreps: four 2-dimensional and four 6-dimensional irreps

Check: $8 \times 1^2 + 4 \times 2^2 + 24 \times 3^2 + 4 \times 6^2 = 384$