

# ***Solar Model Parameters and Direct Measurement of Solar Neutrino Fluxes***

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# ***OUTLINE***

- **Standard Solar Model and its parameters**
  - **Motivations for using neutrinos as probes to SSM parameters**
  - **Neutrino flux – present knowledge and future measurements**
  - **Techniques to determine SSM parameters using neutrino flux measurements**
  - **Results**
  - **Summary**
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# *Standard Solar Model & its parameters*

- Homogeneous cloud of **H**, **He**, small fraction of **heavier elements**
- Mass =  **$M_{\text{sun}}$**
- Evolved throughout a time  **$t = t_{\text{sun}}$**
- To get present day value of  **$L_{\text{sun}}$**  and  **$R_{\text{sun}}$**

**Hydrostatic equilibrium**

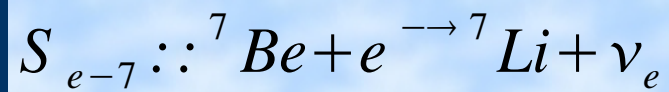
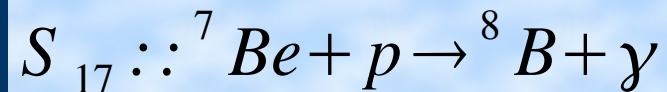
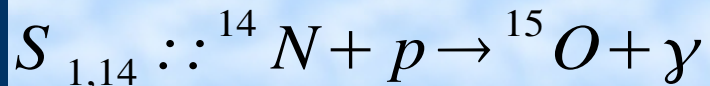
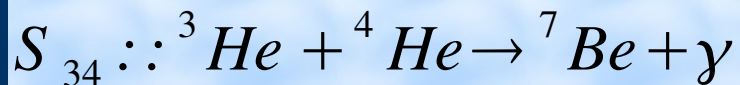
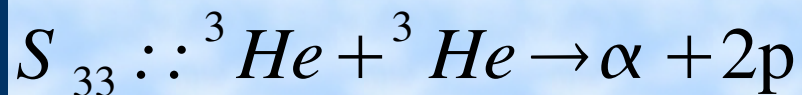
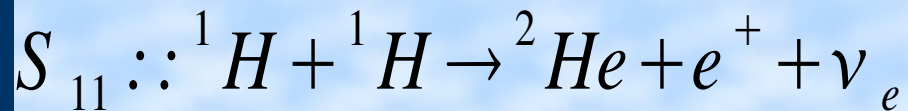
**Energy transport primarily by photon diffusion**

- **Energy generations by nuclear reactions**
- **Abundance changes caused by nuclear reactions**



# SSM & its parameters (contd..)

## Low E Cross Sections



## Astronomical parameters

- **Solar Age**
- **Solar Luminosity**
- **Initial heavy element abundance:  $Z / X$**
- **Diffusion**
- **Opacity**

# Solar Neutrino Flux – present knowledge and future measurements $(^8B)$

- **SSM Prediction:**  $\phi_B = 5.79(1 \pm 0.23) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$
- **SNO NC:**  $\phi_B = 4.94(1 \pm 0.088) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$
- **Global analysis of Solar+KamLAND data:**  $\phi_B = 4.88(1 \pm 0.036) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$

## Data Set

|                  | $\frac{\Delta \phi_B}{\phi_B}$ |
|------------------|--------------------------------|
| <b>Solar</b>     | <b>4.4%</b>                    |
| <b>+ KamLAND</b> | <b>3.6%</b>                    |
| <b>+ SNO-III</b> | <b>3.2%</b>                    |
| <b>+ pp</b>      | <b>2.5%</b>                    |
| <b>+ SPMIN</b>   | <b>1.7%</b>                    |

# Solar Neutrino Flux – present knowledge and future measurements

( ${}^7\text{Be}$ ,  $pp$ )

•  $\phi_{7\text{Be}} / (10^9 \text{ cm}^{-2} \text{ s}^{-1})$

**SSM : 4.16(1 +/- 0.12)**  $\frac{\Delta \phi_{\text{Be}}}{\phi_{\text{Be}}}$

**Solar+KL: worse**

**+Borexino(10%): 10%**

**(+Borexino(5%)):** 5.5%

•  $\phi_{pp} / (10^{10} \text{ cm}^{-2} \text{ s}^{-1})$

**SSM : 5.94(1 +/- 0.01)**  $\frac{\Delta \phi_{pp}}{\phi_{pp}}$

**Solar+KL: worse**

**+ Lum Constraint: 2%**

**+Borexino(5%): 0.5%**

# Dependence of Neutrino flux on SSM

parameters [BP04]

<http://www.sns.ias.edu/~bahcall>

$$\phi_B = \text{Const} \times S_{11}^{-2.59} S_{33}^{-0.40} S_{34}^{+0.81} S_{1,14}^{+0.01} S_{17}^{1.0} S_{e-7}^{-1.0} \\ L^{+6.76} \tau^{+1.28} \text{Opac}^{-2.93} D^{-2.2} (Z/X)^{+1.36}$$

$$\phi_{Be} = \text{Const} \times S_{11}^{-0.97} S_{33}^{-0.43} S_{34}^{+0.86} L^{+3.40} \tau^{+0.69} \text{Opac}^{-1.49} \\ D^{-0.96} (Z/X)^{0.62}$$

$$\phi_{pp} = \text{Const} \times S_{11}^{+0.14} S_{33}^{+0.03} S_{34}^{-0.06} S_{1,14}^{-0.02} L^{+0.73} \tau^{-0.07} \\ \text{Opac}^{+0.14} D^{+0.13} (Z/X)^{-0.08}$$

$$\phi_i = C_i \prod_{\text{all } j} x_j^{\alpha_{ij}}$$

$$\alpha_{ij} = \frac{\partial \ln \phi_i}{\partial \ln x_j}$$

# Determination of SSM parameters using measured neutrino flux

$$\phi_i = C_i \prod_{all\ j} x_j^{\alpha_{ij}} \quad i = \text{pp, pep, hep, B, Be, N, O, F}$$

$K = \#$  of known fluxes       $N =$  total  $\#$  of input parameters

Use the set of  $K$  eqns. to solve for any  $K$  input parameters ( $\mathbf{x}$ ), taking rest ( $N-K$ ) parameters from their laboratory measurements.

$$(\Delta \ln x_j)^2 = \sum_{i=1, K} P_{ir}^2 (\Delta \ln \phi_i)^2 + \sum_{j \neq 1, K} Q_{jr}^2 (\Delta \ln x_j)^2$$
$$\Delta \ln x_r = \ln \left( 1 + \frac{\Delta x_r}{x_r} \right)$$



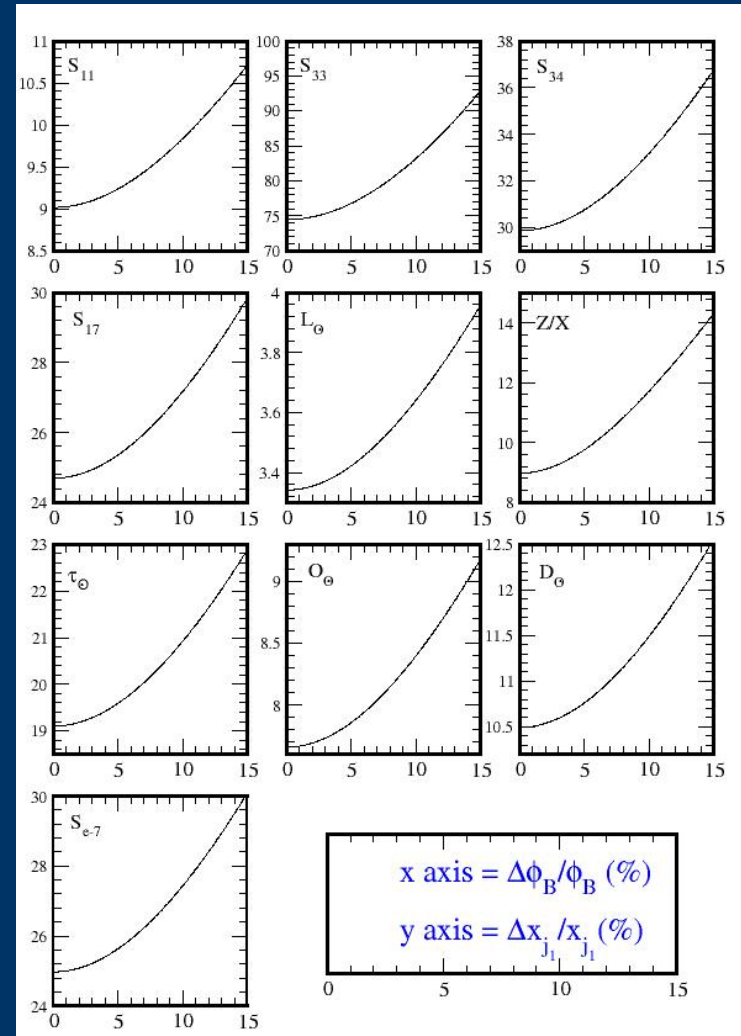
# Results (Measured flux = B)

- **Good precision of  $S_{11}$ ,  $Z/X$ ,  $L$ , Opacity**
- **Uncertainty in  $Z/X$  is smaller than its currently estimated uncertainty in SSM**
- **Poor precision of  $S_{33}$  and  $S_{1,14}$**

| $x_j$     | $\frac{\Delta x_j}{x_j} (\%)$        |                                      |                                      | $\frac{\Delta x_j^0}{x_j^0} (\%)$ | $\frac{x_j}{x_j^0}$ |
|-----------|--------------------------------------|--------------------------------------|--------------------------------------|-----------------------------------|---------------------|
|           | $\frac{\Delta \phi_B}{\phi_B} = 2\%$ | $\frac{\Delta \phi_B}{\phi_B} = 3\%$ | $\frac{\Delta \phi_B}{\phi_B} = 4\%$ |                                   |                     |
| $S_{11}$  | 9.05                                 | 9.10                                 | 9.16                                 | 0.4                               | 1.07                |
| $S_{34}$  | 30.01                                | 30.19                                | 30.44                                | 9.4                               | 0.81                |
| $S_{17}$  | 24.80                                | 24.94                                | 25.12                                | 3.8                               | 0.84                |
| $L_\odot$ | 3.35                                 | 3.37                                 | 3.39                                 | 0.4                               | 0.97                |
| $Z/X$     | 9.11                                 | 9.27                                 | 9.48                                 | 15.0                              | 0.88                |
| Age       | 19.18                                | 19.28                                | 19.42                                | 0.4                               | 0.87                |
| $O_\odot$ | 7.69                                 | 7.73                                 | 7.79                                 | 2.0                               | 1.06                |
| $D_\odot$ | 10.54                                | 10.59                                | 10.66                                | 2.0                               | 1.08                |
| $S_{e-7}$ | 25.08                                | 25.22                                | 25.40                                | 2.0                               | 1.19                |

# Results (Measured flux = B)

- Model parameter uncersts depends strongly on  $\delta B$  flux uncerst if  $\delta B$  flux uncerst  $> 5\%$
- Model parameter uncerst are stable below  $4\%$  of  $\delta B$  uncerst.



# Results (Measured flux = B, Be)

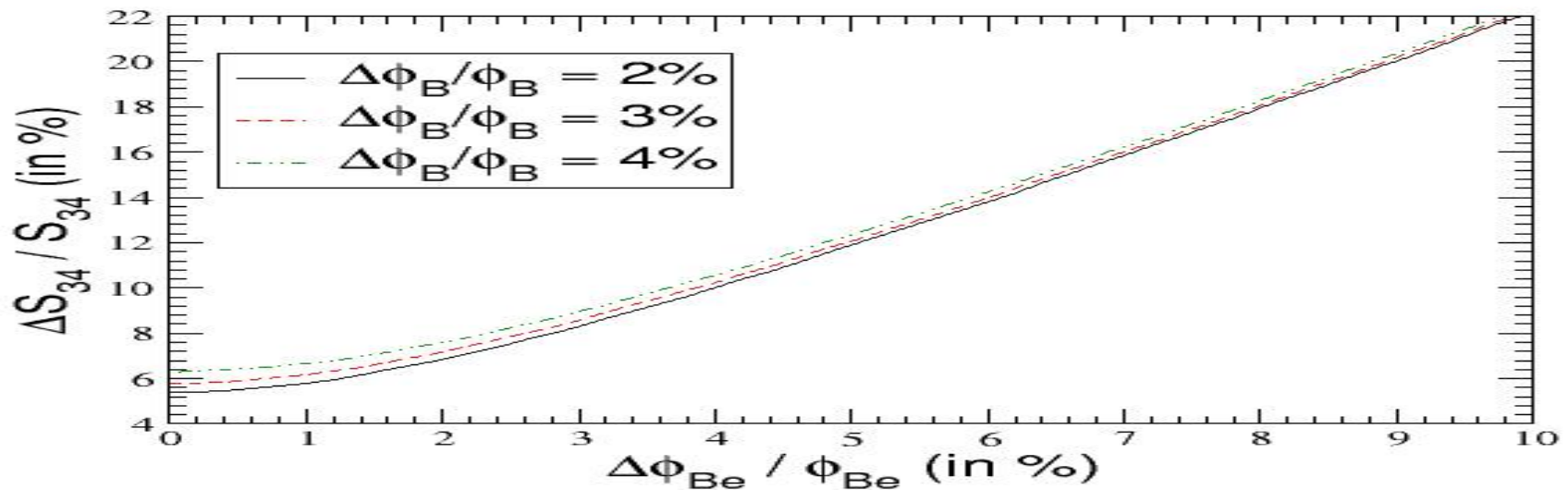
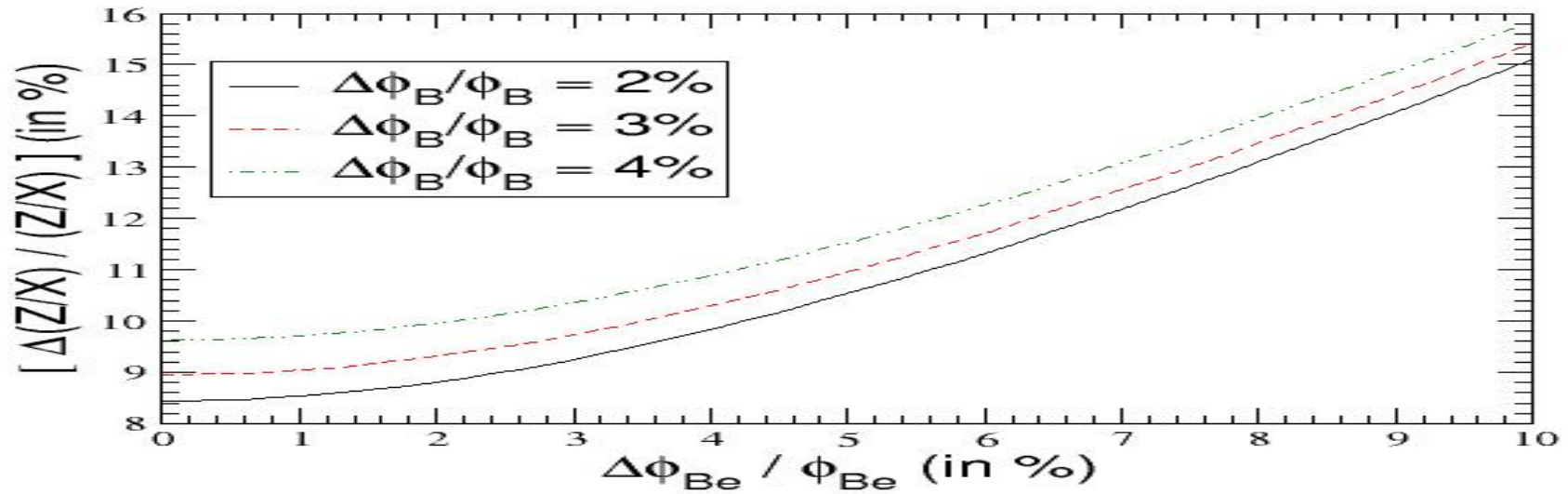
4% Uncert of B flux

6% Uncert of Be flux

- Good precision of S34 when attempted to measure with S11 or Z/X or D
- L is measured with good accuracy when attempted to measure with S17, S34, Se-7

| combinations    |                 | $\frac{\Delta x_{j_1}}{x_{j_2}}$ | $\frac{\Delta x_{j_2}}{x_{j_2}}$ |
|-----------------|-----------------|----------------------------------|----------------------------------|
| $j_1$           | $j_2$           | (%)                              | (%)                              |
| S <sub>34</sub> | S <sub>11</sub> | 12.71                            | 8.81                             |
| S <sub>34</sub> | Z/X             | 14.25                            | 12.26                            |
| S <sub>34</sub> | D <sub>⊙</sub>  | 13.79                            | 11.35                            |

# Results (Measured flux = B, Be)





# Results (Measured flux =B, Be, pp)

Uncerts in fluxes:

B -> 3%

Be -> 4%

pp -> 1%

Some Most effective combinations:

{S<sub>34</sub>, L, S<sub>e-7</sub>} -> S<sub>34</sub> (6.3%)  
(present~9.4%)

{S<sub>17</sub>, L, Z/X} ->Z/X (6.65%)  
(present~15%)

| combinations    |                 |                  | $\frac{\Delta x_{j1}}{x_{j2}}$ | $\frac{\Delta x_{j2}}{x_{j2}}$ | $\frac{\Delta x_{j3}}{x_{j3}}$ (%) |
|-----------------|-----------------|------------------|--------------------------------|--------------------------------|------------------------------------|
| $x_{j1}$        | $x_{j2}$        | $x_{j3}$         | (%)                            | (%)                            |                                    |
| S <sub>11</sub> | S <sub>34</sub> | L <sub>⊙</sub>   | 6.16                           | 7.98                           | 1.62                               |
| S <sub>11</sub> | L <sub>⊙</sub>  | O <sub>⊙</sub>   | 13.67                          | 1.72                           | 12.25                              |
| S <sub>34</sub> | L <sub>⊙</sub>  | Z/X              | 8.87                           | 1.00                           | 8.63                               |
| S <sub>34</sub> | L <sub>⊙</sub>  | O <sub>⊙</sub>   | 10.45                          | 1.69                           | 5.31                               |
| S <sub>34</sub> | L <sub>⊙</sub>  | D <sub>⊙</sub>   | 9.03                           | 1.57                           | 5.77                               |
| S <sub>34</sub> | L <sub>⊙</sub>  | τ <sub>⊙</sub>   | 11.40                          | 1.66                           | 12.86                              |
| S <sub>11</sub> | S <sub>17</sub> | L <sub>⊙</sub>   | 4.73                           | 11.05                          | 1.73                               |
| S <sub>11</sub> | L <sub>⊙</sub>  | S <sub>e-7</sub> | 4.53                           | 1.73                           | 11.05                              |
| S <sub>33</sub> | S <sub>17</sub> | L <sub>⊙</sub>   | 12.40                          | 8.47                           | 1.95                               |
| S <sub>33</sub> | L <sub>⊙</sub>  | S <sub>e-7</sub> | 13.43                          | 1.95                           | 8.47                               |
| S <sub>34</sub> | S <sub>17</sub> | L <sub>⊙</sub>   | 8.61                           | 8.98                           | 1.86                               |
| S <sub>34</sub> | L <sub>⊙</sub>  | S <sub>e-7</sub> | 6.33                           | 1.85                           | 8.98                               |
| S <sub>17</sub> | L <sub>⊙</sub>  | Z/X              | 9.26                           | 1.21                           | 6.65                               |
| S <sub>17</sub> | L <sub>⊙</sub>  | τ <sub>⊙</sub>   | 9.05                           | 1.85                           | 7.34                               |
| S <sub>17</sub> | L <sub>⊙</sub>  | O <sub>⊙</sub>   | 9.21                           | 1.84                           | 3.40                               |
| S <sub>17</sub> | L <sub>⊙</sub>  | D <sub>⊙</sub>   | 9.74                           | 1.74                           | 4.68                               |
| L <sub>⊙</sub>  | Z/X             | S <sub>e-7</sub> | 1.21                           | 6.65                           | 9.88                               |
| L <sub>⊙</sub>  | τ <sub>⊙</sub>  | S <sub>e-7</sub> | 1.85                           | 7.42                           | 9.29                               |
| L <sub>⊙</sub>  | O <sub>⊙</sub>  | S <sub>e-7</sub> | 1.84                           | 3.40                           | 9.48                               |
| L <sub>⊙</sub>  | D <sub>⊙</sub>  | S <sub>e-7</sub> | 1.73                           | 4.68                           | 10.12                              |