Neutrinos and the Stars

Georg Raffelt, MPI Physik, Munich, Germany

Lectures at JIGSAW 07, 12–23 Feb 2007, TIFR, Mumbai, India

Where do Neutrinos Appear in Nature?

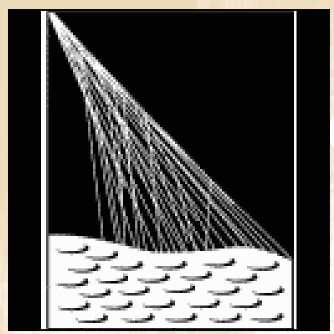
✓ Nuclear Reactors

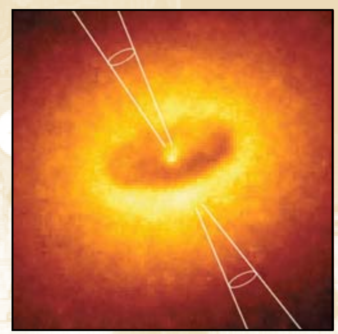
Sun ✓

✓ Particle Accelerators

Supernovae
(Stellar Collapse)
SN 1987A ✓

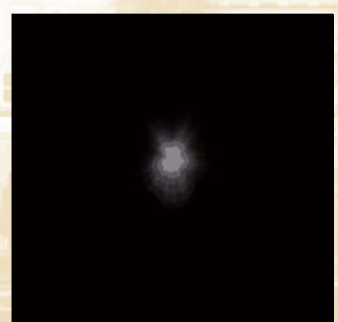
✓ Earth Atmosphere
(Cosmic Rays)





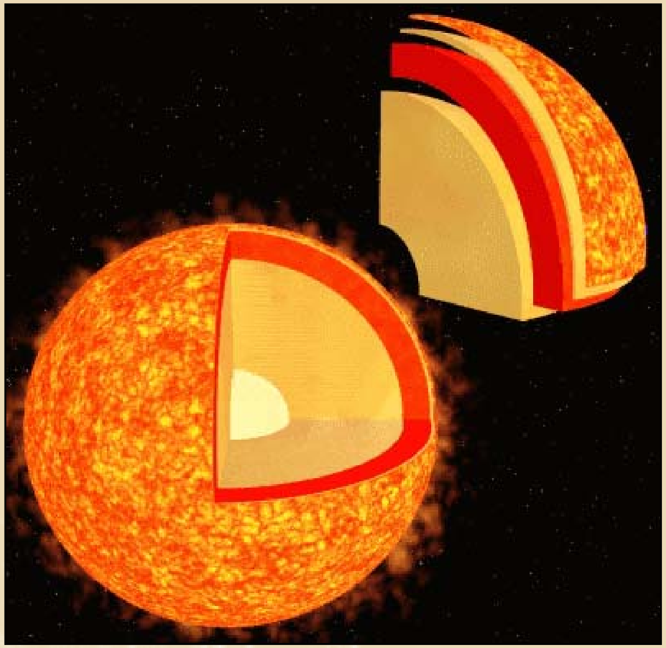
Astrophysical
Accelerators Soon ?

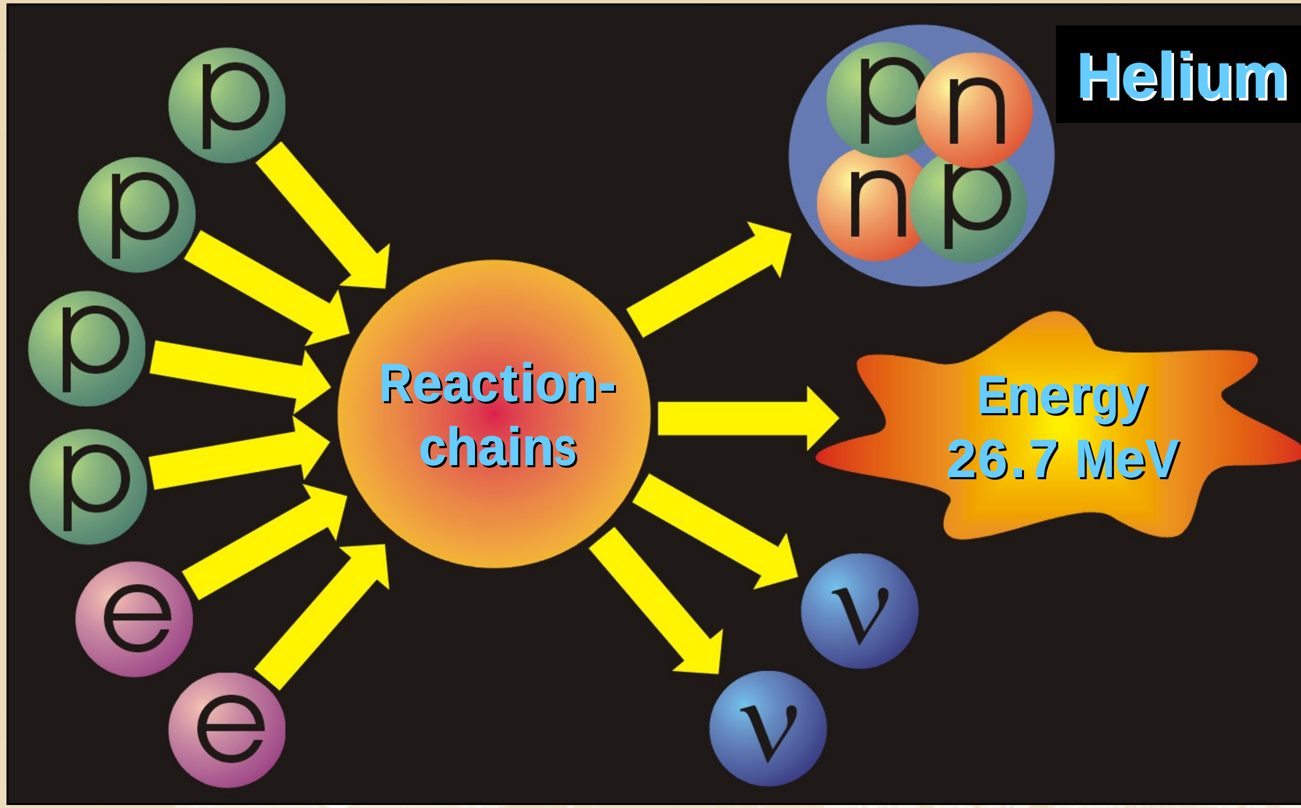
✓ Earth Crust
(Natural
Radioactivity)



Cosmic Big Bang
(Today 330 ν/cm^3)
Indirect Evidence

Neutrinos from the Sun





Solar radiation: 98 % light
2 % neutrinos
At Earth 66 billion neutrinos/cm² sec

Hans Bethe (1906–2005, Nobel prize 1967)
Thermonuclear reaction chains (1938)

Bethe's Classic Paper on Nuclear Reactions in Stars

MARCH 1, 1939 PHYSICAL REVIEW VOLUME 55

Energy Production in Stars*

H. A. Bethe
Cornell University, Ithaca, New York

(Received September 7, 1938)

It is shown that the *most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons.* These reactions form a cycle in which the original nucleus is reproduced, *viz.* $C^{12}+H=N^{13}$, $N^{13}=C^{13}+\epsilon^+$, $C^{13}+H=N^{14}$, $N^{14}+H=O^{15}$, $O^{15}=N^{15}+\epsilon^+$, $N^{15}+H=C^{12}+He^4$. Thus carbon and nitrogen merely serve as catalysts for the combination of four protons (and two electrons) into an α-particle (§7).

The carbon-nitrogen reactions are unique in their cyclical character (§8). For all nuclei lighter than carbon, reaction with protons will lead to the emission of an α-particle so that the original nucleus is permanently destroyed. For all nuclei heavier than fluorine, only radiative capture of the protons occurs, also destroying the original nucleus. Oxygen and fluorine reactions mostly lead back to nitrogen. Besides, these heavier nuclei react much more slowly than C and N and are therefore unimportant for the energy production.

The agreement of the carbon-nitrogen reactions with observational data (§7, 9) is excellent. In order to give the correct energy evolution in the sun, the central temperature of the sun would have to be 18.5 million degrees while integration of the Eddington equations gives 19. For the brilliant star Y Cygni the corresponding figures are 30 and 32. This good agreement holds for all bright stars of the main sequence, but, of course, not for giants.

For fainter stars, with lower central temperatures, the reaction $H+H=D+\epsilon^+$ and the reactions following it, are believed to be mainly responsible for the energy production. (§10)

It is shown further (§5–6) that *no elements heavier than* He^4 *can be built up in ordinary stars.* This is due to the fact, mentioned above, that all elements up to boron are disintegrated by proton bombardment (α-emission!) rather than built up (by radiative capture). The instability of Be^8 reduces the formation of heavier elements still further. The production of neutrons in stars is likewise negligible. The heavier elements found in stars must therefore have existed already when the star was formed.

Finally, the suggested mechanism of energy production is used to draw conclusions about astrophysical problems, such as the mass-luminosity relation (§10), the stability against temperature changes (§11), and stellar evolution (§12).

§1. Introduction

The progress of nuclear physics in the last few years makes it possible to decide rather definitely which processes can and which cannot occur in the interior of stars. Such decisions will be attempted in the present paper, the discussion being restricted primarily to main sequence stars. The results will be at variance with some current hypotheses.

The first main result is that, under present conditions, no elements heavier than helium can be built up to any appreciable extent. Therefore we must assume that the heavier elements were built up *before* the stars reached their present state of temperature and density. No attempt will be made at speculations about this previous state of stellar matter.

The energy production of stars is then due entirely to the combination of four protons and two electrons into an α-particle. This simplifies the discussion of stellar evolution inasmuch as the amount of heavy matter, and therefore the opacity, does not change with time.

The combination of four protons and two electrons can occur essentially only in two ways. The first mechanism starts with the combination of two protons to form a deuteron with positron emission, *viz.*

$$H+H=D+\epsilon^+. \tag{1}$$

The deuteron is then transformed into He^4 by further capture of protons; these captures occur very rapidly compared with process (1). The second mechanism uses carbon and nitrogen as catalysts, according to the chain reaction

$$\begin{array}{ll} C^{12}+H=N^{13}+\gamma, & N^{13}=C^{13}+\epsilon^+ \\ C^{13}+H=N^{14}+\gamma, & \\ N^{14}+H=O^{15}+\gamma, & O^{15}=N^{15}+\epsilon^+ \\ N^{15}+H=C^{12}+He^4. & \end{array} \tag{2}$$

The catalyst C^{12} is reproduced in all cases except about one in 10,000, therefore the abundance of carbon and nitrogen remains practically unchanged (in comparison with the change of the number of protons). The two reactions (1) and

* Awarded an A. Cressy Morrison Prize in 1938, by the New York Academy of Sciences.

No neutrinos from nuclear reactions in 1938 ...

The combination of four p… and two electrons can occur essentially o… two ways. The first mechanism starts with … combination of two protons to form a deuter… with positron emission, *viz.*

$$H+H=D+\epsilon^+. \tag{1}$$

The deuteron is then transformed into He^4 by further capture of protons; these captures occur very rapidly compared with process (1). The second mechanism uses carbon and nitrogen as catalysts, according to the chain reaction

$$\begin{array}{ll} C^{12}+H=N^{13}+\gamma, & N^{13}=C^{13}+\epsilon^+ \\ C^{13}+H=N^{14}+\gamma, & \\ N^{14}+H=O^{15}+\gamma, & O^{15}=N^{15}+\epsilon^+ \\ N^{15}+H=C^{12}+He^4. & \end{array} \tag{2}$$

Gamow & Schoenberg, Phys. Rev. 58:1117 (1940)

The Possible Role of Neutrinos in Stellar Evolution

It can be considered at present as definitely established that the energy production in stars is caused by various types of thermonuclear reactions taking place in their interior. Since these reaction chains usually contain the processes of β-disintegration accompanied by the emission of high speed neutrinos, and since the neutrinos can pass almost without difficulty through the body of the star, we must assume that a certain part of the total energy produced escapes into interstellar space without being noticed as the actual thermal radiation of the star. Thus, for example, in the case of the carbon-nitrogen cycle in the sun, about 7 percent of the energy produced is lost in the form of neutrino radiation. However, since, in such reaction chains, the energy taken away by neutrinos represents a definite fraction of the total energy liberation, these losses are of but secondary importance for the problem of stellar equilibrium and evolution.

We want to indicate here that the situation becomes entirely different in cases where, as the result of the progressive contraction of the star, the density and tempera-

More detailed calculations on this collapse process are now in progress.

G. Gamow

The George Washington University,
Washington, D. C.,

M. Schoenberg*

University of São Paulo,
São Paulo, Brazil,
November 23, 1940.

* Fellow of the Guggenheim Memorial Foundation. Now in Washington, D. C.

Neutrino Theory of Stellar Collapse

G. GAMOW, *George Washington University, Washington, D. C.*
M. SCHOENBERG,* *University of São Paulo, São Paulo, Brazil*

(Received February 6, 1941)

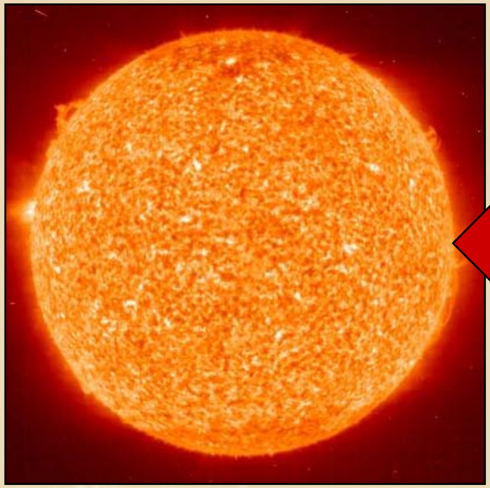
At the very high temperatures and densities which must exist in the interior of contracting stars during the later stages of their evolution, one must expect a special type of nuclear processes accompanied by *the emission of a large number of neutrinos*. These neutrinos penetrating almost without difficulty the body of the star, must carry away very large amounts of energy and prevent the central temperature from rising above a certain limit. This must cause *a rapid contraction of the stellar body* ultimately resulting in a *catastrophic collapse*. It is shown that energy losses through the neutrinos produced in reactions between free electrons and oxygen nuclei can cause a complete collapse of the star within the time period of half an hour. Although the main energy losses in such collapses are due to neutrino emission which escapes direct observation, the heating of the body of a collapsing star must necessarily lead to the *rapid expansion of the outer layers* and the *tremendous increase of luminosity*. It is suggested that stellar collapses of this kind are responsible for the phenomena of *novae* and *supernovae*, the difference between the two being probably due to the difference of their masses.

§1. INTRODUCTION

ONE of the most peculiar phenomena which we encounter in the evolutionary life of stars consists in vast stellar explosions known as "ordinary novae" and "supernovae." It is now well established that, although these two classes of novae possess a great many features in common, they are sharply separated insofar as their maximum luminosities are concerned. The ordi-

ordinary novae, and probably above 30,000°C for supernovae), and the rapid expansion of the stellar atmosphere which is evidently blown up by the increasing radiative pressure. In the case of Nova Aquilae 1918, for example, the star was surrounded by a luminous gas shell expanding with a velocity of 2000 kilometers per second, whereas the gas masses expelled by the galactic supernova of the year A.D. 1054 (observed by Chinese astronomers) form at present an ex-

Sun Glasses for Neutrinos?

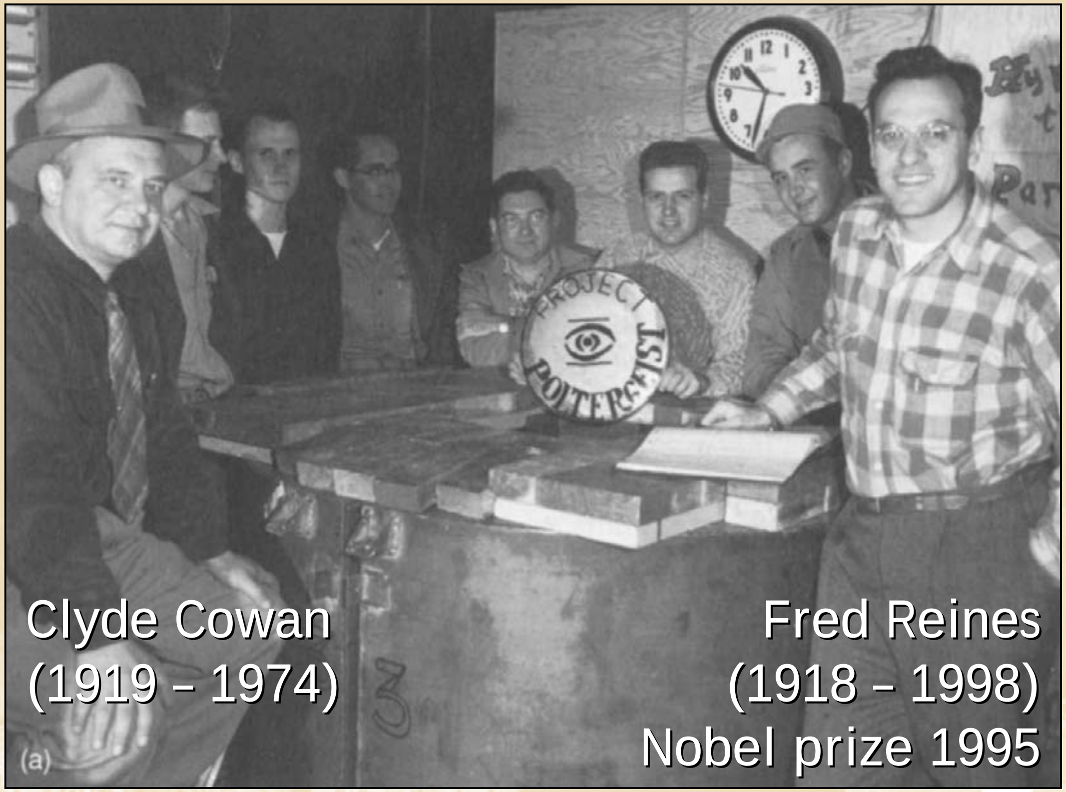


8.3 light minutes

100 light years of lead needed to shield solar neutrinos

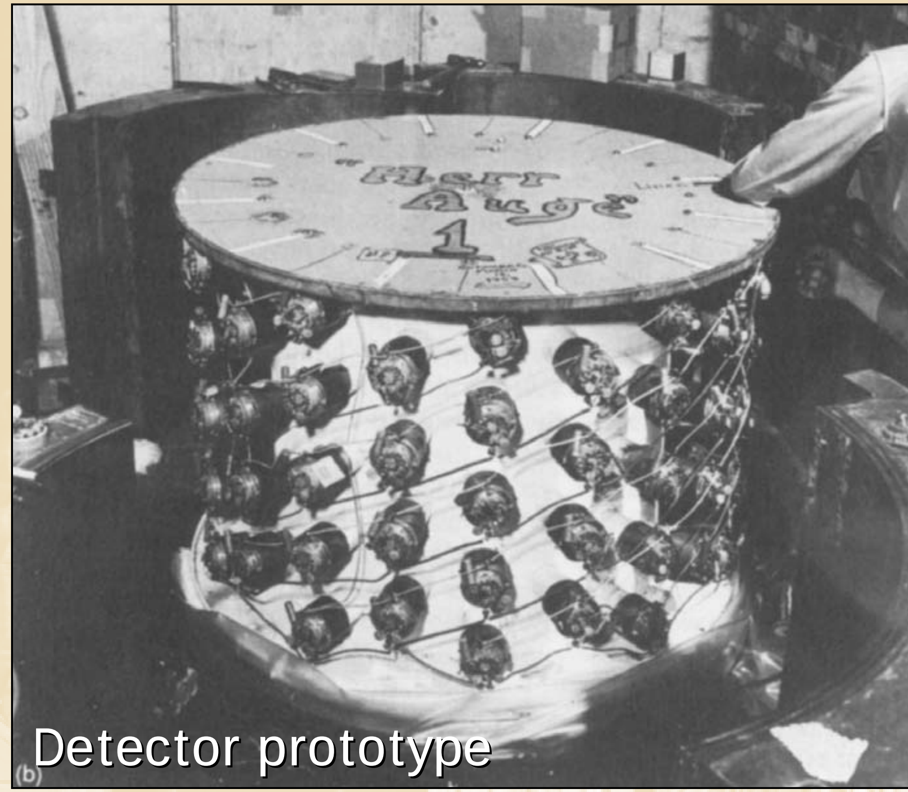
<u>Bethe & Peierls 1934:</u>
"… this evidently means that one will never be able to observe a neutrino."

First Detection (1954 - 1956)



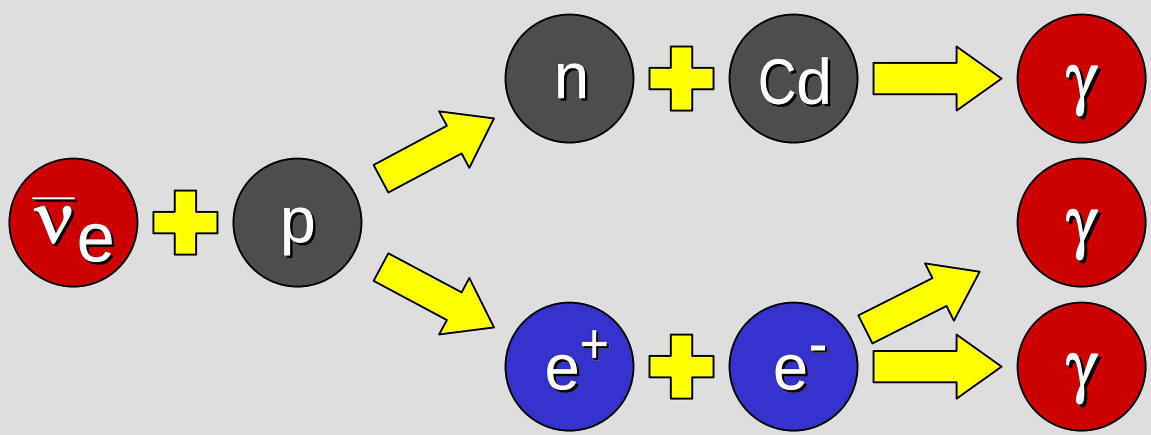
Clyde Cowan
(1919 – 1974)

Fred Reines
(1918 – 1998)
Nobel prize 1995



Detector prototype

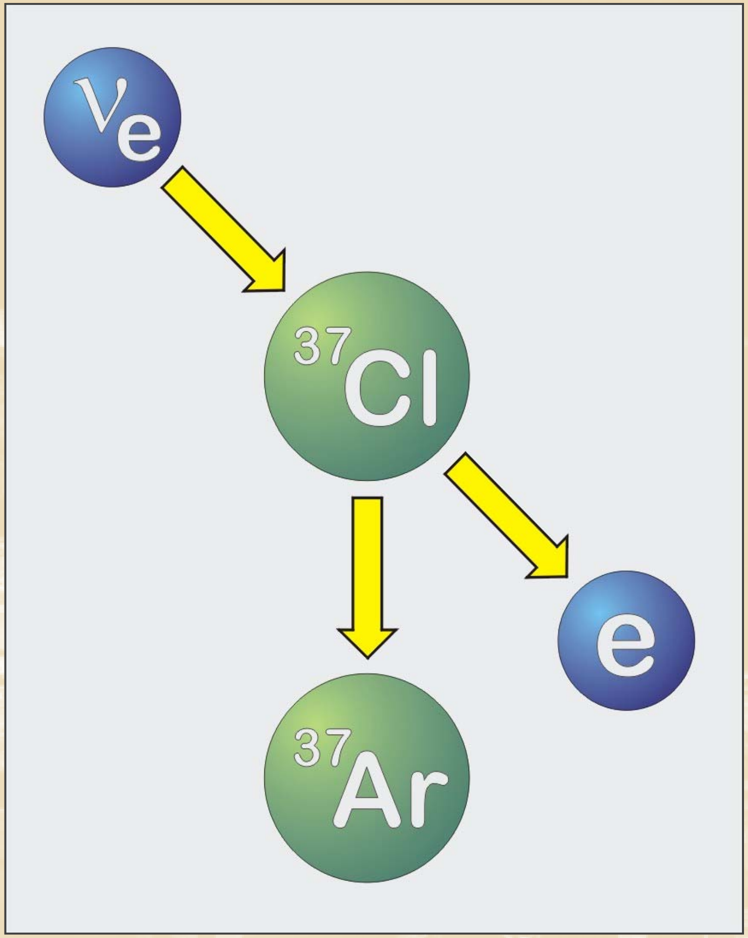
Anti-Electron
Neutrinos
from
Hanford
Nuclear Reactor

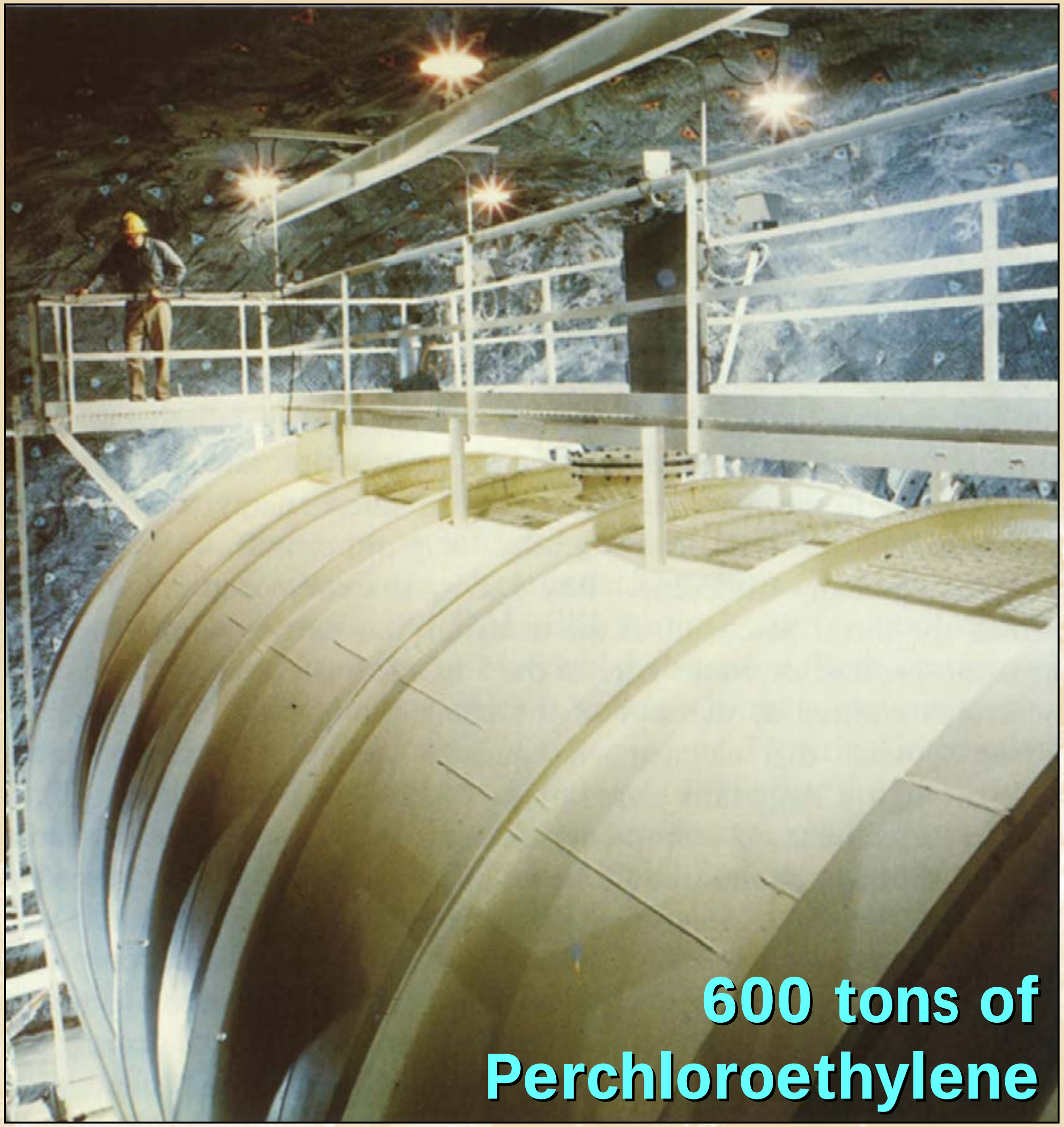


3 Gammas
in coincidence

First Measurement of Solar Neutrinos

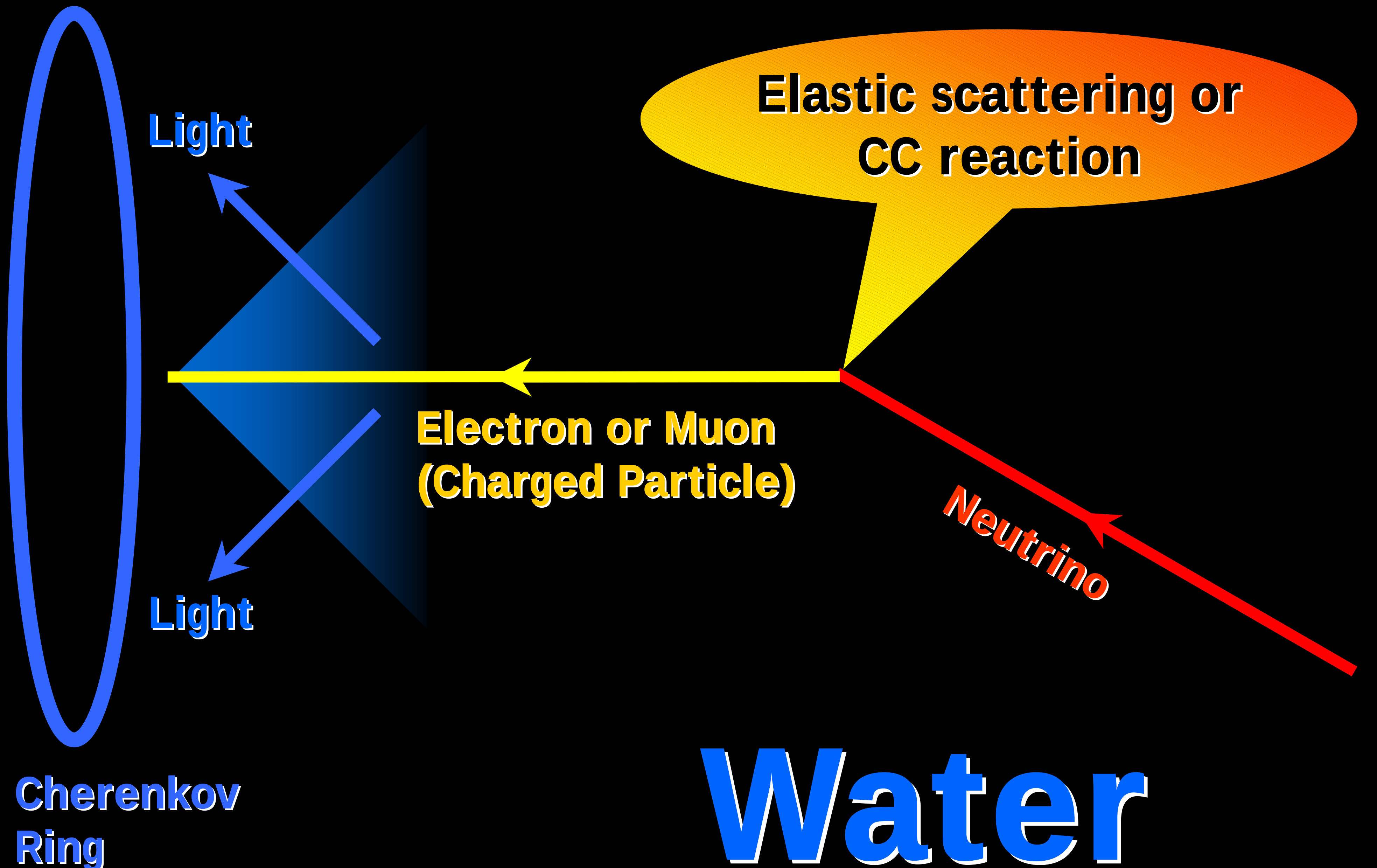
Inverse beta decay of chlorine



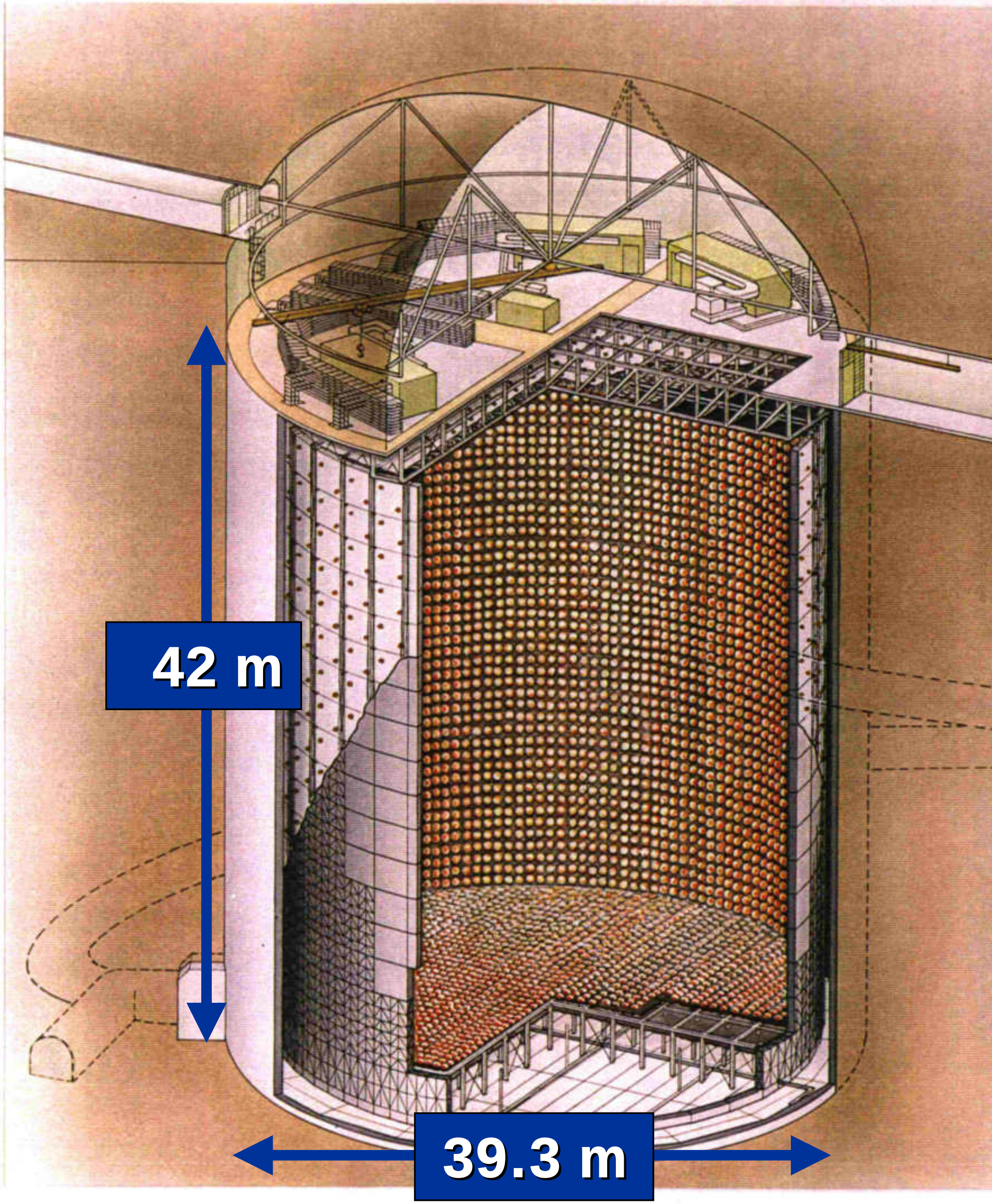


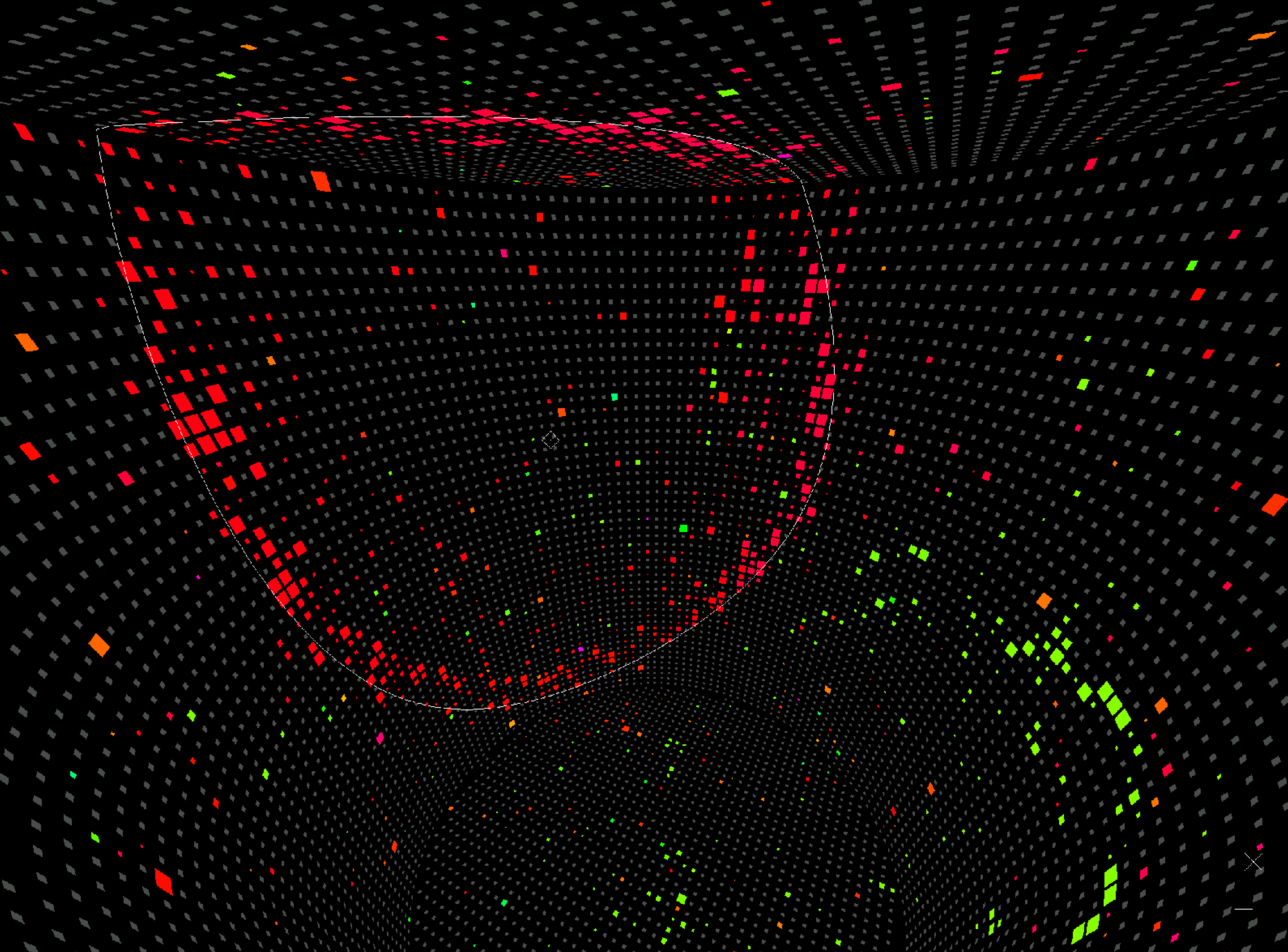
Homestake solar neutrino observatory (1967–2002)

Cherenkov Effect

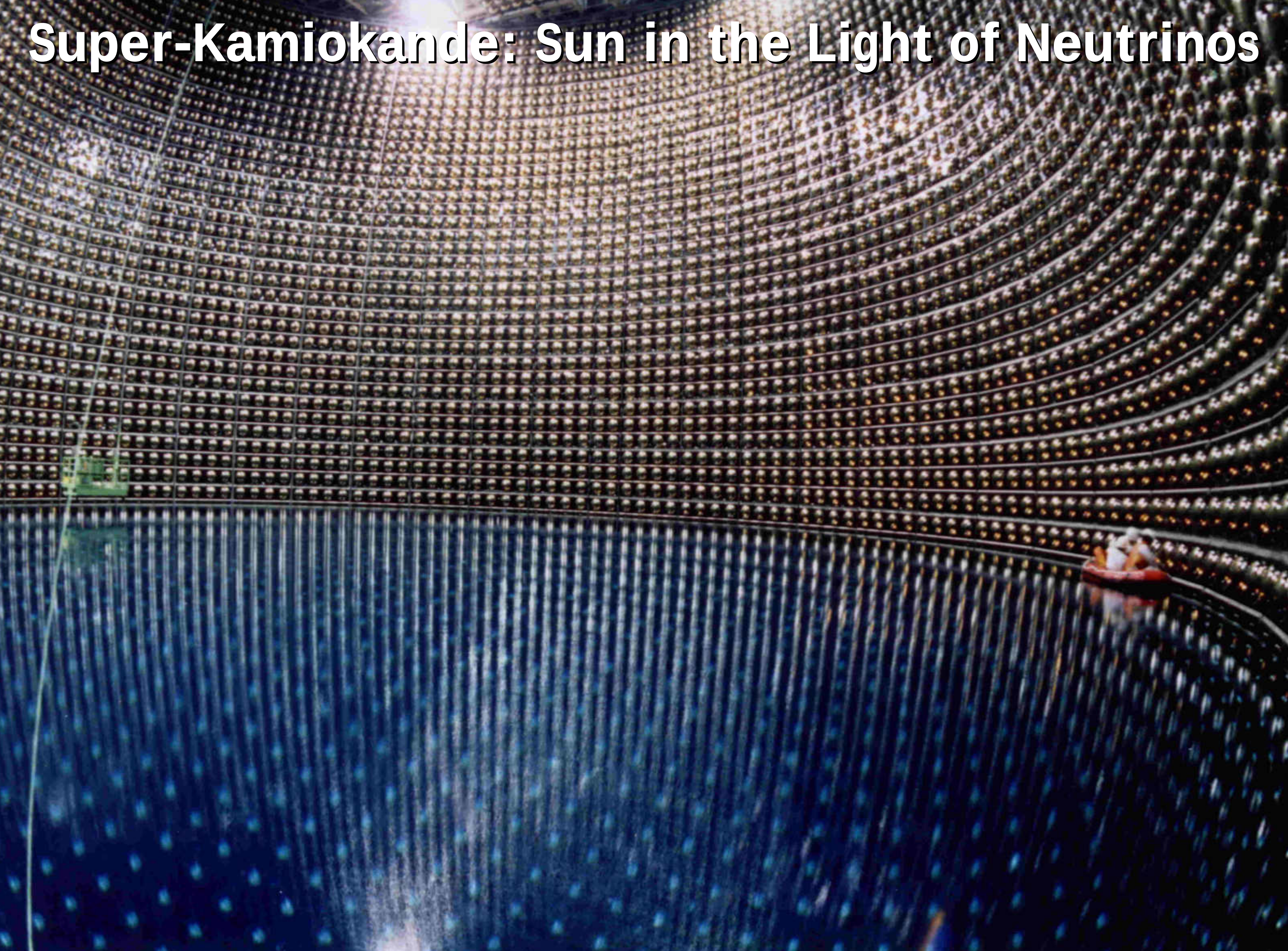


Super-Kamiokande Neutrino Detector

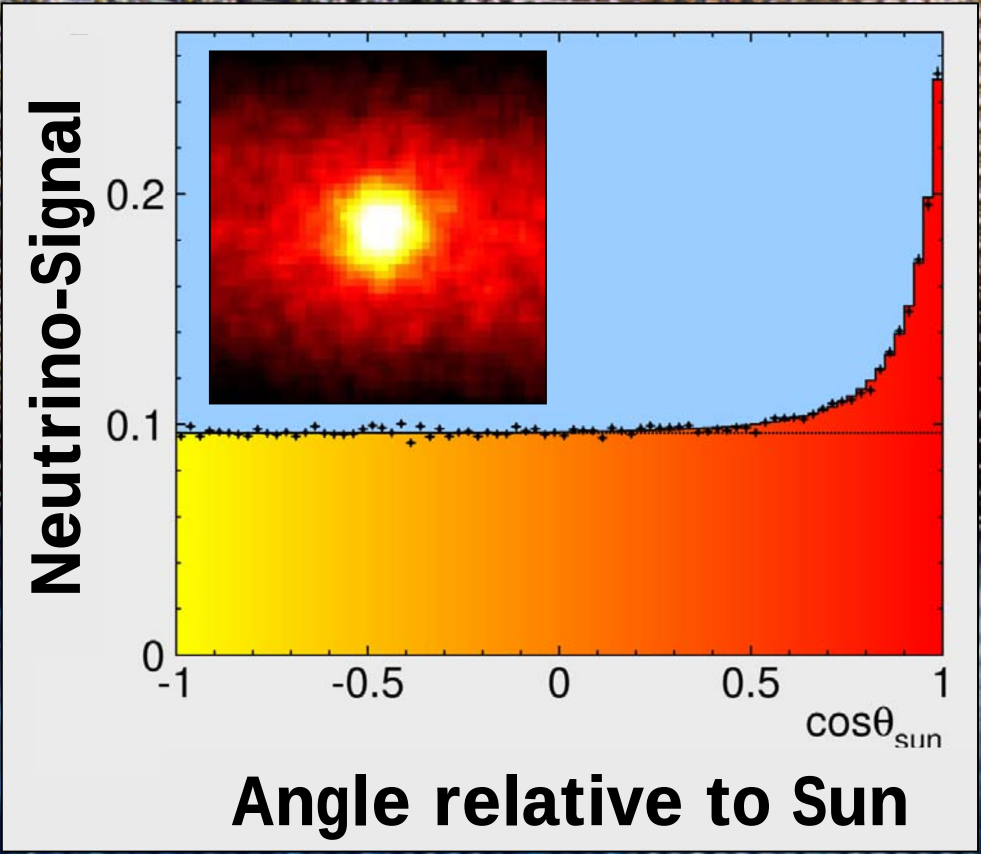


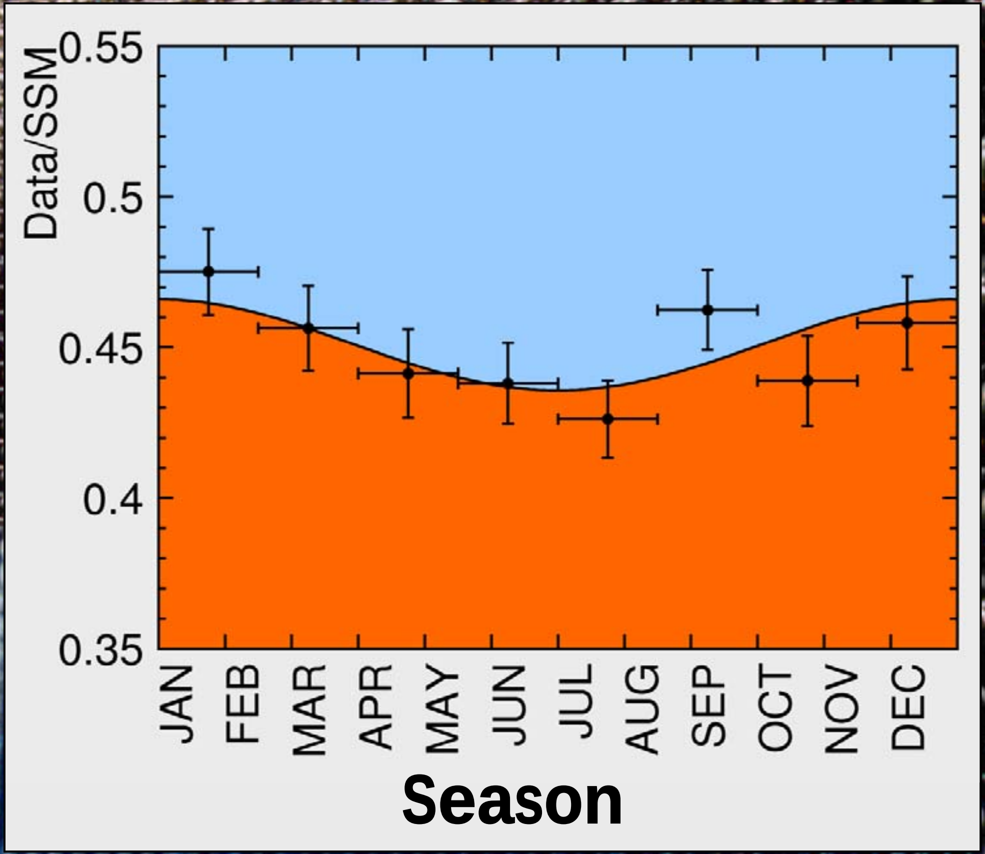


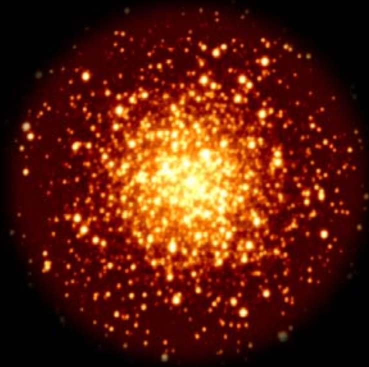
Super-Kamiokande: Sun in the Light of Neutrinos



Super-Kamiokande: Sun in the Light of Neutrinos

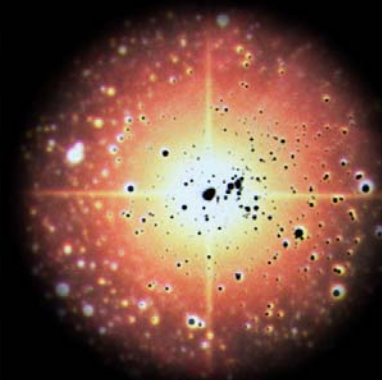






I. Stellar Evolution and Particle Limits

II. Neutrinos and Axions from the Sun



III. Supernova Neutrinos

Basics of Stellar Evolution

Equations of Stellar Structure

Assume spherical symmetry and static structure (neglect kinetic energy)
Excludes: Rotation, convection, magnetic fields, supernova-dynamics, ...

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$
Energy conservation	$\frac{dL_r}{dr} = 4\pi r^2 \varepsilon\rho$
Energy transfer	$L_r = \frac{4\pi r^2}{3\kappa\rho}\frac{d(aT^4)}{dr}$

Literature
- Clayton: Principles of stellar evolution and nucleosynthesis (Univ. Chicago Press 1968)
- Kippenhahn & Weigert: Stellar structure and evolution (Springer 1990)

r	Radius from center
P	Pressure
G_N	Newton's constant
ρ	Mass density
M_r	Integrated mass up to r
L_r	Luminosity (energy flux)
ε	Local rate of energy generation [erg/g/s]
	$\varepsilon = \varepsilon_{nuc} + \varepsilon_{grav} - \varepsilon_\nu$
κ	Opacity
	$\kappa^{-1} = \kappa_\gamma^{-1} + \kappa_c^{-1}$
κ_γ	Radiative opacity
	$\kappa_\gamma \rho = \langle \lambda_\gamma \rangle_{\text{Rosseland}}^{-1}$
κ_c	Electron conduction

Convection in Main-Sequence Stars

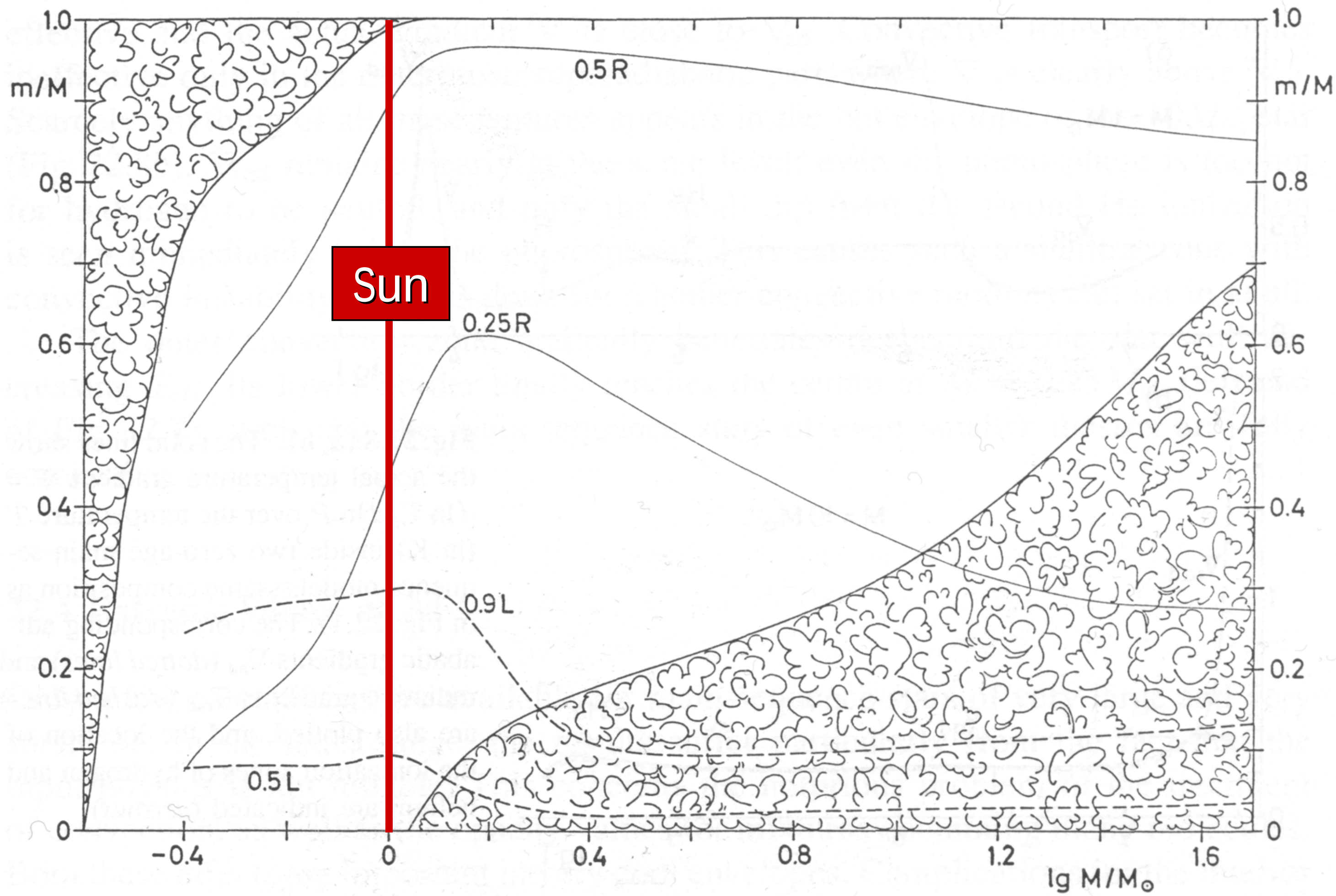


Fig. 22.7. The mass values m from centre to surface are plotted against the stellar mass M for the same zero-age main-sequence models as in Fig. 22.1. "Cloudy" areas indicate the extension of convective zones inside the models. Two solid lines give the m values at which r is 1/4 and 1/2 of the total radius R. The dashed lines show the mass elements inside which 50% and 90% of the total luminosity L are produced

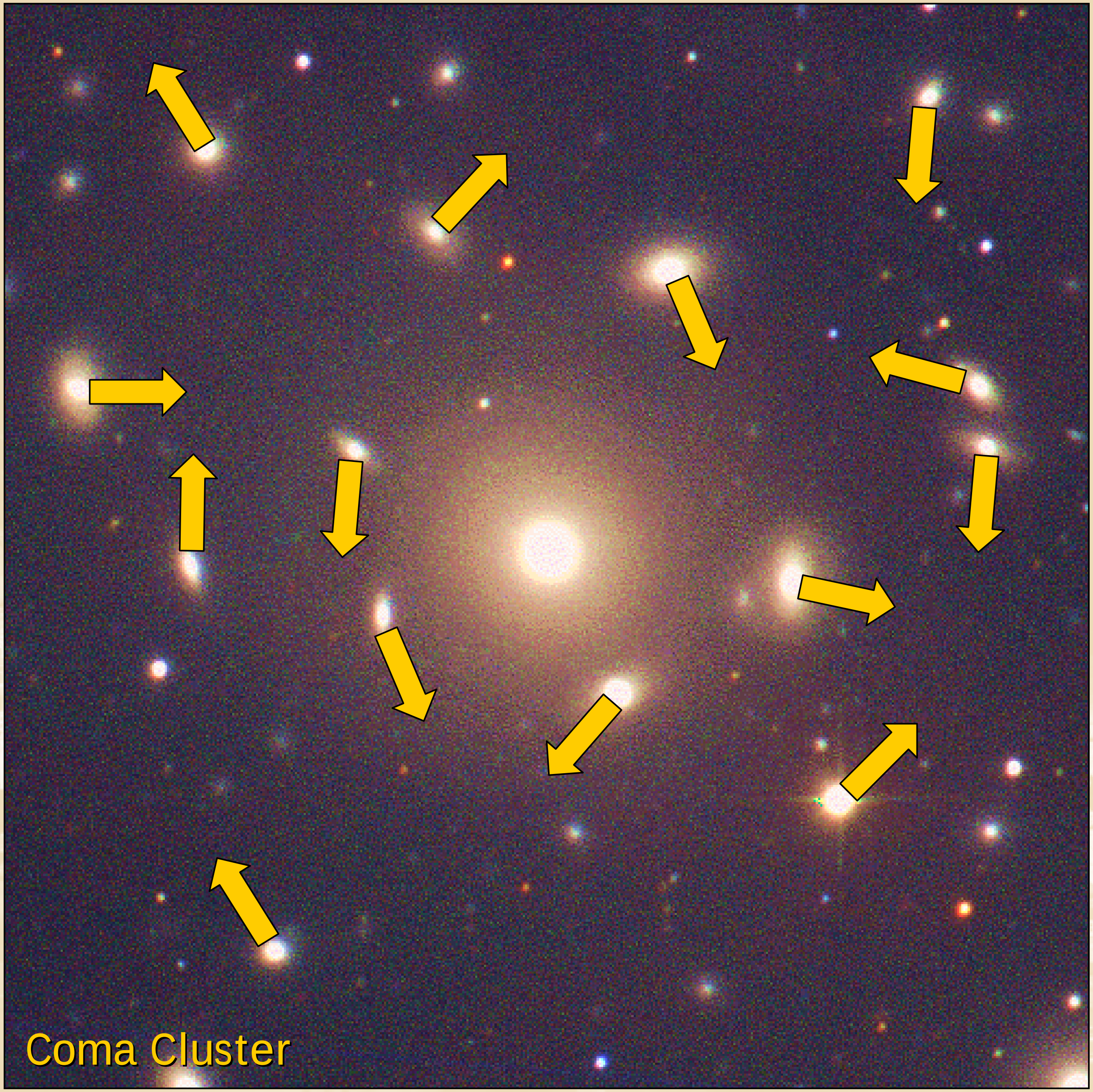
Kippenhahn & Weigert, Stellar Structure and Evolution

Virial Theorem and Hydrostatic Equilibrium

Hydrostatic equilibrium	$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$
Integrate both sides	$\int_0^R dr\, 4\pi r^3 P' = -\int_0^R dr\, 4\pi r^3 \frac{G_N M_r \rho}{r^2}$
L.h.s. partial integration with P = 0 at surface R	$-3\int_0^R dr\, 4\pi r^2 P = E_{grav}^{tot}$
Classical monatomic gas: $P = \frac{2}{3}U$ (U density of internal energy)	$U^{tot} = -\frac{1}{2}E_{grav}^{tot}$
Average energy of single "atoms" of the gas	$\langle E_{kin}\rangle = -\frac{1}{2}\langle E_{grav}\rangle$ — Virial Theorem

Most important tool to understand self-gravitating systems

Dark Matter in Galaxy Clusters



A gravitationally bound system of many particles obeys the virial theorem

$$2\langle E_{kin}\rangle = -\langle E_{grav}\rangle$$

$$2\left\langle \frac{mv^2}{2} \right\rangle = \left\langle \frac{G_N M_r m}{r} \right\rangle$$

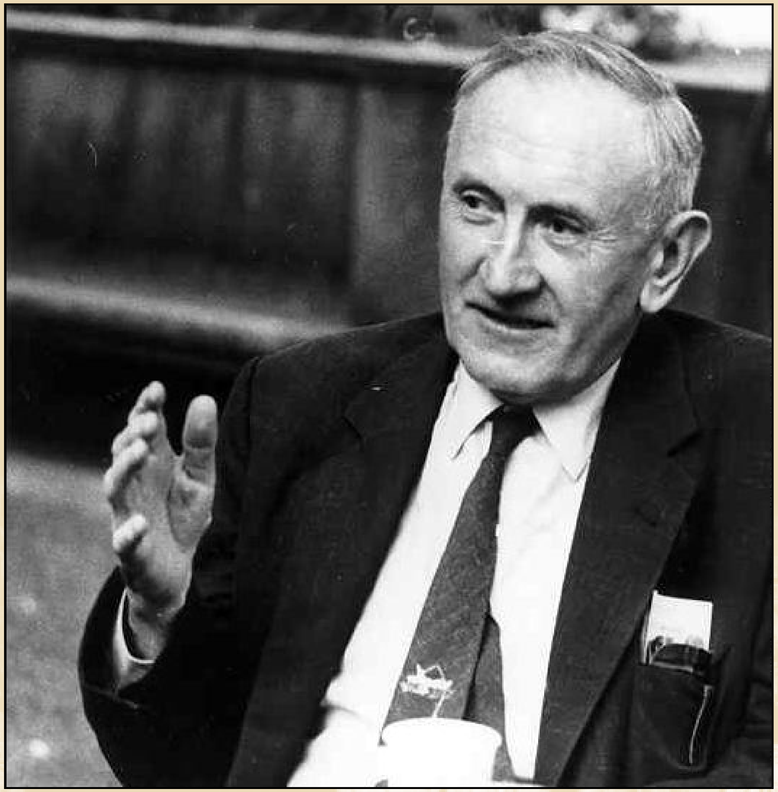
$$\langle v^2 \rangle \approx G_N M_r \langle r^{-1} \rangle$$

Velocity dispersion from Doppler shifts and geometric size

Total Mass

Dark Matter in Galaxy Clusters

Fritz Zwicky:
Die Rotverschiebung von Extragalaktischen Nebeln
(The redshift of extragalactic nebulae)
Helv. Phys. Acta 6 (1933) 110



In order to obtain the observed average Doppler effect of 1000 km/s or more, the average density of the Coma cluster would have to be at least 400 times larger than what is found from observations of the luminous matter.
Should this be confirmed one would find the surprising result that dark matter is far more abundant than luminous matter.

Virial Theorem Applied to the Sun

$$\langle E_{kin} \rangle = -\frac{1}{2}\langle E_{grav} \rangle$$

Virial Theorem

Approximate Sun as a homogeneous sphere with

Mass $M_{sun} = 1.99 \times 10^{33}\,g$

Radius $R_{sun} = 6.96 \times 10^{10}\,cm$

Gravitational potential energy of a proton near center of the sphere

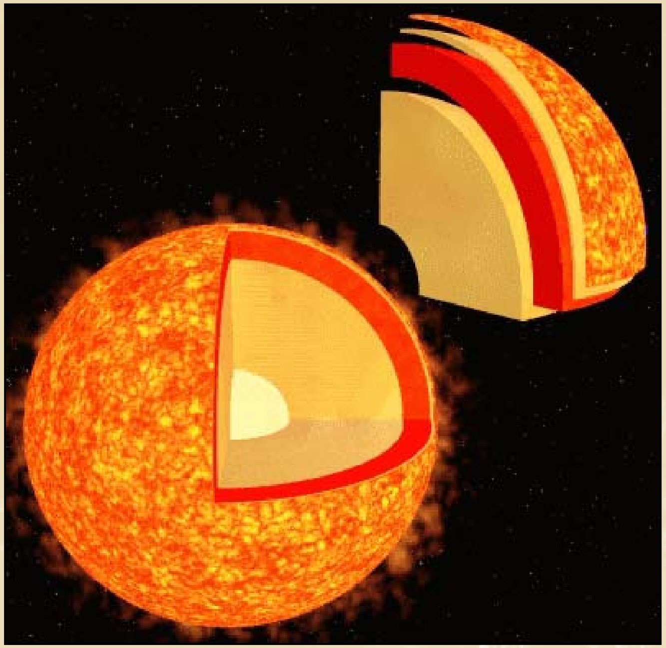
$$\langle E_{grav} \rangle = -\frac{3}{2}\frac{G_N M_{sun} m_p}{R_{sun}} = -3.2\ \mathrm{keV}$$

Thermal velocity distribution

$$\langle E_{kin} \rangle = \frac{3}{2} k_B T = -\frac{1}{2}\langle E_{grav} \rangle$$

Estimated temperature

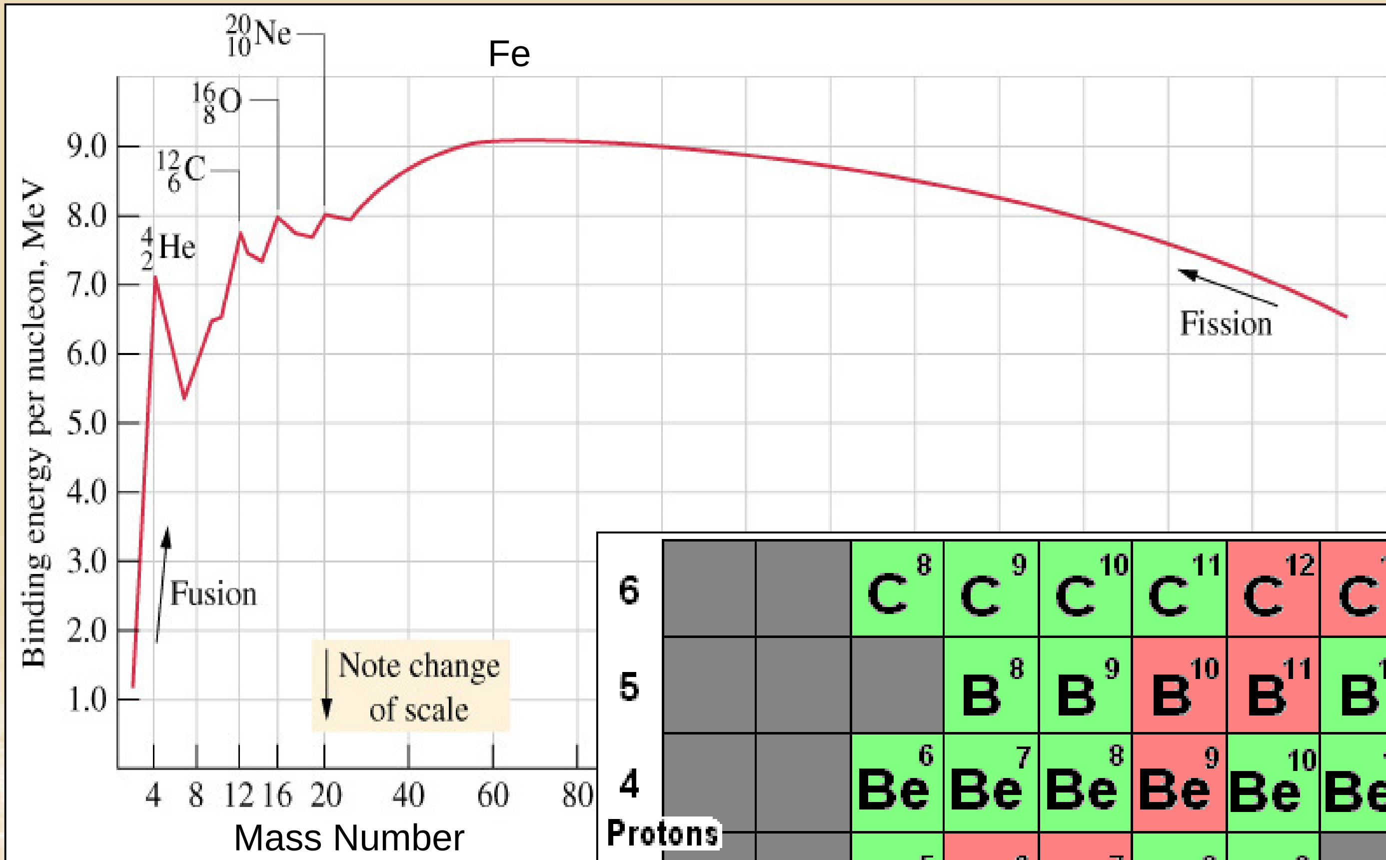
$T = 1.1\ \mathrm{keV}$

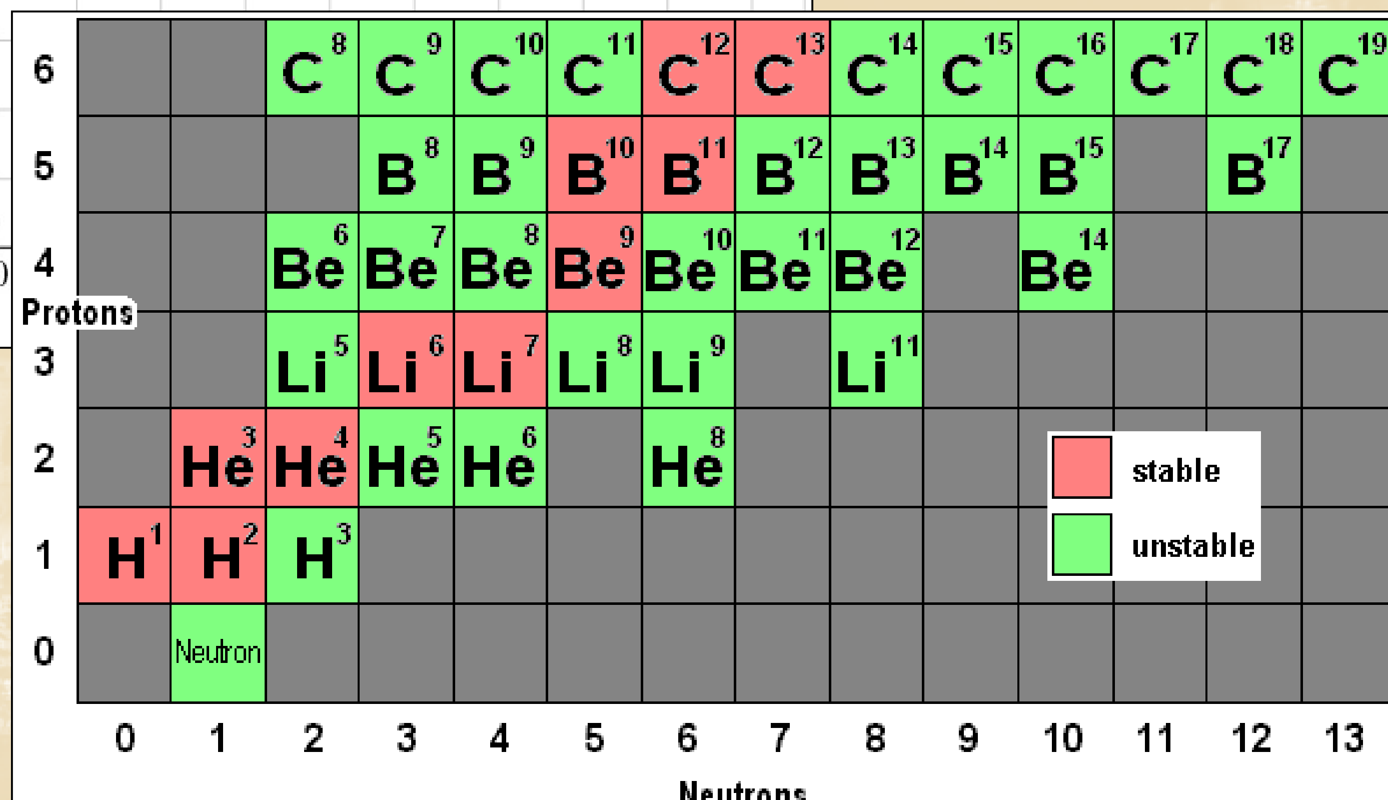


Central temperature from standard solar models

$$T_c = 1.56 \times 10^7\,K$$
$$= 1.34\ \mathrm{keV}$$

Nuclear Binding Energy





Hydrogen burning: Proton-Proton Chains

$p + p \to {}^2H + e^+ + \nu_e$ — < 0.420 MeV

$p + e^- + p \to {}^2H + \nu_e$ — 1.442 MeV

100% | 0.24%

${}^2H + p \to {}^3He + \gamma$

85% | 15% | (hep)

PPI: ${}^3He + {}^3He \to {}^4He + 2p$

${}^3He + {}^4He \to {}^7Be + \gamma$

hep: ${}^3He + p \to {}^4He + e^+ + \nu_e$ — < 18.8 MeV

90% | 10% | 0.02%

${}^7Be + e^- \to {}^7Li + \nu_e$ — 0.862 MeV

${}^7Be + e^- \to {}^7Li^* + \nu_e$ — 0.384 MeV

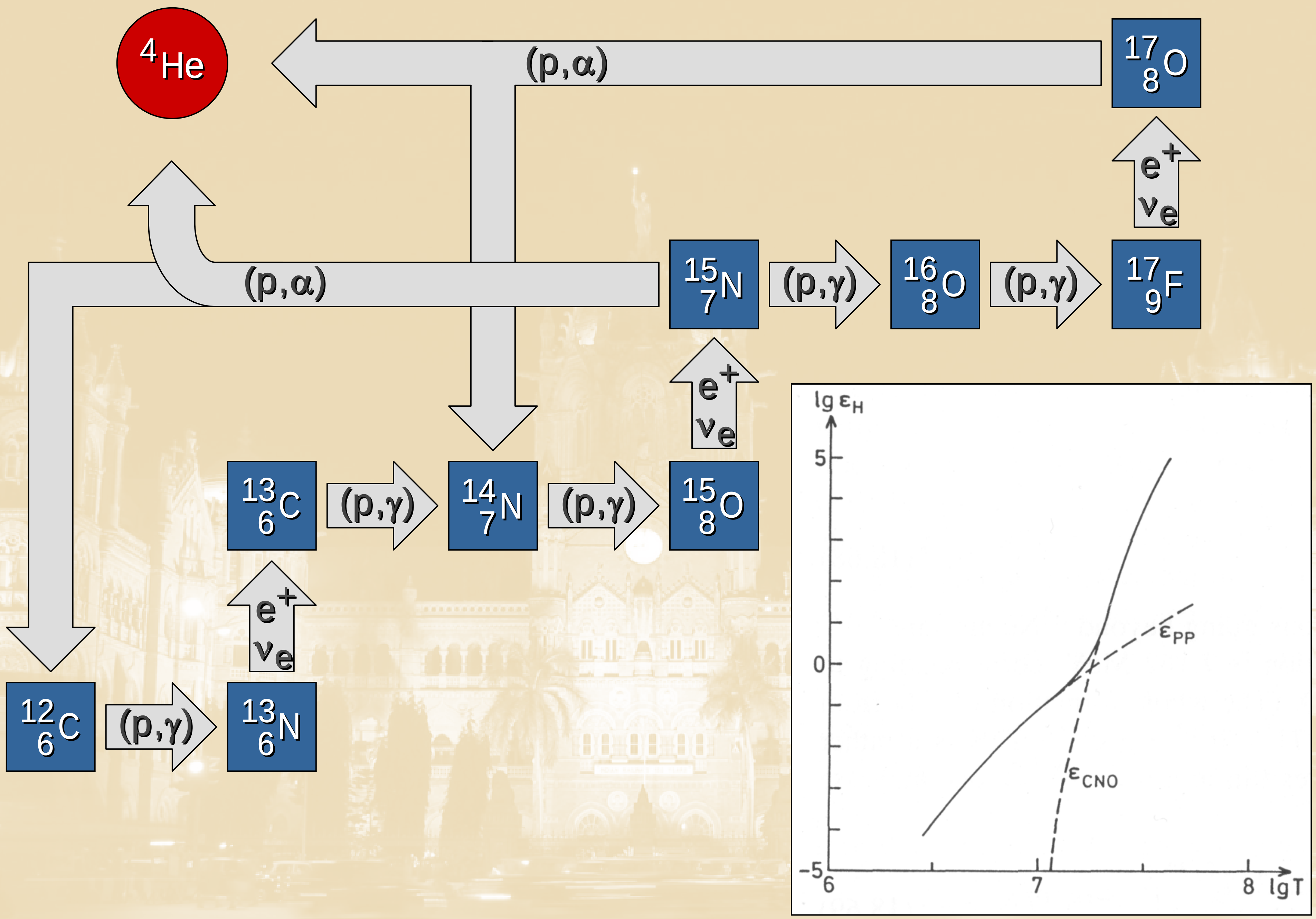
${}^7Be + p \to {}^8B + \gamma$

${}^8B \to {}^8Be^* + e^+ + \nu_e$ — < 15 MeV

PPII: ${}^7Li + p \to {}^4He + {}^4He$

PPIII: ${}^8Be^* \to {}^4He + {}^4He$

Hydrogen Burning: CNO Cycle



Thermonuclear Reactions and Gamow Peak

Coulomb repulsion prevents nuclear reactions, except for Gamow tunneling

Tunneling probability

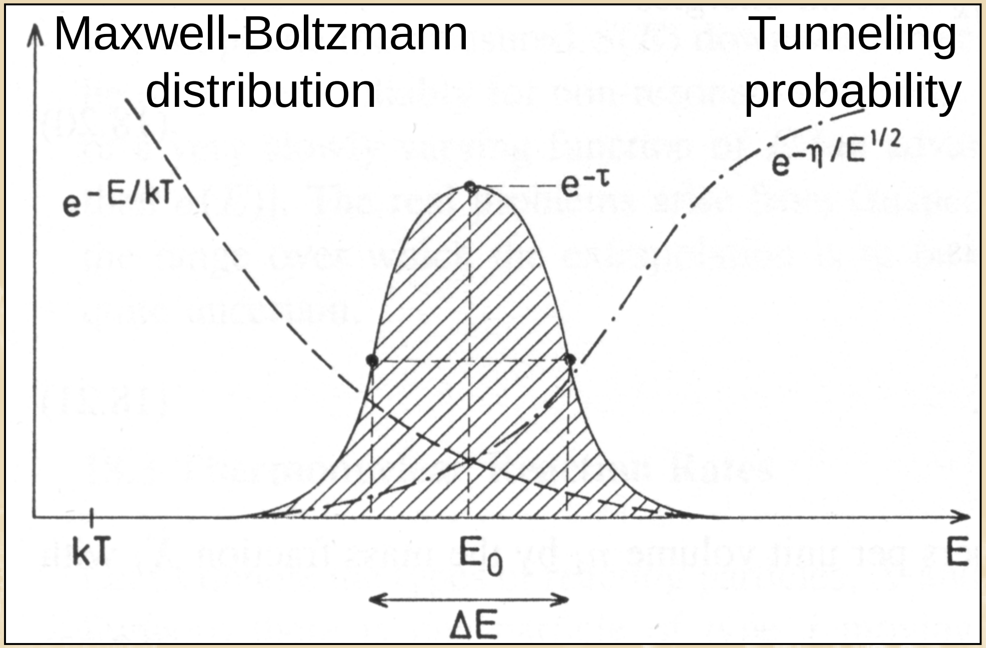
$$p \propto E^{-1/2} e^{-2\pi\eta}$$

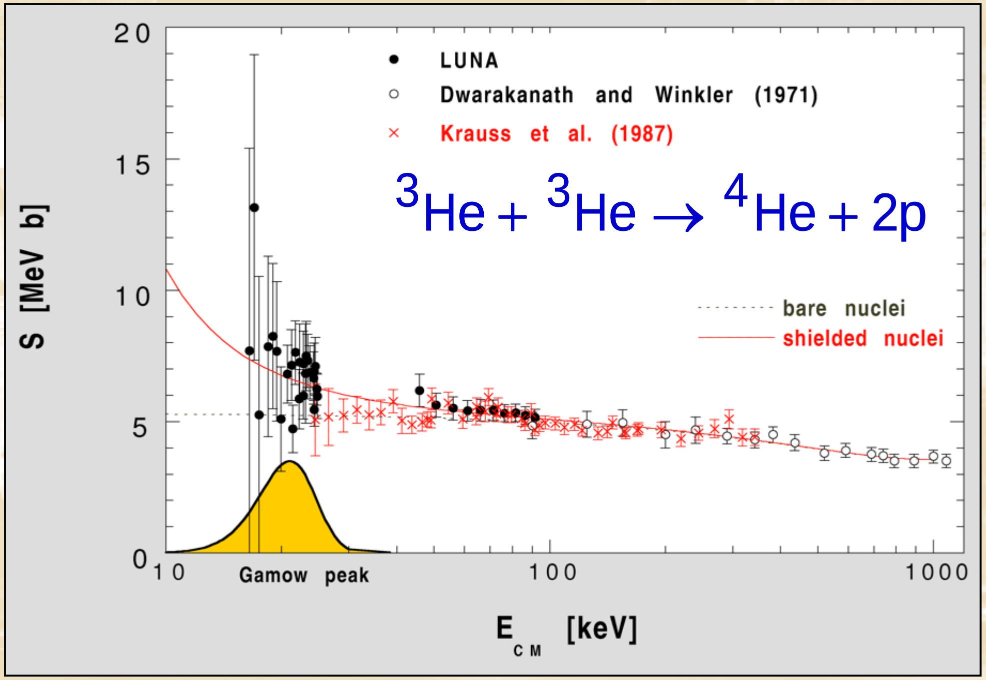
With Sommerfeld parameter

$$\eta = \left(\frac{m}{2E}\right)^{1/2} Z_1 Z_2 e^2$$

Parameterize cross section with astrophysical S-factor

$$S(E) = \sigma(E)\, E\, e^{2\pi\,\eta(E)}$$





LUNA Collaboration, nucl-ex/9902004

Main Nuclear Burnings

Hydrogen burning $4p + 2e^- \to {}^4He + 2\nu_e$

- Proceeds by pp chains and CNO cycle
- No higher elements are formed because no stable isotope with mass number 8
- Neutrinos from $p \to n$ conversion
- Typical temperatures: 10^7 K (~1 keV)

Helium burning

$${}^4He + {}^4He + {}^4He \leftrightarrow {}^8Be + {}^4He \to {}^{12}C$$

"Triple alpha reaction" because ${}^8Be$ unstable, builds up with concentration ~ 10^{-9}

$${}^{12}C + {}^4He \to {}^{16}O$$

$${}^{16}O + {}^4He \to {}^{20}Ne$$

Typical temperatures: 10^8 K (~10 keV)

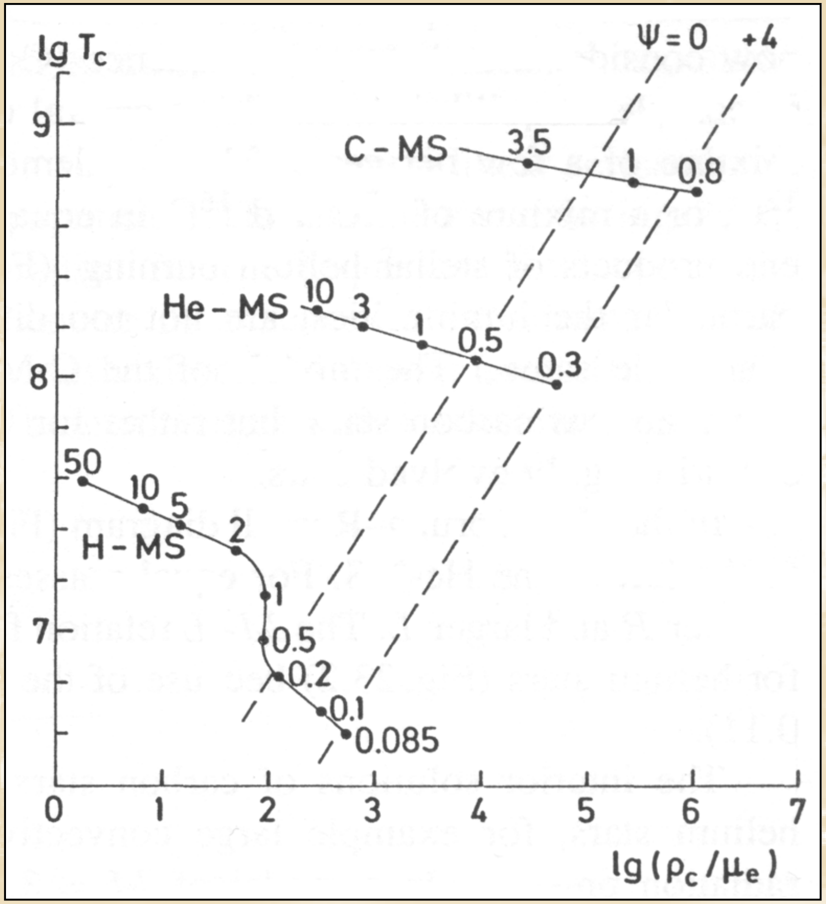
Carbon burning

Many reactions, for example

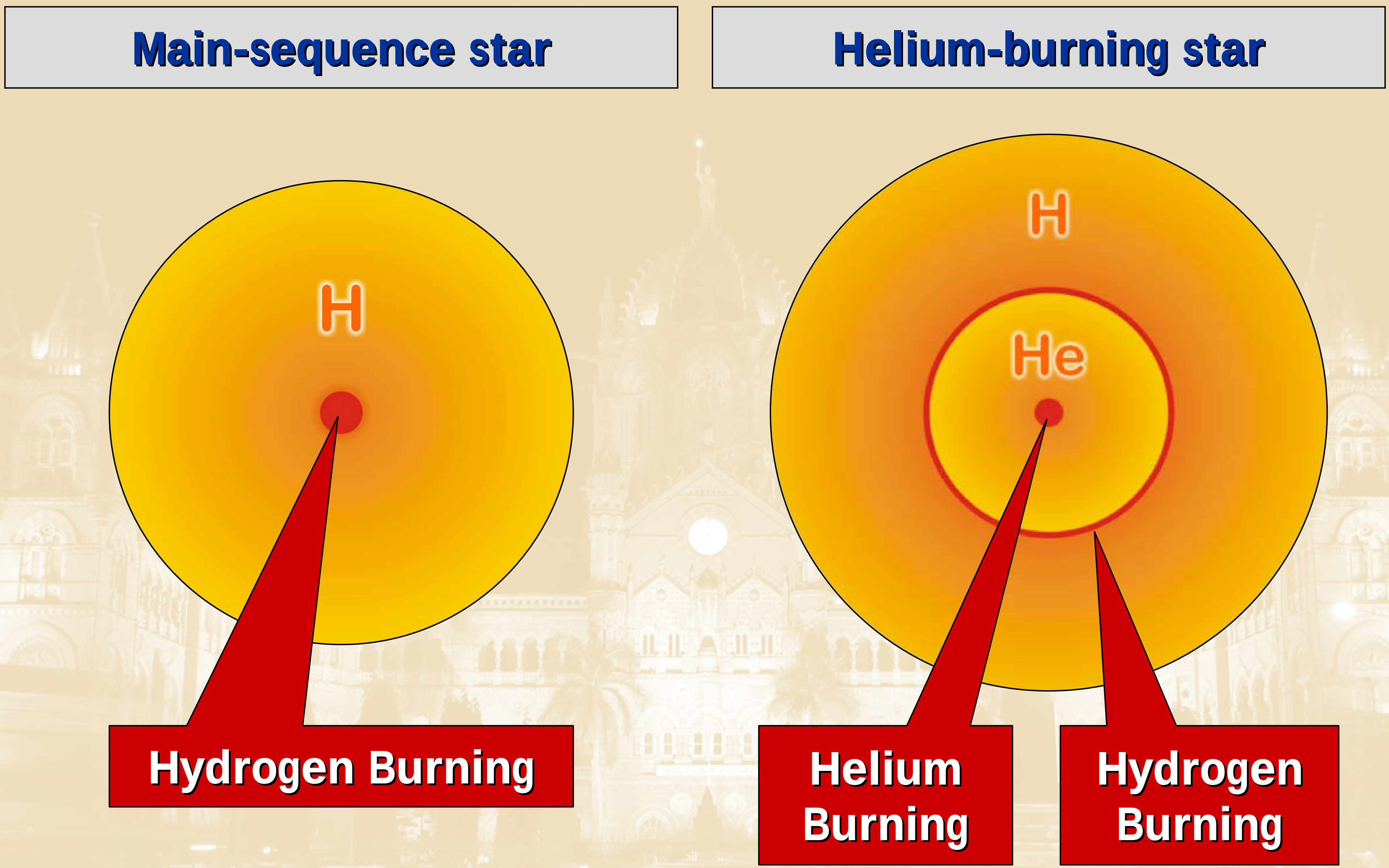
$${}^{12}C + {}^{12}C \to {}^{23}Na + p \quad \text{or} \quad {}^{20}Ne + {}^4He \quad \text{etc}$$

Typical temperatures: 10^9 K (~100 keV)

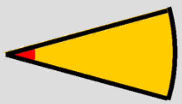
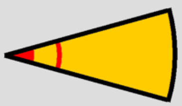
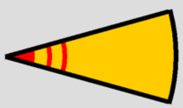
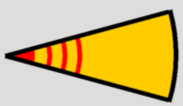
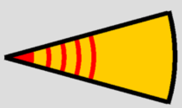
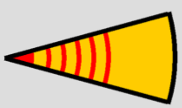
- Each type of burning occurs at a very different T but a broad range of densities
- Never co-exist in the same location



Hydrogen Exhaustion



Burning Phases of a 15 Solar-Mass Star

Burning Phase		Dominant Process	T_c [keV]	ρ_c [g/cm^3]	L_γ [10^4 L_{sun}]	L_ν/L_γ	Duration [years]
	Hydrogen	H → He	3	5.9	2.1	–	1.2×10^7
	Helium	He → C, O	14	1.3×10^3	6.0	1.7×10^{-5}	1.3×10^6
	Carbon	C → Ne, Mg	53	1.7×10^5	8.6	1.0	6.3×10^3
	Neon	Ne → O, Mg	110	1.6×10^7	9.6	1.8×10^3	7.0
	Oxygen	O → Si	160	9.7×10^7	9.6	2.1×10^4	1.7
	Silicon	Si → Fe, Ni	270	2.3×10^8	9.6	9.2×10^5	6 days

Neutrinos from Thermal Plasma Processes

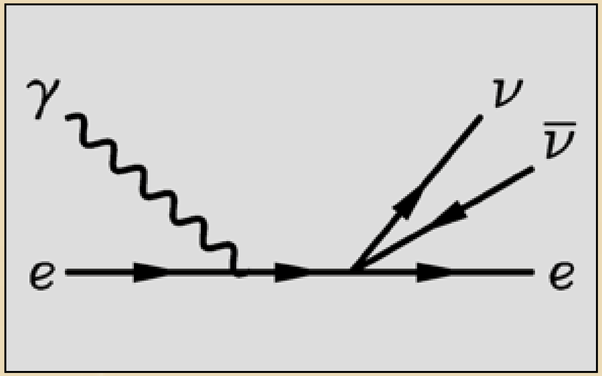
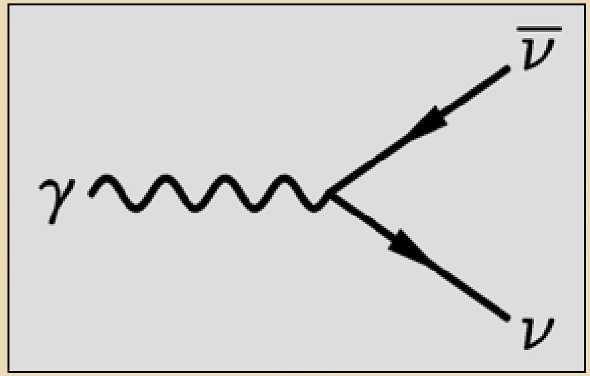
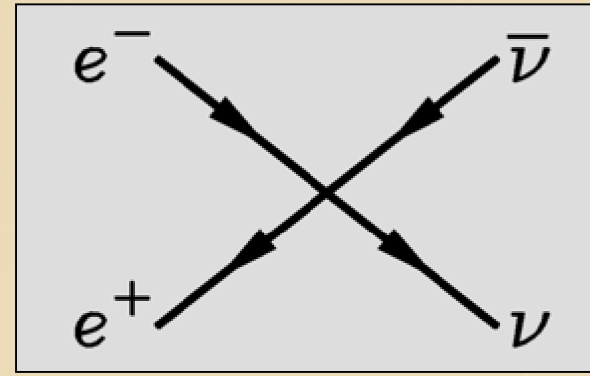


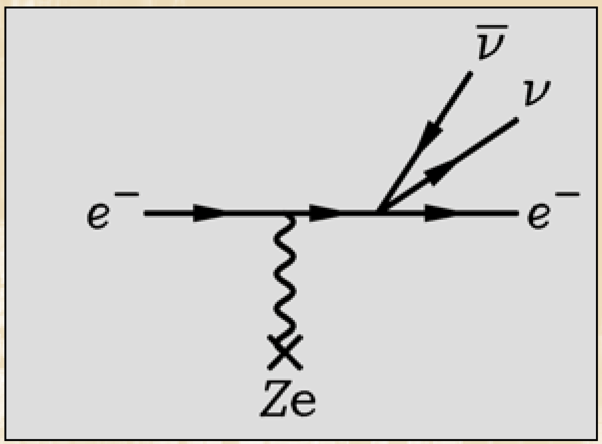
Photo (Compton)



Plasmon decay

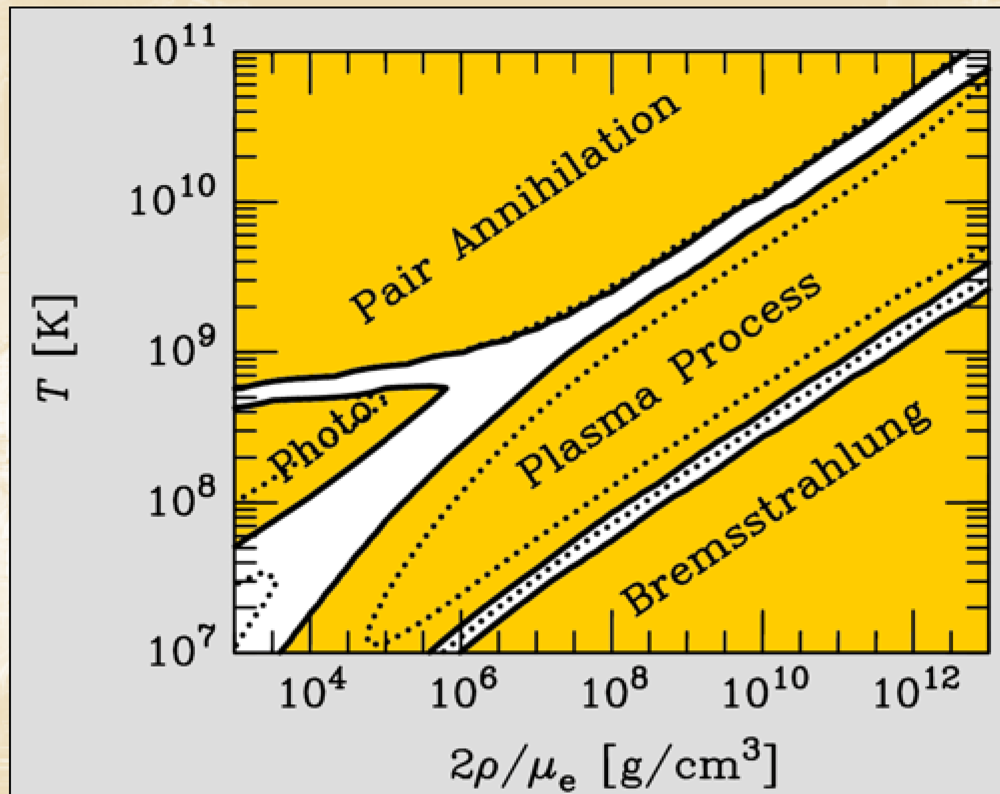


Pair annihilation



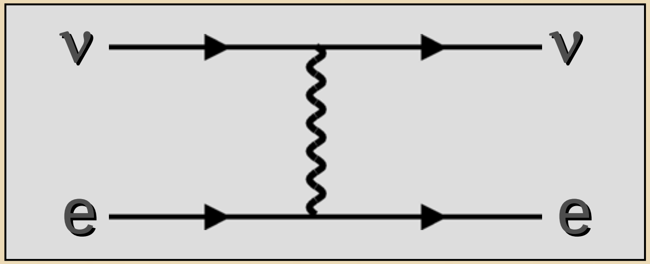
Bremsstrahlung

These processes first discussed in 1961-63 after V–A theory

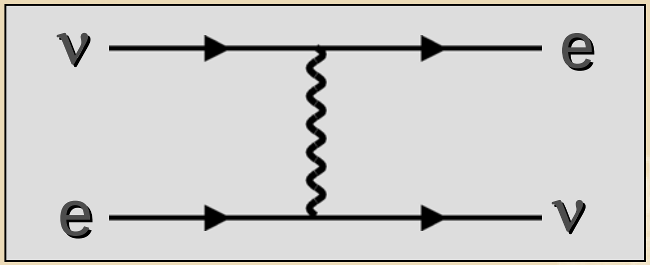


Effective Neutrino Neutral-Current Couplings

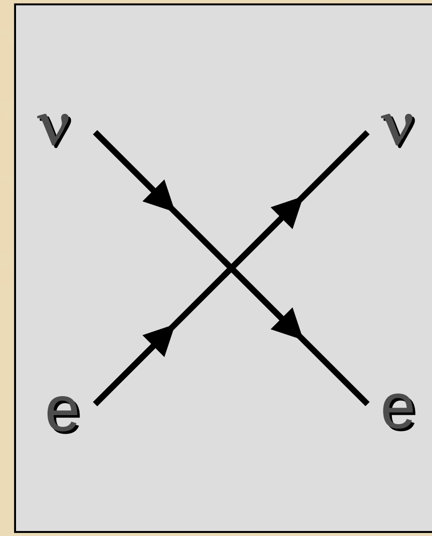
Neutral current



Charged current



$E \ll M_{W,Z}$



Effective four fermion coupling

$$H_{int} = \frac{G_F}{\sqrt{2}} \overline{\Psi}_f \gamma_\mu (C_V - C_A \gamma_5) \Psi_f \; \overline{\Psi}_\nu \gamma^\mu (1-\gamma_5) \Psi_\nu$$

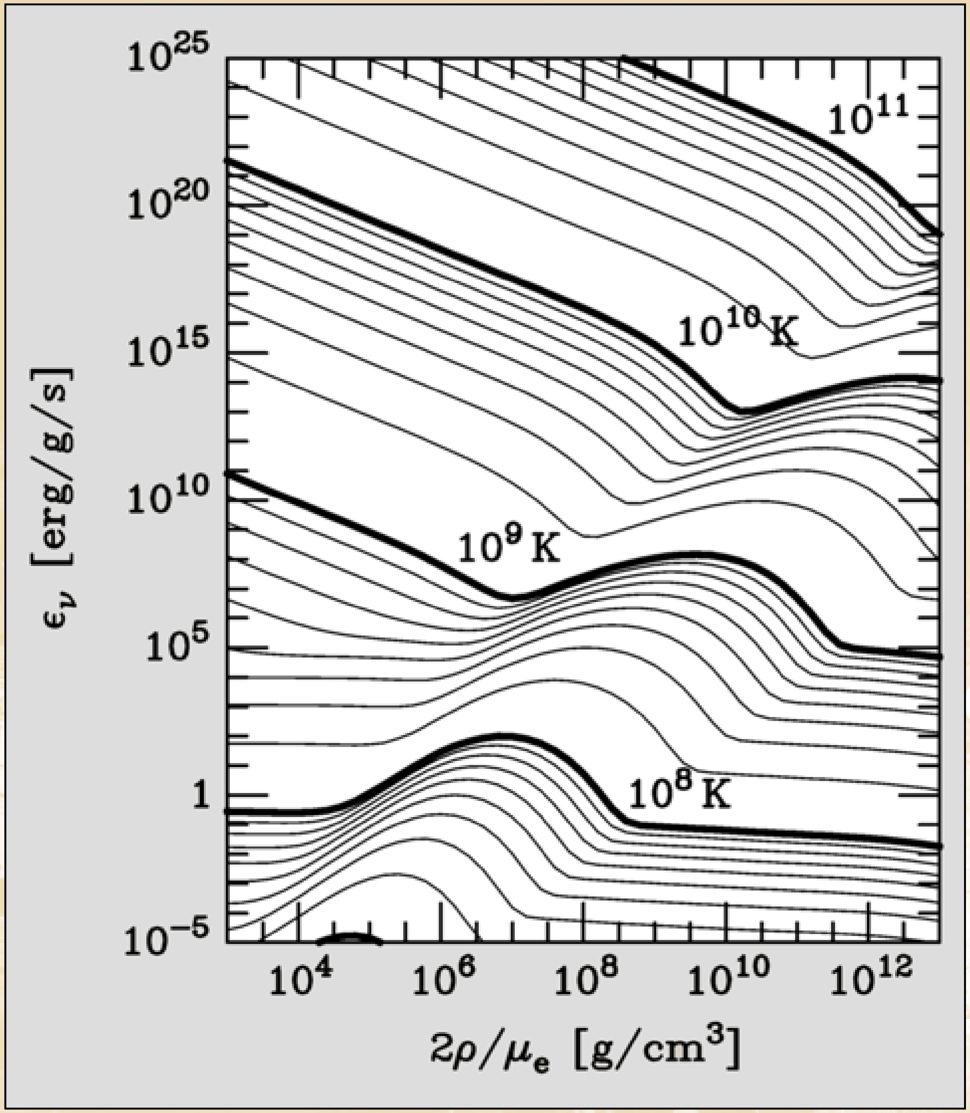
Neutrino	Fermion	C_V	C_A
ν_e	Electron	$+\frac{1}{2} + 2\sin^2\Theta_W \approx 1$	$+\frac{1}{2}$
ν_μ, ν_τ		$-\frac{1}{2} + 2\sin^2\Theta_W \approx 0$	$-\frac{1}{2}$
ν_e, ν_μ, ν_τ	Proton	$+\frac{1}{2} - 2\sin^2\Theta_W \approx 0$	$+\frac{1.26}{2}$
	Neutron	$-\frac{1}{2}$	$-\frac{1.26}{2}$

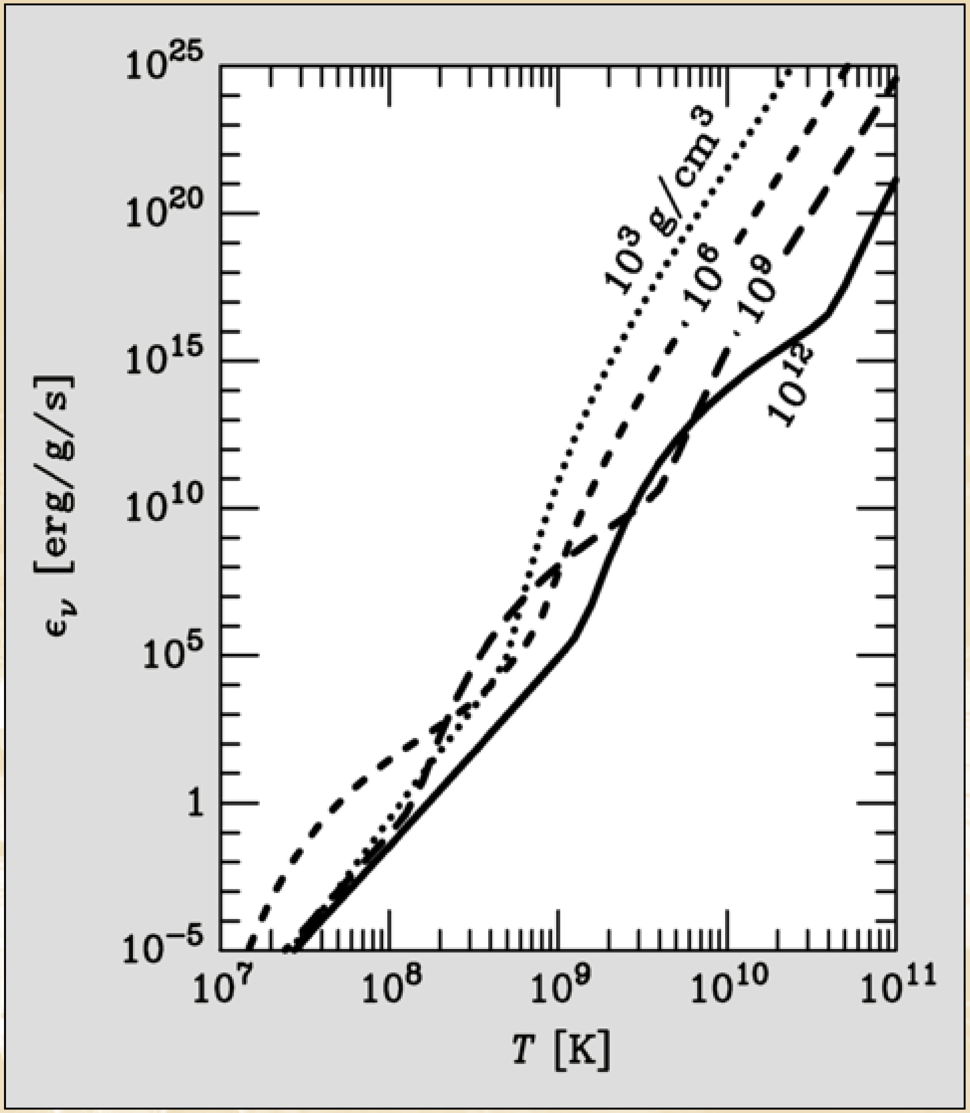
$G_F =$

$1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

$\sin^2\Theta_W = 0.231$

Neutrino Energy Loss Rates





Existence of Direct Neutrino-Electron Coupling

VOLUME 24, NUMBER 10 PHYSICAL REVIEW LETTERS 9 MARCH 1970

ASTROPHYSICAL DETERMINATION OF THE COUPLING CONSTANT FOR THE ELECTRON-NEUTRINO WEAK INTERACTION

Richard B. Stothers*
Goddard Institute for Space Studies, National Aeronautics and Space Administration, New York, New York 10025
(Received 22 December 1969)

The existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ weak interaction is confirmed by the results of nine astrophysical tests. The value of the coupling constant is equal to, or close to, the coupling constant of beta decay, namely, $g^2 = 10^{0\pm 2} g_\beta{}^2$.

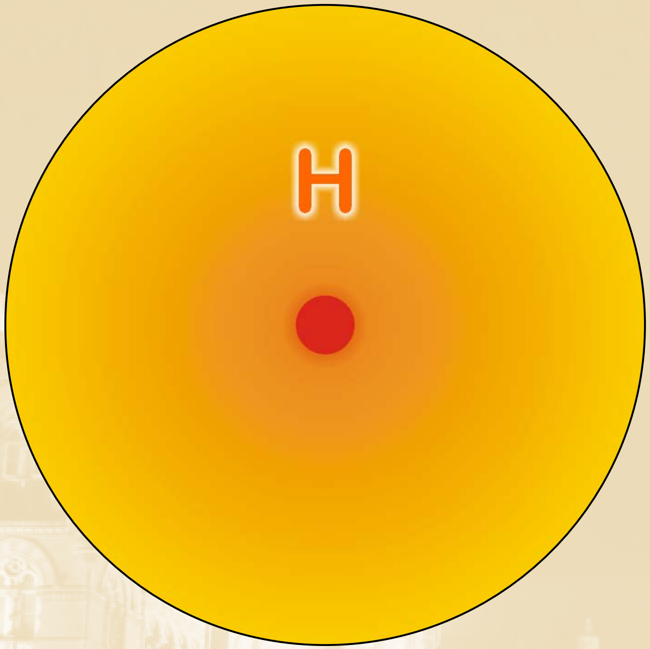
Of all the astrophysical tests applied so far for the inference of a direct electron-neutrino interaction in nature, none has unambiguously provided a useful upper limit on the coupling constant, which in the $V-A$ theory of Feynman and Gell-Mann[1] is taken to be equal to the "universal" weak-interaction coupling constant measured from beta decays (called g_β hereafter). However, it is important to point out that these tests, made by the author and his colleagues during the past eight years, do provide a nonzero lower limit, and therefore establish at least the existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction. It should be emphasized, nonetheless, that all of these tests rely on the validity of various stellar model calculations. These models, while not subject to scrutiny in the same sense as a laboratory ex-

relative theoretical lifetimes, calculated with and without the inclusion of neutrino emission. In this Letter, the unmodified term "luminosity" will mean the photon luminosity L radiated by the star. The "neutrino luminosity" will be designated L_ν. Quantities referring to the sun are subscripted with an encircled dot.

The most accurate available data on white dwarfs are those collected by Eggen[7] for the two clusters Hyades and Pleiades and for the nearby general field. Of chief interest here are the hot white dwarfs, for which the observational data[7,8] have been reduced following the procedure of Van Horn.[9] The resulting luminosities are estimated to have a statistical accuracy of ±0.1 in $\log(L/L_\odot)$, which is adequate here.

Models of cooling white dwarfs have been con-

Self-Regulated Nuclear Burning



Main-Sequence Star

Virial Theorem $\langle E_{kin} \rangle = -\frac{1}{2} \langle E_{grav} \rangle$

Small Contraction
$\rightarrow$ Heating
$\rightarrow$ Increased nuclear burning
$\rightarrow$ Increased pressure
$\rightarrow$ Expansion

Additional energy loss (“cooling”)
$\rightarrow$ Loss of pressure
$\rightarrow$ Contraction
$\rightarrow$ Heating
$\rightarrow$ Increased nuclear burning

Hydrogen burning at a nearly fixed T
$\rightarrow$ Gravitational potential nearly fixed:
$G_N M/R \sim$ constant
$\rightarrow$ $R \propto M$ (More massive stars bigger)

Modification of Stellar Properties by Particle Emission

Homologous changes of stellar structure

Assume that some small perturbation (e.g. axion emission) leads to "homologous" modification of stellar structure, i.e. every point is mapped to a new position $r' = yr$

Requires power-law relations for constitutive relations

- Nuclear burning rate $\varepsilon \propto \rho^n T^m$
- Mean opacity $\kappa \propto \rho^s T^t$

Implies for other quantities:

- Density $\rho'(r') = y^{-3} \rho(r)$
- Pressure $p'(r') = y^{-4} p(r)$
- Temperature gradient $dT'(r')/dr' = y^{-2}\, dT(r)/dr$

Impact of small exotic energy loss

Modified nuclear burning rate $\varepsilon \propto (1-\delta_x)\,\varepsilon_{nuc}$

Assume Kramers opacity law $s = 1, \quad t = -3.5$

Hydrogen burning $n = 1, \quad m = 4-6$

$$\frac{\delta R}{R} = \frac{-2\delta_x}{2m+5} \qquad \frac{\delta L_\gamma}{L_\gamma} = \frac{\delta_x}{2m+5} \qquad \frac{\delta T}{T} = \frac{\delta_x}{2m+5}$$

Degenerate Stars ("White Dwarfs")

Assume T very small
→ No thermal pressure
→ Electron degeneracy is pressure source

Pressure ~ Momentum density x Velocity

- Electron density $n_e = p_F^3 / (3\pi^2)$
- Momentum p_F (Fermi momentum)
- Velocity $v \propto p_F / m_e$
- Pressure $\boxed{P \propto p_F^5 \propto \rho^{5/3} \propto M^{5/3} R^{-5}}$
- Density $\rho \propto M R^{-3}$

(Stellar mass M and radius R)

Hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

With $dP/dr \sim -P/R$ we have approximately

$$\boxed{P \propto G_N M \rho R^{-1} \propto G_N M^2 R^{-4}}$$

Inverse mass-radius relationship for degenerate stars: $R \propto M^{-1/3}$

$$R = 10{,}500\ \mathrm{km} \left(\frac{0.6\, M_{sun}}{M}\right)^{1/3} (2Y_e)^{5/3}$$

(Y_e electrons per nucleon)

For sufficiently large mass, electrons become relativistic

- Velocity = speed of light
- Pressure

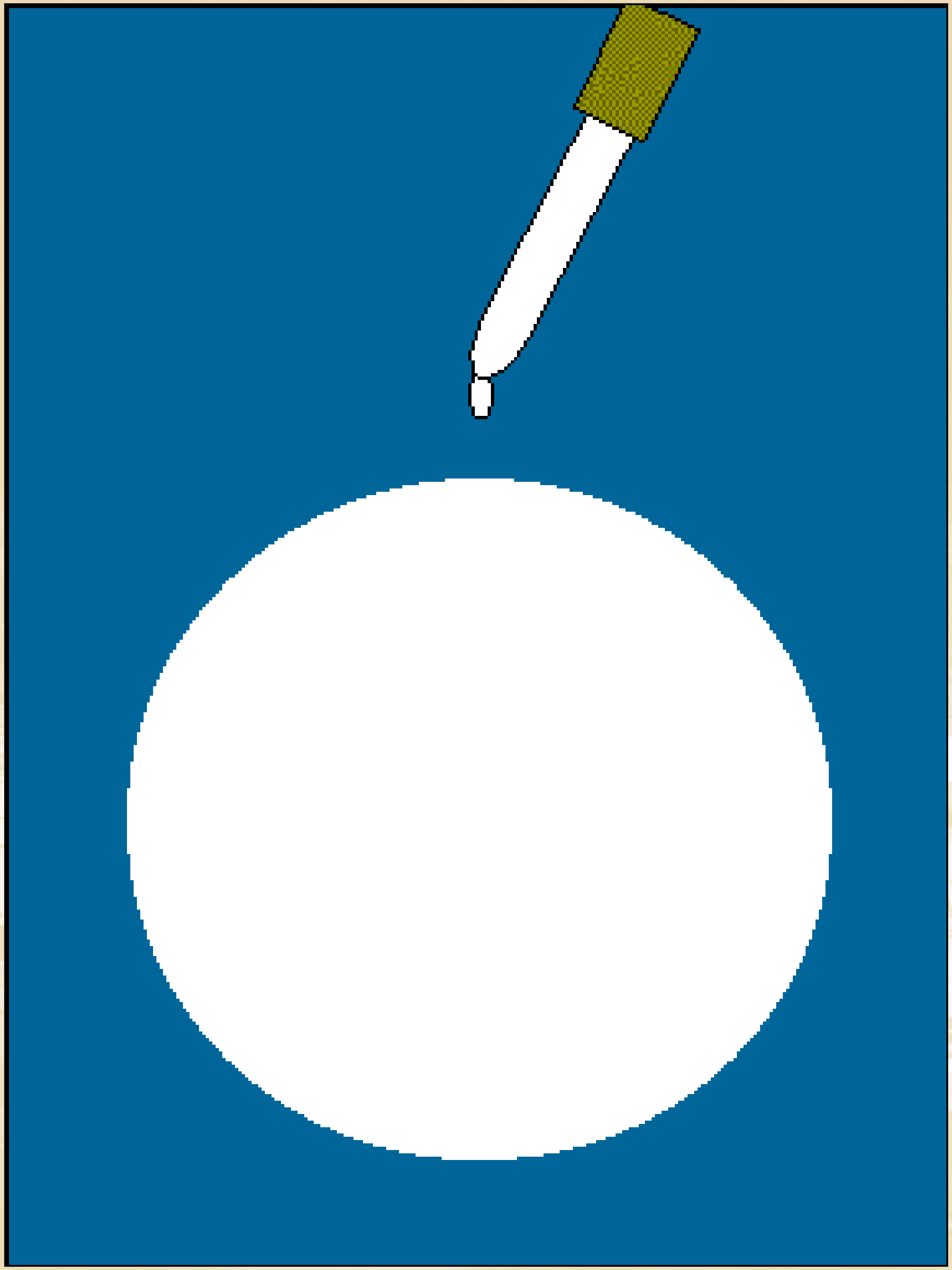
$$P \propto p_F^4 \propto \rho^{4/3} \propto M^{4/3} R^{-4}$$

No stable configuration

Chandrasekhar mass limit

$$M_{Ch} = 1.457\, M_{sun} (2Y_e)^2$$

Degenerate Stars ("White Dwarfs")

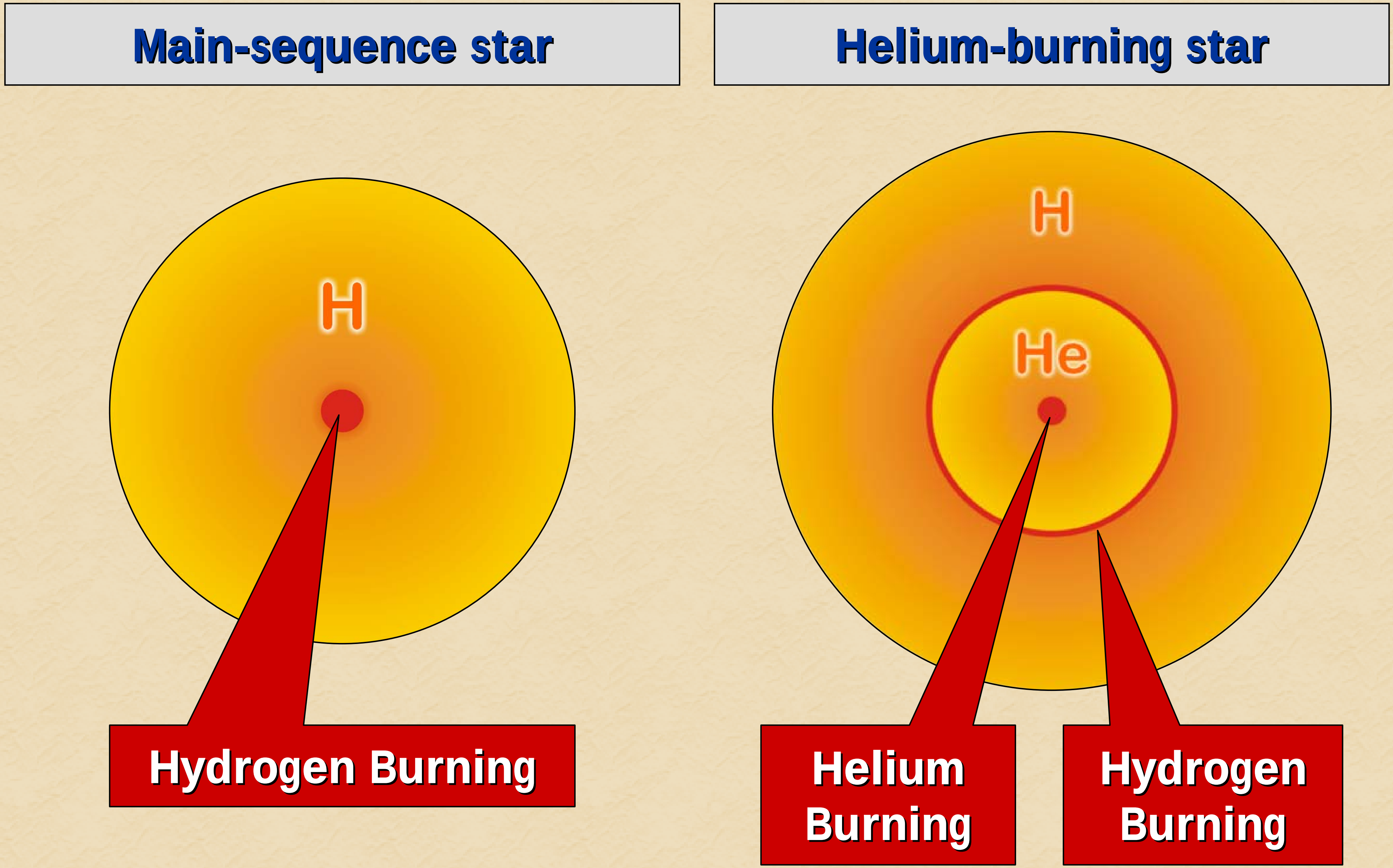


Inverse mass-radius relationship for degenerate stars: $R \propto M^{-1/3}$

Chandrasekhar mass limit

$M_{Ch} = 1.457\ M_{sun}\,(2Y_e)^2$

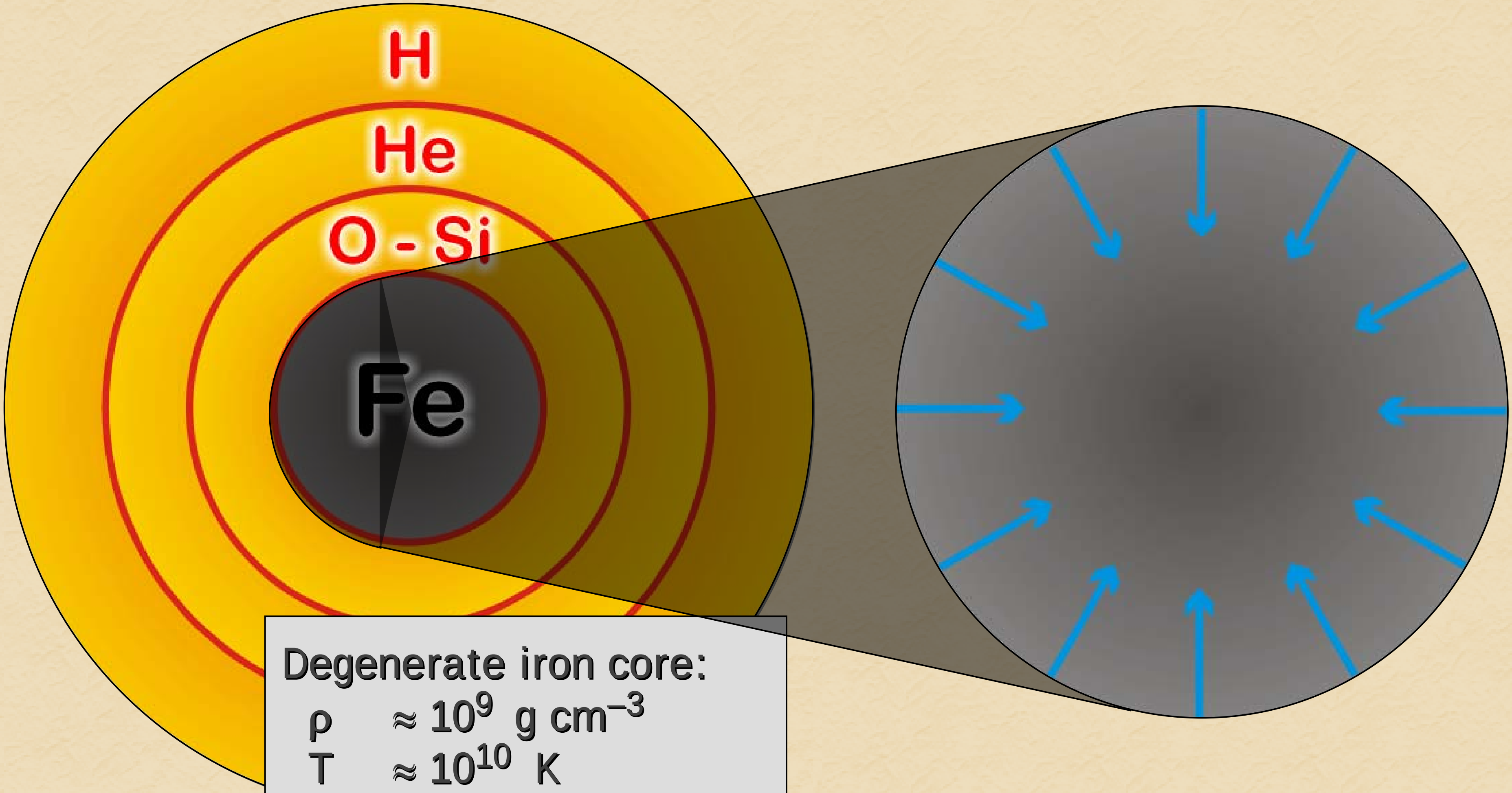
Stellar Collapse



Stellar Collapse

Onion structure

Collapse (implosion)



Degenerate iron core:

$\rho \approx 10^9$ g cm^{-3}

$T \approx 10^{10}$ K

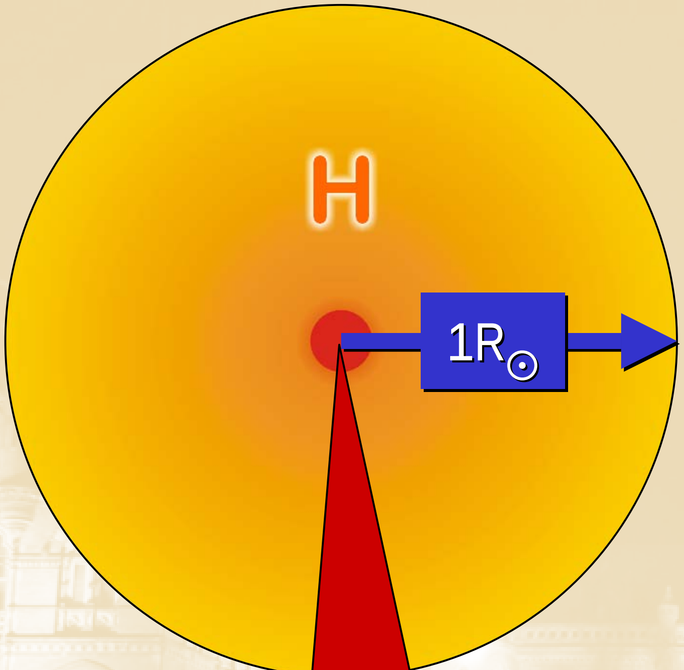
$M_{Fe} \approx 1.5\ M_{sun}$

$R_{Fe} \approx 8000$ km

Giant Stars

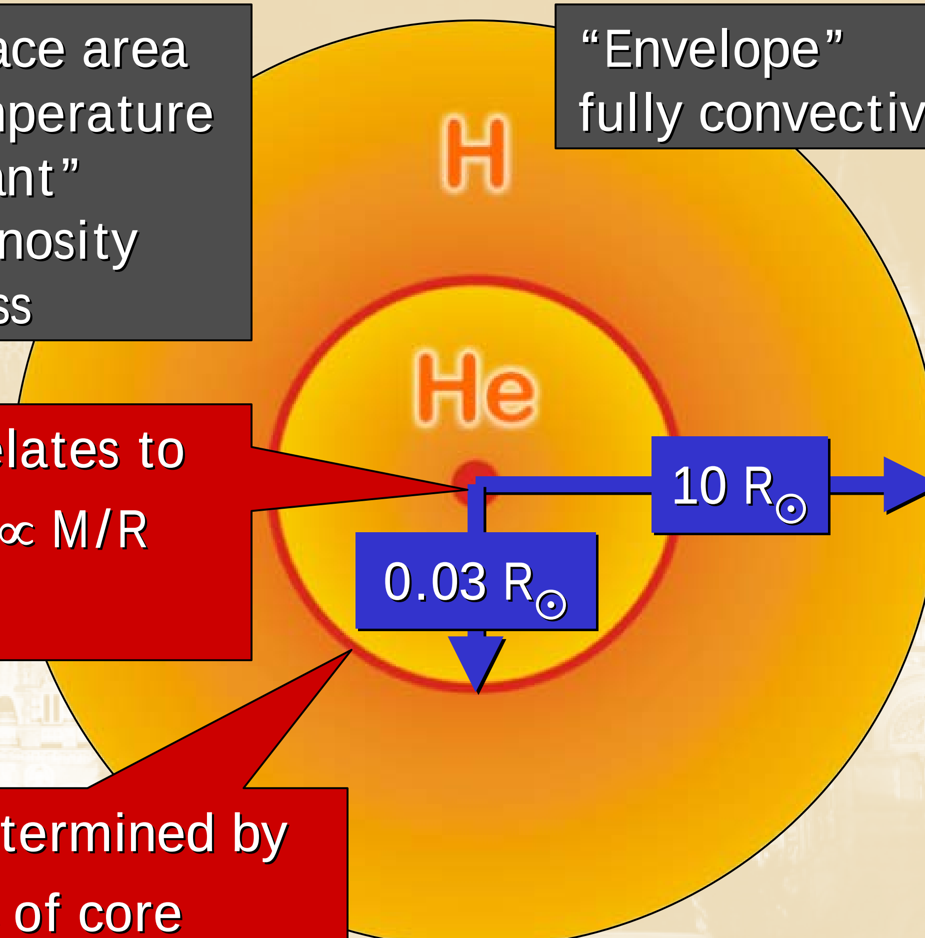
Main-sequence star $1M_\odot$ (Hydrogen burning)

Helium-burning star $1M_\odot$



Large surface area
$\to$ low temperature
$\to$ "red giant"
Large luminosity
$\to$ mass loss

"Envelope"
fully convective

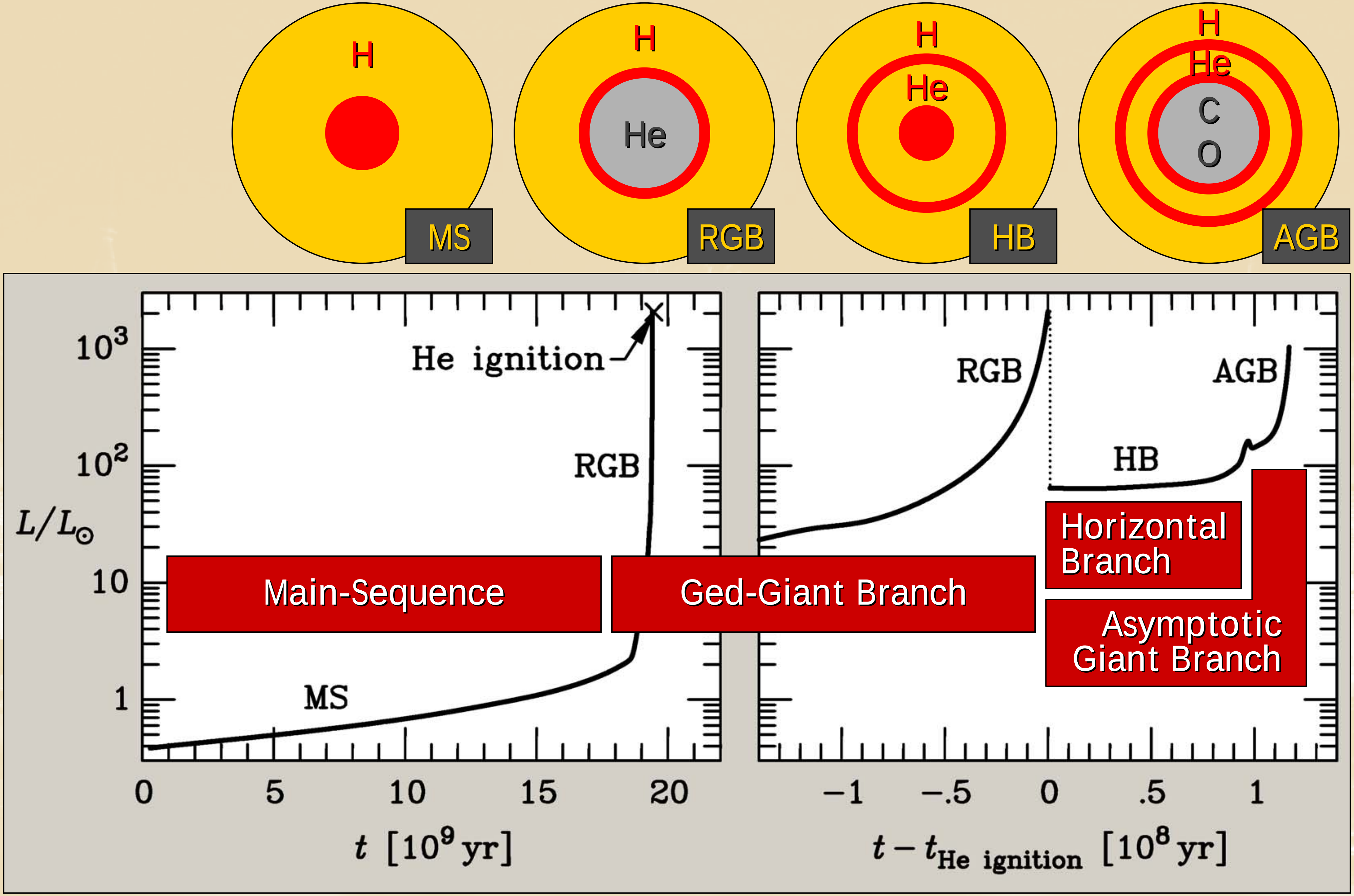


ε_{nuc}(He) relates to
$T \propto \Phi_{grav} \propto M/R$
of core

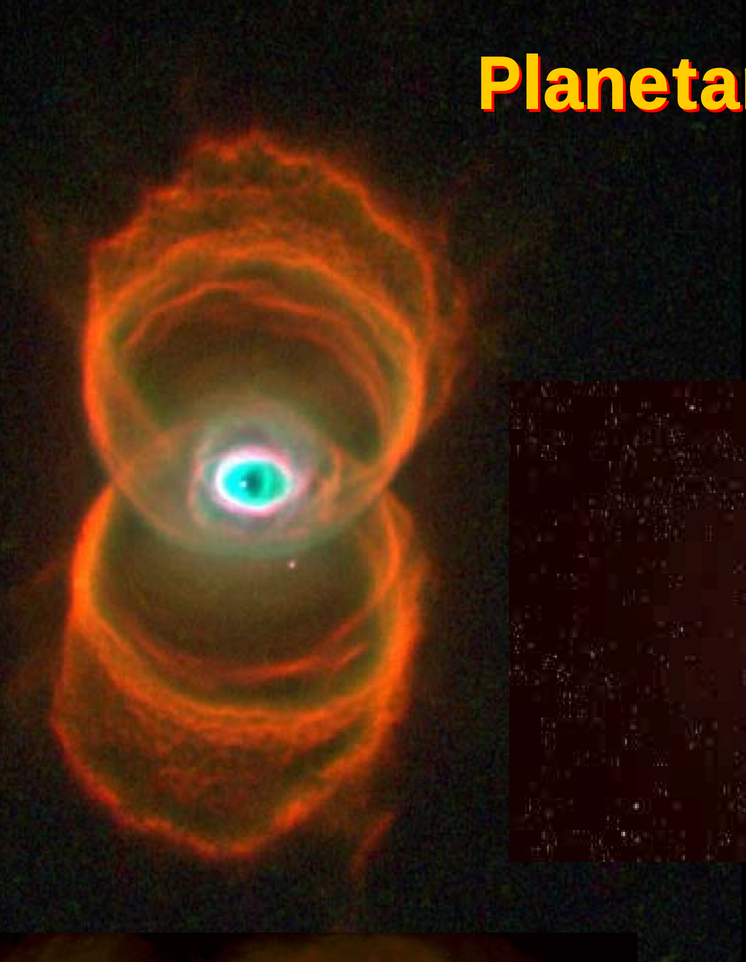
ε_{nuc}(H) relates to
$T \propto \Phi_{grav} \propto M/R$
of full star

ε_{nuc}(H) determined by
$T \propto \Phi_{grav}$ of core
$\to$ huge L(H)

Evolution of a Low-Mass Star



Planetary Nebulae



Hour
Glass
Nebula

Planetary
Nebula IC 418

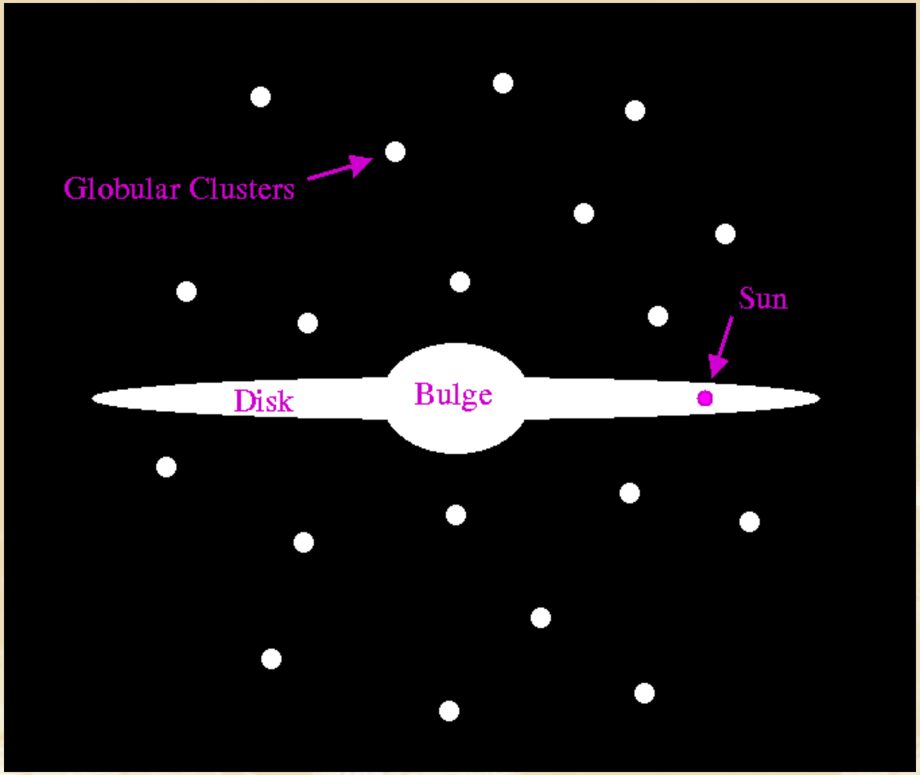
Eskimo
Nebula

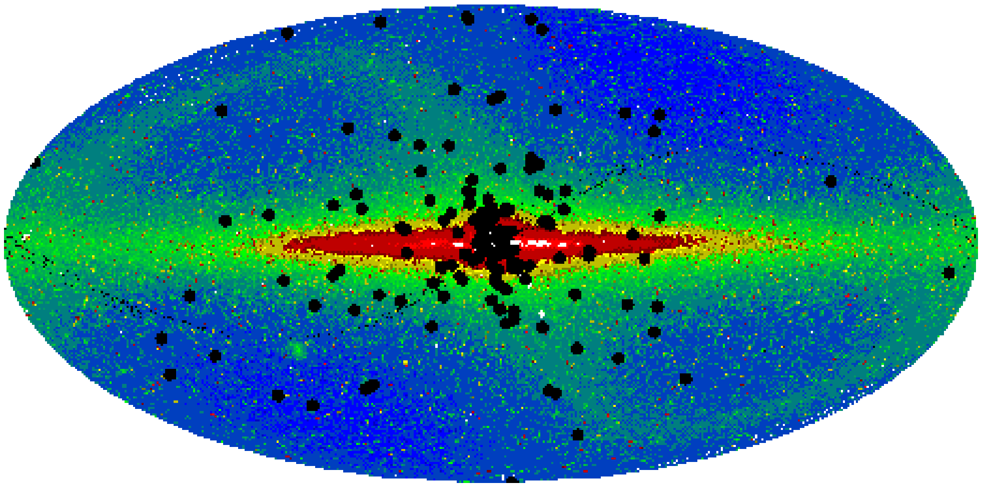
Planetary
Nebula NGC 3132

Evolution of Stars

$M < 0.08\ M_{sun}$	Never ignites hydrogen → cools ("hydrogen white dwarf")	Brown dwarf
$0.08 < M \lesssim 0.8\ M_{sun}$	Hydrogen burning not completed in Hubble time	Low-mass main-squence star
$0.8 \lesssim M \lesssim 2\ M_{sun}$	Degenerate helium core after hydrogen exhaustion	• Carbon-oxygen white dwarf • Planetary nebula
$2 \lesssim M \lesssim 5–8\ M_{sun}$	Helium ignition non-degenerate	
$5–8\ M_{sun} \lesssim M < ???$	All burning cycles → Onion skin structure with degenerate iron core; Core collapse supernova	• Neutron star (often pulsar) • Sometimes black hole? • Supernova remnant (SNR), e.g. crab nebula

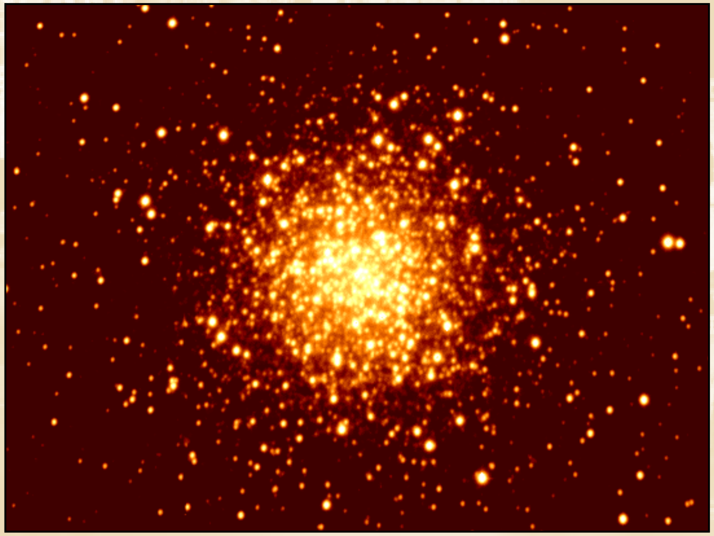
Globular Clusters of the Milky Way





http://www.dartmouth.edu/~chaboyer/mwgc.html

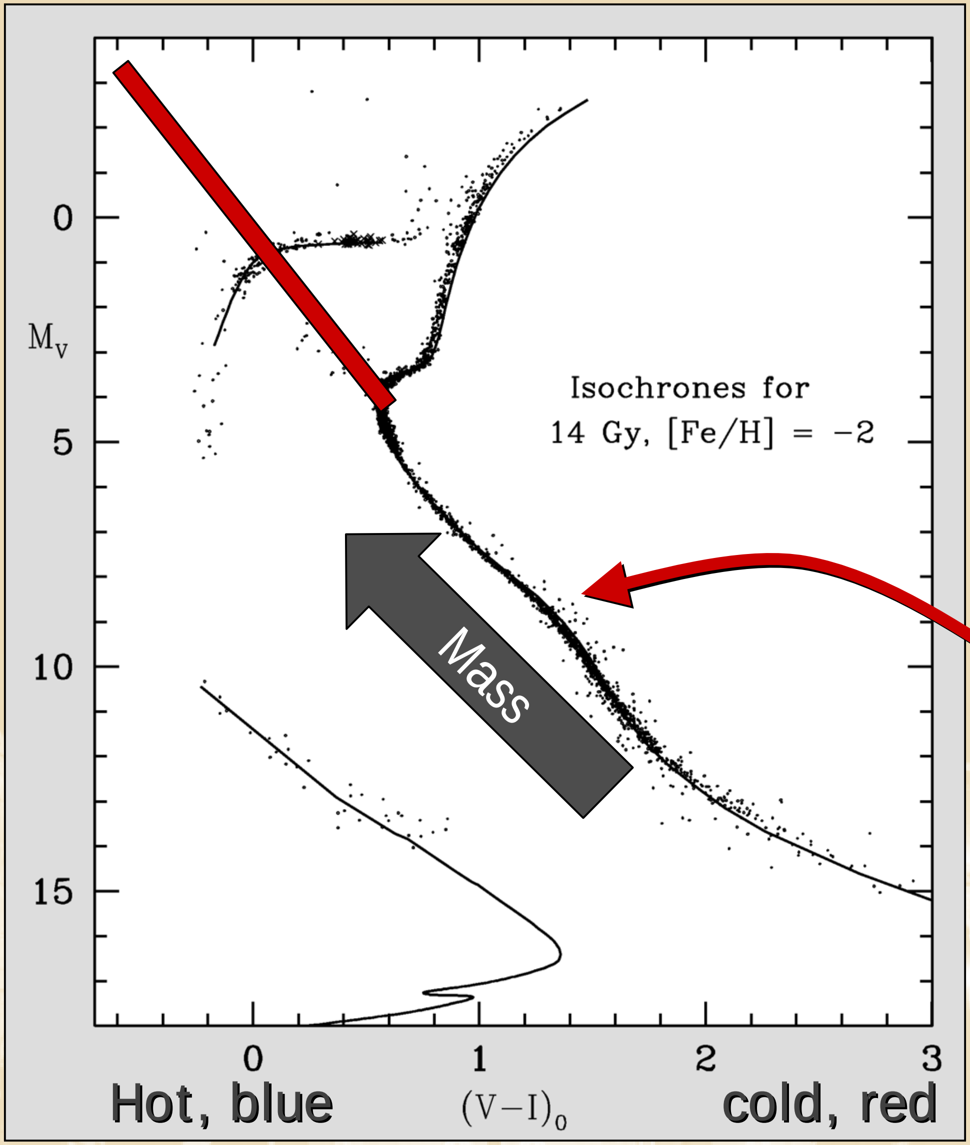
Globular clusters on top of the FIRAS 2.2 micron map of the Galaxy

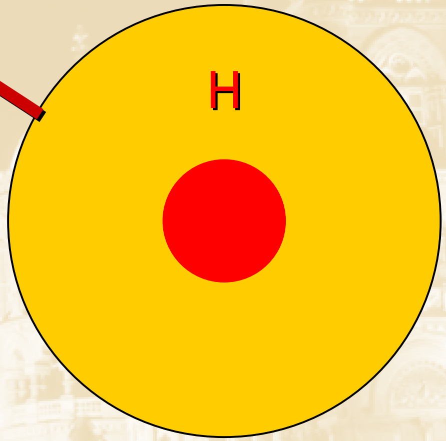


The galactic globular cluster M3

Color-Magnitude Diagram for Globular Clusters

- Stars with M so large that they have burnt out in a Hubble time
- No new star formation in globular clusters

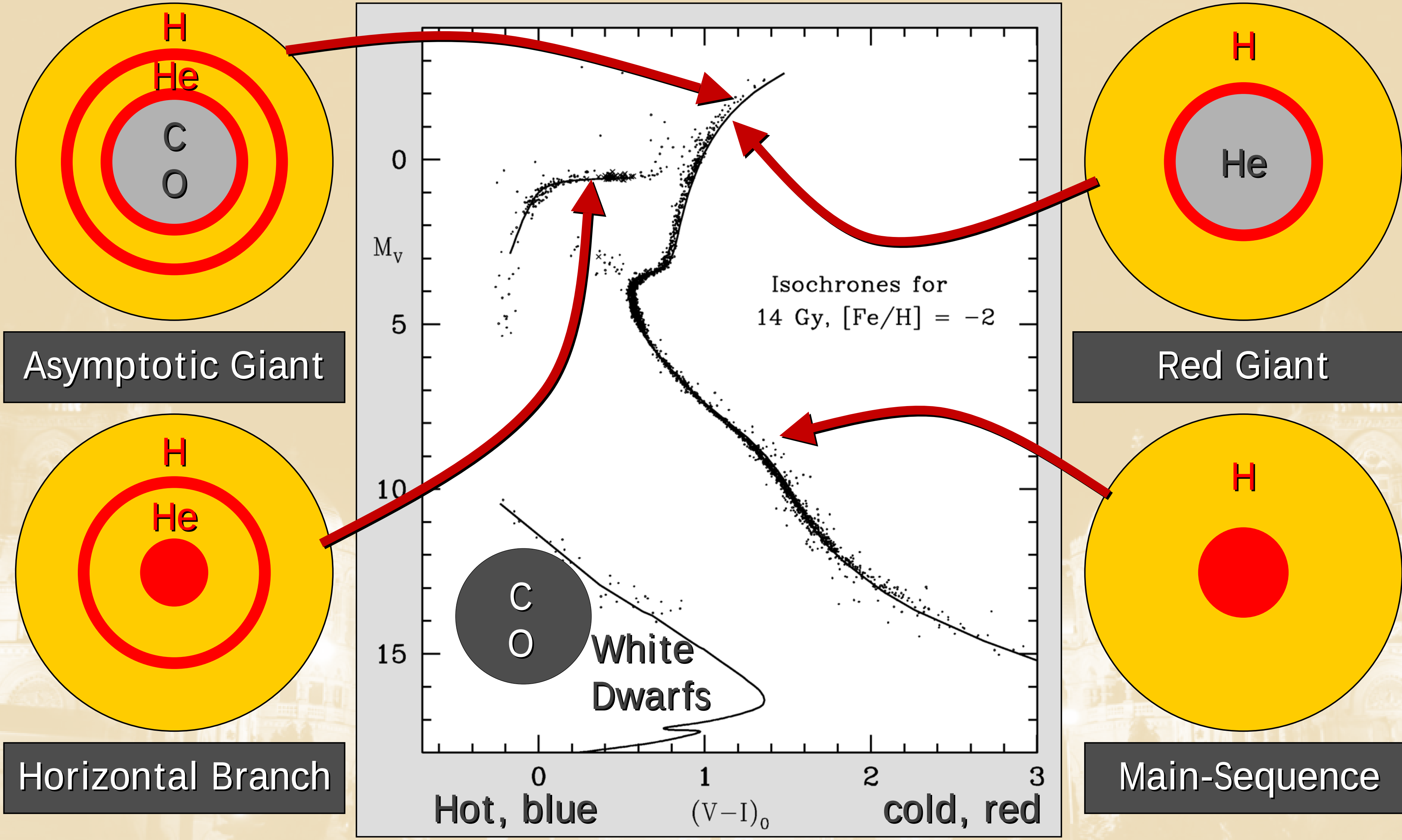




Main-Sequence

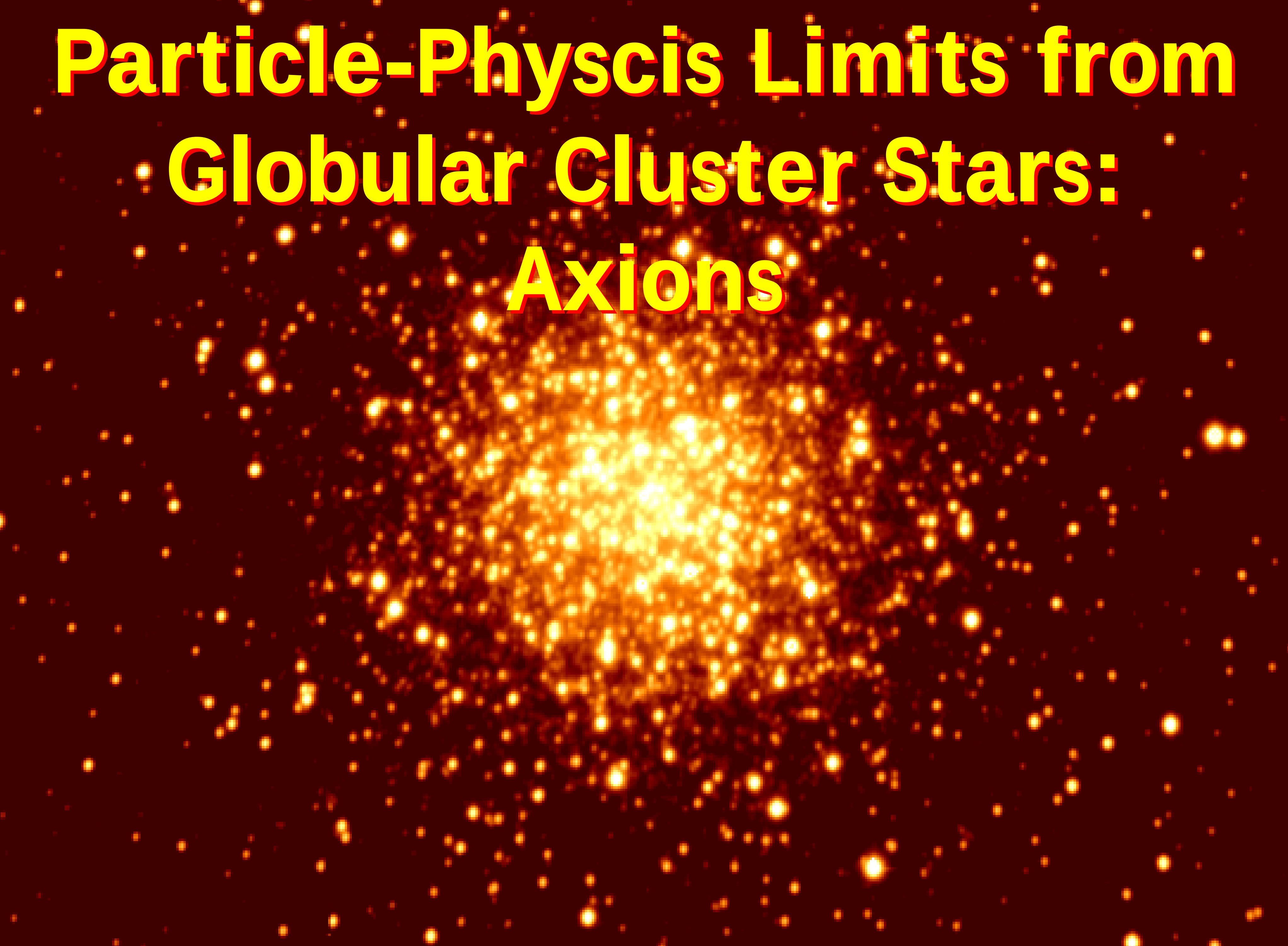
Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Color-Magnitude Diagram for Globular Clusters

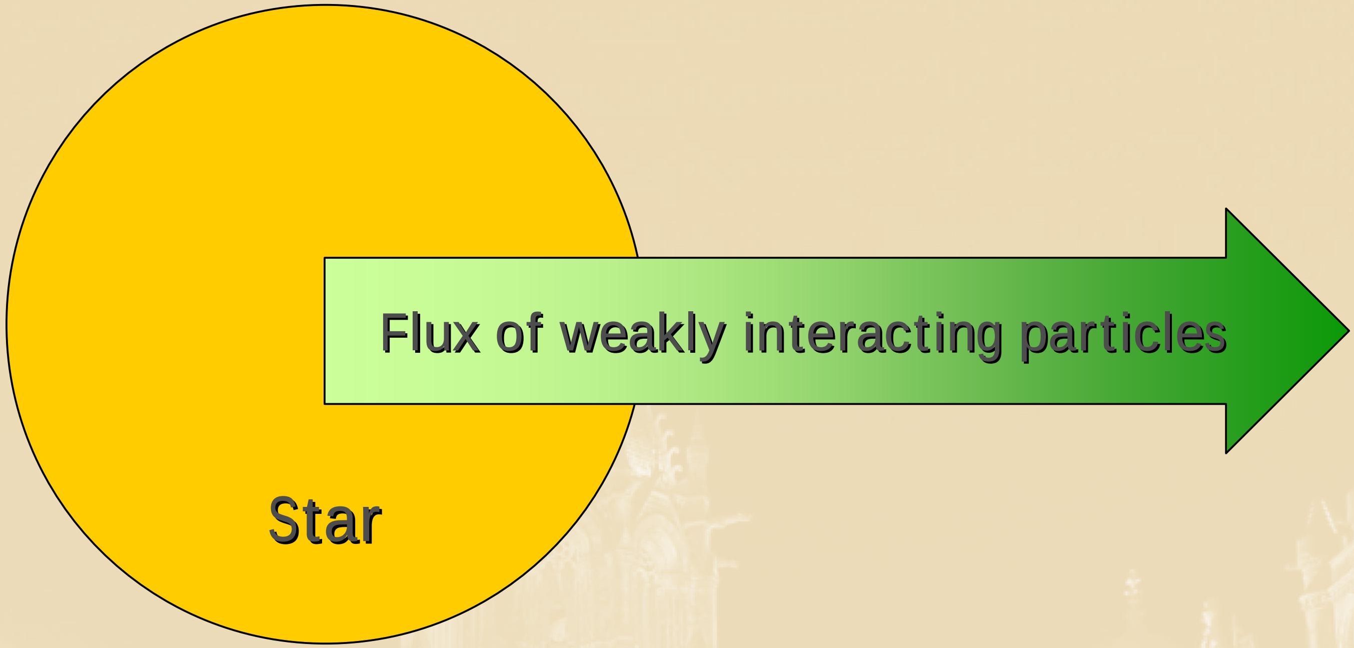


Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Particle-Physcis Limits from Globular Cluster Stars: Axions



Basic Argument



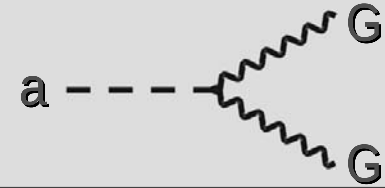
- Invisible axions have very small mass
- Emission from stellar plasma not suppressed by threshold effects (analogous to neutrinos)
- New energy-loss channel
- Back-reaction on stellar properties and evolution

- What are the emission processes?
- What are the observable consequences?

Axion Properties

Gluon coupling (Generic property)

$$L_{aG} = \frac{\alpha_s}{8\pi f_a} G\tilde{G}a$$



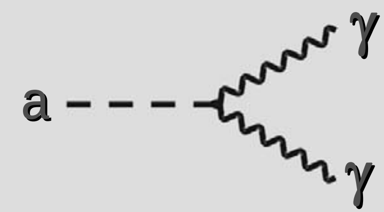
Mass

$$m_a = \frac{0.6\,\mathrm{eV}}{f_a / 10^7\,\mathrm{GeV}} \approx \frac{m_\pi f_\pi}{f_a}$$

Photon coupling

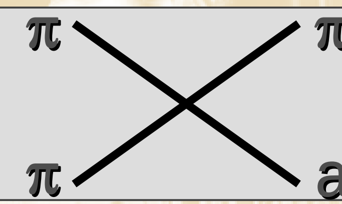
$$L_{a\gamma} = -\frac{g_{a\gamma}}{4} F\tilde{F}a = g_{a\gamma}\vec{E}\cdot\vec{B}\,a$$

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92\right)$$



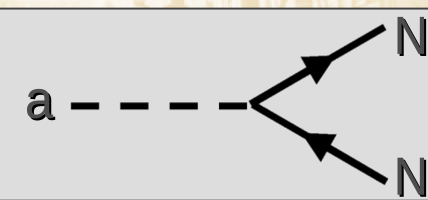
Pion coupling

$$L_{a\pi} = \frac{C_{a\pi}}{f_a f_\pi}\left(\pi^0\pi^+\partial_\mu\pi^- + \ldots\right)\partial^\mu a$$



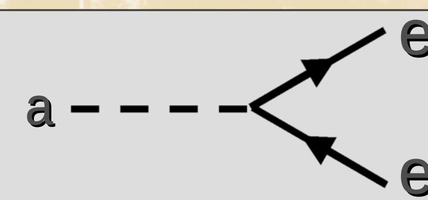
Nucleon coupling (axial vector)

$$L_{aN} = \frac{C_N}{2f_a}\overline{\Psi}_N\gamma^\mu\gamma_5\Psi_N\,\partial_\mu a$$



Electron coupling (optional)

$$L_{ae} = \frac{C_e}{2f_a}\overline{\Psi}_e\gamma^\mu\gamma_5\Psi_e\,\partial_\mu a$$

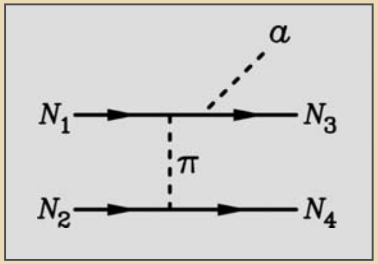


Axion or Graviton Emission Processes in Stars

Nucleons

$$\frac{C_N}{2f_a}\,\overline{\Psi}_N\gamma_\mu\gamma_5\Psi_N\,\partial^\mu a$$

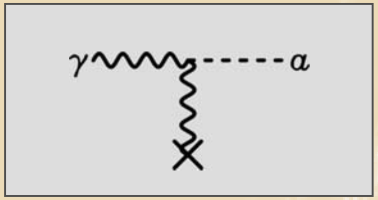
- Nucleon Bremsstrahlung



Photons

$$C_\gamma\,\frac{\alpha}{2\pi f_a}\,\vec{E}\cdot\vec{B}\,a$$

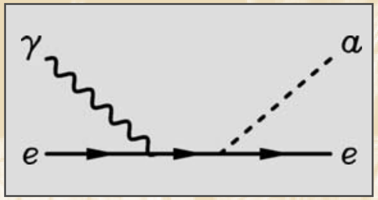
- Primakoff



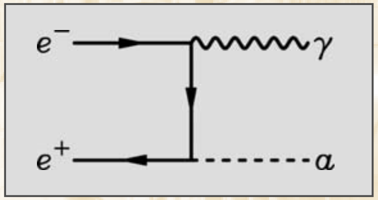
Electrons

$$\frac{C_e}{2f_a}\,\overline{\Psi}_e\gamma_\mu\gamma_5\Psi_e\,\partial^\mu a$$

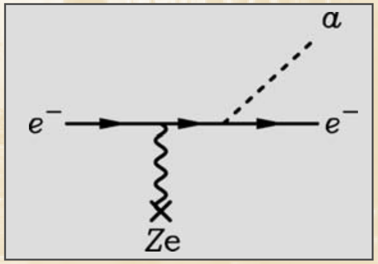
- Compton



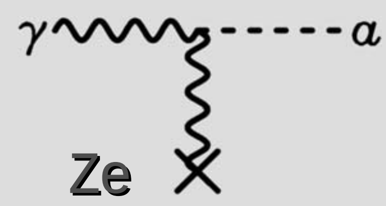
- Pair Annihilation



- Electromagnetic Bremsstrahlung



Primakoff Process in Stars

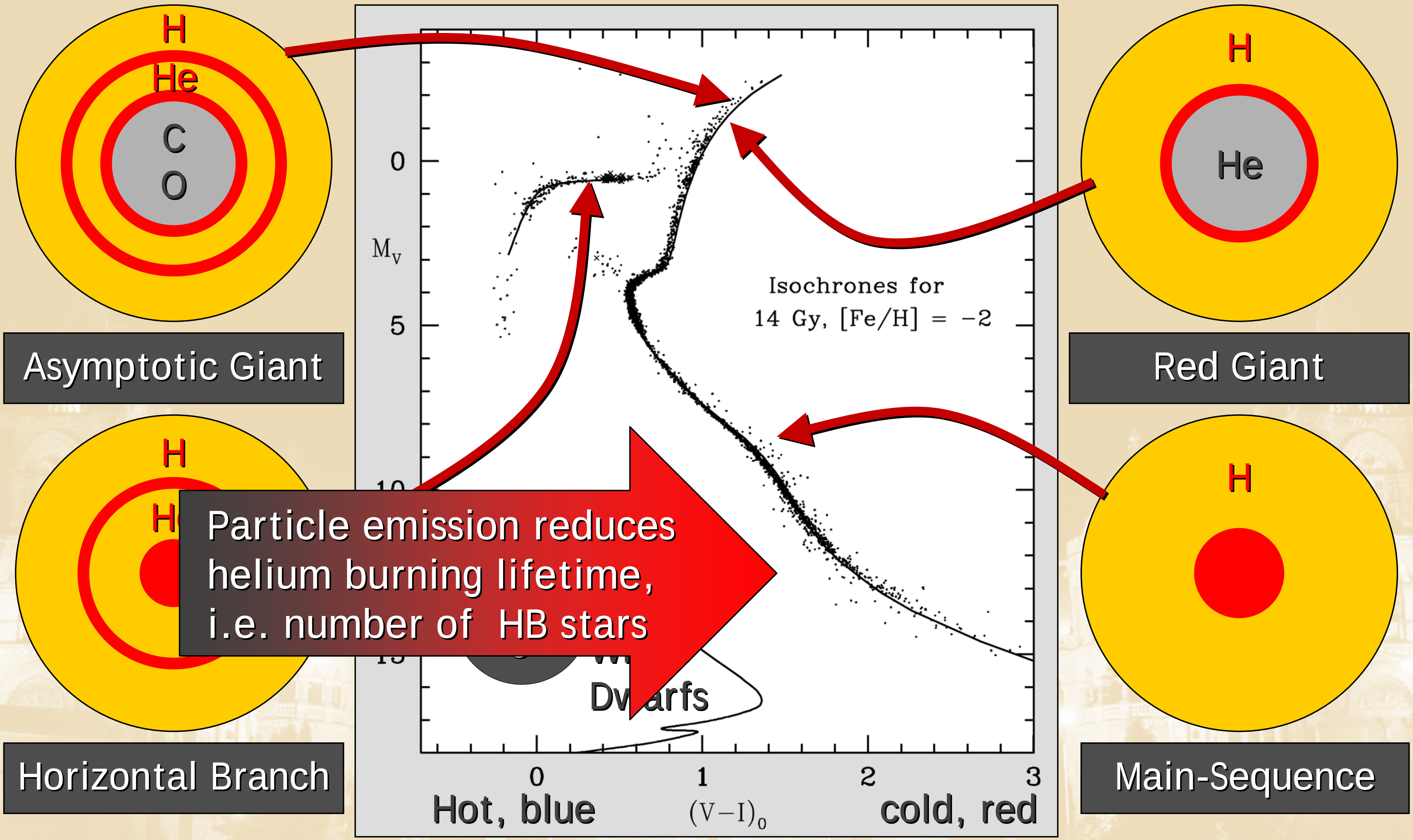
Interaction Lagrangian	$L_{a\gamma} = -\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} a = g_{a\gamma} \vec{E} \cdot \vec{B}\, a$
Primakoff cross section	$\frac{d\sigma_{\gamma \to a}}{d\Omega} = \frac{g_{a\gamma}^2 Z^2 \alpha}{8\pi} \frac{\left\lvert \vec{k}_a \times \vec{k}_\gamma \right\rvert^2}{\left\lvert \vec{k}_a - \vec{k}_\gamma \right\rvert^4}$ 
Conversion rate (screening effects, no nuclear recoil)	$\Gamma_{\gamma \to a} = \frac{g_{a\gamma}^2 T k_S^2}{32\pi} \left[\left(1 + \frac{k_S^2}{4E^2}\right) \ln\left(1 + \frac{4E^2}{k_S^2}\right) - 1 \right]$
Screening scale (non-relativistic non-degenerate)	$\kappa_S^2 = \frac{k_S^2}{4T^2} = \frac{\pi\alpha}{T^3} n_B \left(Y_e + \sum_j Z_j^2 Y_j \right)$ Sun $\kappa_S^2 \approx 12$; HB Star $\kappa_S^2 \approx 2.5$

- G. Raffelt, "Astrophysical axion bounds diminished by screening effects", Phys. Rev. D 33 (1986) 897 (Part of GR's Ph.D. Thesis)
- Consistent with results from FTD methods, see Altherr, Petitgirard & del Rio Gaztelurrutia, Astropart. Phys. 2 (1994) 175

Energy-Loss Rate of the Sun

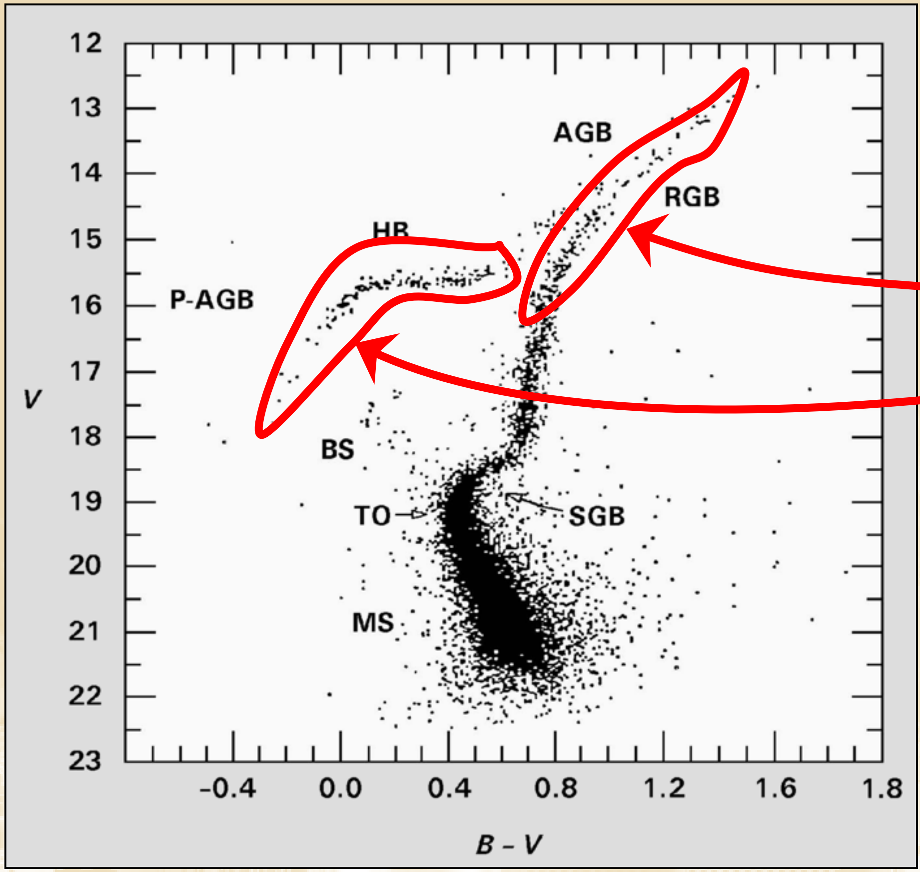
Conversion rate	$\Gamma_{\gamma\to a} = \frac{g_{a\gamma}^2 T k_S^2}{32\pi}\left[\left(1+\frac{k_S^2}{4E^2}\right)\ln\left(1+\frac{4E^2}{k_S^2}\right)-1\right]$ $\approx g_{10}^2\, 10^{-15}\,\mathrm{s}^{-1}$ for few keV-energy photons (Sun) $g_{10} = \frac{g_{a\gamma}}{10^{-10}\,\mathrm{GeV}^{-1}}$
Energy-Loss Rate	$Q = \int \frac{2d^3\vec{k}_\gamma}{(2\pi)^3}\frac{\Gamma_{a\to\gamma}E}{e^{E/T}-1} = \frac{g_{a\gamma}^2 T^7}{4\pi}F(\kappa_S^2)$ $F(\kappa_S^2) = \frac{\kappa_S^2}{2\pi^2}\int_0^\infty dx\left[(x^2+\kappa_S^2)\ln\left(1+\frac{x^2}{\kappa_S^2}\right)-x^2\right]\frac{x}{e^x-1}$
Solar Axion Luminosity	$L_a = g_{10}^2\, 1.85\times 10^{-3} L_{sun}$

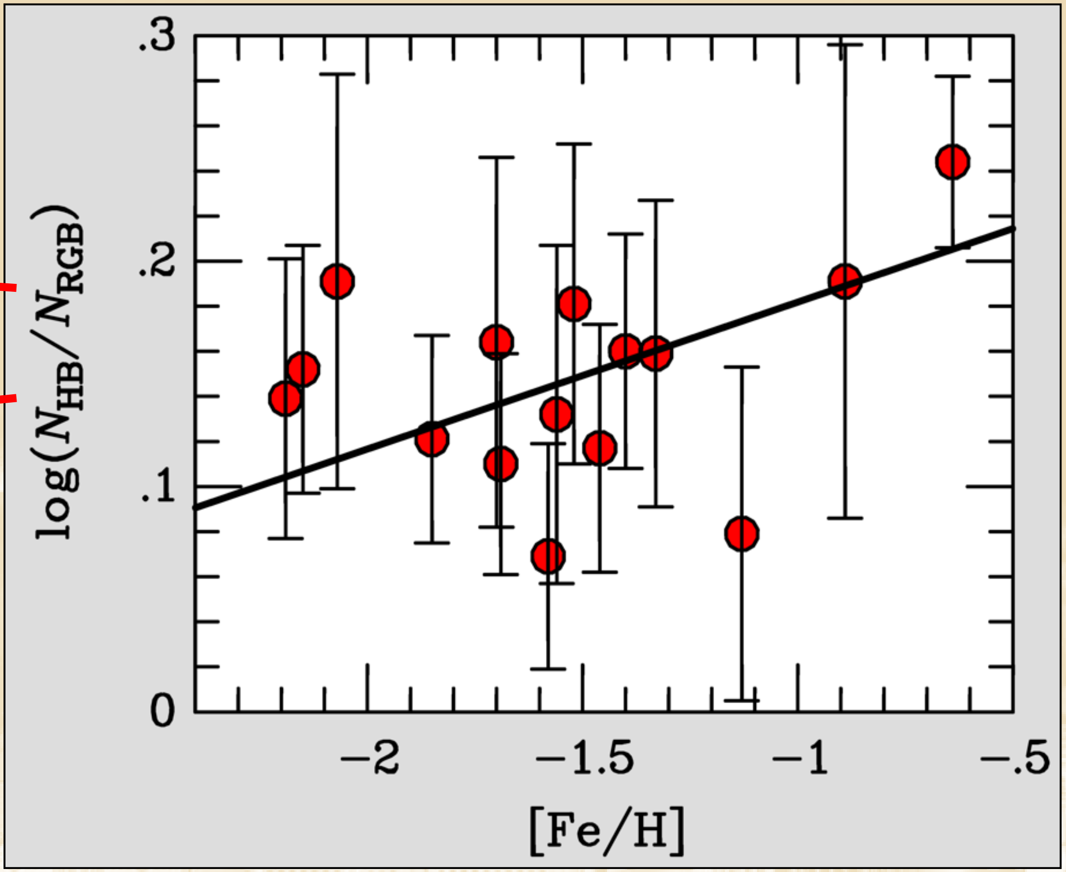
Color-Magnitude Diagram for Globular Clusters



Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Helium-Burning Lifetime of Horizontal-Branch Stars





Number ratio of HB-Stars/Red Giants in 15 galactic globular clusters
(Buzzoni et al. 1983)

Helium-burning lifetime established within ±10%

Globular Cluster Data

Astron. Astrophys. 128, 94–101 (1983)

ASTRONOMY AND ASTROPHYSICS

Helium abundance in globular clusters: the R-method

A. Buzzoni[1], F. Fusi Pecci[1], R. Buonanno[2], and C. E. Corsi[2]

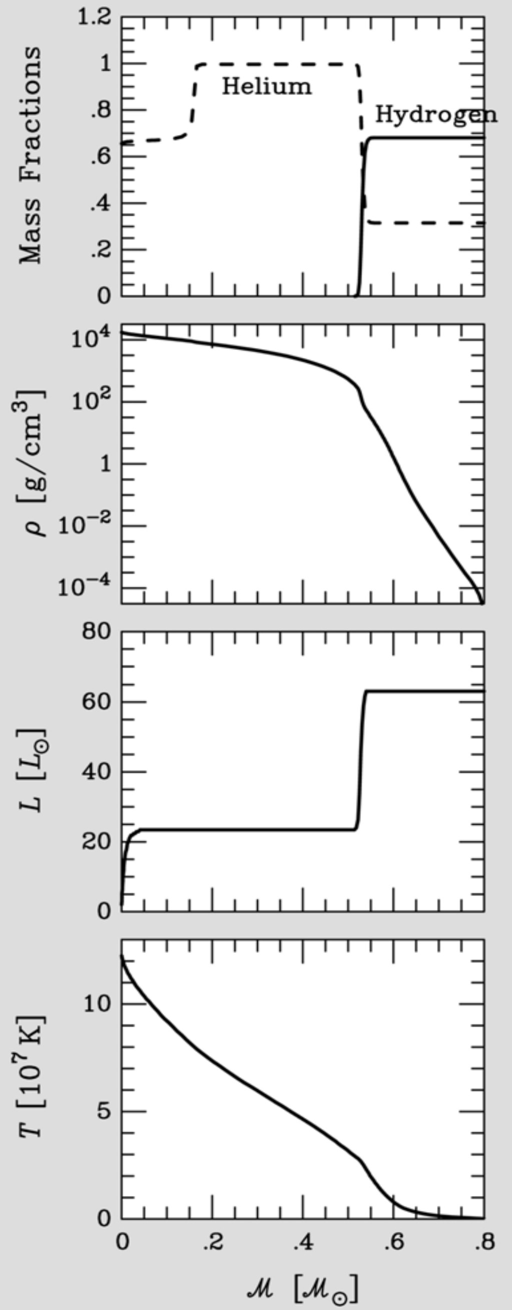
[1] Osservatorio Astronomico Universitario, C.P. 596, I-40100 Bologna, Italy
[2] Osservatorio Astronomico su Monte Mario, Roma, Italy

Received April 13, accepted July 4, 1983

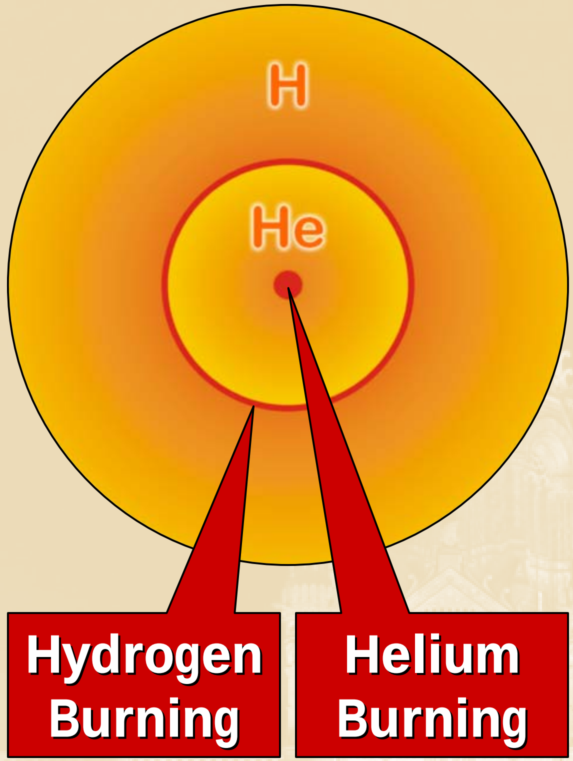
Table 1. Data for galactic globular clusters

Cluster	N_{HB+RR}	N_{RR}	N_{RGB}	N_{AGB}	R	R'	Y(R)	Y(R)	Y(R')	Y(R')	R1	References
104 47Tuc^{+}	365	0	208	45	1.75±0.21	1.44±0.17	0.27±0.02	0.27	0.27±0.02	0.27	0.22±0.05	Lee 1977a
362	78	4	65	13	1.20 0.28	1.00 0.23	0.21 0.04	0.21	0.21 0.04	0.20	0.20 0.08	Harris 1982
1851	101	7	70	15	1.44 0.32	1.19 0.25	0.24 0.03	0.24	0.24 0.04	0.24	0.21 0.08	Stetson 1981
3201	175	60	121	19	1.45 0.24	1.25 0.20	0.24 0.03	0.24	0.25 0.03	0.25	0.16 0.05	Lee 1977c
4147	59	14	38	7	1.55 0.45	1.31 0.37	0.25 0.05	0.25	0.26 0.05	0.26	0.18 0.10	Sandage and Walker 1955
5272 M3	183	83	142	28	1.29 0.20	1.08 0.16	0.22 0.02	0.22	0.22 0.03	0.22	0.20 0.05	Sandage and Katem 1982
5904 M5	164	40	140	31	1.16 0.19	0.96 0.15	0.20 0.03	0.20	0.20 0.03	0.20	0.22 0.06	Buonanno et al.1981
6121 M4	148	38	113	20	1.31 0.23	1.11 0.19	0.22 0.03	0.22	0.23 0.03	0.23	0.18 0.06	Lee 1977b
6171 M107	45	8	29	6	1.55 0.52	1.29 0.41	0.25 0.05	0.25	0.25 0.06	0.25	0.21 0.12	Dickens and Rolland 1972
6218 M12	80	0	59	11	1.36 0.33	1.14 0.26	0.23 0.04	0.23	0.23 0.04	0.23	0.19 0.08	Racine 1971
6254 M10	70	0	48	11	1.46 0.39	1.19 0.30	0.24 0.04	0.24	0.24 0.05	0.24	0.23 0.10	Harris et al.1976
6341 M92	117	7	85	21	1.38 0.28	1.10 0.21	0.23 0.03	0.23	0.22 0.02	0.22	0.25 0.08	Buonanno et al.1983b
6752	97	2	64	13	1.52 0.34	1.26 0.27	0.25 0.03	0.25	0.25 0.04	0.25	0.20 0.08	Cannon and Lee 1981
6809 M55	209	7	158	45	1.32 0.20	1.03 0.14	0.22 0.02	0.22	0.21 0.02	0.21	0.28 0.06	Lee 1977d
7078 M15	152	33	107	22	1.42 0.25	1.18 0.20	0.23 0.03	0.23	0.24 0.03	0.24	0.21 0.07	Buonanno et al.1983a

Globular-Cluster Limit on Axion-Photon Coupling



Helium-burning star

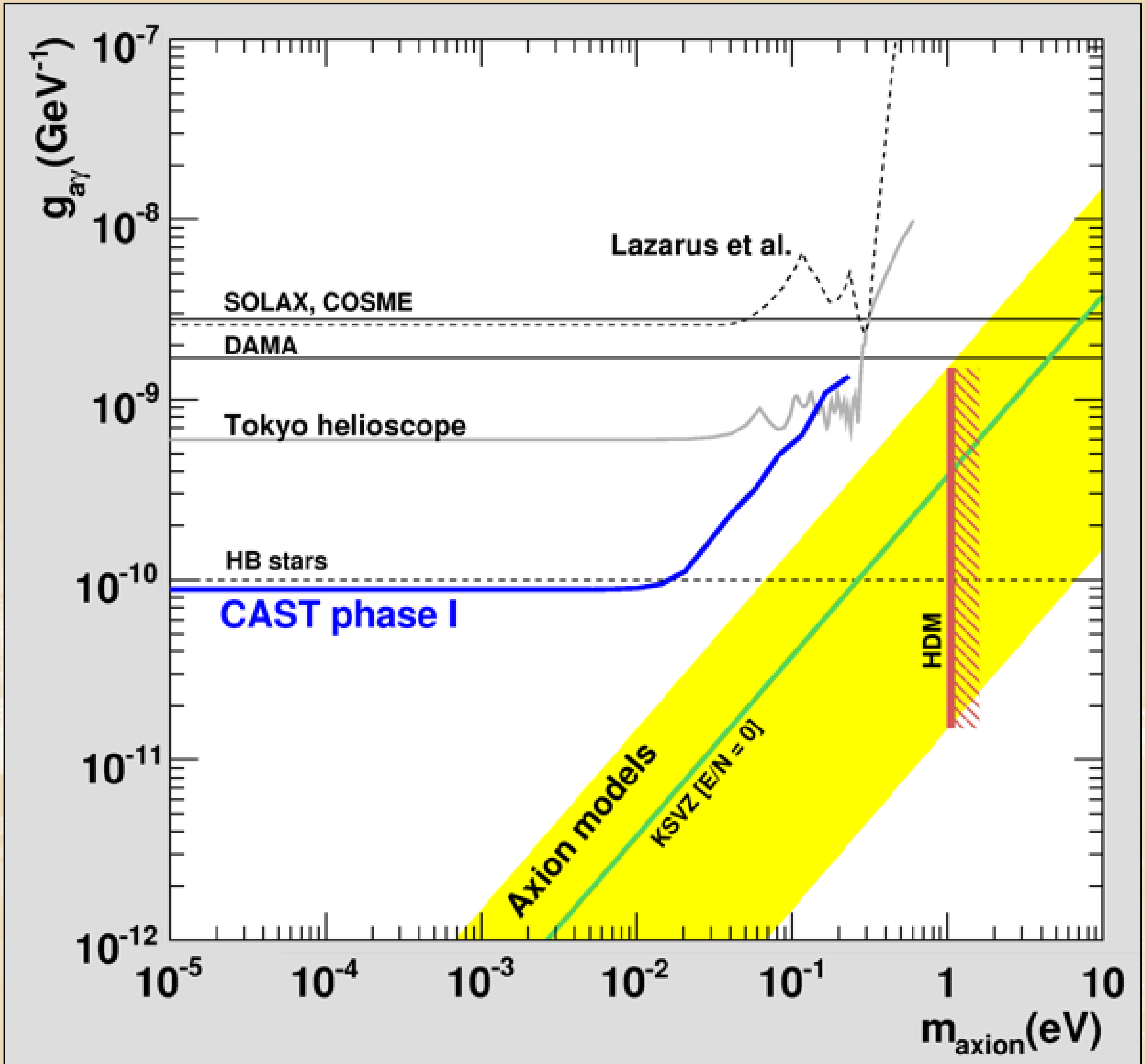


- Helium-burning luminosity
  $L_{3\alpha} \approx 20\ L_\odot$
  $T \approx 10$ keV
  $\rho \approx 10^4$ g cm^{-3}
- Core-average nuclear energy generation rate
  $\varepsilon_{3\alpha} \approx 80$ erg g^{-1} s^{-1}
- Core-average Primakoff emission rate
  $\varepsilon_{\rm Primakoff} \approx g_{10}^2\ 30$ erg g $^{-1}$ s^{-1}
- Reduction of helium-burning lifetime

$$\frac{\tau}{\tau_0} \approx \frac{1}{1+0.4\,g_{10}^2}$$

- Adopt nominal limit $g_{10} < 1$
  (More restrictive limit if using 10% precision for helium burning lifetime)

CAST Phase I Results (2003–2004)



CAST Collaboration:

An improved limit on the axion-photon coupling from the CAST experiment

hep-ex/0702006

Free Streaming vs Trapping of New Particles

Free Streaming Mean Free Path ≫ Stellar Radius	Trapping Mean Free Path ≪ Stellar Radius

Energy conservation

$$\frac{dL_r}{dr} = 4\pi r^2 \varepsilon\rho$$

Hydrostatic Equilibrium

$$\frac{dP}{dr} = -\frac{G_N M_r \rho}{r^2}$$

Energy transfer

$$L_r = \frac{4\pi r^2}{3\kappa\rho}\frac{d(aT^4)}{dr}$$

Weakly interacting particles constitute a new energy-loss channel in addition to neutrinos and thus violate "energy conservation," reducing the available nuclear energy

$$\varepsilon = \varepsilon_{nuc} - \varepsilon_\nu - \varepsilon_x$$

Strong effect on stellar evolution when

ε_x comparable to ε_{nuc}

Weakly interacting particles achieve local thermal equilibrium and thus contribute an energy-transfer channel in addition to photons and conduction

$$\kappa^{-1} = \kappa_c^{-1} + \kappa_\gamma^{-1} + \kappa_x^{-1}$$

Relation to average mean free path

$$(\kappa_\gamma \rho)^{-1} = \langle \lambda_\gamma \rangle_{\mathrm{Rosseland}}$$

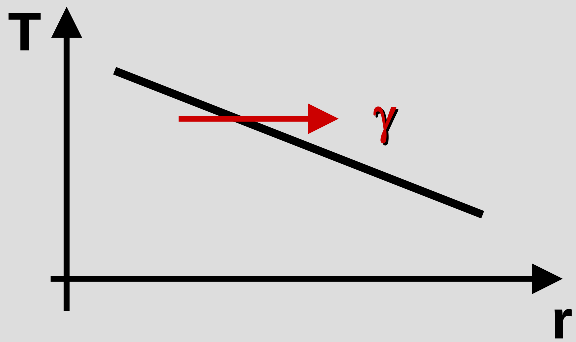
Strong effect on stellar structure when $\lambda_x \gtrsim \lambda_\gamma$

Strongest effect of new particles when mean free path ~ stellar radius

What if axion-like particles are "trapped"?

Radiative energy transfer

Photons transport energy over a distance ~ 1 mean free path (mfp)



To be harmless, a "trapped" low-mass particle species, e.g., axion-like particles, must have a mfp approximately less than photons (in the Sun ~ few cm)

A new low-mass particle has the strongest effect on a star
when its mfp is of order the geometric dimensions of the star!

Raffelt & Starkman, "Stellar energy transfer by keV-mass scalars",
Phys. Rev. D 40, 942 (1989)

Particle-Physcis Limits from Globular Cluster Stars: Neutrino Dipole Moments

Neutrinos from Thermal Plasma Processes

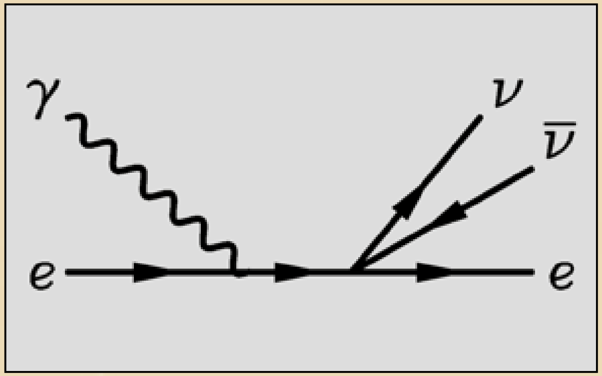
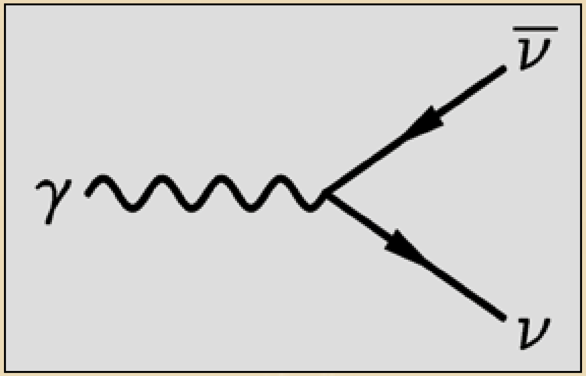
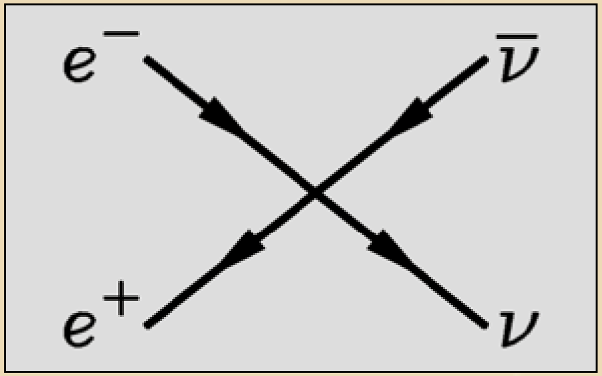


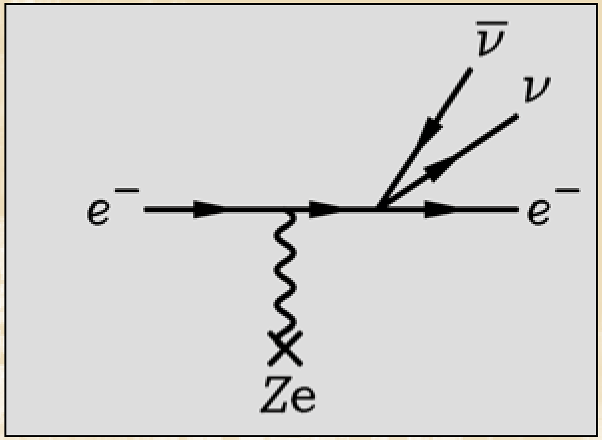
Photo (Compton)



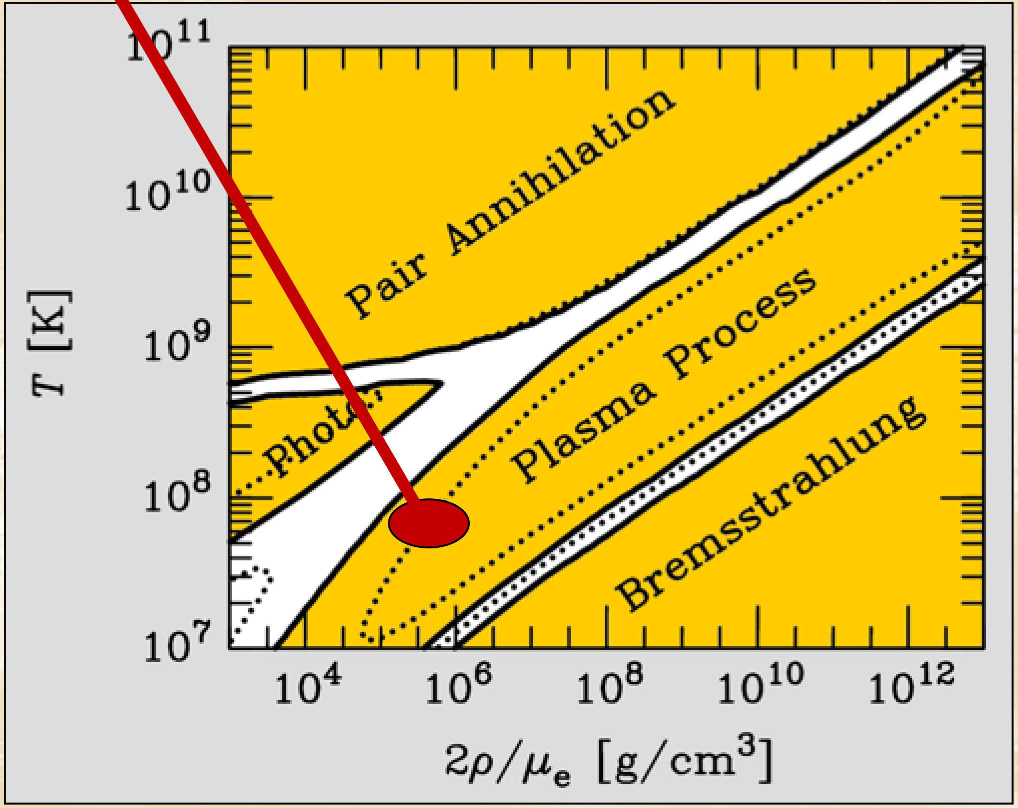
Plasmon decay



Pair annihilation

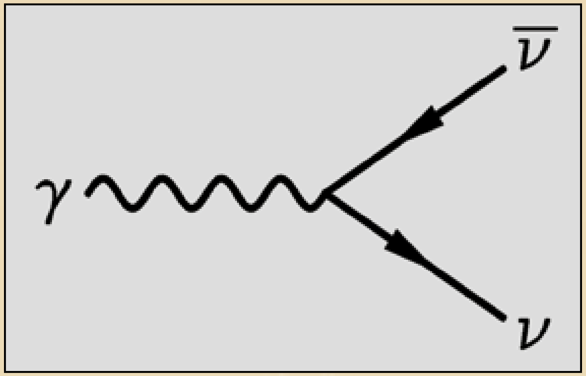


Bremsstrahlung



Plasmon Decay in Neutrinos

Vacuum:
- Photon massless
- Can not decay into other particles, even if they themselves are massless



Plasmon decay

Vacuum:
- Massless neutrinos do not couple to photons
- May have dipole moments or even "millicharges"

Propagation in a medium:
- Photon acquires a "refractive index"
- In a non-relativistic plasma (e.g. Sun, white dwarfs, core of red giant before helium ignition, ...) behaves like massive particle:

$$\omega^2 - k^2 = \omega_{pl}^2$$

Plasma frequency (electron density n_e) $\quad \omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$

- Degenerate helium core $\omega_{pl} = 18\ \mathrm{keV}$ ($\rho = 10^6\ \mathrm{g\ cm^{-3}}$, $T = 8.6\ \mathrm{keV}$)

In a medium:
- Neutrinos interact coherently with the charged particles which themselves couple to photons
- Induces an "effective charge"
- In a degenerate plasma (electron Fermi energy E_F and Fermi momentum p_F)

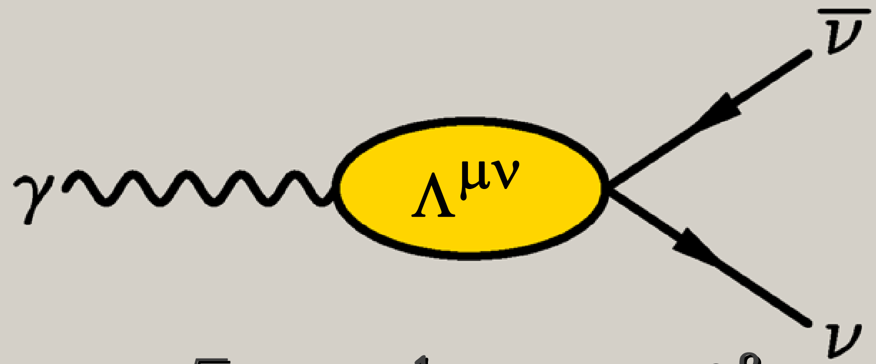
$$\frac{e_\nu}{e} = 16\sqrt{2}\, C_V G_F E_F p_F$$

- Degenerate helium core (and $C_V = 1$)

$$e_\nu = 6 \times 10^{-11} e$$

Neutrino-Photon-Coupling in a Plasma

Neutrino effective in-medium coupling



$$L_{\rm eff} = -\sqrt{2}G_F\overline{\Psi}\gamma_\alpha\frac{1}{2}(1-\gamma_5)\Psi\Lambda^{\alpha\beta}A_\beta$$

For vector-current analogous to photon polarization tensor

$$\Lambda_V^{\mu\nu}(K) = 4eC_V\int\frac{d^3\vec{p}}{2E(2\pi)^3}\left[f_{e^-}(\vec{p})+f_{e^+}(\vec{p})\right]\frac{(PK)^2g^{\mu\nu}+K^2P^\mu P^\nu-(PK)(P^\mu K^\nu+K^\mu P^\nu)}{(PK)^2-\frac{1}{4}(K^2)^2}$$

$$= \frac{C_V}{e}\Pi_V^{\mu\nu}(K)$$

$$\Lambda_A^{\mu\nu}(K) = 2ieC_A\varepsilon^{\mu\nu\alpha\beta}\int\frac{d^3\vec{p}}{2E(2\pi)^3}\left[f_{e^-}(\vec{p})-f_{e^+}(\vec{p})\right]\frac{K^2P_\alpha K_\beta}{(PK)^2-\frac{1}{4}(K^2)^2}$$

Usually negligible

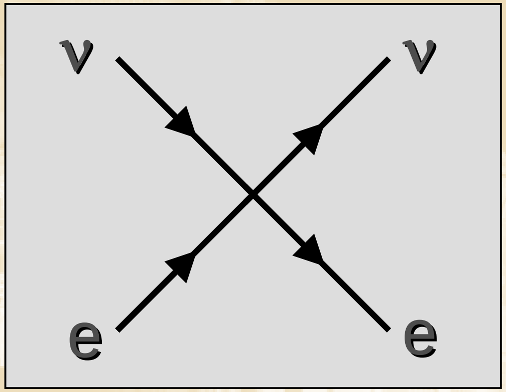
Neutral-Current Couplings and Plasmon Decay

Standard-model plasmon decay process $\propto C_V^2$

$\sin^2\Theta_W \approx \frac{1}{4}$

Standard-model plasmon decay produces almost exclusively $\nu_e\overline{\nu}_e$

A neutral-current process that was never useful for “neutrino counting” unlike big-bang nucleosynthesis (of course today Z^0-decay width fixes $N_\nu = 3$)



$$H_{\rm int} = \frac{G_F}{\sqrt{2}}\,\overline{\Psi}_f\gamma_\mu(C_V - C_A\gamma_5)\Psi_f\;\overline{\Psi}_\nu\gamma^\mu(1-\gamma_5)\Psi_\nu$$

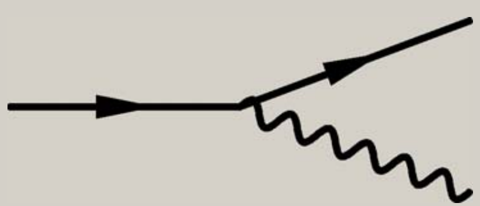
$G_F =$

$1.166\times10^{-5}\,\mathrm{GeV}^{-2}$

$\sin^2\Theta_W = 0.231$

Neutrino	Fermion	C_V	C_A
ν_e	Electron	$+\frac{1}{2}+2\sin^2\Theta_W \approx 1$	$+\frac{1}{2}$
ν_μ, ν_τ	Electron	$-\frac{1}{2}+2\sin^2\Theta_W \approx 0$	$-\frac{1}{2}$
ν_e, ν_μ, ν_τ	Proton	$+\frac{1}{2}-2\sin^2\Theta_W \approx 0$	$+\frac{1.26}{2}$
ν_e, ν_μ, ν_τ	Neutron	$-\frac{1}{2}$	$-\frac{1.26}{2}$

Plasmon Decay vs. Cherenkov Effect

Photon dispersion in a medium can be	"Time-like" $\omega^2 - k^2 > 0$	"Space-like" $\omega^2 - k^2 < 0$
Refractive index n ($k = n\,\omega$)	$n < 1$	$n > 1$
Example	• Ionized plasma • Normal matter for large photon energies	Water ($n \approx 1.3$), air, glass for visible frequencies
Allowed process that is forbidden in vacuum	Plasmon decay to neutrinos $\gamma \to \nu\overline{\nu}$ 	Cherenkov effect $e \to e + \gamma$ 

Neutrino Electromagnetic Form Factors

Effective coupling of electromagnetic field to a neutral fermion

$$\begin{aligned}L_{eff} = &-F_1\overline{\psi}\gamma_\mu\psi A^\mu \\ &-G_1\overline{\psi}\gamma_\mu\gamma_5\psi\partial_\nu F^{\mu\nu} \\ &-\tfrac{1}{2}F_2\overline{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} \\ &-\tfrac{1}{2}G_2\overline{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}\end{aligned}$$

Charge $e_\nu = F_1(0) = 0$

Anapole moment $G_1(0)$

Magnetic dipole moment $\mu = F_2(0)$

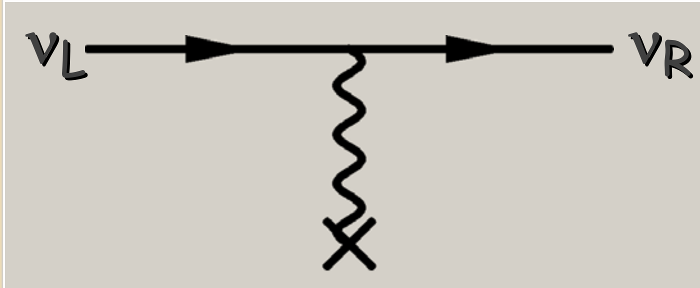
Electric dipole moment $\varepsilon = G_2(0)$

- Charge form factor $F_1(q^2)$ and anapole $G_1(q^2)$ are short-range interactions if charge $F_1(0) = 0$
- Connect states of equal chirality
- In standard model they represent radiative corrections to weak interaction

- Dipole moments connect states of opposite chirality
- Violation of individual flavor lepton numbers (neutrino mixing)
  $\to$ Magnetic or electric dipole moments can connect different flavors or different mass eigenstates ("Transition moments")
- Usually measured in "Bohr magnetons" $\mu_B = e/(2m_e)$

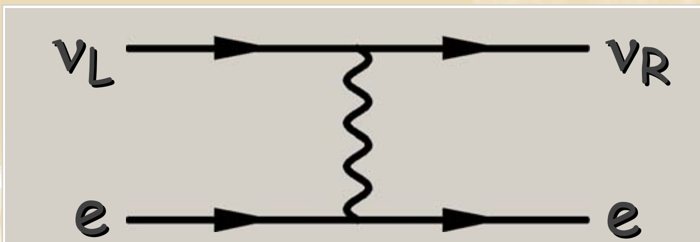
Consequences of Neutrino Dipole Moments

Spin precession in external E or B fields



$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \begin{pmatrix}0 & \mu_\nu B_T\\ \mu_\nu B_T & 0\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}$$

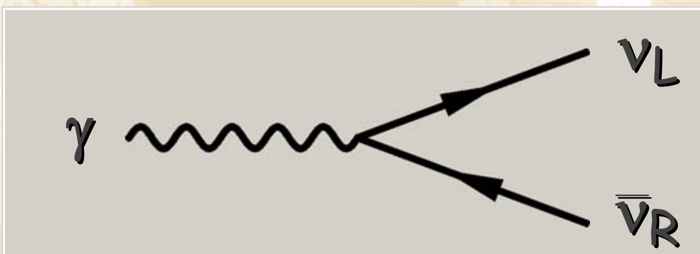
Scattering



$$\frac{d\sigma}{dT} = \frac{G_F^2 m_e}{2\pi}\left[(C_V + C_A)^2 + (C_V - C_A)^2\left(1-\frac{T}{E_\nu}\right)^2 + (C_V^2 - C_A^2)\frac{m_e T}{E_\nu^2}\right] + \alpha\mu_\nu^2\left[\frac{1}{T} - \frac{1}{E_\nu}\right]$$

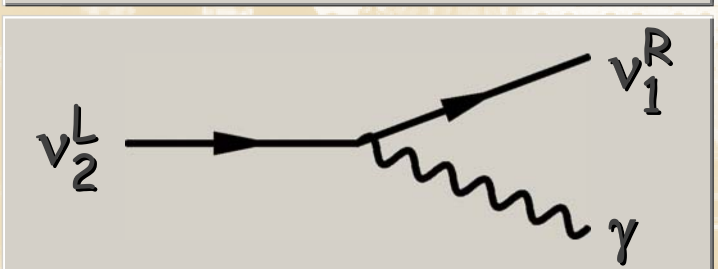
T electron recoil energy

Plasmon decay in stars



$$\Gamma = \frac{\mu_\nu^2}{24\pi}\omega_{pl}^3$$

Decay or Cherenkov effect



$$\Gamma = \frac{\mu_\nu^2}{8\pi}\left(\frac{m_2^2 - m_1^2}{m_2}\right)^3$$

Plasmon Decay And Stellar Energy Loss Rates

Assume photon dispersion relation like a massive particle (nonrelativistic plasma)

$$E_\gamma^2 - p_\gamma^2 = \omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$$

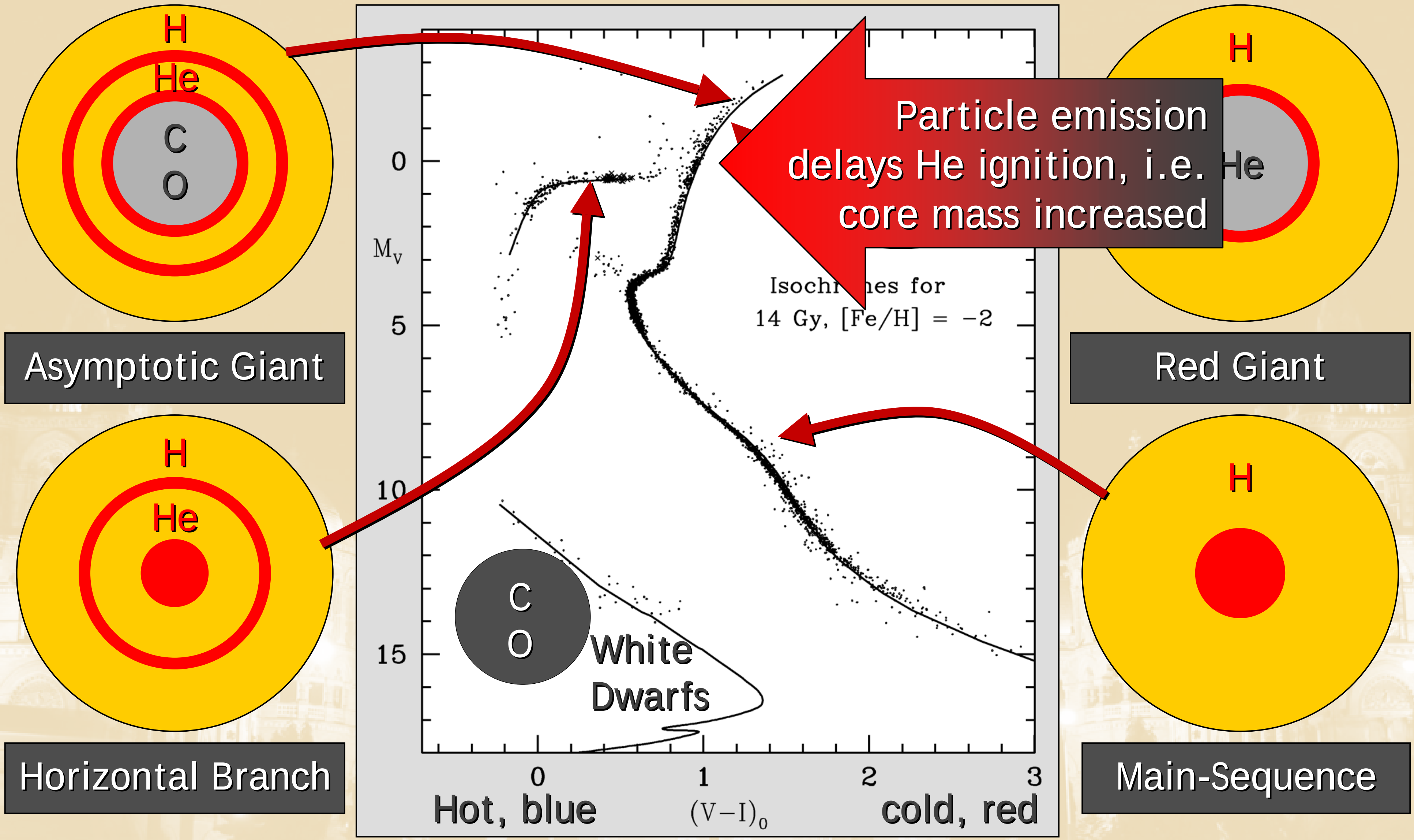
Decay rate of photon (transverse plasmon) with energy E_γ

$$\Gamma(\gamma \to \nu\bar{\nu}) = \frac{4\pi}{3}\frac{1}{E_\gamma} \times \begin{cases} \alpha_\nu \left(\omega_{pl}^2/4\pi\right) & \text{Millicharge} \\ \frac{\mu_\nu^2}{2}\left(\omega_{pl}^2/4\pi\right)^2 & \text{Dipole moment} \\ \frac{C_V^2 G_F^2}{\alpha}\left(\omega_{pl}^2/4\pi\right)^3 & \text{Standard model} \end{cases}$$

Energy-loss rate of stellar plasma (temperature T and plasma frequency ω_{pl})

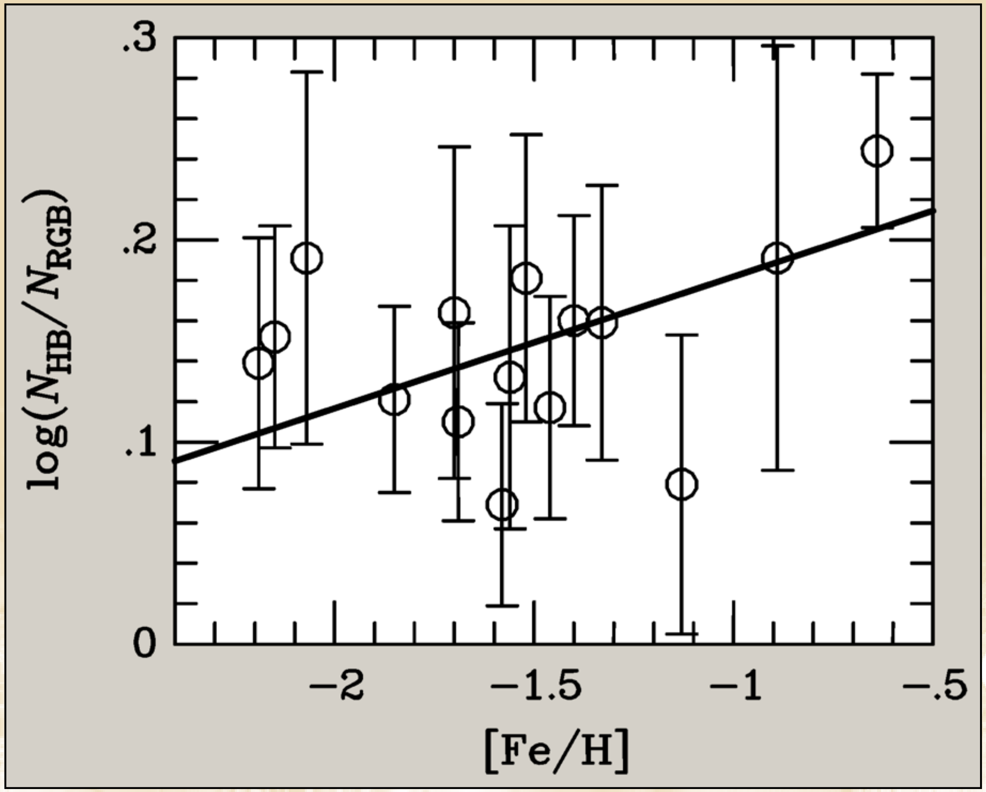
$$Q(\gamma \to \nu\bar{\nu}) = \int \frac{2d^3\vec{p}}{(2\pi)^3}\frac{E_\gamma \Gamma}{e^{E_\gamma/T}-1} = \frac{8\zeta_3}{3\pi}T^3 \times \begin{cases} \alpha_\nu \left(\omega_{pl}^2/4\pi\right) \\ \frac{\mu_\nu^2}{2}\left(\omega_{pl}^2/4\pi\right)^2 \\ \frac{C_V^2 G_F^2}{\alpha}\left(\omega_{pl}^2/4\pi\right)^3 \end{cases}$$

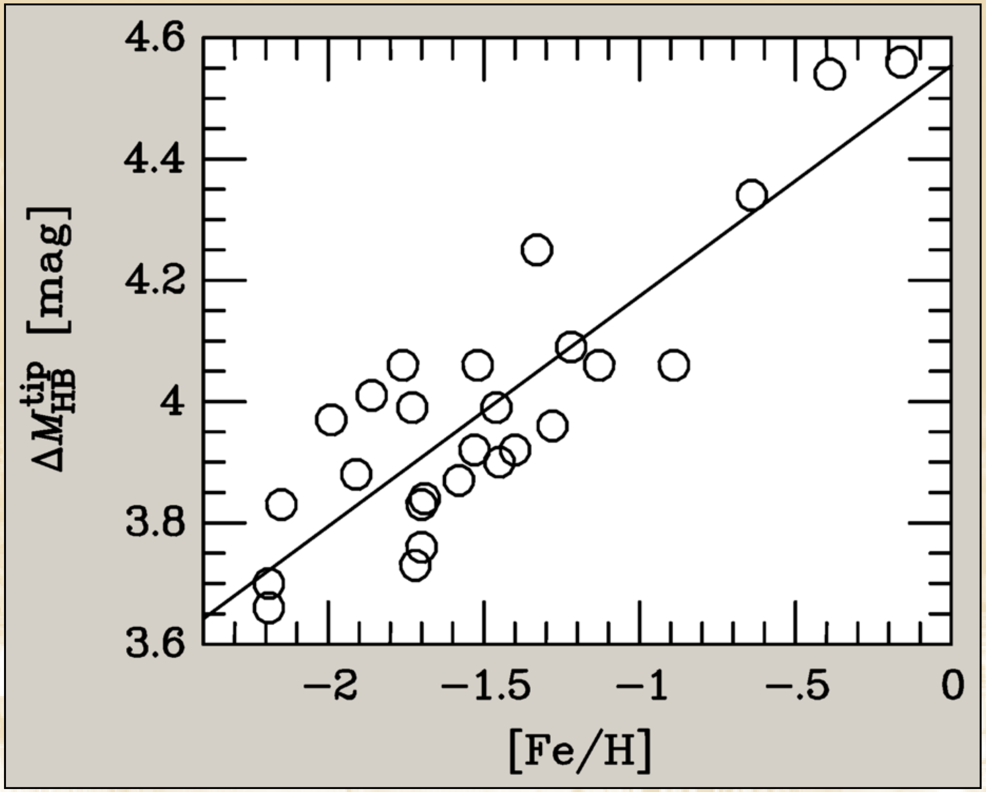
Color-Magnitude Diagram for Globular Clusters



Color-magnitude diagram synthesized from several low-metallicity globular clusters and compared with theoretical isochrones (W.Harris, 2000)

Measurements of Globular Cluster Observables

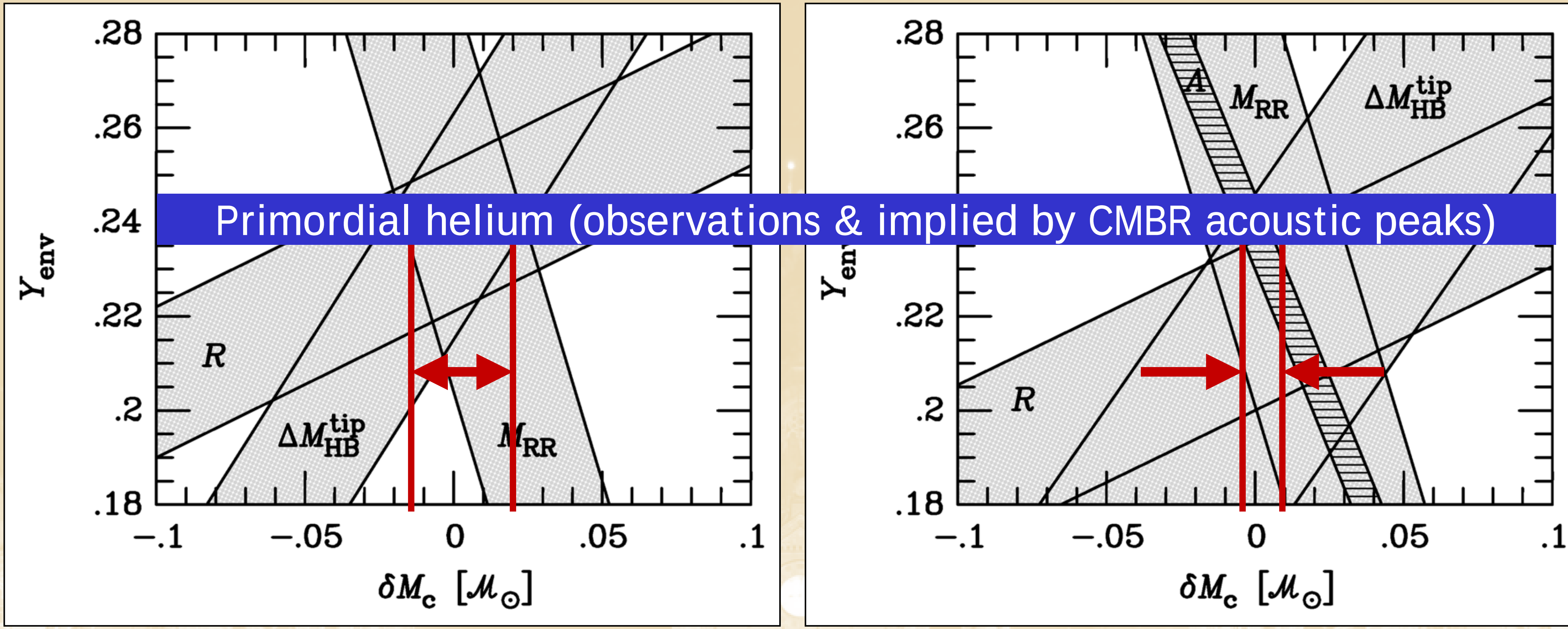




Number ratio of HB vs. RGB stars in 15 globular clusters

Brightness difference between HB (RR Lyrae stars) and brightest red giant in 26 globular clusters

Core-Mass at Helium Ignition



G.Raffelt, Stars as Laboratories for Fundamental Physics (1996)

Catalan et al., astro-ph/9509062

Core mass at helium ignition established to ± 0.02 M_{sun} or ± 4%

Globular Cluster Limits on Neutrino Dipole Moments

Compare magnetic-dipole plasma emission with standard case

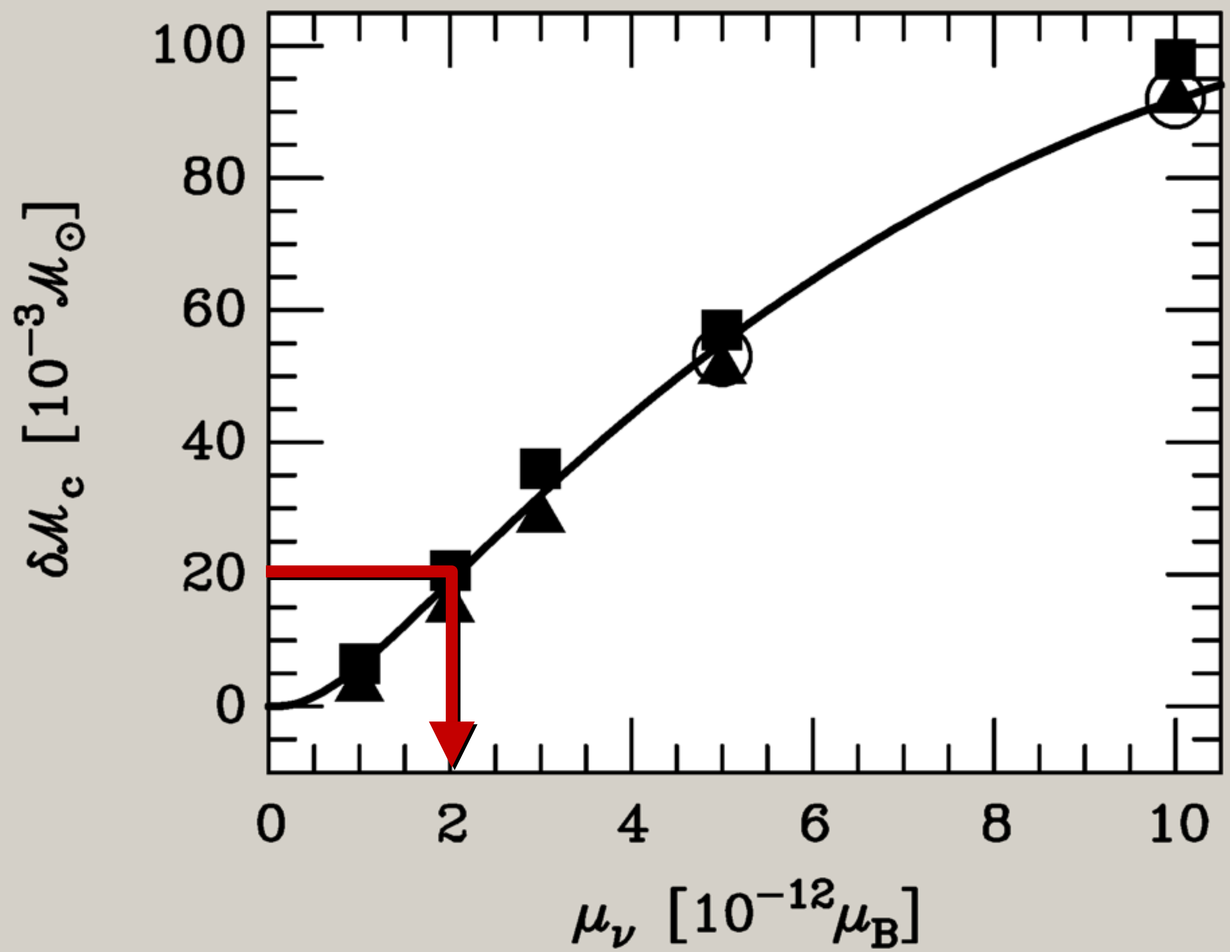
$$\frac{Q_\mu}{Q_{SM}} = \frac{2\pi\alpha\mu_\nu^2}{C_V^2 G_F^2 \omega_{pl}^2}$$

For red-giant core before helium ignition $\omega_{pl} = 18$ keV

$$\frac{Q_\mu}{Q_{SM}} = 9\times10^{22}\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

Require this to be < 1

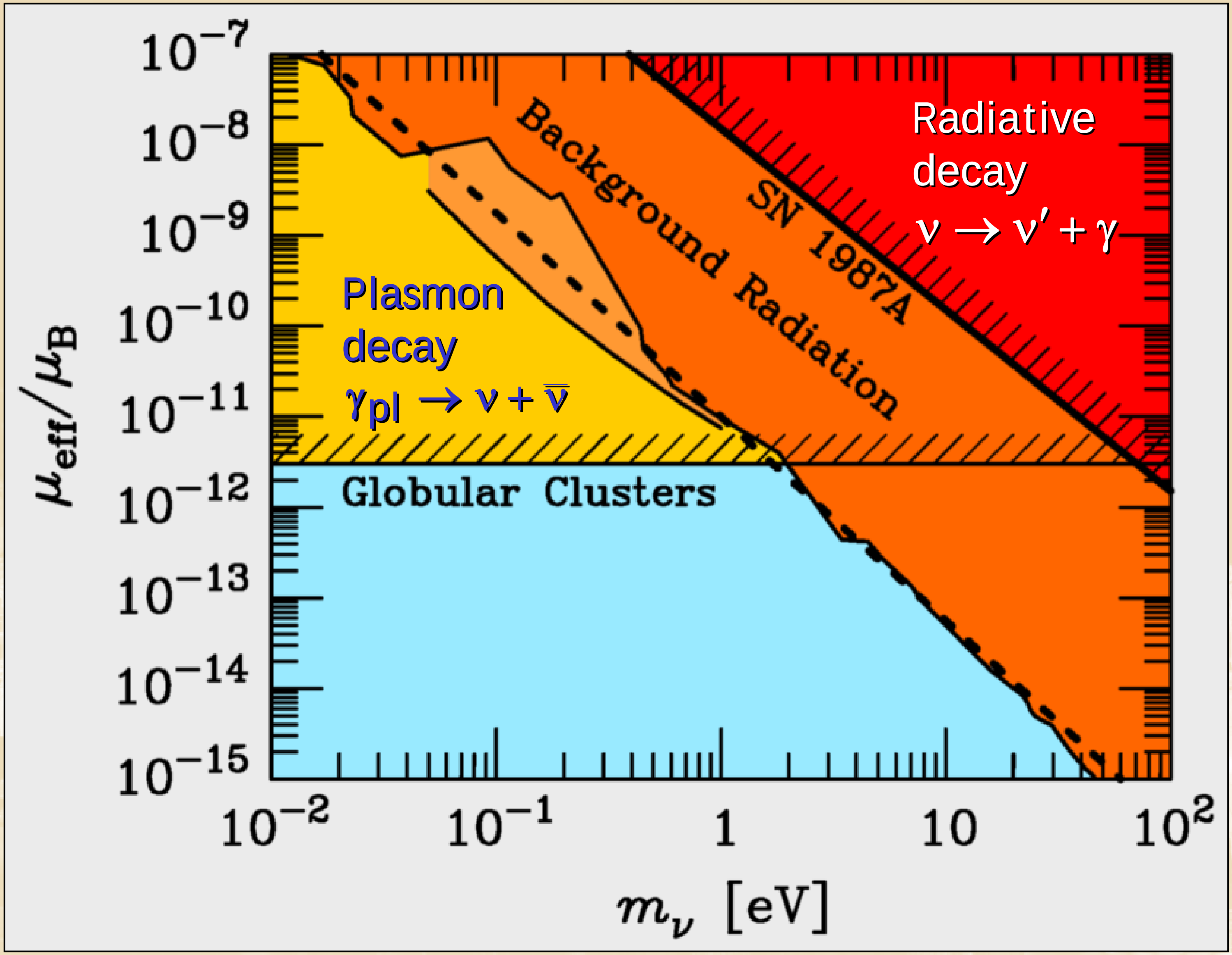
$$\mu_\nu < 3\times10^{-12}\mu_B$$



Globular-cluster limit on neutrino dipole moment

$$\mu_\nu < 2\times10^{-12}\mu_B$$

Neutrino Radiative Lifetime Limits



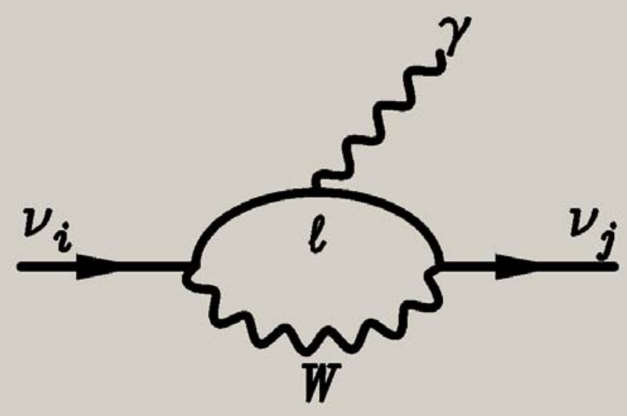
$$\Gamma_{\nu\to\nu'\gamma} = \frac{\mu_{eff}^2}{8\pi} m_\nu^3$$

$$\Gamma_{\gamma\to\nu\overline{\nu}} = \frac{\mu_{eff}^2}{24\pi} \omega_{pl}^3$$

For low-mass neutrinos, plasmon decay in globular cluster stars yields most restrictive limits

Standard Dipole Moments for Massive Neutrinos

In standard electroweak model,
neutrino dipole and
transition moments
are induced at higher order



Massive neutrinos ν_i (i = 1,2,3),
mixed to form weak eigenstates

$$\nu_\ell = \sum_{i=1}^{3} U_{\ell i} \nu_i$$

Explicit evaluation for Dirac
neutrinos
(Magnetic moments μ_{ij}
electric moments ε_{ij})

$$\mu_{ij} = \frac{e\sqrt{2}G_F}{(4\pi)^2}(m_i + m_j) \sum_{\ell=e,\mu,\tau} U_{\ell j} U^*_{\ell i}\, f\left(\frac{m_\ell}{m_W}\right)$$

$$\varepsilon_{ij} = \ldots (m_i - m_j) \ldots$$

$$f\left(\frac{m_\ell}{m_W}\right) = -\frac{3}{2} + \frac{3}{4}\left(\frac{m_\ell}{m_W}\right)^2 + O\left(\left(\frac{m_\ell}{m_W}\right)^4\right)$$

Standard Dipole Moments for Massive Neutrinos

Diagonal case
(Magnetic moments of Dirac neutrinos)

$$\mu_{ii} = \frac{3e\sqrt{2}G_F}{(4\pi)^2} m_i = 3.20\times 10^{-19}\mu_B \frac{m_i}{\mathrm{eV}} \qquad \mu_B = \frac{e}{2m_e}$$

$$\varepsilon_{ii} = 0$$

Off-diagonal case
(Transition moments)

First term in $f(m_\lambda / m_W)$ does not contribute ("GIM cancellation")

$$\mu_{ij} = \frac{3e\sqrt{2}G_F}{4(4\pi)^2}(m_i + m_j)\left(\frac{m_\tau}{m_W}\right)^2 \sum_{\ell=e,\mu,\tau} U_{\ell j}U^*_{\ell i}\left(\frac{m_\ell}{m_\tau}\right)^2$$

$$= 3.96\times 10^{-23}\mu_B \frac{m_i+m_j}{\mathrm{eV}} \sum_{\ell=e,\mu,\tau} U_{\ell j}U^*_{\ell i}\left(\frac{m_\ell}{m_\tau}\right)^2$$

$$\varepsilon_{ij} = \ldots(m_i - m_j)\ldots$$

Largest neutrino mass eigenstate   0.05 eV < m < 0.7 eV
For Dirac neutrino expect

$$1.6\times 10^{-20}\mu_B < \mu_\nu < 2.2\times 10^{-19}\mu_B$$

Limits on Milli-Charged Particles

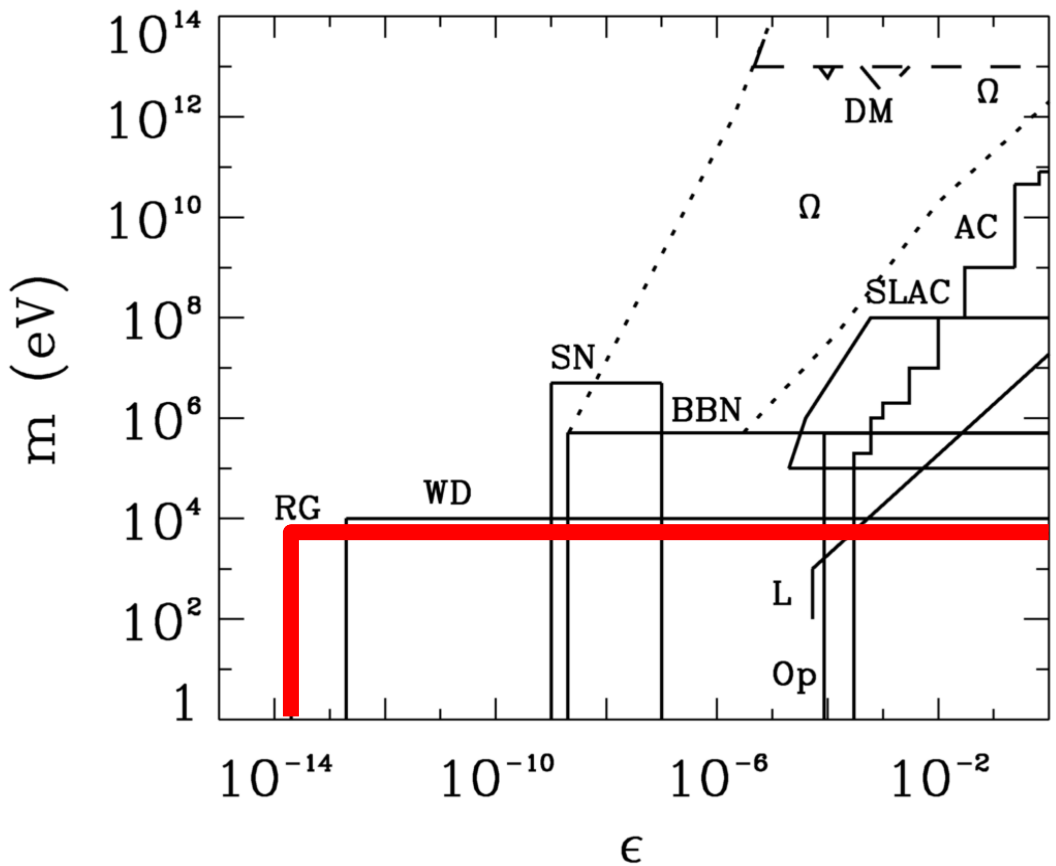


Figure 1: Regions of mass-charge space ruled out for milli-charged particles. The solid and dashed lines apply to the model with a paraphoton; solid and dotted lines apply in the absence of a paraphoton. The bounds arise from the following constraints: AC — accelerator experiments; Op — the Tokyo search for the invisible decay of ortho-positronium [27]; SLAC — the SLAC milli-charged particle search [28]; L — the Lamb shift; BBN — nucleosynthesis; Ω — $\Omega < 1$; RG — plasmon decay in red giants; WD — plasmon decay in white dwarfs; DM — dark matter searches; SN — Supernova 1987A.

Davidson, Hannestad & Raffelt
JHEP 5 (2000) 3

Globular cluster limit most restrictive for small masses

PHYSICAL REVIEW VOLUME 132, NUMBER 3 1 NOVEMBER 1963

Electromagnetic Properties of the Neutrino

JEREMY BERNSTEIN* AND MALVIN RUDERMAN†
Department of Physics, New York University, New York, New York

AND

GERALD FEINBERG‡
Department of Physics, Columbia University, New York, New York
(Received 11 June 1963)

In this note we make a detailed survey of the experimental information on the neutrino charge, charge radius, and magnetic moment. Both weak-interaction data and astrophysical results can be used to give precise limits to these quantities, independent of the supposition that the weak interactions are charge conserving.

I. INTRODUCTION

MOST physicists now accept the prospect that there are two neutrinos—ν_e and ν_μ—identical except for interaction (ν_e couples weakly with electrons and ν_μ with muons) and that these neutrinos have the simplest properties compatible with existing experimental evidence; i.e., zero mass, charge, electric, and magnetic dipole moments. However, the weak interactions have produced so many surprises that it is worthwhile, from time to time, to study the *experimental* limits that have been set on these quantities. In this note we present a systematic survey of the properties of the two neutrinos that can be inferred from experiment.

II. PROPERTIES

We begin by listing the properties of the neutrinos to

tritium experiments give

$$m_{\nu_e} < 200 \text{ eV}, \tag{2}$$

and the experiments are consistent with $m_{\nu_e} = 0$.

(2) ν_μ: The mass of the muon neutrino is the least well known of the parameters associated with either neutrino. The best measurements of it come from the energy-momentum balance in π decay. The experiment of Barkas *et al.*[3] gives[4]

$$m_{\nu_\mu} < 3.5 \text{ MeV}. \tag{3}$$

The reason for this uncertainty lies in the kinematic fact that the small neutrino mass is given as the difference between measured quantities of order 1. In the $\pi \rightarrow \mu + \nu$ decay, the accuracy with which the neutrino mass can be determined is given by

Further Reading on Particle Limits from Stars

Georg Raffelt:

Stars as Laboratories for Fundamental Physics
(University of Chicago Press, 1996)

Particle Physics from Stars
Annu. Rev. Nucl. Part. Sci. 49 (1999) 163-216
[hep-ph/9903472]

Astrophysical Methods to Constrain Axions
and Other Novel Particle Phenomena
Phys. Rept. 198 (1990) 1-113