

The Jigsaw of Neutrino Mass and Mixing Current Status

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Parameters of Neutrino Mass Matrix

- 9 unknown parameters in the light 3-neutrino mass matrix
 - 3 masses, m_1 , m_2 and m_3
 - 3 mixing angles, 1 Dirac CP phase, 2 Majorana CP phases

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Oscillation experiments sensitive to
 - 2 mass squared differences (Δm^2_{21} , $\Delta m^2_{31} \approx \Delta m^2_{32}$)
 - 3 mixing angles (θ_{12} , θ_{13} , θ_{23})
 - 1 Dirac CP phase
- Majorana CP phases observable only in $\Delta L = 2$ processes
- Absolute neutrino mass scale comes from
 - Tritium beta decay
 - Neutrino-less double beta decay
 - Cosmology

ν Oscillation: experimental evidences ...

- Atmospheric Neutrino data from SuperKamiokande
- Solar Neutrino data from Homestake, SAGE, Gallex, GNO, Kamiokande, SuperKamiokande, SNO (Phase-I, Phase-II)
- Data from long baseline accelerator based experiment K2K
- Long baseline reactor experiment KamLAND
- Data from long baseline accelerator based experiment MINOS
- Accelerator based oscillation experiment LSND
 - not confirmed by Karmen
 - Miniboone will provide independent check

Plan of Talk

- Determination of Δm_{21}^2 and θ_{12}
Global Solar SNO,SK,GNO,SAGE,Homestake and KamLAND data
- Determination of Δm_{32}^2 and θ_{23}
SK atmospheric Neutrino data, K2K, first results from MINOS
- Determination of θ_{13}
Bounded by Global data
- Absolute Neutrino Masses
Constrained by Tritium Beta decay, Neutrino less double beta decay, Cosmology
- Three or more Neutrinos
LSND, MiniBoone
- Status of Mass Models

Oscillation Analysis – Ingredients

Experimental Data

- statistical error
- systematic errors and their correlations

Theoretical Predictions

- the fluxes and their uncertainties ← input from Astro Physics
- the interaction cross-sections and their uncertainties ← input from Nuclear Physics
- the oscillation probabilities (depends on the density profile of the propagating medium, Δm^2 , θ , E_ν ) ← Input from Particle Physics

rate = flux \times cross – section \times probability

Minimisation of χ_{global}^2

Best-fit values of parameters Δm^2 , $\sin^2 \theta$...

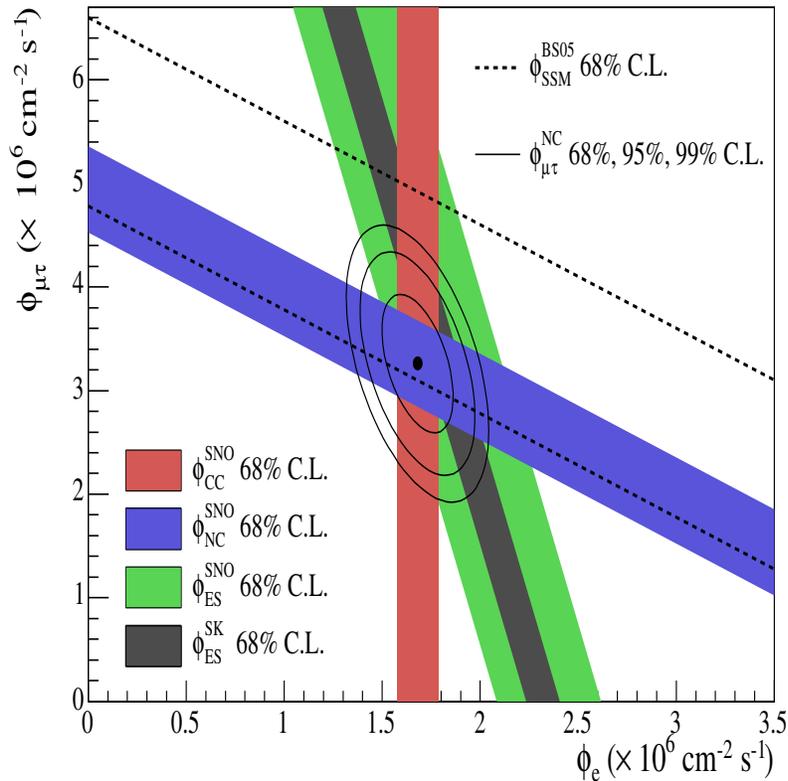
Solar Neutrinos: Experimental Results

- The ratio of the observed solar neutrino rates to the SSM predictions.

Homestake	$^{37}\text{Cl} + \nu_e \rightarrow ^{37}\text{Ar} + e^-$	0.337 ± 0.029
Gallex/GNO/SAGE	$^{71}\text{Ga} + \nu_e \rightarrow ^{71}\text{Ge} + e^-$	0.553 ± 0.034
SK	$e^- + \nu_x \rightarrow e^- + \nu_x$	0.465 ± 0.014
SNO CC	$d + \nu_e \rightarrow p + p + e^-$	0.309 ± 0.19
SNO ES	$e^- + \nu_x \rightarrow e^- + \nu_x$	0.429 ± 0.058
SNO NC	$d + \nu_x \rightarrow p + n + \nu_x$	1.012 ± 0.09

- Observed ν_e flux < Theoretical Prediction \rightarrow solar Neutrino Problem
- $\frac{CC}{NC} = \frac{\nu_e}{\nu_e + \nu_\mu + \nu_\tau} < 1 \rightarrow$ indicative of flavour conversion
- $\frac{CC}{ES} = \frac{\nu_e}{\nu_e + 0.15(\nu_\mu + \nu_\tau)} < 1 \rightarrow$ indicative of flavour conversion
- "Neutrino Oscillation" \rightarrow Neutrino Mass and Mixing
- Standard Solar Model is correct

The Solar 8B flux measured in SNO and SK



SK:

• $\phi_{ES} = 2.35 \pm 0.09 [\times 10^6 / cm^2 / s]$



SNO

• $\phi_{CC} = 1.68 \pm 0.01 [\times 10^6 / cm^2 / s]$



SNO

• $\phi_{NC} = 4.94 \pm 0.43 [\times 10^6 / cm^2 / s]$



SSM (BP04)

• $\phi_{SSM} = 5.82 \pm 1.33 [\times 10^6 / cm^2 / s]$

SNO-II 391d, nucl-ex/0502021

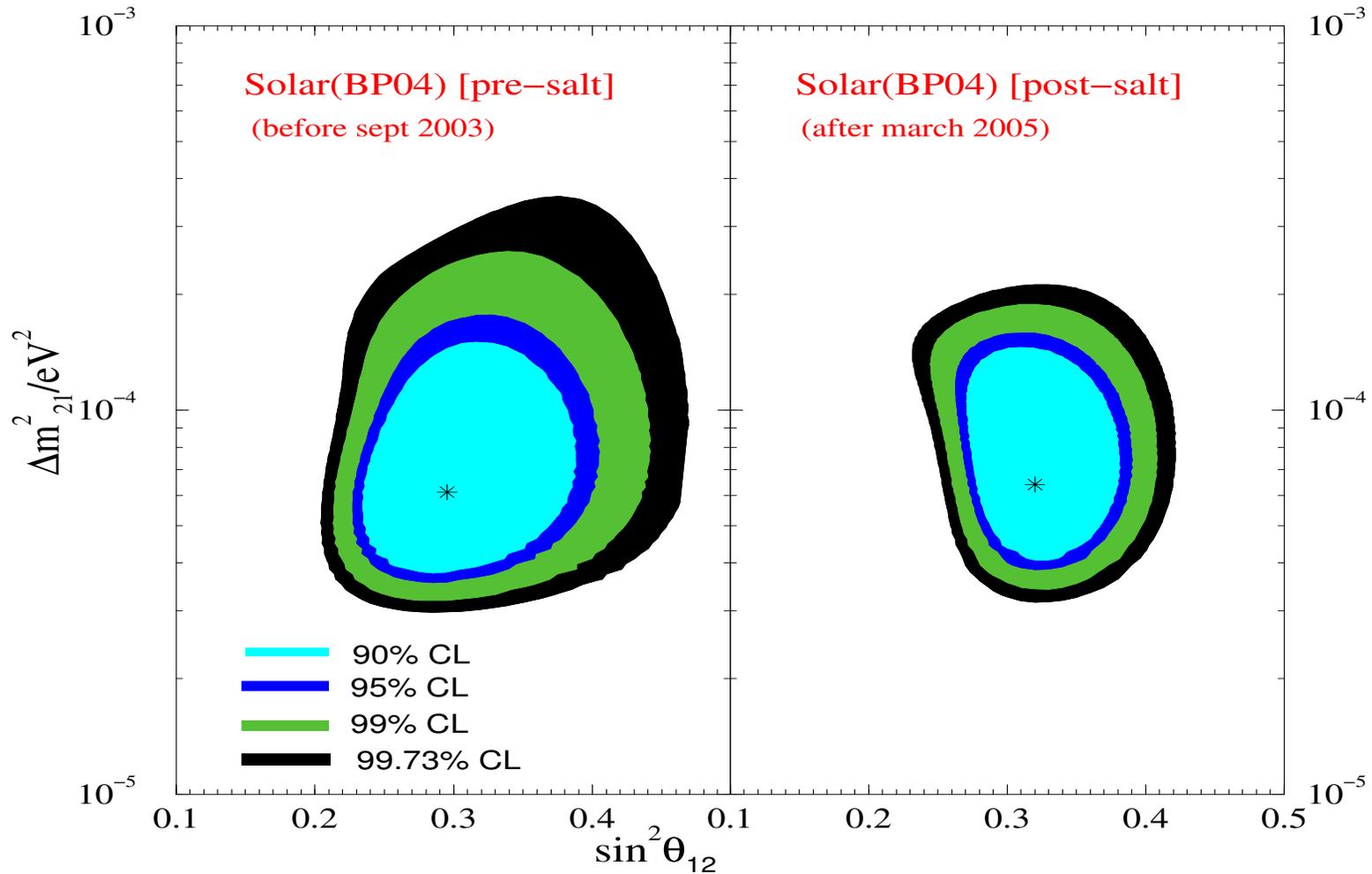
$$\frac{CC}{NC} = 0.340 \pm 0.023 \pm 0.030$$

7 σ evidence for a non-zero $\nu_{\mu,\tau}$ flux from
SUN

Solar Neutrinos: Status of Oscillation Parameters

■ $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2$, $\theta_{\odot} \equiv \theta_{12}$ from Global Solar Data

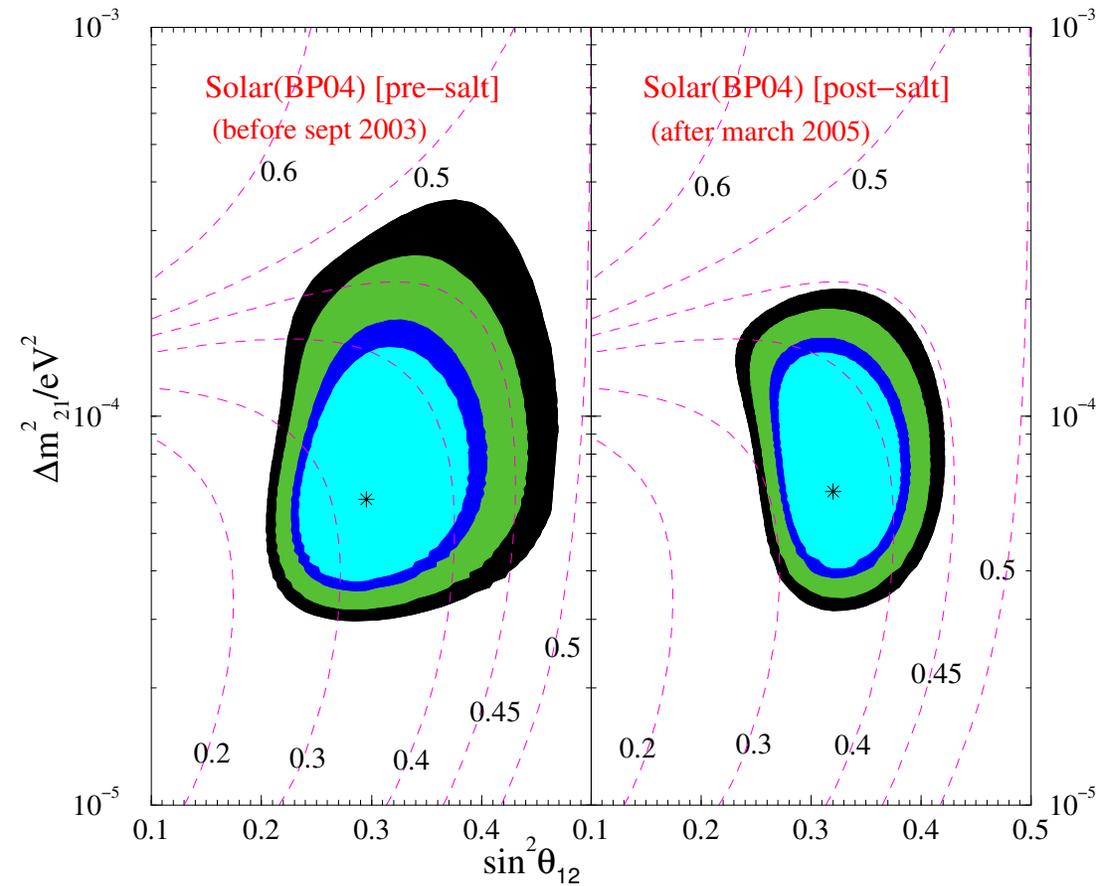
■ Including the Spectral data from SNO and SK



Solar Neutrinos: Status of Oscillation Parameters

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Solar Neutrinos: Status of Oscillation Parameters

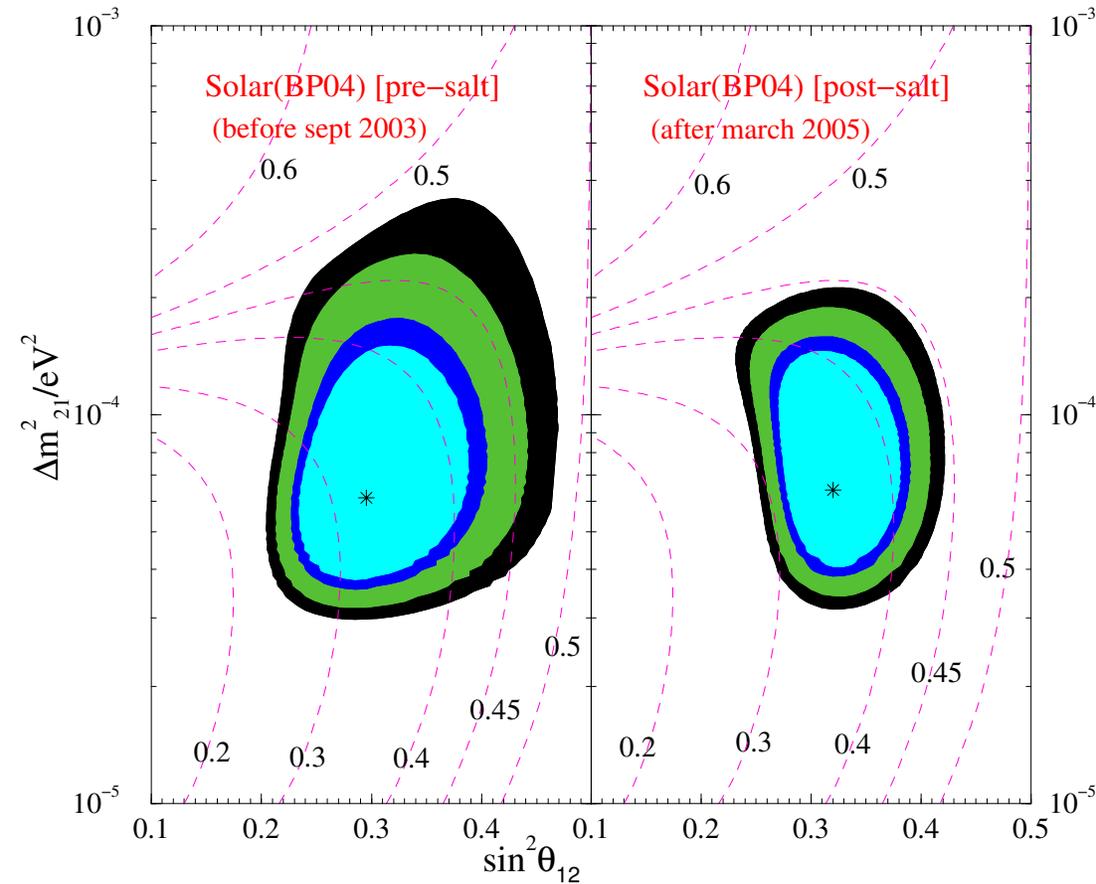
 $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2$, $\theta_{\odot} \equiv \theta_{12}$ from **Global Solar Data**

 Including the Spectral data from SNO and SK

● **Best fit**

$$\Delta m_{21}^2 = 6.4 \times 10^{-5} \text{eV}^2$$

$$\sin^2 \theta_{12} = 0.33 \quad f_B = 0.84$$



Solar Neutrinos: Status of Oscillation Parameters

● $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2$, $\theta_{\odot} \equiv \theta_{12}$ from **Global Solar Data**

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● **Best fit**

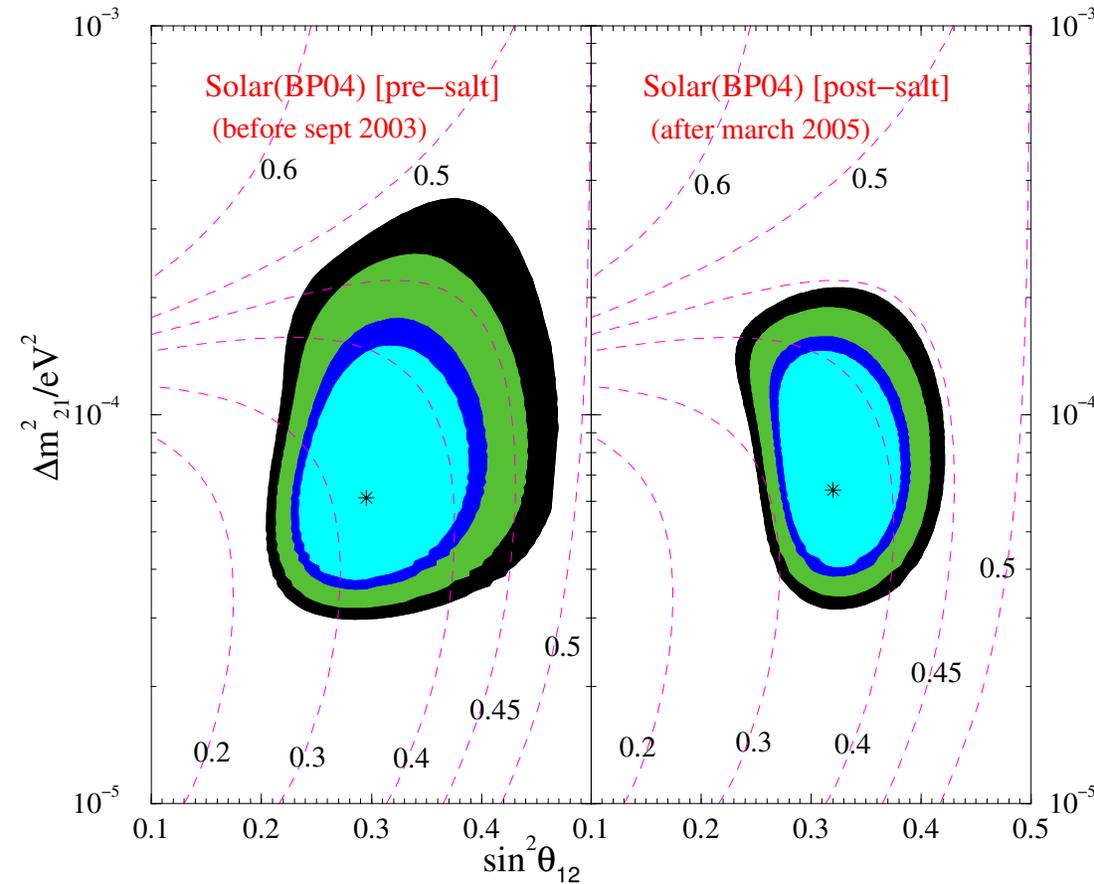
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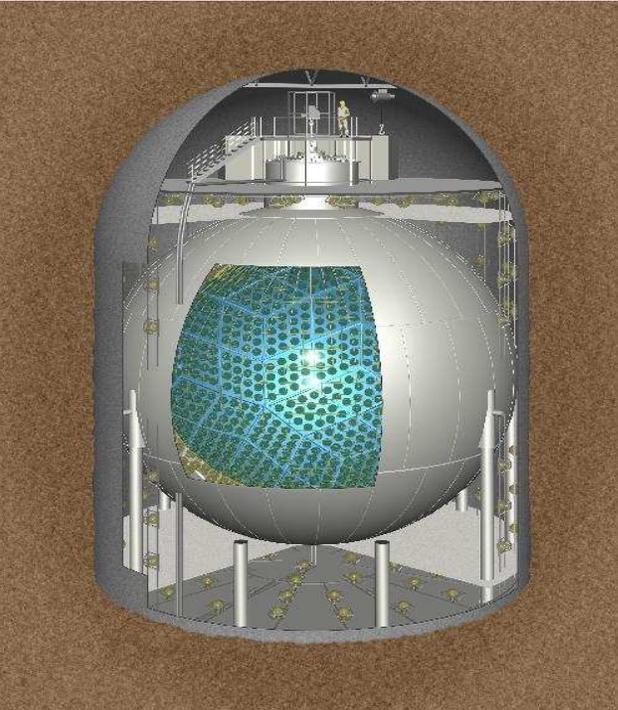
$$\sin^2 \theta_{12} = 0.33 \quad f_B = 0.84$$

● **3 σ range** ($\Delta\chi^2 = 11.83$)

$$\Delta m_{21}^2 = (3.0 - 17) \times 10^{-5} \text{eV}^2$$

$$\sin^2 \theta_{12} = 0.21 - 0.39$$



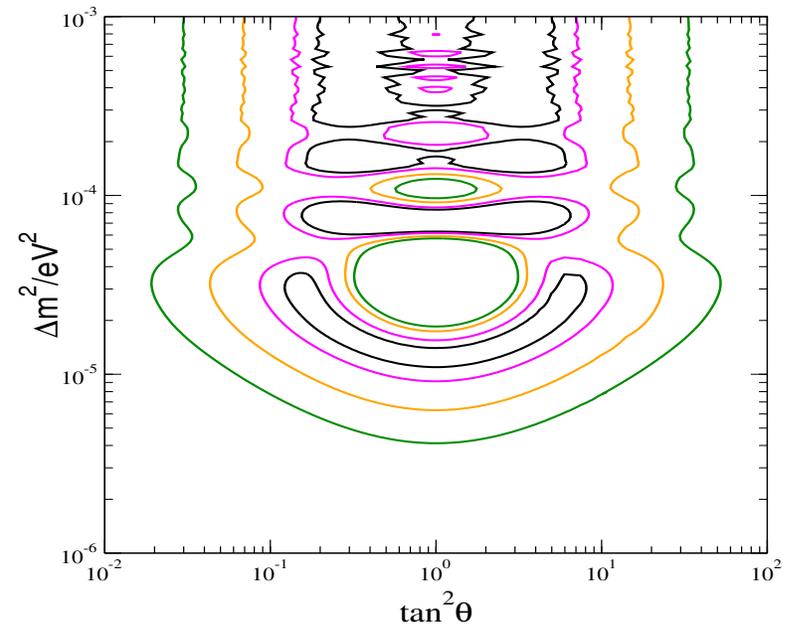
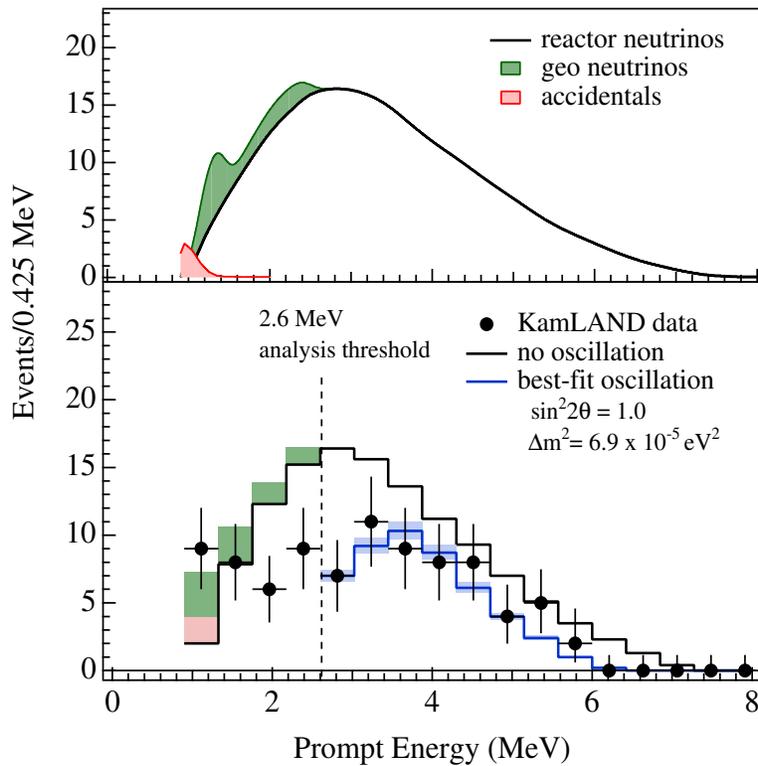


- 1 kton liquid scintillator neutrino detector at Kamioka
- $\bar{\nu}_e$ source: 16 main reactors at distances 81-824 km
- most powerful reactors are at 160 km
- detects reactor antineutrinos through: $\bar{\nu}_e + p \rightarrow n + e^+$
- $E_\nu \sim 3$ MeV, $L \sim 1.8 \times 10^5$ m ,
 $\Delta m^2 \sim 1.6 \times 10^{-5}$ eV²

Impact of KamLAND on Oscillation Parameters

First KamLAND results

 Evidence of $\bar{\nu}_e$ disappearance



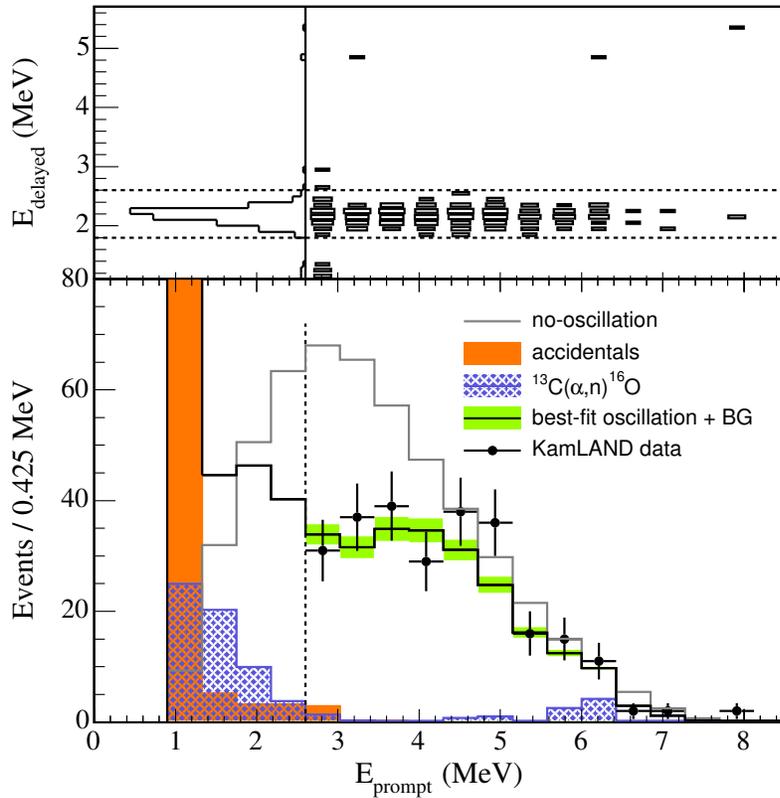
KamLAND Collaboration, PRL 2003

Bandyopadhyay et al, PLB, 2003

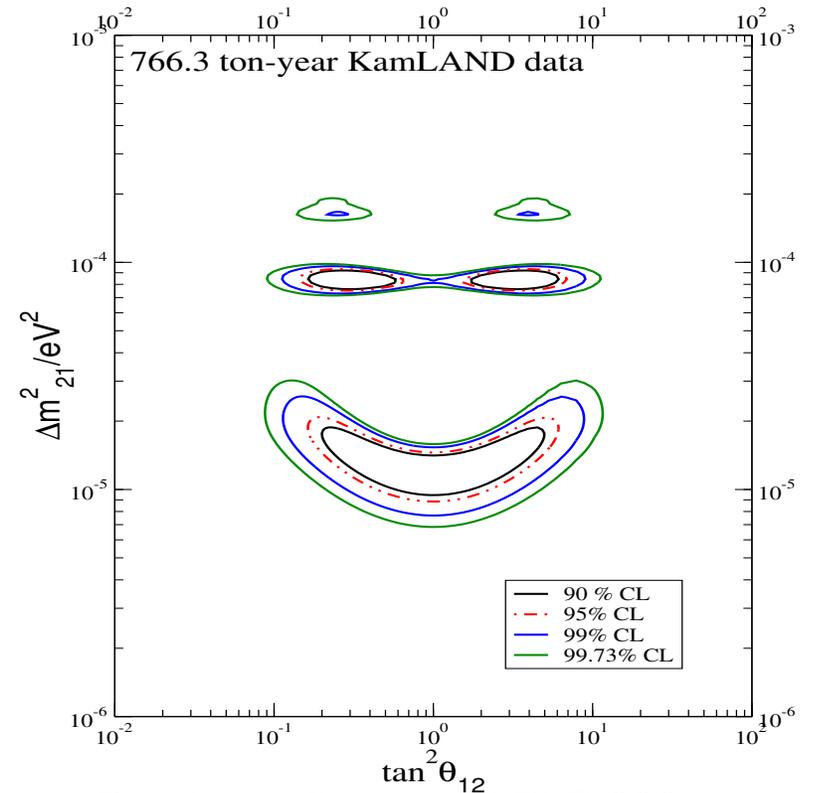
Impact of KamLAND on Oscillation Parameters

Second KamLAND results

 Evidence for **spectral distortion** at 99.6% significance



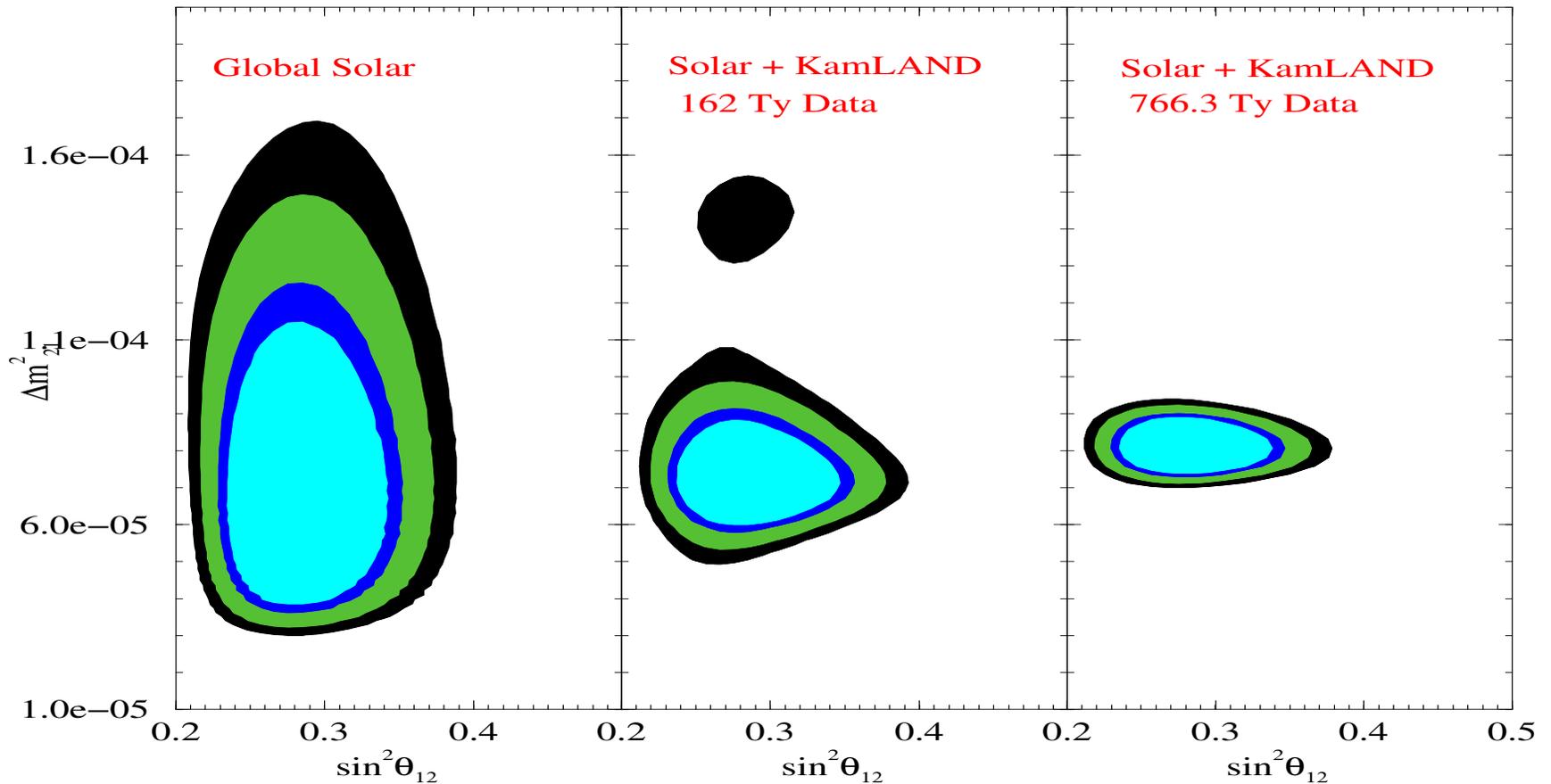
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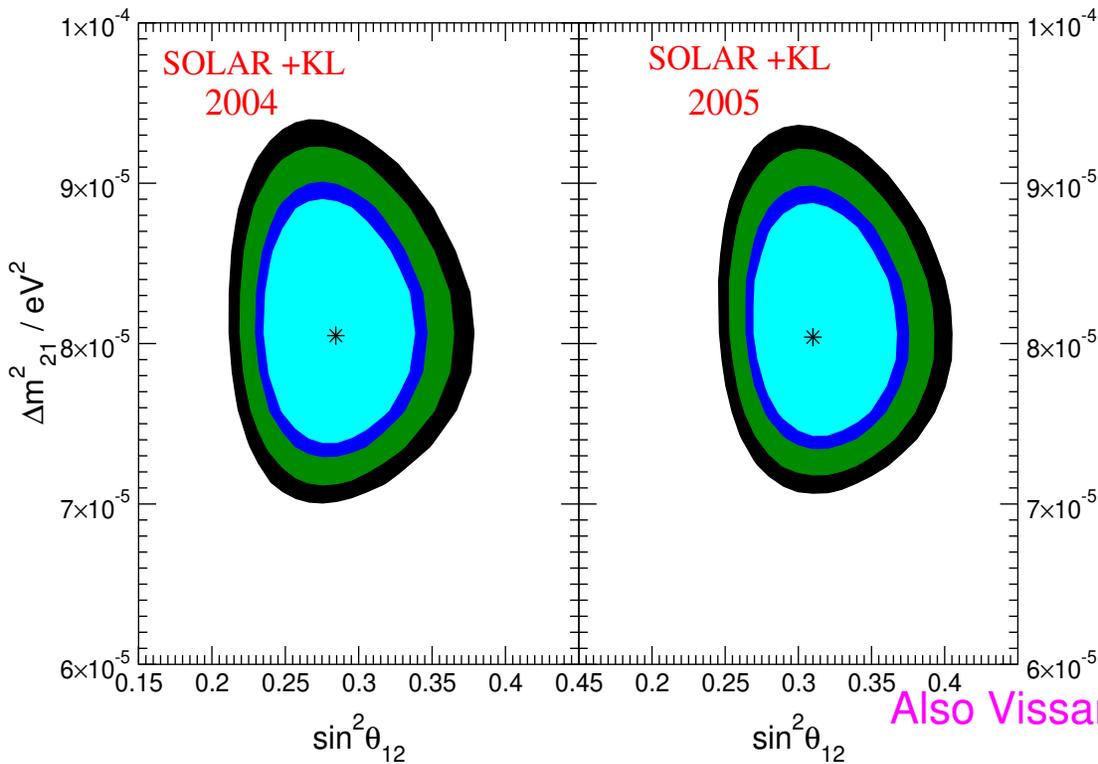
Impact of KamLAND on Oscillation Parameters

- Survival Probability : $P(\bar{\nu}_e \leftrightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{1.27 \Delta m^2 L}{E_\nu} \right)$
- Sensitive to **LMA** region (assuming CPT conservation)



- Solar data disallows $\theta > \pi/4$ (Dark-Side) solutions

Status of Solar Neutrino Oscillation Parameters



Best-fit (Solar+KamLAND)

$$\Delta m_{21}^2 = 8.0 \times 10^{-5} \text{ eV}^2$$

$$\sin^2 \theta_{12} = 0.31$$

3σ range (Solar+KamLAND)

$$\Delta m_{21}^2 = (7.0-9.3) \times 10^{-5} \text{ eV}^2$$

$$\sin^2 \theta_{12} = 0.24 - 0.41$$

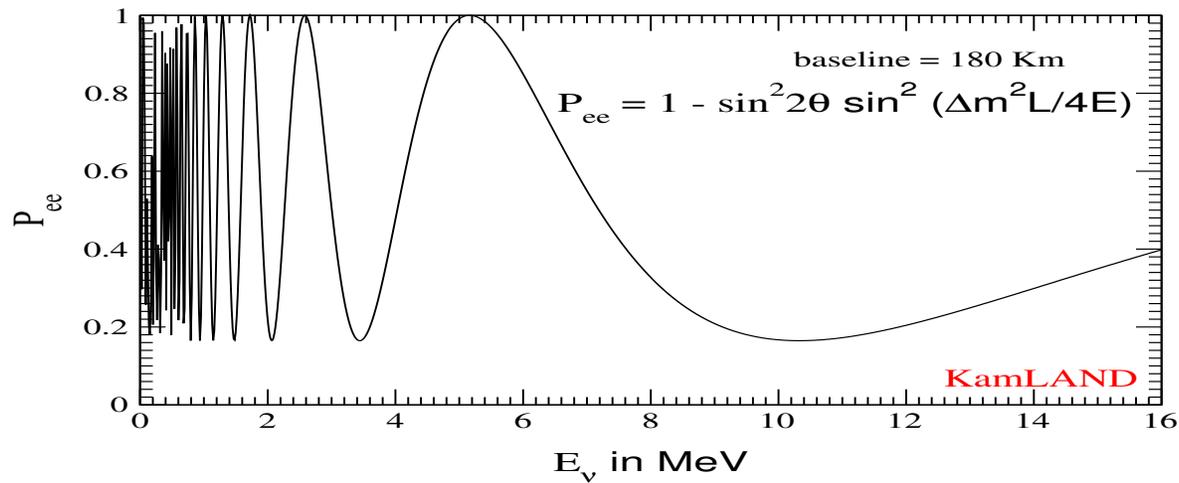
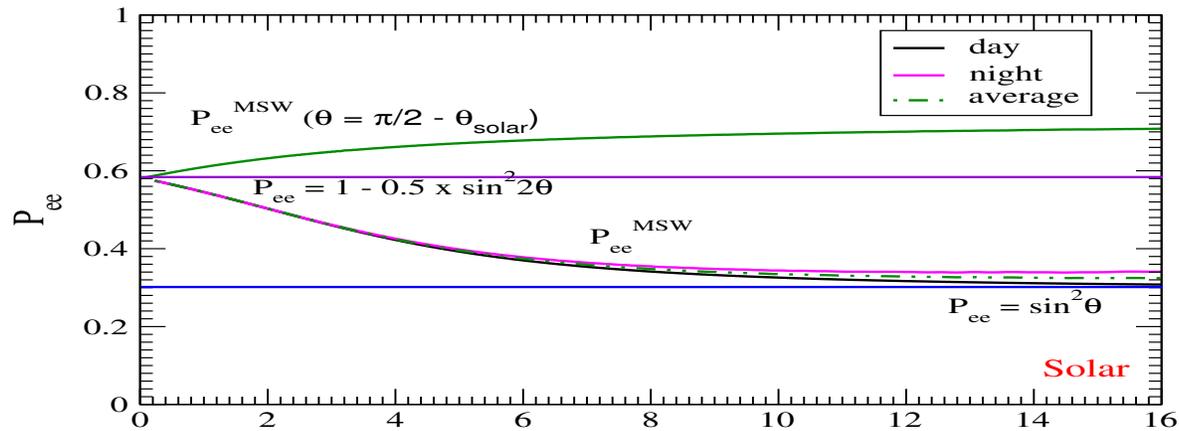
Bandyopadhyay et al, 2005

Also Vissani and Strumia, Fogli et. al, Maltoni et al.

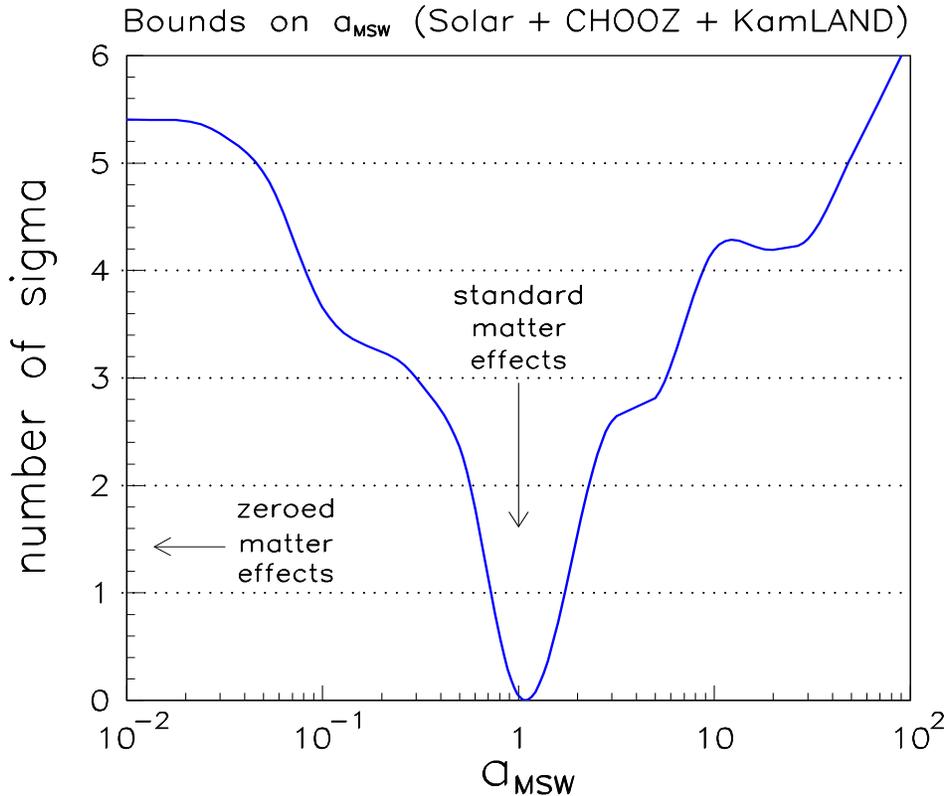


Maximal mixing ruled at almost 6σ

Survival Probabilities for solar and KamLAND



Evidence for MSW effect in Sun



$$V_{MSW} = \sqrt{2}G_F n_e$$

$$V_{MSW} \rightarrow \alpha_{MSW} \cdot V$$

■ "No MSW" rejected at several σ

Fogli et al. 2005

$$\tan 2\theta_{12}^m = \frac{\Delta m_{21}^2 \sin 2\theta_{12}}{\Delta m_{21}^2 \cos 2\theta_{12} - 2\sqrt{2}G_F n_e l}$$

■ For Resonance $\Delta m_{21}^2 \cos 2\theta_{12} > 0$

$$\text{■ } P_{ee}^{solar}({}^8B) = 0.5 (1 - \cos 2\theta_{12})$$

$$\text{■ Solar data} \Rightarrow P_{ee}^{solar}({}^8B) < 0.5$$

$$\text{■} \Rightarrow \cos 2\theta_{12} > 0 \text{ (Dark-Side gone)}$$

$$\text{■} \Rightarrow \Delta m_{21}^2 > 0$$

On the precision of ν_{\odot} oscillation parameters

$$\text{spread} = \frac{a_{max} - a_{min}}{a_{max} + a_{min}} \times 100 \%$$

Data set used	Range* of $\Delta m^2_{21} \times 10^{-5} \text{ eV}^2$	spread in Δm^2_{21}	Range* of $\sin^2 \theta_{12}$	spread in $\sin^2 \theta_{12}$
only sol	3.2 - 14.9	65%	0.22 - 0.37	25%
sol+162 Ty KL	5.2 - 9.8	31%	0.22 - 0.37	25%
sol+ 766.3 Ty KL	7.3 - 9.4	13%	0.22 - 0.36	24%
sol2005+766.3 Ty KL	7.2 - 9.2	12%	0.25 - 0.39	22%

* 99% C.L. ($\Delta\chi^2 = 9.21$, 2 parameters)

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 KamLAND has tremendous sensitivity to Δm^2_{21}

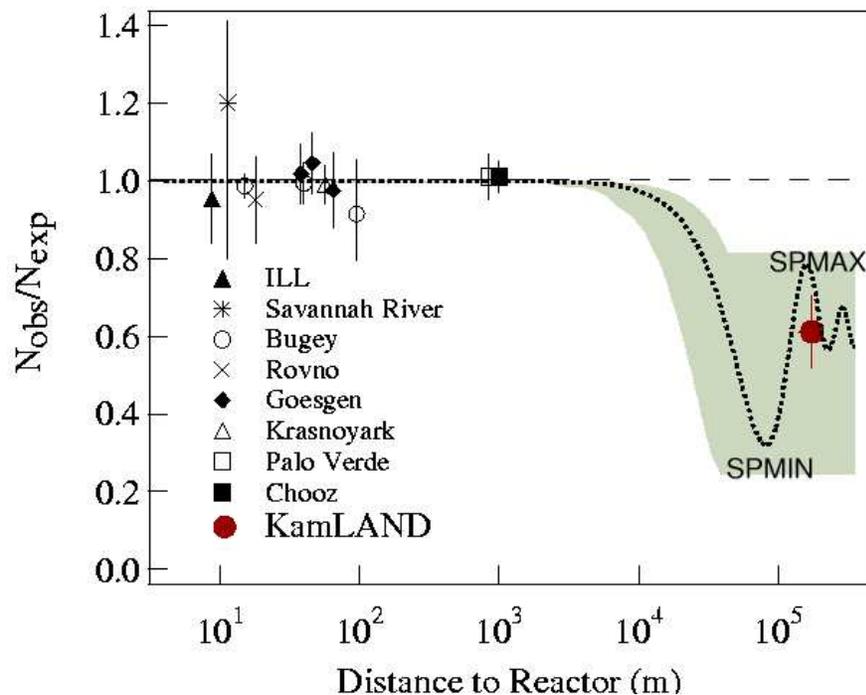
 Does not constrain θ_{12} much better than the current set of solar experiments

Sensitivity of KamLAND for θ_{12}


 $P_{ee}^{vac} = 1 - \sin^2 2\theta_{12} \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)$


 $\sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \rightarrow 1 \Rightarrow \text{SPMIN}, \quad \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \rightarrow 0 \Rightarrow \text{SPMAX}$


SPMIN best for $\sin^2 \theta_{12}$ for $\sin^2 \theta_{\odot} \lesssim 0.4$



Best-Fit: $\sin^2 \theta_{12} = 0.3$
 Range: $0.25 < \sin^2 \theta_{\odot} < 0.39$



-  KamLAND is at **SPMAX**
-  The θ_{12} sensitivity gets smothered
-  KamLAND is not at best position for θ_{12}
-  **SPMIN** at $L \sim 60 \text{ km}$ for present best-fit Δm_{21}^2

A. Bandyopadhyay, S.Choubey, S.G. PRD, 2003

(KamLAND Collaboration, hep-ex/0212021)

Atmospheric neutrinos ...

➤ Main goal: Study oscillation pattern in atmospheric neutrino events.

Cosmic Ray + A_{air} $\rightarrow \pi^+$ + ...

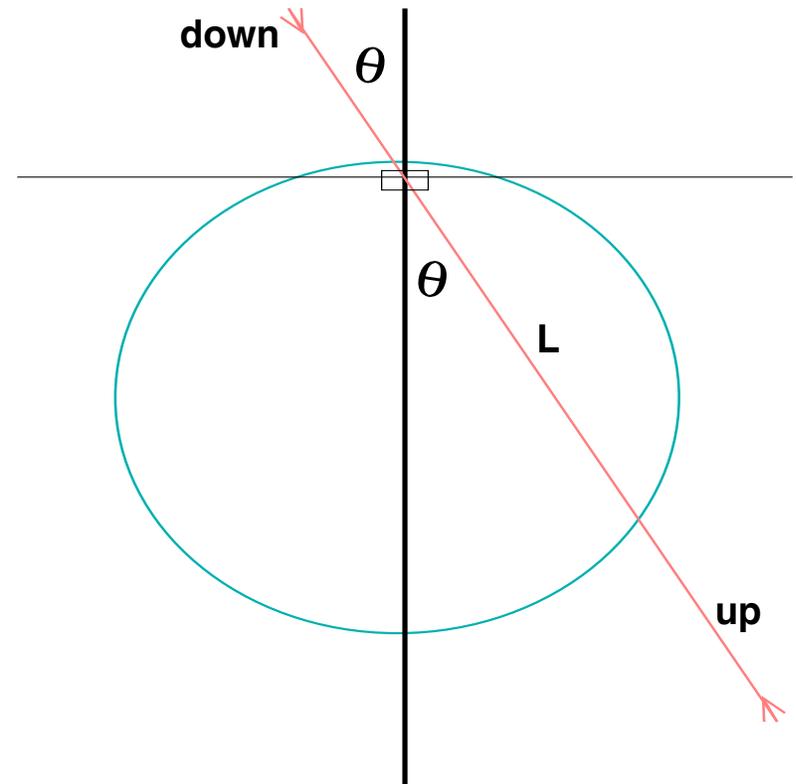
π^+ $\rightarrow \mu^+$ + ν_μ

μ^+ $\rightarrow e^+$ + $\bar{\nu}_\mu$ + ν_e

$\nu_\mu : \nu_e = 2:1$ (expected)

$\nu_\mu/\nu_e \sim 0.9 - 1$ (observed)

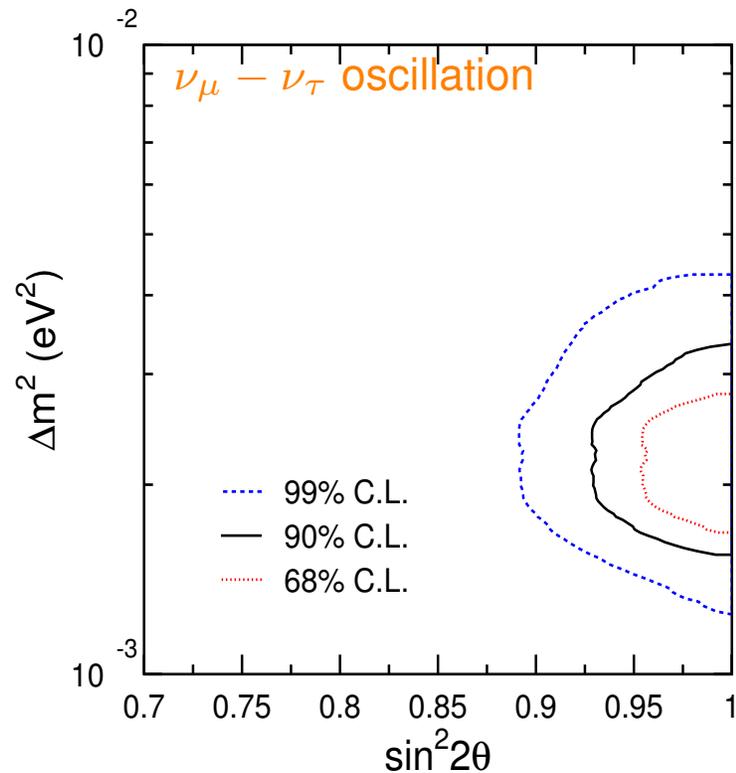
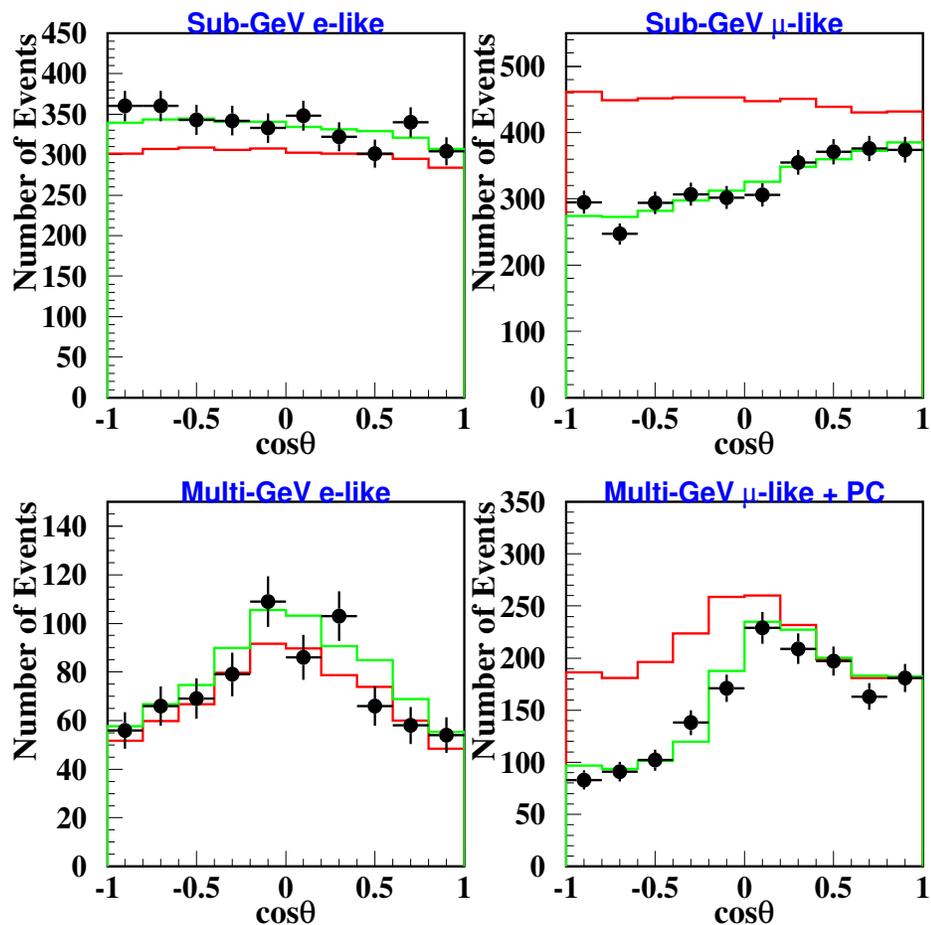
Up-Down asymmetry



Oscillation Parameters from Atmospheric Neutrinos



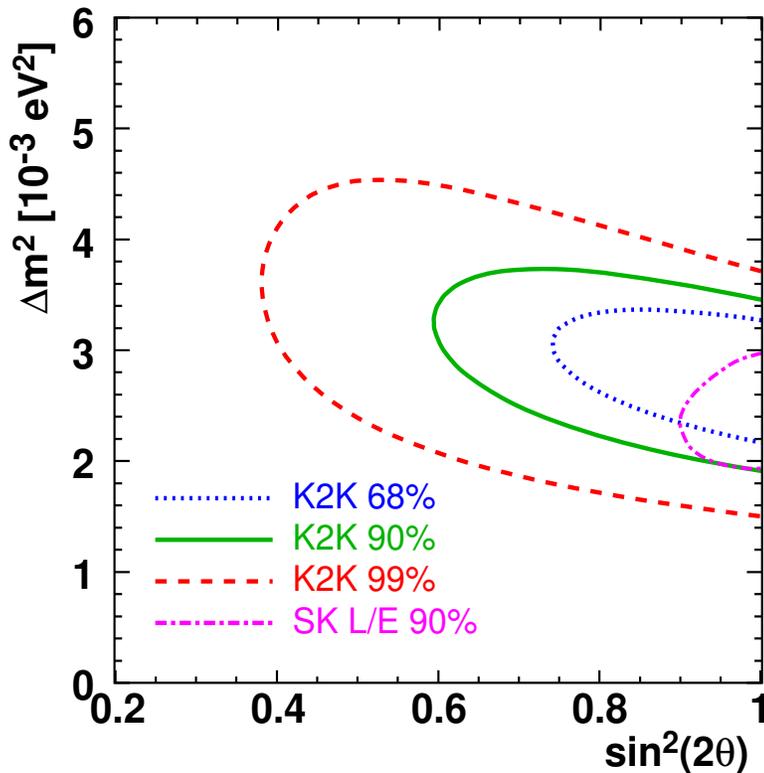
SK-I 1489 days zenith angle spectrum



$$\Delta m_{atm}^2 = 2.0 \times 10^{-3} \text{ eV}^2,$$

$$\sin^2 2\theta_{atm} = 1.0$$

K2K



● $L \sim 250 \text{ km}, E_\nu \sim 1.3 \text{ GeV}$

● $P_{\mu\mu} = 1 - \sin^2 2\theta_{atm} \sin^2 \left(\frac{\Delta m_{atm}^2 L}{4E} \right)$

● Best-fit

$\Delta m_{atm}^2 = 2.8 \times 10^{-3} \text{ eV}^2$
 $\sin^2 2\theta_{atm} = 1.0$

(Main Injector Neutrino Oscillation Search)



- NuMI beam produced at Fermilab
- 92.9% $\bar{\nu}_\mu$, 1.2% ν_e , 0.1% $\bar{\nu}_e$
- Mean Energy ~ 3 GeV
- Far detector at Soudan Mine (L = 735 km)
- 5.5 kT steel-scintillator tracking calorimeters with toroidal magnetic fields ~ 1.3 T
- Oscillation Maximum at ~ 1.5 GeV for $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$

First Results from MINOS

data taken from 20 May to 6 Dec 2005

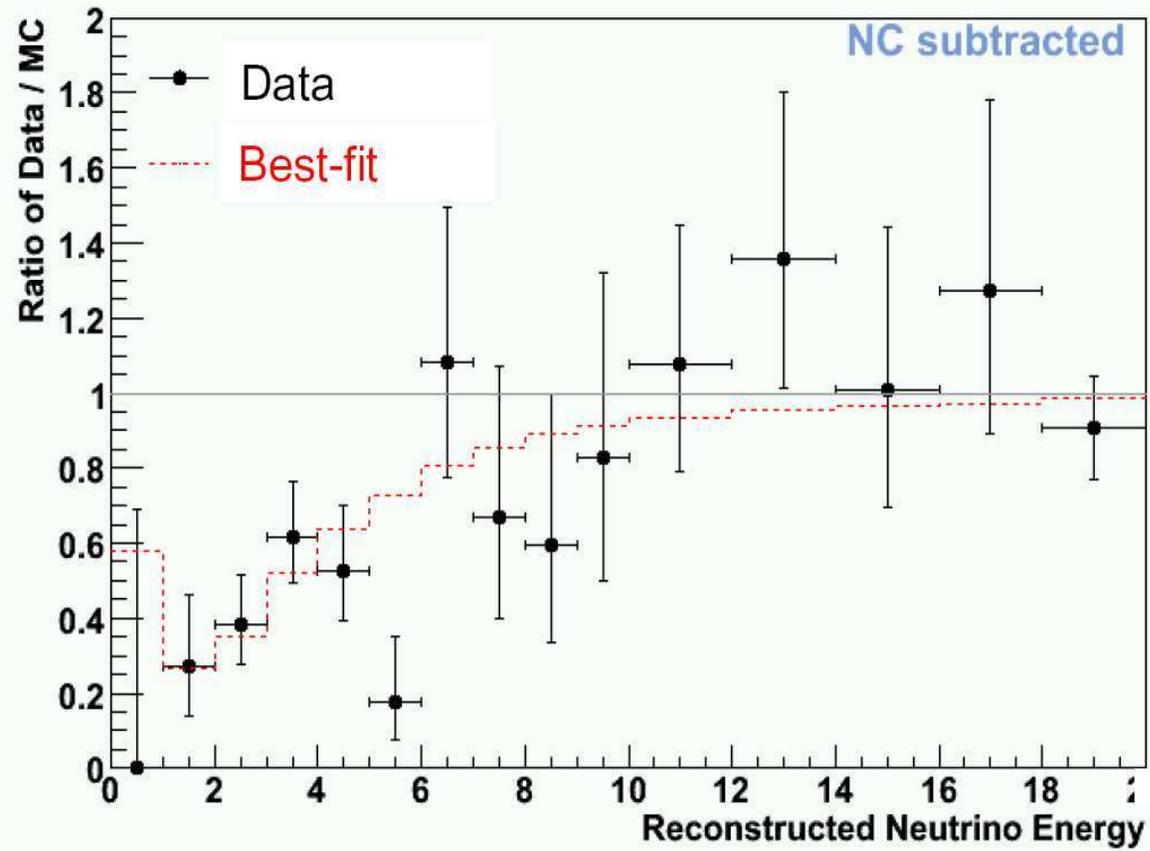
(0.93×10^{20} p.o.t.)

presented in a talk at Fermilab on 30 March 2006

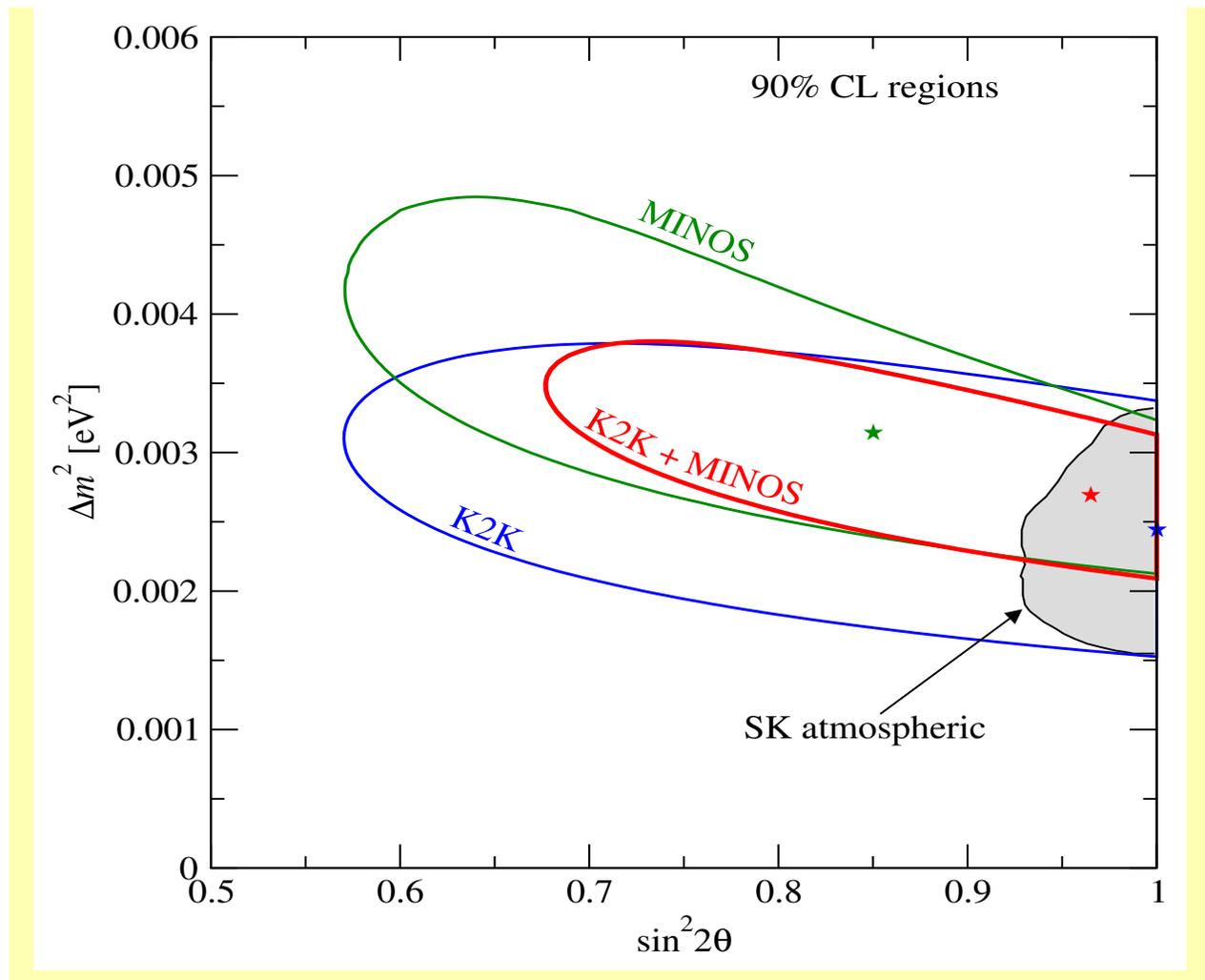
<http://www-numi.fnal.gov/talks/results06.html>

Data sample	observed	expected	ratio	significance
All CC-like events ($\nu_{\mu} + \bar{\nu}_{\mu}$)	204	298 ± 15	0.69	4.1σ
ν_{μ} only (<30 GeV)	166	249 ± 14	0.67	4.0σ
ν_{μ} only (<10 GeV)	92	177 ± 11	0.52	5.0σ

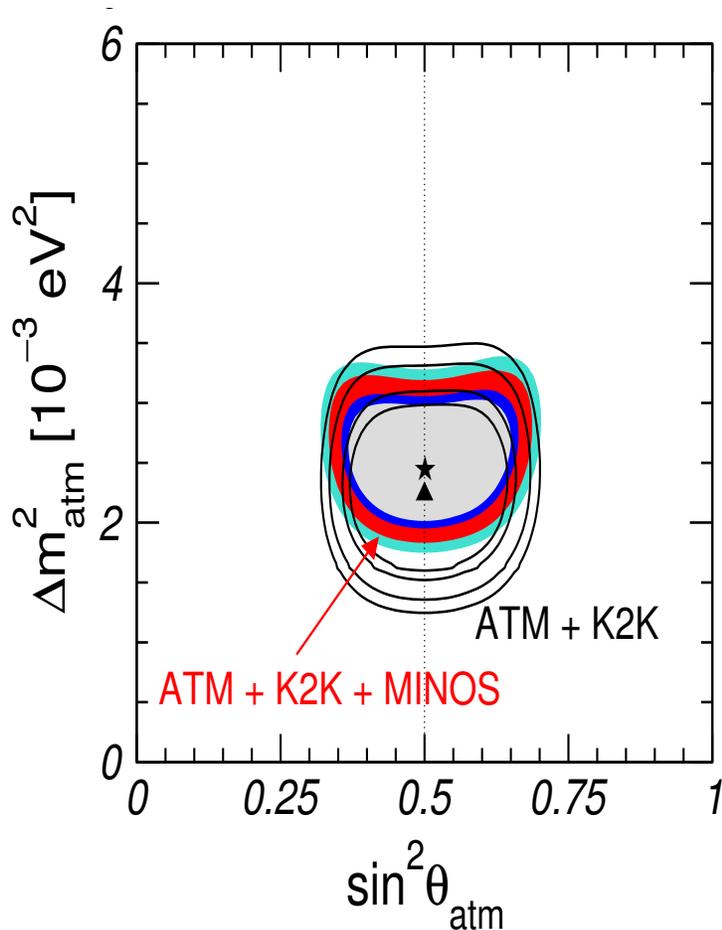
First Results from MINOS



Atmospheric parameters from MINOS



Status of Atmospheric parameters



<i>Exp</i>	<i>bf</i>	3σ range
	$\Delta m_{31}^2 [10^{-3} \text{ eV}^2]$	
<i>SK</i>	2.0	1.0 – 3.8
<i>SK + K2K</i>	2.4	1.4 – 3.3
<i>SK + K2K + MINOS</i>	2.5	1.8 – 3.3
	$\sin^2 \theta_{23}$	
<i>SK</i>	0.5	0.34 – 0.68

Thomas Schwetz, 2006

- Precision

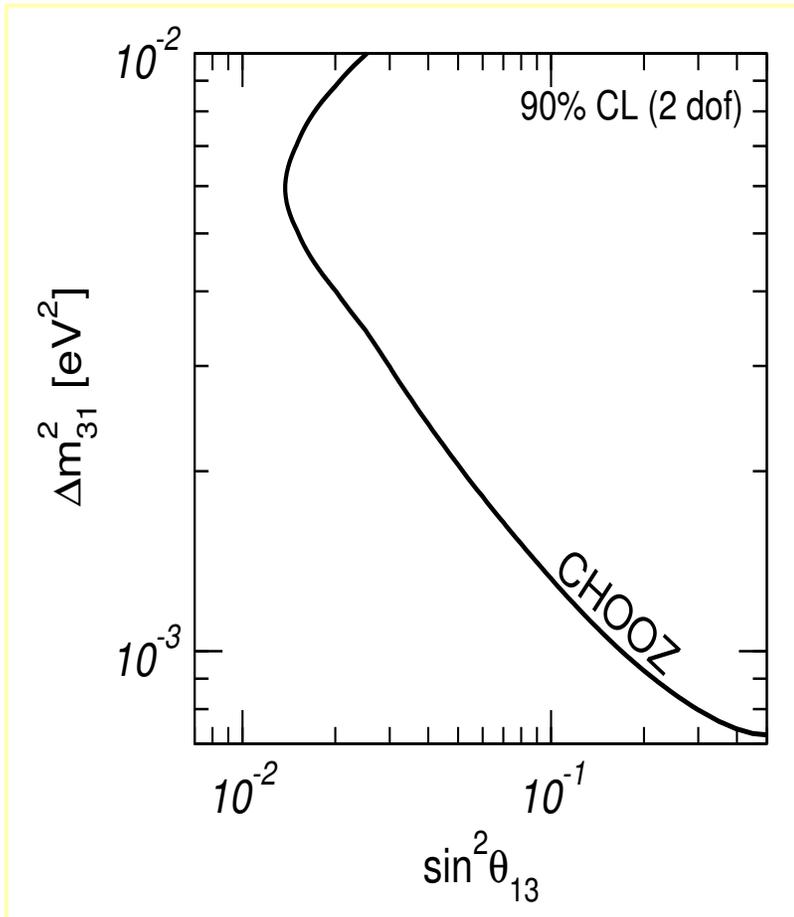
$\Delta m_{31}^2 \sim 29\%$ $\sin^2 \theta_{23} \sim 33\%$

Status of θ_{13}

- Non-zero θ_{13} couples solar and atmospheric
- Constrained by Global data

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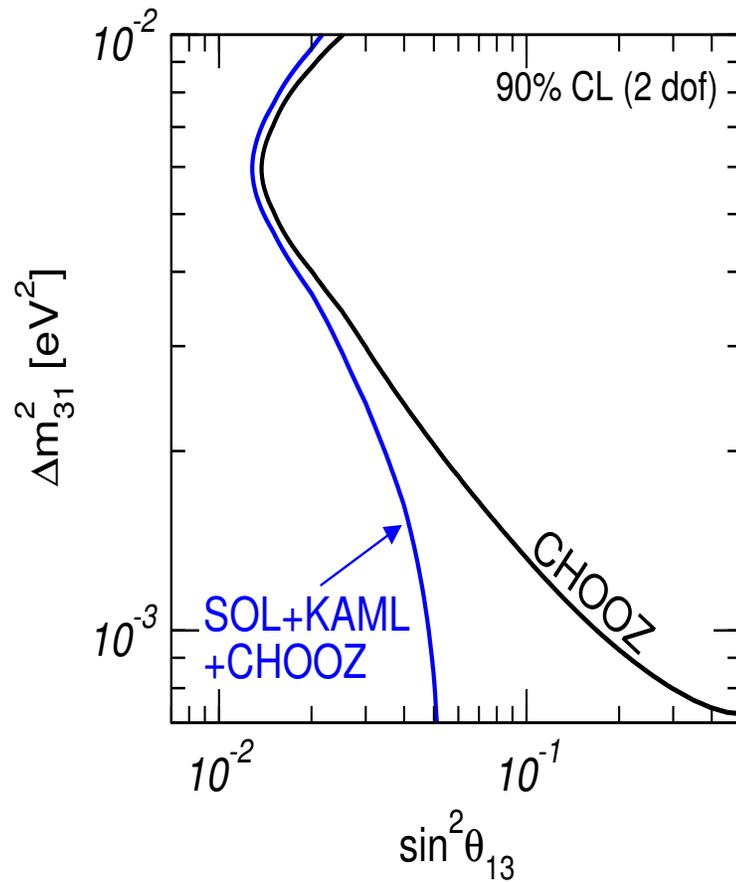
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$$P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta m_{31}^2 L/4E$$

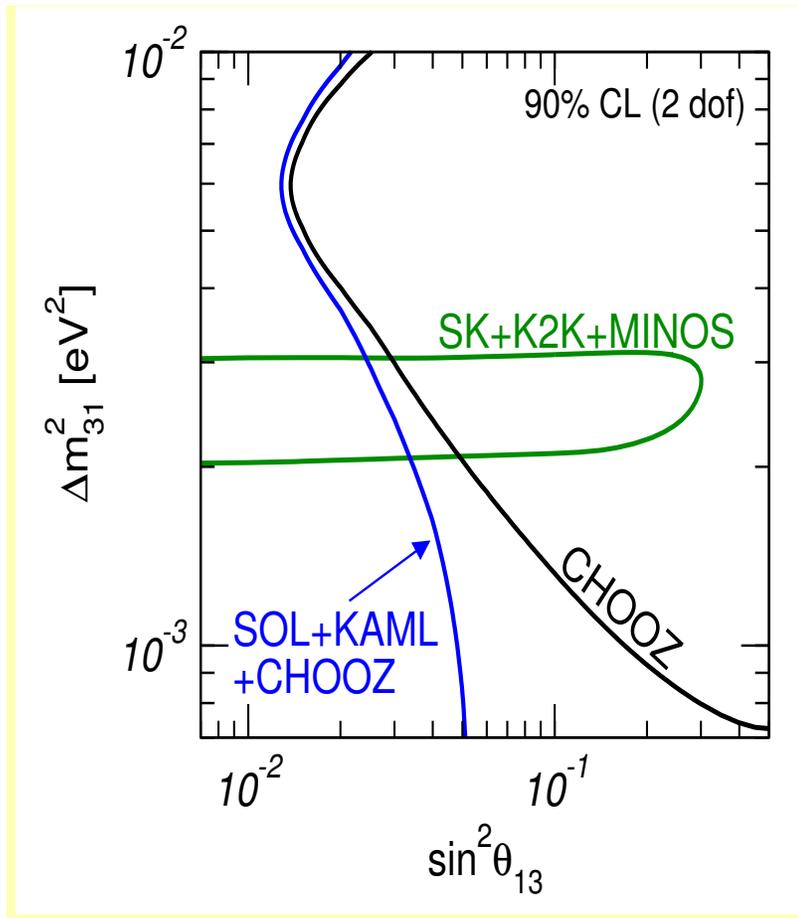
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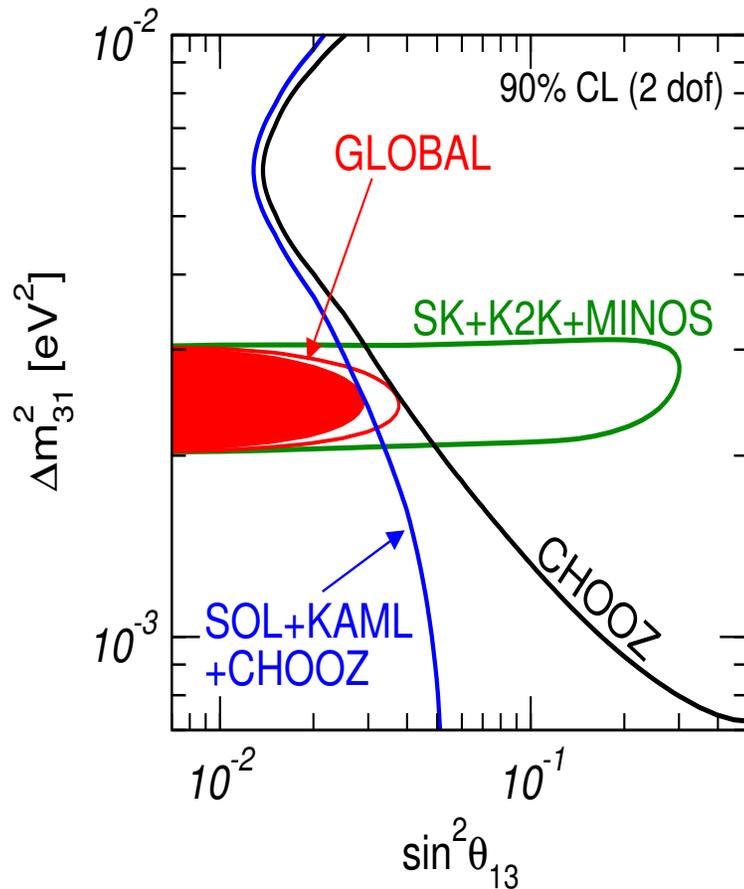
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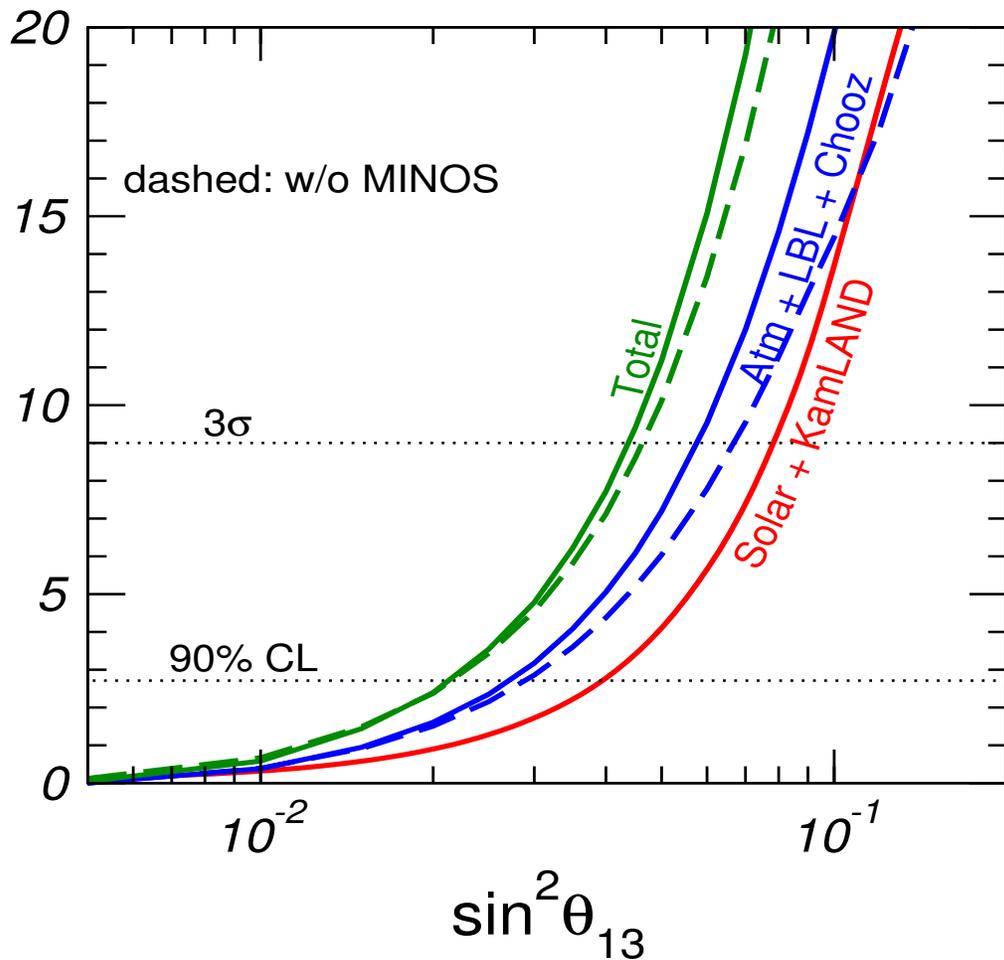
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Thomas Schwetz, 2006, Maltoni *et al.* hep-ph/0405172

Status of θ_{13}



3σ bounds
with (w/o) MINOS:

Solar+KamLAND:
 $\sin^2 \theta_{13} < 0.079$

Atm+Chooz+LBL:
 $\sin^2 \theta_{13} < 0.058$ (0.067)

Global:
 $\sin^2 \theta_{13} < 0.044$ (0.046)

Thomas Schwetz, 2006, Maltoni *et al.* hep-ph/0405172

Absolute Neutrino Masses

Tritium β -decay

$$\begin{aligned} m_{\nu_e} &= \left(\sum |U_{ei}|^2 m_i^2 \right)^{\frac{1}{2}} \\ &= \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2 \right]^{\frac{1}{2}} \end{aligned}$$

Neutrinoless double beta decay

$$\begin{aligned} m_{ee} &= \left| \sum U_{ei}^2 m_i \right| \\ &= \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3} \right| \end{aligned}$$

Cosmology

$$m_{\text{cosmo}} = \sum m_i$$

Bound on Neutrino Mass from Cosmology

■ Different upper bounds from different groups

- Some recent bounds (95% C.L.)

Reference	upper bound on m_{cosmo}
Elgaroy <i>etal.</i> , 02	1.8 eV
Danchez <i>etal.</i> , 05	1.2 eV
Goobar <i>etal.</i> , 06	0.62 eV
Fukugita <i>etal.</i> , 06	2.0 eV
Spergel <i>etal.</i> , 06	0.68eV
Seljak <i>etal.</i> , 06	0.17eV
Kristiansen <i>etal.</i> , 06	1.4eV

Elgaroy et al., 2006

■ The bound varies depending on the cosmological model and data sets used

Bound on Neutrino Mass from Cosmology

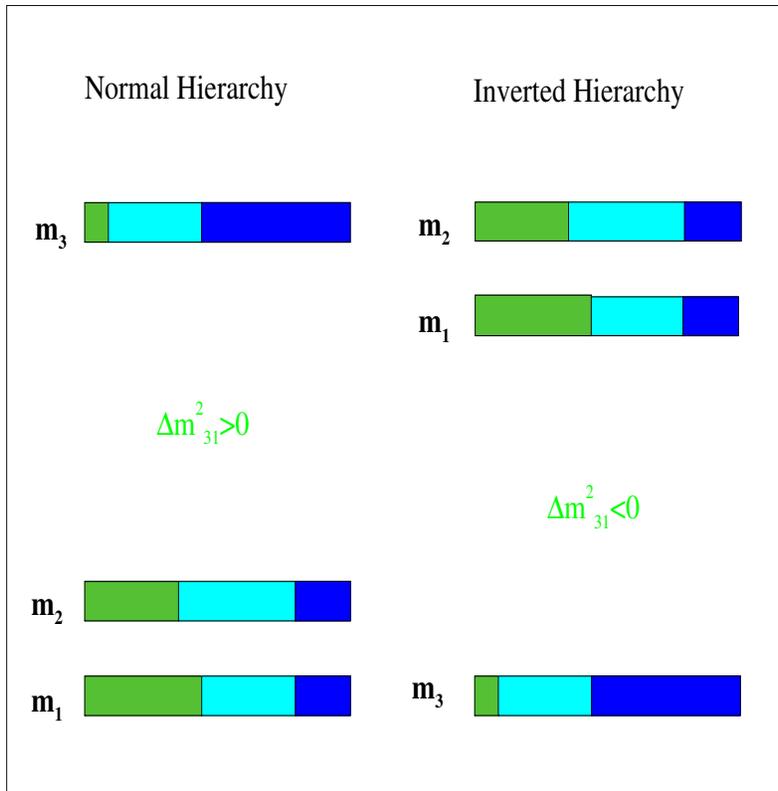
Dependence of the bound on different combination of data sets used

Fogli *et al.*, 2006

TABLE II: Input cosmological data sets for seven representative cases considered in this work, together with their 2σ (95% C.L.) constraints on the sum of neutrino masses Σ .

Case	Cosmological data set	Σ bound (2σ)
1	WMAP	< 2.3 eV
2	WMAP + SDSS	< 1.2 eV
3	WMAP + SDSS + SN _{Riess} + HST + BBN	< 0.78 eV
4	CMB + LSS + SN _{Astier}	< 0.75 eV
5	CMB + LSS + SN _{Astier} + BAO	< 0.58 eV
6	CMB + LSS + SN _{Astier} + Ly- α	< 0.21 eV
7	CMB + LSS + SN _{Astier} + BAO + Ly- α	< 0.17 eV

Mass Squared Differences and Absolute Masses



Normal Hierarchy :

$$m_3^2 = m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2$$

$$m_2^2 = m_1^2 + \Delta m_{21}^2$$

$$m_3^2 \approx \Delta m_{atm}^2 \gg m_2^2 \approx \Delta m_{\odot}^2 \gg m_1^2$$

Inverted Hierarchy :

$$m_2^2 = m_3^2 + \Delta m_{23}^2$$

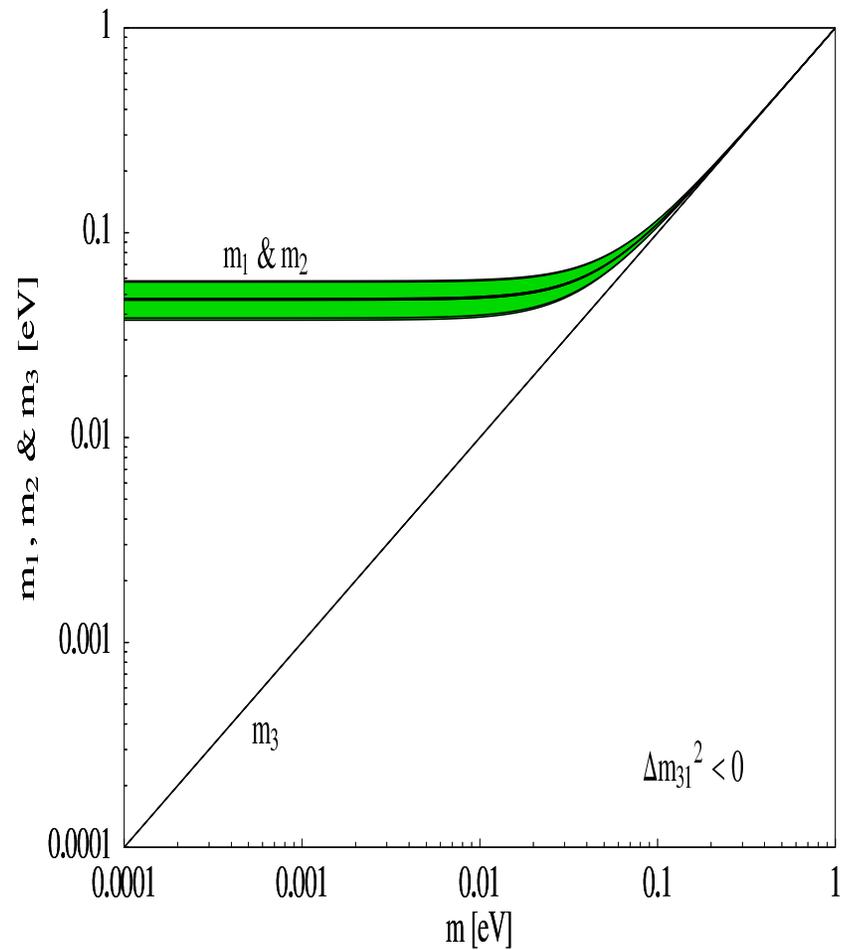
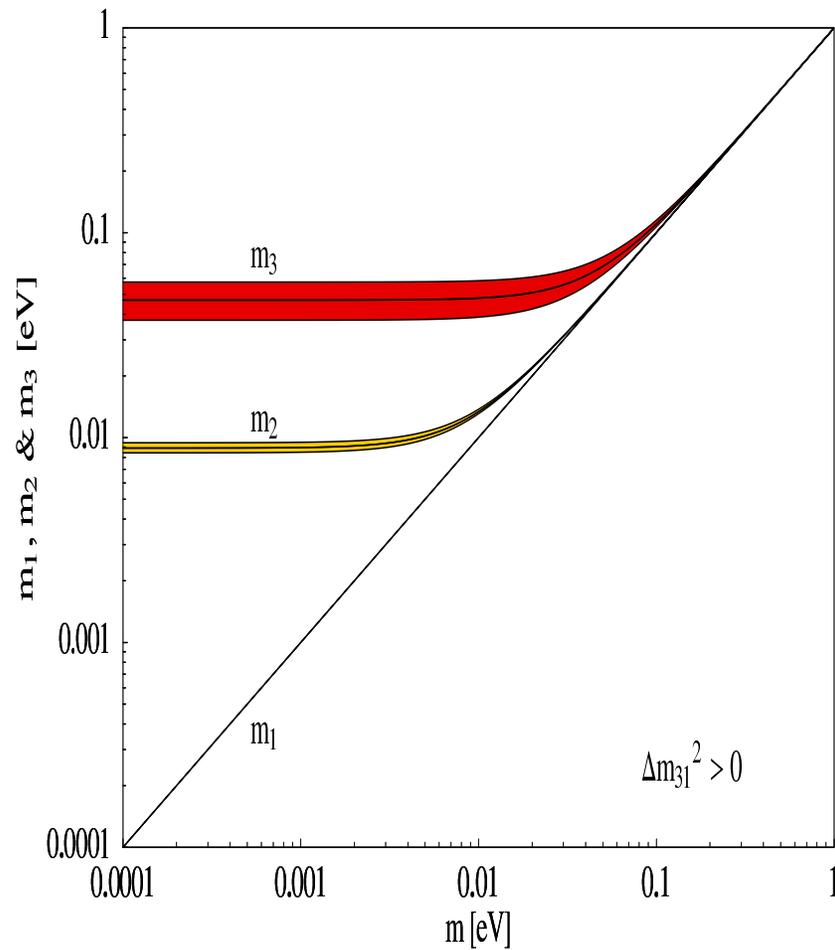
$$m_1^2 = m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2$$

$$m_2^2 \approx \Delta m_{atm}^2 \approx m_3^2 \gg m_1^2$$

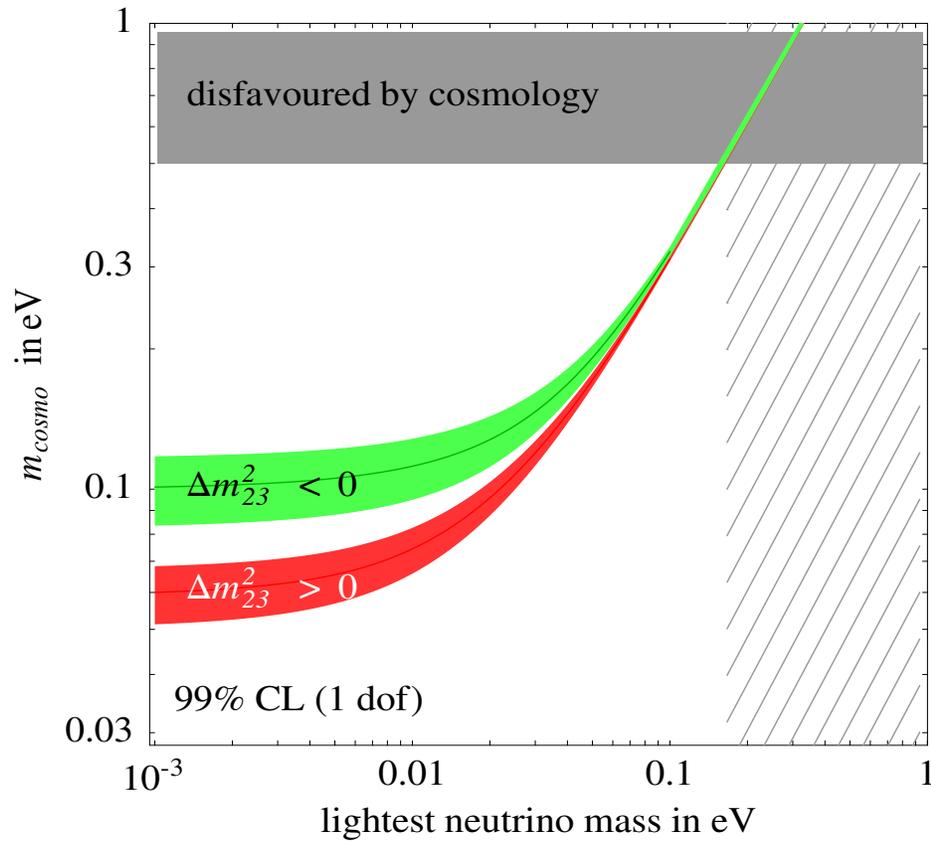
Quasi-Degenerate

$$m_3 \approx m_2 \approx m_1 \gg \sqrt{\Delta m_{atm}^2}$$

Mass Squared Differences and Absolute Masses

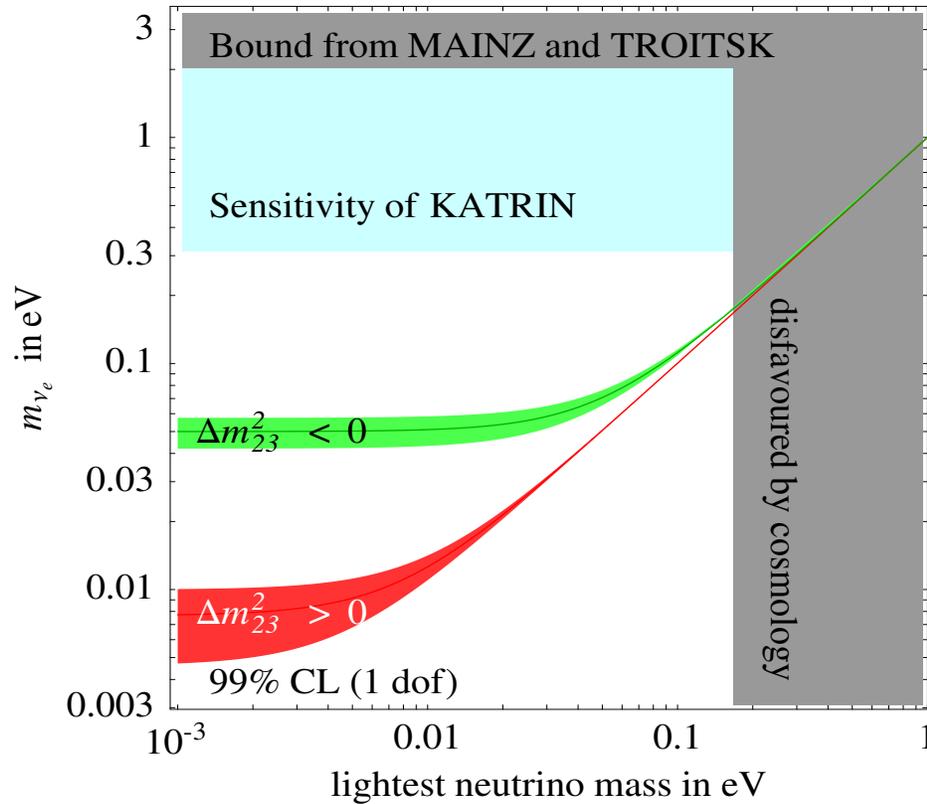


Sum of 3-neutrino masses vs the lightest mass



Vissani and Strumia, 2005

Tritium -decay and 3-neutrino masses



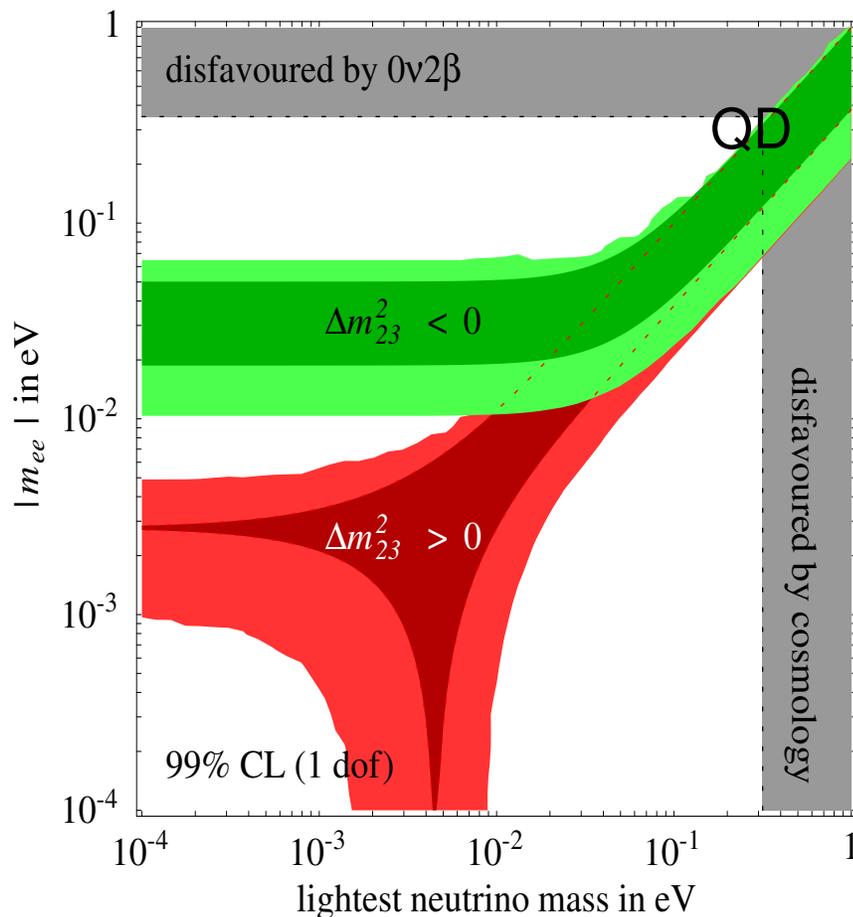
Vissani and Strumia, 2005

Neutrino Less Double Beta Decay

Can establish Majorana nature of Neutrinos

$$\langle m \rangle = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\phi_1} + m_3|U_{e3}|^2 e^{i\phi_2}|$$

- ν Mass Spectrum
- Absolute ν Mass Scale
- CP phases



NH: $m_1 \ll m_2 \ll m_3$

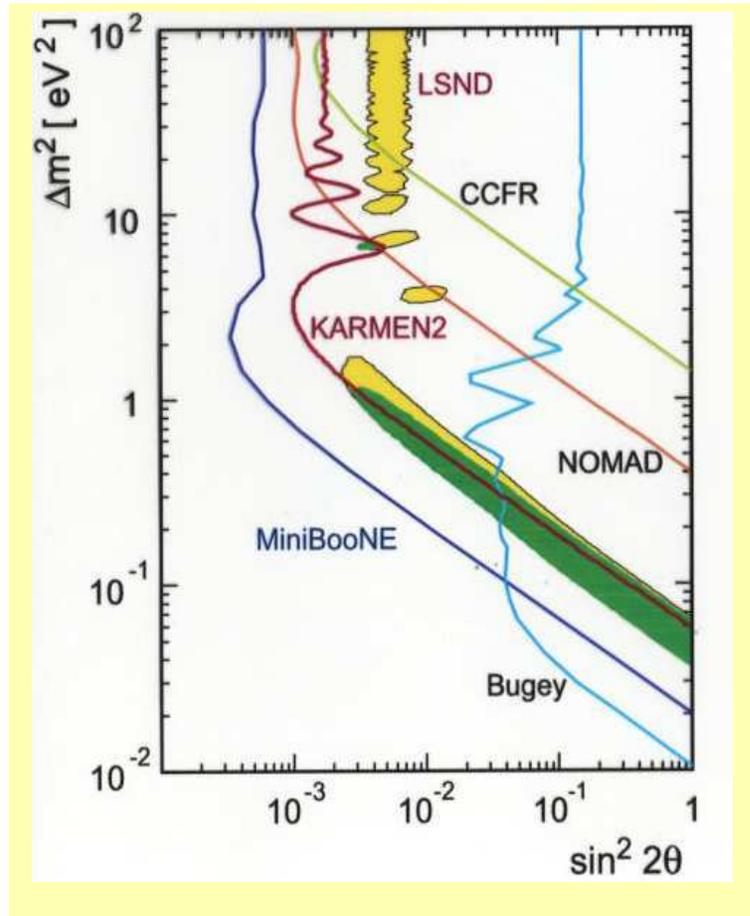
IH: $m_3 \ll m_1 \approx m_2$

QD: $m_1 \approx m_2 \approx m_3$

- $\langle m \rangle$ is in the range of sensitivity of upcoming $0\nu\beta\beta$ experiments for IH and QD

- Uncertainties coming from Nuclear matrix elements

The LSND Puzzle



- LSND : Liquid Scintillator Neutrino Detector

- $\pi^+ \rightarrow \mu^+ \nu_\mu$
 $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \rightarrow$ decay at rest

- $E \sim 36 - 60$ MeV, $L \sim 30m$

- $\bar{\nu}_e$ is detected by correlating in position and time with a γ produced by $\bar{\nu}_e + p \rightarrow e^+ + n$
 $np \rightarrow d\gamma$ (2.2 MeV)

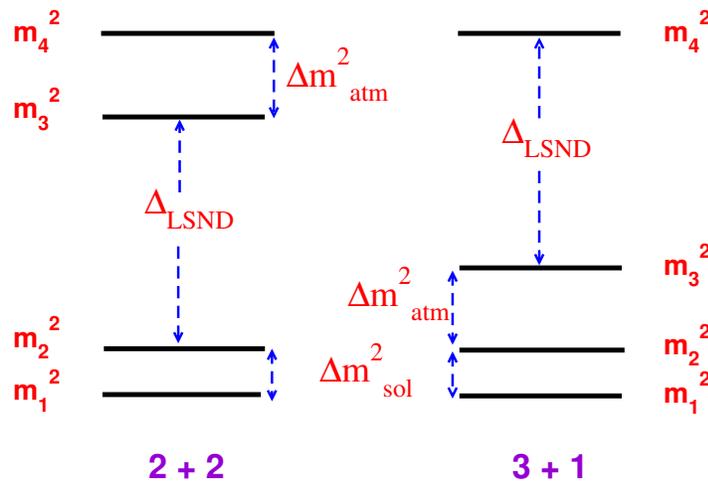
- Positive evidence of $\bar{\nu}_\mu - \bar{\nu}_e$ oscillation

$87.9 \pm 22.4 \pm 6.0$ excess $\bar{\nu}_e$ events

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 0.264 \pm 0.067 \pm 0.045$$

Accommodating the LSND Signal

- Evidence for oscillations for $\Rightarrow \Delta m^2 \sim \text{eV}^2$
- Adding extra sterile neutrinos



One additional sterile neutrino
2+2 and 3+1 mass schemes

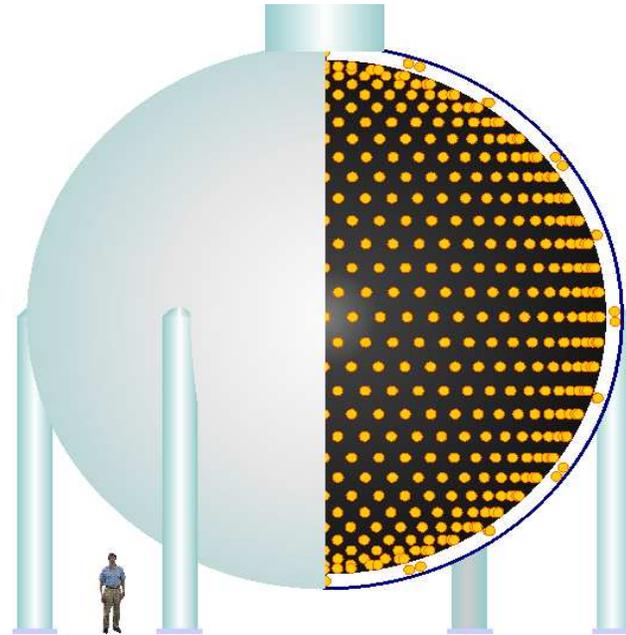
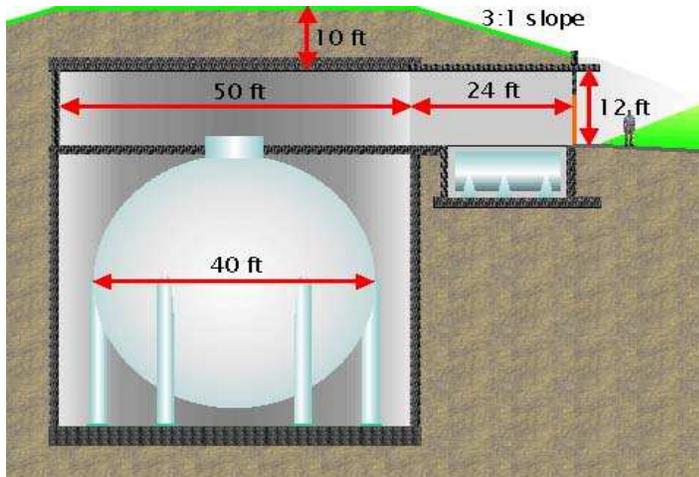
2+2 ruled out by solar and atmospheric data

3+1 disfavoured by SBL experiment

Confirmation of LSND by Miniboone
need new ideas

MiniBoone

- Look for ν_e events in a pure ν_μ beam
- Confirm or refute the LSND evidence with higher statistics and different systematics



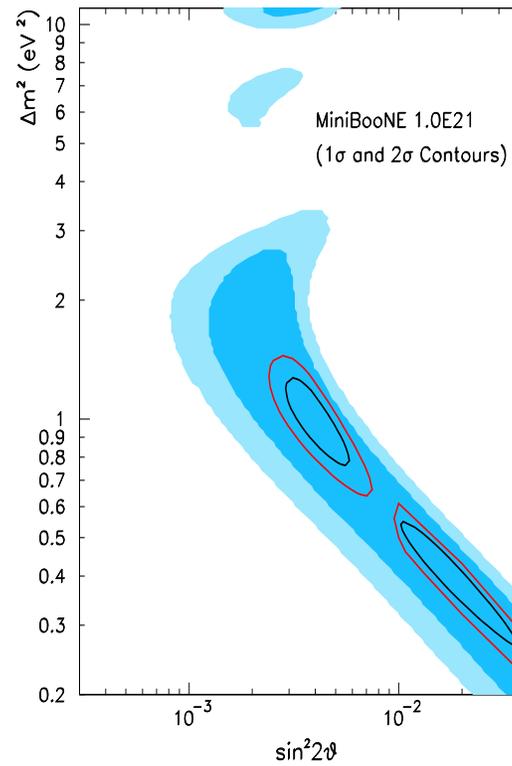
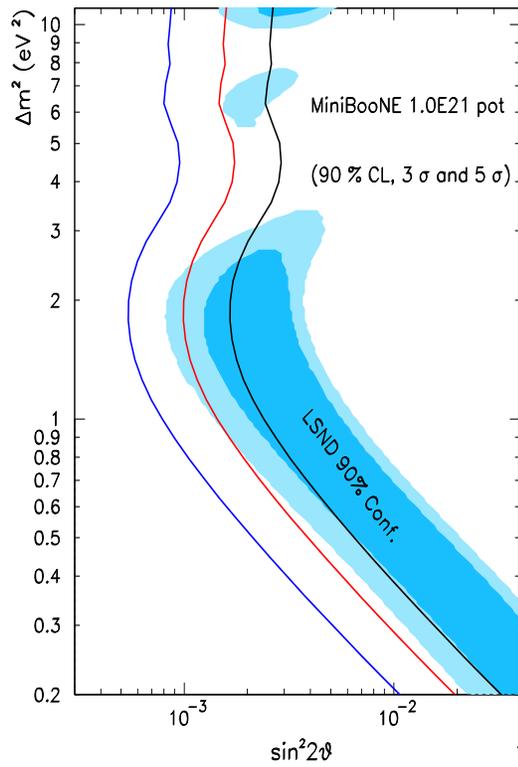
- Use protons from the 8 GeV booster, $\langle E_\nu \rangle \sim 1\text{GeV}$

Detector: 12m Diameter Sphere filled with mineral oils and PMTs and 500m from source

MiniBoone

- 10²¹ p.o.t to reach coverage of LSND signal (~ July 2005)
- L/E ~ 1 MeV/m – same as LSND by increasing both L and E
- This changes signature of signal and major backgrounds in the detector
- MiniBoone detects neutrinos through
 - QE scattering (CCQE)
 - 1 Pion reactions (CCPiP)
- Miniboone collaboration has presented preliminary results for CCPiP/CCQE ratio
- Check the possibility of ν_μ disappearance
- $\nu_\mu - \nu_e$ oscillation analysis –Results ??

MiniBoone Sensitivity



Tri-BiMaximal Mixing

- Present data implies tri-bimaximal mixing
(Harrison, Perkins and Scott)

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i\alpha_3/2}),$$

- $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{12} = 0.33$, $\sin^2 \theta_{13} = 0$ Two large and one small mixing
- Key Question : Is there some underlying Symmetry or accidental ?
- Hierarchy of neutrino masses not strong : $m_3/m_2 \leq 6$
- completely different from quark sector

Quark-Lepton Complimentarity

Present data also imply

$$\theta_{12} + \theta_c \approx \pi/4$$

$\theta_c \rightarrow$ Cabbibo Angle

\rightarrow Quark-Lepton Complimentarity

 QLC1

$$U = U_l^\dagger U_\nu$$

$$U_l = V_{CKM}$$

$$U_\nu = U_{bimax}$$

$$\theta_{12} \approx 35.4^\circ$$

$$\theta_{23} \approx 42.5^\circ$$

$$\theta_{13} \approx 8.9^\circ$$

 QLC2

Raidal, Minakata and Smirnov

$$U = U_l^\dagger U_\nu$$

$$U_l = U_{bimax}^T$$

$$U_\nu = V_{CKM}^\dagger$$

$$\theta_{12} \approx 32.4^\circ$$

$$\theta_{12} \approx 32.4^\circ$$

$$\theta_{23} \approx 42.7^\circ$$

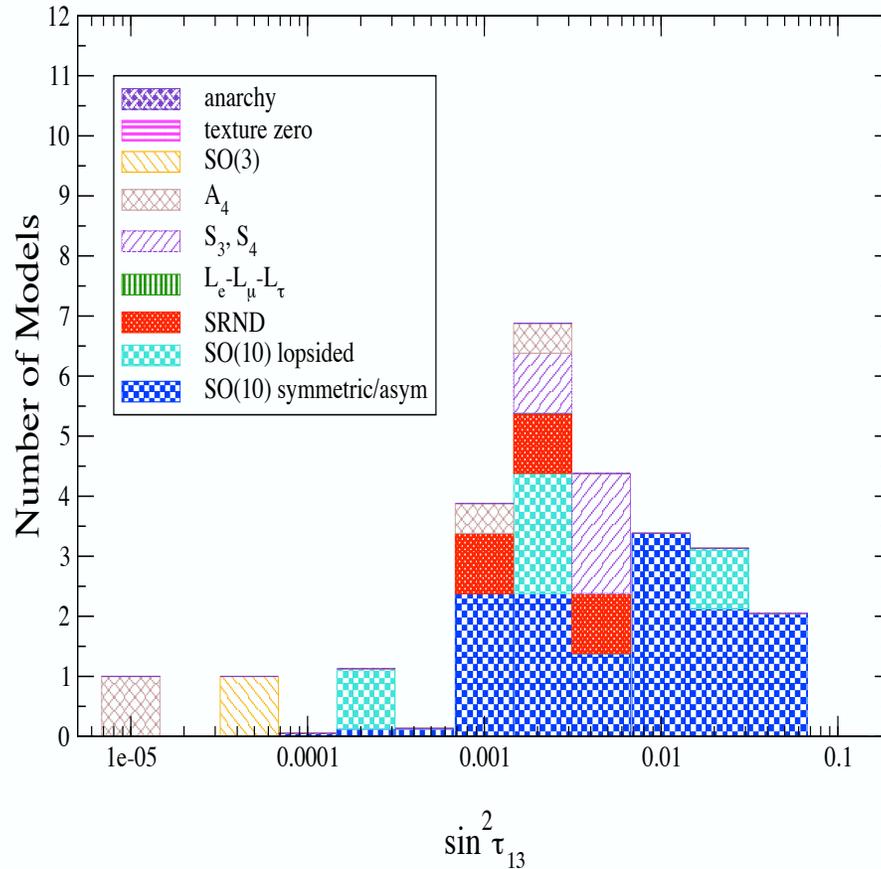
$$\theta_{13} \approx 0$$

Different Mechanisms to generate Neutrino Mass

- Anarchy – No flavour symmetry
- Flavour Symmetry
 - $\mu - \tau$ exchange symmetry
 - $L_e - L_\mu - L_\tau$ symmetry
 - $L_\mu - L_\tau$ symmetry
 - S_3, A_4, S_4, D_5
 $A_4 \rightarrow$ Can give tri-bimaximal mixing
 - SO(3) and SU(3) Flavour Symmetries
- SRND
- GUTS : natural framework for Seesaw
Most Popular candidate : SO(10)

Theoretical Models and their Predictions

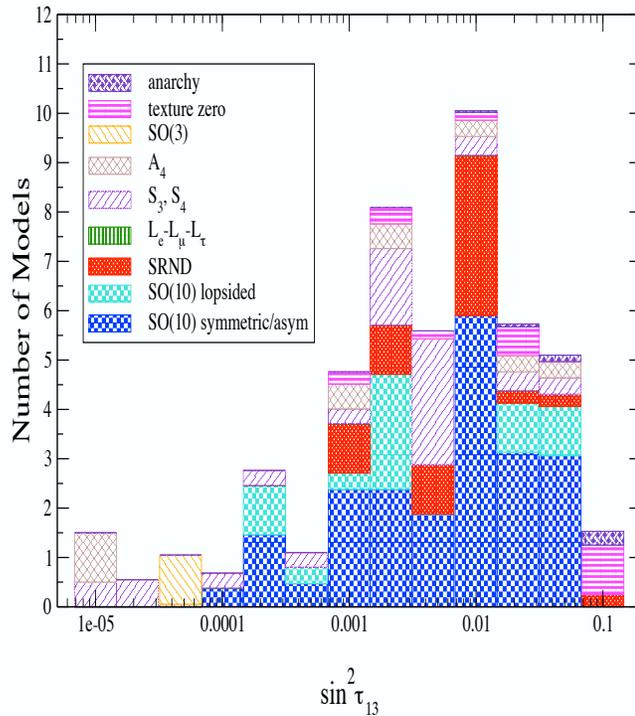
Models that Predict All 3 Angles



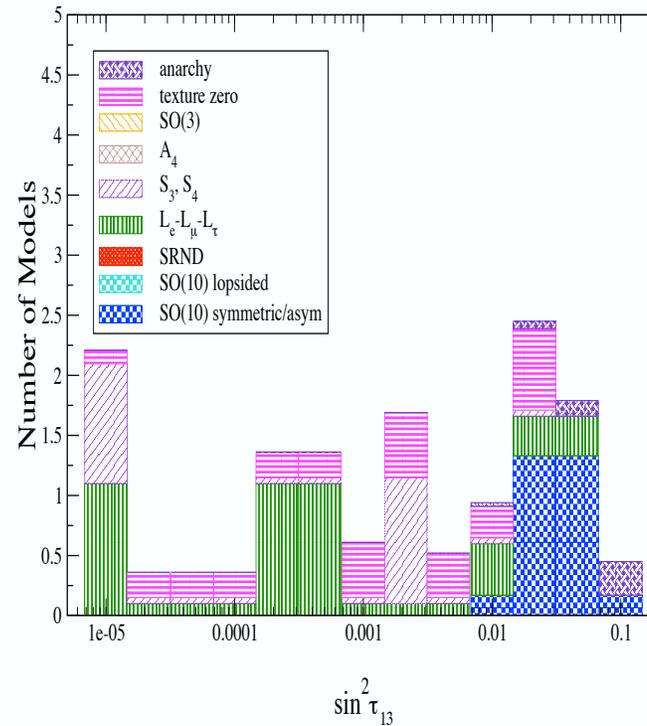
27 models predict
all 3 mixing angles
correctly $\sin^2 \theta_{13} =$
 $0.001 - 0.04$
C. Albright *et al.*, hep-ph/0608137

Theoretical Models and their Predictions

Models with Normal Hierarchy



Models with Inverted Hierarchy



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Three times more models predict Normal Hierarchy and $\sin^2 \theta_{13} = 0.001 - 0.04$

C. Albright *et al.*, hep-ph/0608137

Summary

Known Parameters

Parameters	Best – fit	3σ range
Δm_{21}^2	$8 \times 10^{-5} \text{ eV}^2$	$(7.0-9.3) \times 10^{-5} \text{ eV}^2$
Δm_{31}^2	$2.5 \times 10^{-3} \text{ eV}^2$	$(1.8-3.3) \times 10^{-3} \text{ eV}^2$
$\sin^2 \theta_{23}$	0.5	0.34 – 0.68
$\sin^2 \theta_{12}$	0.31	0.24 – 0.41
$\sin^2 \theta_{23}$	0.5	0.34 – 0.68

Bounded Parameters

$$\sin^2 \theta_{13} \leq 0.044, \Sigma m_i \leq (0.17 - 2.0) \text{ eV}$$

No information on

$\text{sgn}(\Delta m_{31}^2)$ (nature of mass spectrum), Octant of θ_{23} , CP phases

LSND

(2+2) ruled out, 3+1 strongly disfavoured

Confirmation of LSND results by Miniboone will need new ideas

Conclusion and Outlook

- The **small observed** neutrino masses **cannot** be explained by Standard Model
- The **only** observational signal of physics **Beyond the Standard Model**
- Theoretical challenges
 - Why $m_{\nu_i} \ll m_l, m_u, m_d$
 - Why two **large** and one **small** angle in neutrino sector while all the angles in quark sector are small
 - Hierarchy for neutrino masses not strong
Can even be degenerate!
- Hierarchy, θ_{13} , deviation from maximality \Rightarrow **good discriminator of mass Models**
- Future neutrino experiments can help in choosing between various alternatives