Determination of Neutrino Mass Hierarchy with Long Baseline Experiments

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(to be submitted)

Solar and atmospheric neutrino problems can both be explained in terms of three flavour neutrino oscillations.

The survival/oscillation probabilities depend on two mass-squared differences, Δ_{21} and Δ_{31}, three mixing angles, θ_{12}, θ_{13} and θ_{23}, and a CP violating phase δ.

Solution to solar neutrino problem requires the smaller mass-squared difference, which is commonly taken to be Δ_{21}, should be positive.

Atmospheric neutrino data and the data from accelerator experiments, K2K and MINOS, determine the magnitude of Δ_{31} but not its sign.

CHOOZ reactor experiment and solar neutrino (and KamLAND) data lead to the constraint $\theta_{13} \leq 14^\circ$.

Mohan Narayan, G. Rajasekaran and S. Uma Sankar (hep-ph/9712409), Phys. Rev. D **58**, 031301 (1998).

A. Bandyopadhyaya *et al*, Phys. Lett. B **608**, 115 (2005).

Efforts are on to measure non-zero value of θ_{13} with both reactor neutrinos (Double CHOOZ and Daya Bay) and accelerator neutrinos (T2K and Noνa).

Question: How can we measure the other unknown quantities, sign of Δ_{31} and the CP violating phase δ?

Sign of Δ_{31} is also called neutrino mass hierarchy.

$m_3 \gg m_2 > m_1$ (normal hierarchy) OR $m_2 > m_1 \gg m_3$ (inverted hierarchy)

Consider the case $\theta_{13} = 0$. Then it can be shown that solar neutrino oscillations are $\nu_e \leftrightarrow \nu_\mu/\nu_\tau$ and atmospheric neutrino oscillations are $\nu_\mu \leftrightarrow \nu_\tau$.

ν_e, as they propagate through matter, undergo elastic forward scattering off electrons. This scattering is parametrized by the matter (Wolfenstein) term $A = 2\sqrt{2}EG_F N_e$. This term is absent for other flavours.

This difference in the propagation of different flavours interferes with the vacuum oscillations which are driven by mass-squared difference. Thus the matter term leads to modification of oscillation probabilities, which depend on the interference between Δ and A.

The matter term is a function of neutrino energy. Therefore the modifications induced by it are also energy dependent.

The electron neutrino survival probability in solar neutrino problem has a particular energy dependence, which is reproduced by matter modified neutrino oscillations only for the case Δ_{21} positive.

The muon neutrino survival probability in atmospheric neutrinos (and also accelerator neutrinos) in the limit $\theta_{13} = 0$ is given by

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta_{31} L}{E}\right),$$

which is the same for both signs of Δ_{31}.

In the limit of $\theta_{13} = 0$, matter effects have no role here because these oscillations do not involve ν_e.

θ_{13} is a measure of the ν_e component of the mass eigenstate ν_3 ($\sin\theta_{13} = U_{e3}$). Non-zero value of θ_{13} implies that ν_e also has a role in atmospheric neutrino oscillations.

Matter effects modify this angle which links ν_e to other flavours at the atmospheric neutrino energy scale ($\sim$ GeV).

The expression of matter modified θ_{13} is given by

$$\sin 2\theta_{13}^m = \sin 2\theta_{13}\Delta_{31}/\Delta_{31}^m,$$

where

$$\Delta_{31}^m = \sqrt{(\Delta_{31}\cos 2\theta_{13} - A)^2 + (\Delta_{31}\sin\theta_{13})^2}.$$

Modification of atmospheric neutrino oscillation/survival probabilities due to matter effects must necessarily be proportional to θ_{13}.

Question: What is the smallest value of θ_{13} for which these matter effects can be measured and the neutrino mass hierarchy can be determined?

Answer (obviously) depends on what kind of experiments will be performed.

Here I will confine my attention to long baseline experiments which are being constructed (Double CHOOZ and T2K) and are likely to be constructed (Daya Bay and Noνa). One important other possibility is INO (which I will not consider here).

Firstly, we need an experiments which are sensitive only to non-zero value of θ_{13} but not matter effects. Reactor neutrino experiments satisfy this constraint.

Double CHOOZ and Daya Bay experiments are designed to have the least possible systematic errors, so that a good precision in θ_{13} can be achieved. They will measure non-zero value for θ_{13} if $\theta_{13} \geq 5^\circ$.

Lindner *et al* have made a proposal *Triple CHOOZ* to improve the situation even further
(hep-ph/0601266) JHEP 0605 (2006) 072.

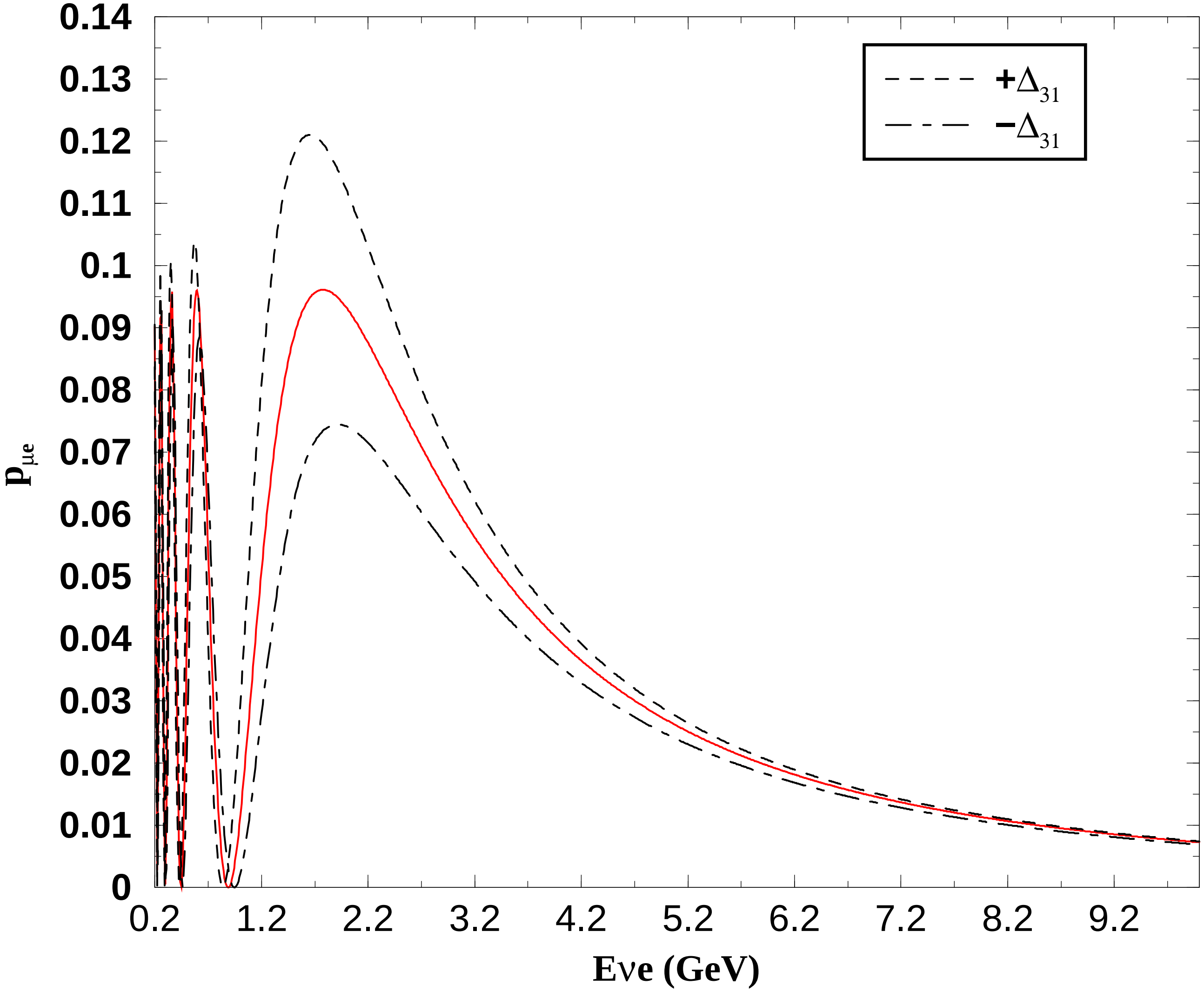
With a non-zero θ_{13} in hand, one can determine the mass hierarchy by measuring $\nu_\mu \rightarrow \nu_e$ oscillation probability, which gets modified by matter effects, in an accelerator experiment.

This oscillation probability is given, in the limit $\Delta_{21} = 0$, by

$$P^m(\nu_\mu \rightarrow \nu_e) = \sin^2\theta_{23} \sin^2 2\theta_{13}^m \sin^2\left(1.27\frac{\Delta_{31}^m L}{E}\right).$$

For longer baseline, this probability achieves a miximum value at larger E.

The matter term is proportional to E. Hence to discern the effect of the matter term should have the oscillation probability maximum at large energy and hence must have large baseline length L.



0.14
0.13
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
$p_{\mu e}$
0.2
1.2
2.2
3.2
4.2
5.2
6.2
7.2
8.2
9.2
Eνe (GeV)
$+\Delta_{31}$
$-\Delta_{31}$

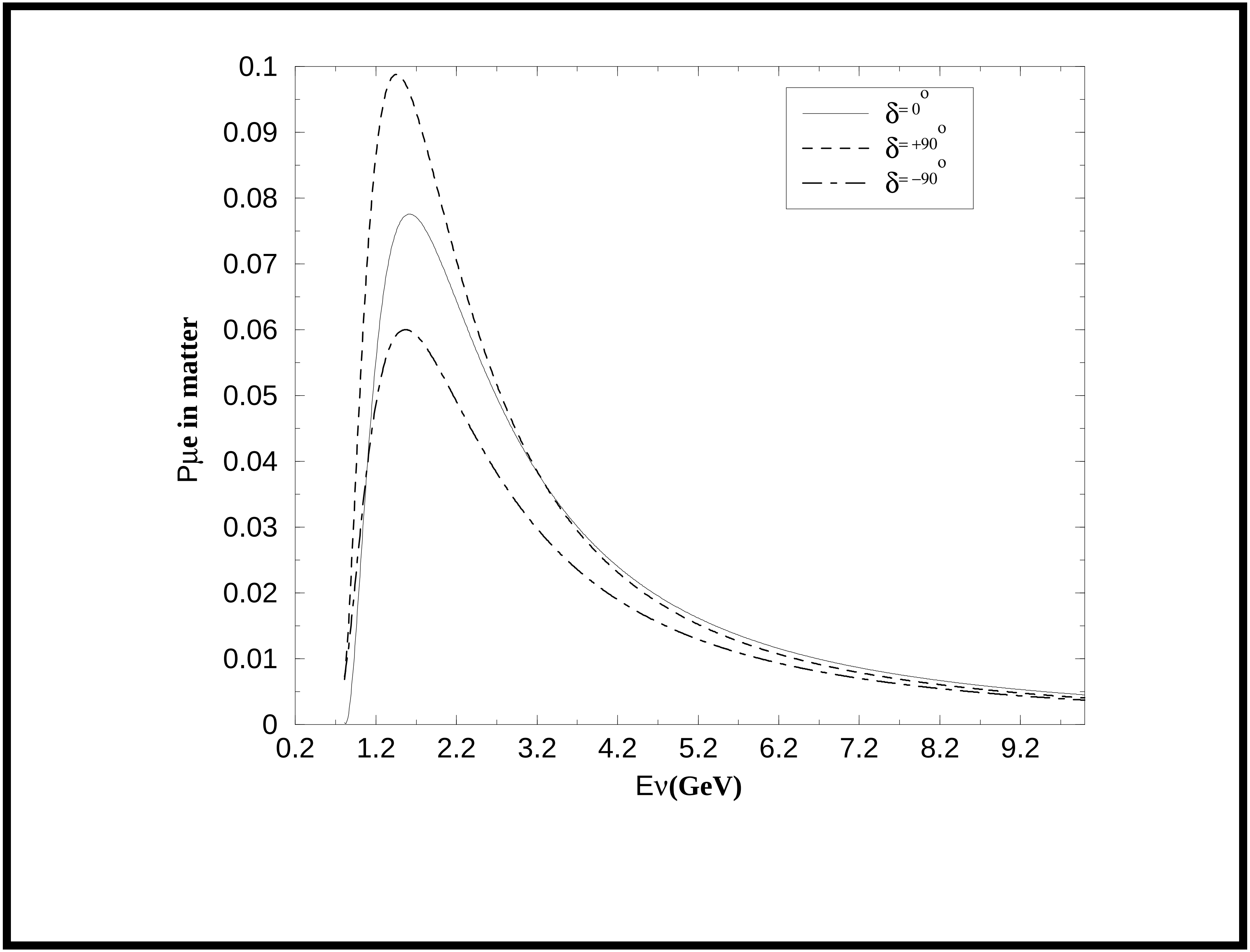
For Δ_{31} positive this probability is higher and for Δ_{31} negative this probability is lower than the vacuum expectation. Changing the sign of Δ_{31} is leading to a change of about 25% in the oscillation probability.

If we keep $\Delta_{21} \neq 0$, then the matter modified oscillation probability becomes

$$
\begin{aligned}
P^m(\nu_\mu \to \nu_e) \; = \; & \sin^2\theta_{23} \sin^2 2\theta_{13} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
& +\alpha \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \\
& [\sin\delta \sin(\Delta) + \cos\delta \cos(\Delta)] \\
& \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin((1-\hat{A})}{(1-\hat{A})} \\
& +\alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{(\hat{A}\Delta)^2},
\end{aligned}
$$

where $\alpha = \Delta_{21}/\Delta_{31}$, $\hat{A} = A/\Delta_{31}$ and $\Delta = 1.27\Delta_{31} L/E$.

The second term, which is the leading term in Δ_{21}, also contains the CP violating phase δ. There is no information on this phase and we must consider variation in its full range $-\pi$ to π. When δ is varied over its full range, the second term leads to 25% change in $P^m(\nu_\mu \rightarrow \nu_e)$. This is illustrated in the next figure.



0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.2
1.2
2.2
3.2
4.2
5.2
6.2
7.2
8.2
9.2
Pμe in matter
Eν(GeV)
δ= 0°
δ= +90°
δ= −90°

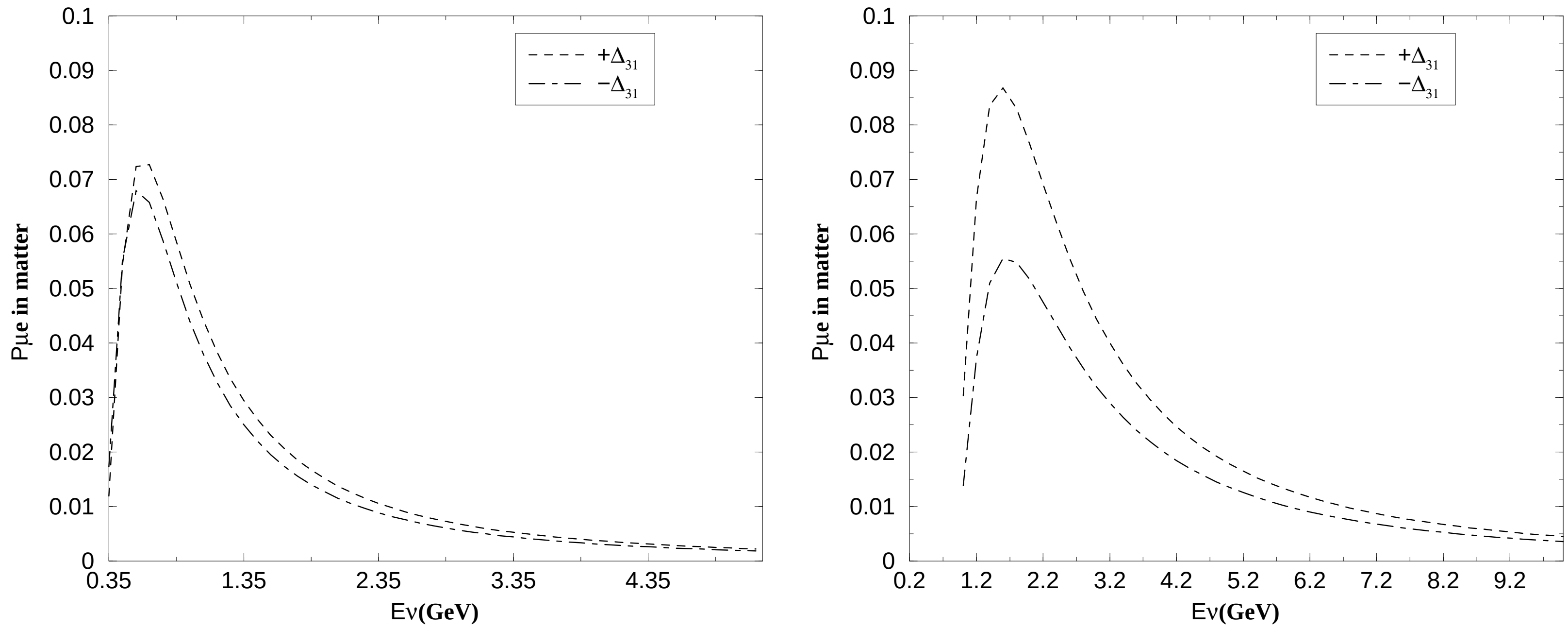
If we have the data on $P^m(\nu_\mu \to \nu_e)$ from one experiment, there are two possible solutions to it: (a) The true hierarchy and the true value of δ and (b) the wrong hierarchy and a wrong value of δ (which is about $\pi/4$ to π away from the true value).

Because of the dependence of $P^m(\nu_\mu \to \nu_e)$ on δ, data from a single long baseline experiment can't determine the mass hierarchy.

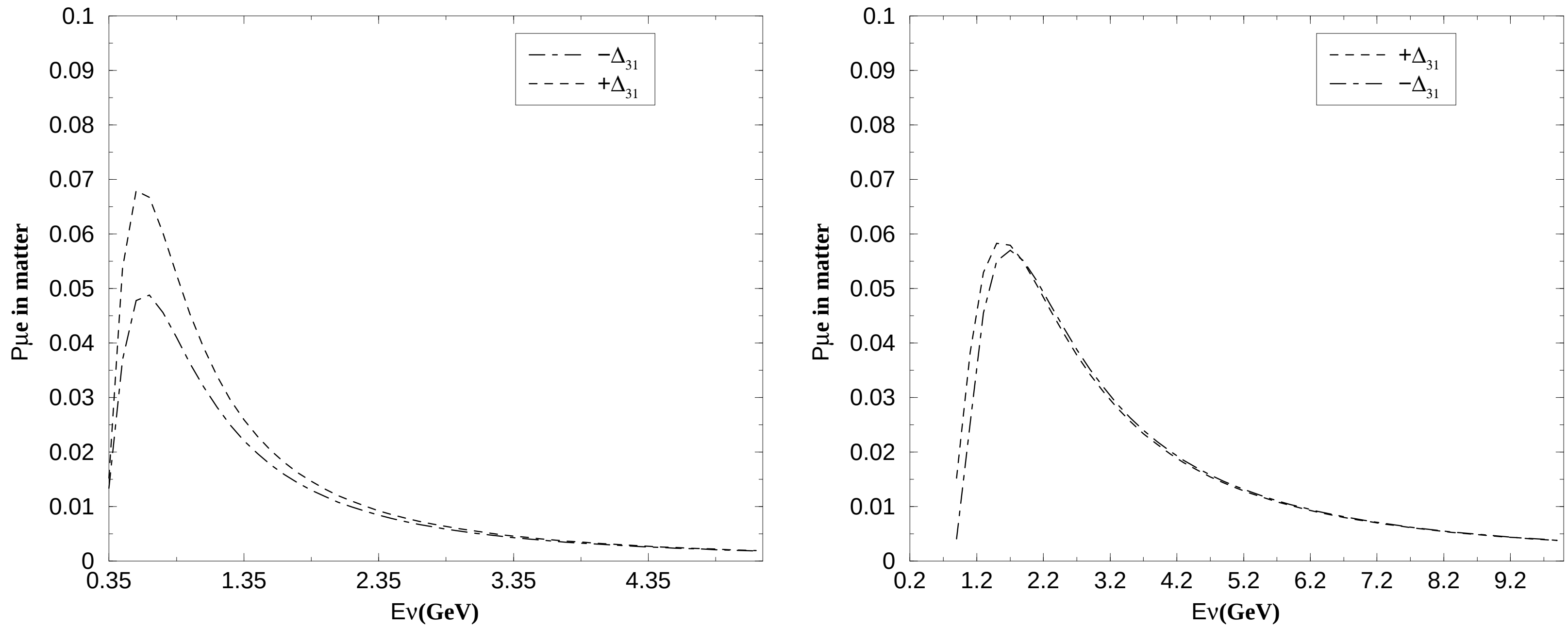
The change induced by the matter term is energy dependent. Whereas the change induced by δ is energy independent provided (L/E) is kept fixed.

Suppose we have data on $P^m(\nu_\mu \to \nu_e)$ from two different experiments with two different baselines. Then the change induced by the matter term in the two experiments will be different. But the change induced by the CP phase δ will roughly be the same.

In such a situation, *the wrong hierarchy along with a single spurious value of* δ can't account for both sets of data. This is illustrated in the next figure for two experiments with baselines $L_1 = 295$ Km (T2K) and $L_2 = 810$ Km (Noνa).



The figure on the left is for T2K and figure on the right is for Noνa. Δ_{31} positive curve has $\delta = 30^\circ$ and Δ_{31} negative curve has $\delta = 75^\circ$. In both cases $|\Delta_{31}| = 2.5 \times 10^{-3}$ eV^2 and $\theta_{13} = 10^\circ$.



The figure on the left is for T2K and figure on the right is for Noνa. Δ_{31} positive curve has $\delta = -90^\circ$ and Δ_{31} negative curve has $\delta = 90^\circ$. In both cases $|\Delta_{31}| = 2.5 \times 10^{-3}$ eV^2 and $\theta_{13} = 10^\circ$.

So here is the strategy we adopt. The three experiments, Double CHOOZ, T2K and Noνa, give us the event spectra in the case of no oscillations.

We assume three years of running of Double CHOOZ (hep-ex/0405032) and five years of running of T2K in neutrino mode (hep-ex/0106019). In the case of Noνa also, we assume five years of running in neutrino mode with POT of 7.3×10^{20} per year (hep-ex/0503053).

We take Δ_{31} to be positive and select a *true value* of θ_{13} and a *true value* of δ from their currently allowed ranges $0 \leq \theta_{13} 14^\circ$ and $-\pi \leq \delta \leq \pi$. For other neutrino parameters, $|\Delta_{31}|$, Δ_{21}, θ_{12} and θ_{23}, we take their current best values.

With these values we compute $P(\bar{\nu}_e \to \bar{\nu}_e)$ (which is relevant for Double CHOOZ) and $P^m(\nu_\mu \to \nu_e)$ (which is relevant for T2K and Noνa) as functions of energy.

We multiply the event spectra in the case of no oscillations with the survival/oscillation probabilities to obtain the event spectra with oscillations driven by the above given neutrino parameters.

Each detector has an uncertainty in measuring the energy of an event. This leads to an distortion of the above spectrum. We compute the distorted spectrum by convoluting the previous spectrum with a Gaussian energy smearing function.

In the case of Double CHOOZ, the uncertainty in the measurement of the energy is much smaller than the bin size. Therefore no energy smearing is done.

For T2K the uncertainty in the measurement of energy is $\sigma_E = 100$ MeV (hep-ex/0106019). For Noνa the energy uncertainty is taken to $\sigma_E/E = 0.1/\sqrt{E}$, where E is in GeV (hep-ex/0503053).

We take these smeared event spectra to be our ”experimental data”. We have 36 data points from Double CHOOZ, 18 from T2K and 46 from Noνa for a total of 100 data points.

We now calculate ”theoretical event spectra” by assuming the wrong hierarchy (in this case negative Δ_{31}) and all allowed values of θ_{13} and δ.

For each ”test value” of θ_{13} and δ (and the wrong hierarchy), we calculate the ”theoretical values” of the above 100 measurables.

We then compute

$$\chi^2(\theta_{13}^{test}, \delta^{test}) = \sum_{i=1}^{100} (N_i - N_i^{test})^2 / N_i,$$

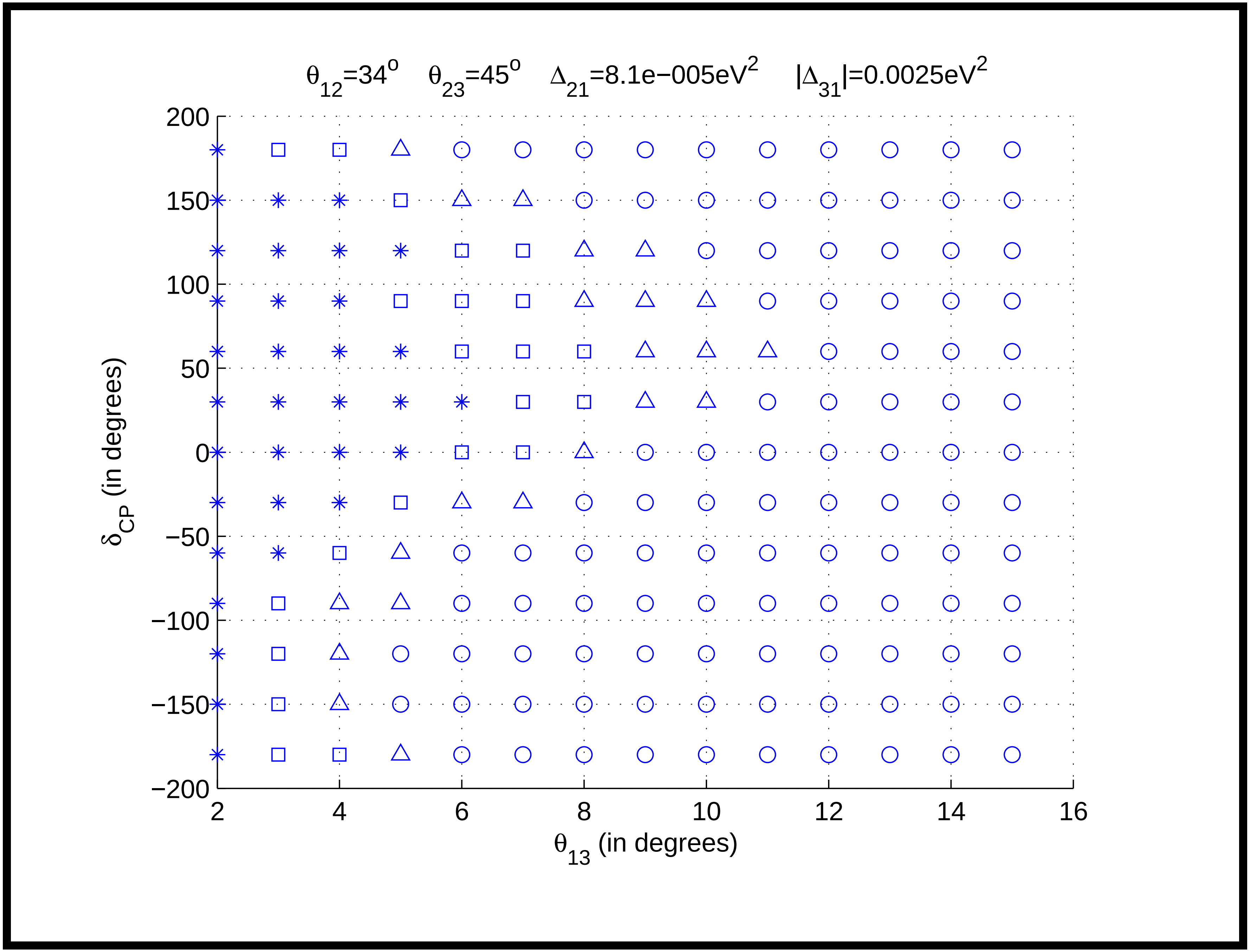
where N_i are the ”experimental data” for each measurable and N_i^{test} are the ”theoretical values” for them, which are functions of θ_{13}^{test} and δ^{test}.

χ^2 will be minimum for some value of θ_{13}^{test} and δ^{test}.

If this minimum value is greater than 4, then the wrong hierarchy can be ruled out at 95% **confidence, for the initially chosen "true" values of** θ_{13} **and** δ.

Now we vary these "true" values of θ_{13} and δ over their allowed ranges (again $0 \leq \theta_{13} 14^\circ$ and $-\pi \leq \delta \pi$) and ask what is the minimum "true" value of θ_{13} for which the **minimum of** χ^2 **will be greater than 4**, *irrespective of the "true" value of* δ.

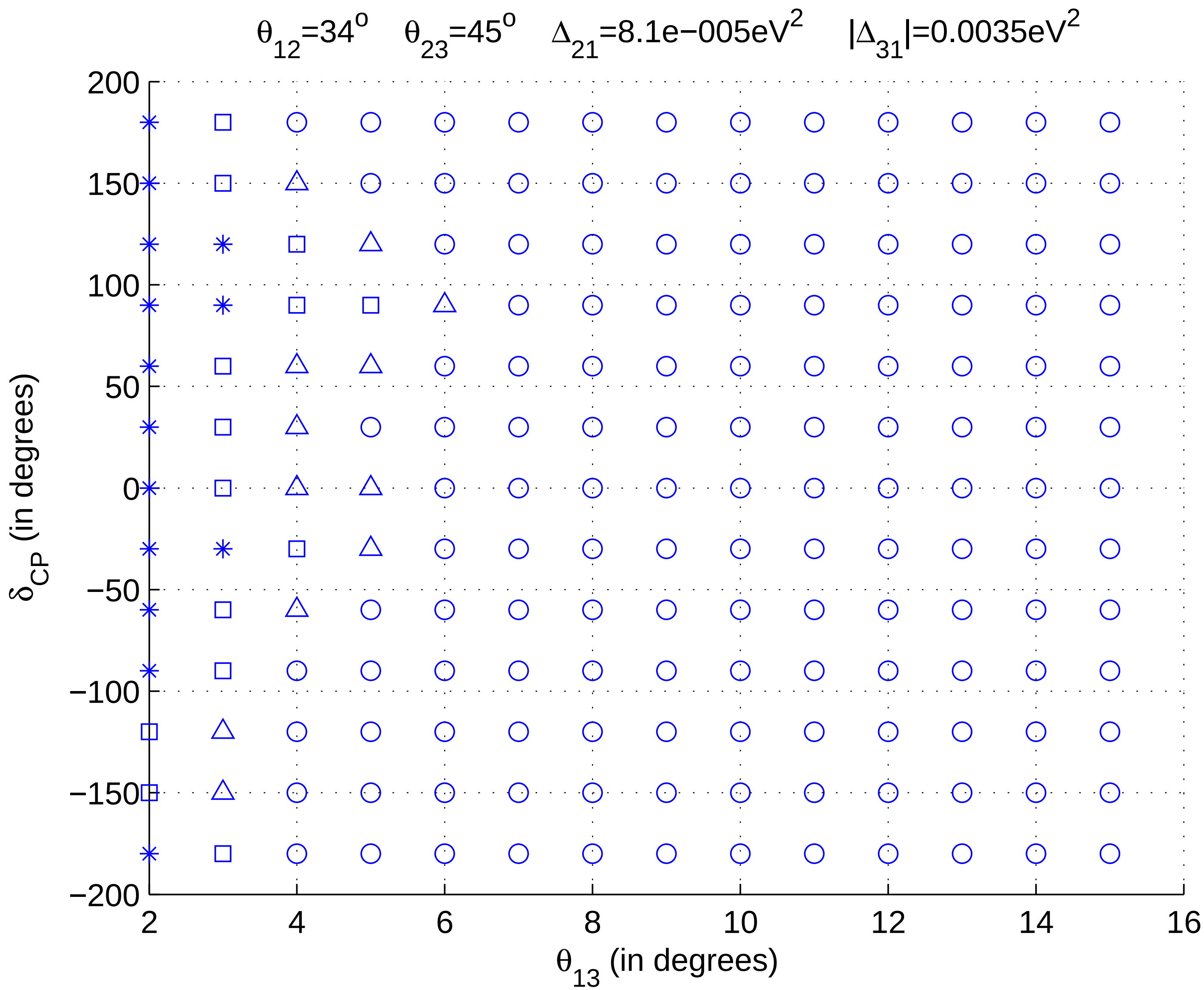
This is shown in the following figure for $\Delta_{31} = 2.5 \times 10^{-3}$ eV^2.



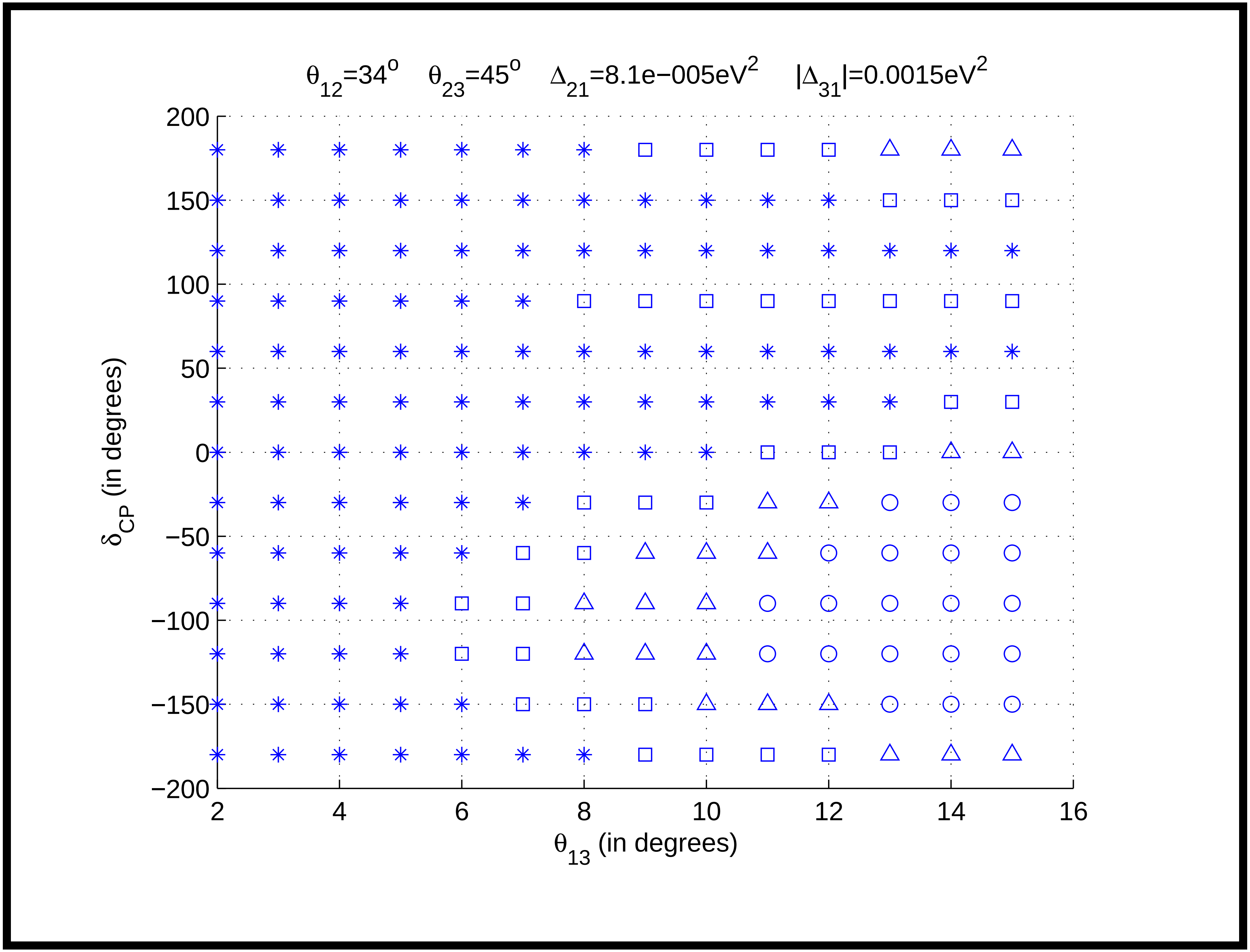
θ12=34° θ23=45° Δ21=8.1e−005eV² |Δ31|=0.0025eV²
δCP (in degrees)
θ13 (in degrees)
200
150
100
50
0
−50
−100
−150
−200
2
4
6
8
10
12
14
16

We find this minimum θ_{13}, for which the matter hierarchy can be resolved at 95% confidence, to be about 6°.

MINOS experiment allows the maximum value of $|\Delta_{31}|$ to be about 3.5×10^{-5} eV^2. For this value of $|\Delta_{31}|$ we get the following result of minimum $\theta_{13} = 4^\circ$.


θ12=34o θ23=45o Δ21=8.1e−005eV2 |Δ31|=0.0035eV2
δCP (in degrees)
θ13 (in degrees)
200
150
100
50
0
−50
−100
−150
−200
2
4
6
8
10
12
14
16

The minimum allowed value of $|\Delta_{31}|$ by Super-Kamiokande is 1.5×10^{-3} eV^2. For this small a value, the ability of current long baseline experiments to distinquish the matter hierarchy is quite bad. We get minimum $\theta_{13} = 16^\circ$.



θ12=34o θ23=45o Δ21=8.1e−005eV2 |Δ31|=0.0015eV2
δCP (in degrees)
θ13 (in degrees)
200
150
100
50
0
−50
−100
−150
−200
2
4
6
8
10
12
14
16