LEPTOGENESIS, DARK ENERGY, DARK MATTER, AND THE NEUTRINOS

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Neutrino Masses

In the standard model, neutrinos are massless

An effective operator: $\mathcal{L}_{eff} = \frac{1}{M_{\not{L}}} \ell_L \ell_L \phi\phi$

gives neutrino masses: $M_\nu = Const. \frac{\langle\phi\rangle^2}{M_{\not{L}}}$

ϕ: **Higgs doublet;** ℓ_L: **leptons;** $M_{\not{L}}$: **L violating scale.**

Different realizations of this operator are:

- **Models with right-handed neutrinos**
- **Models with triplet Higgs scalars**

Models with right-handed neutrinos

Introduce $\mathrm{SU(2)_L}$ singlet right-handed neutrinos:

$$\mathbf{N_{iR}, i = 1, 2, 3.}$$

Interactions of the right-handed neutrinos are:

$$\mathcal{L}_N = \mathbf{h}_{i\alpha}\ \bar{\mathbf{N}}_{Ri}\ \phi\ \ell_{L\alpha} + \mathbf{M}_{ij}\ \overline{(N_{Ri})^c}\ \mathbf{N}_{Rj}.$$

The mass terms now become

$$\begin{aligned}\mathcal{L}_{\mathrm{mass}} &= \mathbf{m}_{D\alpha i}\ \nu_\alpha\ \mathbf{N}_i^c + \mathbf{M}_i\ \mathbf{N}_i^c\ \mathbf{N}_i^c \\ &= \left(\nu_\alpha \quad \mathbf{N}_i^c\right)\begin{pmatrix} 0 & \mathbf{m}_{D\alpha i} \\ \mathbf{m}_{Di\alpha} & \mathbf{M}_i \end{pmatrix}\begin{pmatrix} \nu_\alpha \\ \mathbf{N}_i^c \end{pmatrix}.\end{aligned}$$

where $(\mathbf{N}_{iR})^c = \mathbf{N^c}_{iL}$ and $\mathbf{m}_D = \mathbf{m}_{D\alpha i} = \mathbf{h}_{\alpha i}\langle\phi\rangle$ **is the Dirac mass term.**

The neutrino mass matrix

$$\mathbf{M}_\nu = \begin{pmatrix} \mathbf{0} & \mathbf{m_D} \\ \mathbf{m_D} & \mathbf{M} \end{pmatrix},$$

can be diagonalized to get the physical states

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu \\ \mathbf{N^c} \end{pmatrix},$$

where $\tan\theta = \frac{\mathbf{2M}}{\mathbf{m_D}}$. **with mass eigenvalues**

$$\mathbf{m_1} = -\frac{\mathbf{m_D^2}}{\mathbf{M}} \quad \text{and} \quad \mathbf{m_2} = \mathbf{M}.$$

m_1 **is now see-saw suppressed and tiny.**

ψ_1 **are the observed light physical neutrinos.**

Triplet Higgs model

Extend standard model with a $SU(2)_L$ triplet Higgs field

$$\xi \equiv [\xi^{++}, \xi^{+}, \xi^{0}] \equiv (\mathbf{1}, \mathbf{3}, -\mathbf{2}).$$

Allowed couplings

$$\mathcal{L} = \mathbf{f_{ij}}\, \xi\, \ell_{\mathbf{i}}\, \ell_{\mathbf{j}} + \mu\, \xi^{\dagger}\, \phi\, \phi.$$

give Majorana mass to the neutrinos if $\langle \xi^0 \rangle \neq 0$:

$$m_{\nu ij} = f_{ij} \langle \xi^0 \rangle$$

Case I: $\mu = 0$ **implies L conservation – original model.**

Case II: $\mu \neq 0$ **implies explicit L violation.**

In both cases a tiny vev of ξ^0 is allowed.

In *case I* $\langle \xi^0 \rangle \neq 0$ will imply spontaneous lepton number violation

Non-observation of the Goldstone boson – Majoron in Z-decay rules out the model.

In *case II* since lepton number is broken explicitly, minimization of the potential will give $\langle \xi^0 \rangle \neq 0$.

At present this is fully consistent.

Consider Higgs potential with a doublet and a triplet Higgs

$$\mathbf{V} = \frac{1}{2}m^2\phi^\dagger\phi + M^2\xi^\dagger\xi + \frac{1}{4}\lambda_1(\phi^\dagger\phi)^2 + \frac{1}{4}\lambda_2(\xi^\dagger\xi)^2 + \frac{1}{2}\lambda_3(\xi^\dagger\xi)(\phi^\dagger\phi) + \frac{1}{2}\mu\xi^\dagger\phi\phi.$$

For $\mu \neq 0$, explicit lepton number violation

and for $\mathrm{M} \sim \mu \gg \; <\phi> = \mathrm{v}$, minimization gives

$$\langle\xi\rangle = \mathbf{u} = \frac{-\mu\mathbf{v^2}}{\mathbf{M^2}}$$

so that

$$\mathbf{m^\nu_{ij}} = \mathbf{f_{ij}u} = -\mathbf{f_{ij}}\frac{\mu\mathbf{v^2}}{\mathbf{M^2}}.$$

$M \sim \mu \sim M_{\not{L}}$ **is the lepton number violating scale.**

Leptogenesis with right-handed neutrinos

Mass terms with Right-handed neutrinos $(\mathrm{N_{Ri}}, \mathbf{i} = \mathrm{e}, \mu, \tau)$:

$$\mathcal{L}_{\mathrm{int}} = \mathrm{h}_{\alpha \mathrm{i}}\ \overline{\ell_{L\alpha}}\phi\ \mathrm{N_{Ri}} + \mathrm{M_i}\ \overline{(N_{Ri})^c}\ \mathrm{N_{Ri}}$$

Assume, $\mathrm{M_i}$ **diagonal and** $\mathrm{M_3} > \mathrm{M_2} > \mathrm{M_1}$**.**

Majorana mass of N **allows L-violating decays,**

$$\begin{aligned} \mathrm{N_{Ri}} &\rightarrow \ell_{\mathrm{jL}} + \bar{\phi}, \\ &\rightarrow {\ell_{\mathrm{jL}}}^{\mathrm{c}} + \phi. \end{aligned}$$

CP violation in the lepton number violating decays of N come from complex $h_{\alpha i}$ **and Majorana phases in** $M_{\alpha\beta}$**.**

The amount of lepton asymmetry in N_1 decay is given by,

$$\delta = -\frac{1}{8\pi}\frac{M_1}{M_2}\frac{\text{Im}[\sum_\alpha (h^*_{\alpha 1} h_{\alpha 2}) \sum_\beta (h^*_{\beta 1} h_{\beta 2})]}{\sum_\alpha |h_{\alpha 1}|^2}$$

The out-of-equilibrium condition

$$\frac{|\mathbf{h}_{\alpha 1}|^2}{16\pi}\mathbf{M}_1 < 1.7\sqrt{g_*}\frac{\mathbf{T}^2}{\mathbf{M}_\mathbf{P}} \qquad \text{at} \quad \mathbf{T} = \mathbf{M}_1,$$

is satisfied for $M_1 > 10^8$ GeV and sphalerons convert this asymmetry into a baryon asymmetry

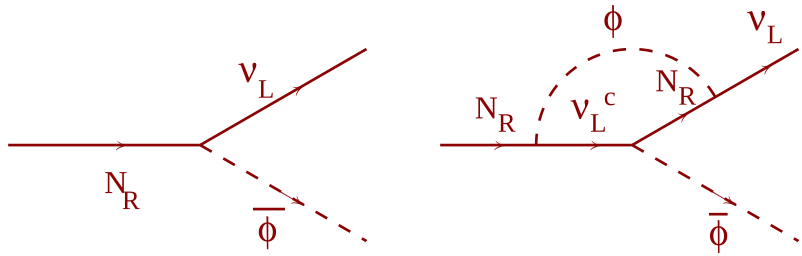
$$\frac{n_B}{s} = \frac{24 + 4n_H}{66 + 13n_H}\frac{n_{B-L}}{s}$$

which is consistent with the BBN and WMAP

$$\frac{n_B}{n_\gamma} = (6.15 \pm 0.25) \times 10^{-10} \qquad \text{with} \quad s = 7.04 n_\gamma$$

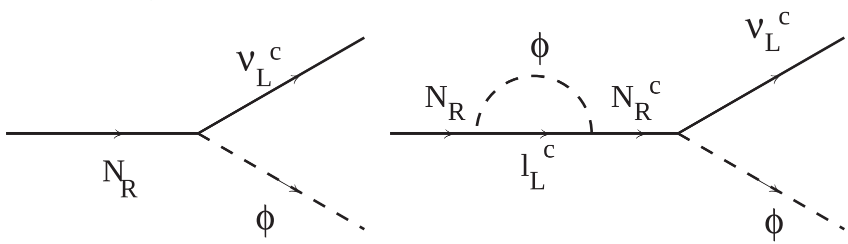
There are two sources of CP violation:

Vertex type diagrams interfering with tree level diagram. This is similar to the CP violation coming from the penguin diagram in K–decays (direct CP violation).



Self energy diagram interfering with tree level diagram. This is similar to CP violation in $K - \bar{K}$ oscillation (indirect CP violation entering in the mass matrix).

[M. Flanz, E.A. Paschos and US, PLB 95]

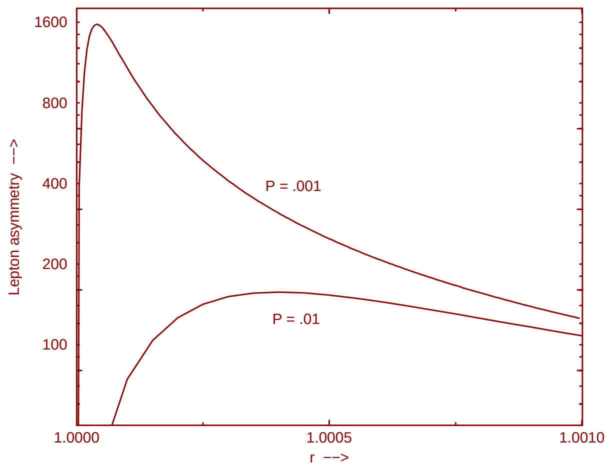


Resonant Leptogenesis

[M. Flanz, E.A. Paschos, J. Weiss and US, PLB 96]

For CP violation of self-energy type, there is a resonant effect for small mass difference between

N_{1R} **and** N_{2R}.



Leptogenesis with triplet higgs scalar

[E. Ma and US, PRL 98]

With $\mathrm{SU(2)_L}$ triplet Higgs scalars $\xi_\mathrm{a} \equiv (1, 3, -2)$, a=1,2, the Yukawa couplings and the mass terms are

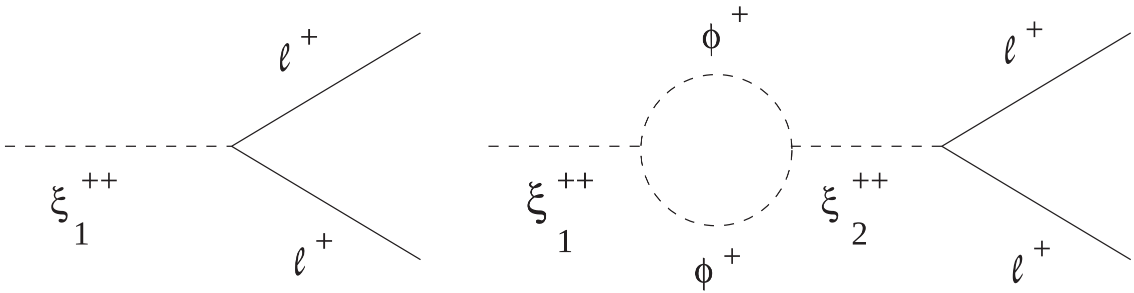
$$\mathcal{L} = \mathrm{f_{aij}}\, \xi_\mathrm{a}\, \ell_\mathrm{i}\, \ell_\mathrm{j} + \mu_\mathrm{a}\, \xi_\mathrm{a}^\dagger\, \phi\, \phi + \mathrm{M_{ab}}\, \xi_\mathrm{a}^\dagger\, \xi_\mathrm{b}$$

For $\mu \neq 0$, Lepton number violating decay modes are,

$$\xi_\mathrm{a}^{++} \rightarrow \begin{cases} l_i^+\, l_j^+ & (L = -2) \\ \phi^+\, \phi^+ & (L = 0) \end{cases}$$

CP violation comes from interference of tree level and one-loop self-energy type diagram (there are no vertex diagrams).

CP violation requires two triplet Higgs



The tree level and the self energy type diagrams interfere to generate a lepton asymmetry of the universe, given by,

$$\delta \simeq \frac{\mathrm{Im}\left[\mu_1 \mu_2^* \sum_{k,l} f_{1kl} f_{2kl}^*\right]}{8\pi^2 (M_1^2 - M_2^2)} \left[\frac{M_1}{\Gamma_1}\right].$$

Neutrino mass of the order of a few eV, and baryogenesis $\sim 10^{-10}$ are consistent with $M_1 \sim 10^{10}$ GeV.

Resonant leptogenesis is possible.

CP violation:

CP violation in models with right-handed neutrinos:

a) Vertex: $\Gamma(N_R \to \ell \phi^\dagger) \neq \Gamma(N_R^c \to \ell^c \phi)$

Similar to CP violation in K-decays (penguin diagrams)

This was considered by Sakharov and all earlier works.

b) Self-energy: $\Gamma(N_R \to N_R^c) \neq \Gamma(N_R^c \to N_R)$

Similar to $K^\circ \overline{K^\circ}$ oscillation (box diagrams)

CP violation in models with triplet Higgs scalars:

$$\Gamma[\xi_a \to \xi_b] \neq \Gamma[\xi_b \to \xi_a]$$

Similar to $\nu-$oscillations: $\Gamma[\nu_a \to \nu_b] \neq \Gamma[\nu_b \to \nu_a]$

DARK ENERGY:

Mass Varying Neutrinos (MaVaNs) postulates m_ν to be a dynamical field, or m_ν depends on acceleron field $\mathcal{A}$ and $\partial m_\nu / \partial \mathcal{A} \neq 0$.

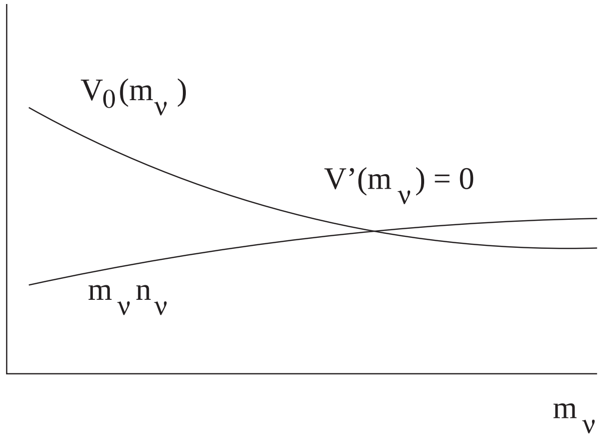
In NR limit, m_ν depends on n_ν (total density of the thermal background neutrinos).

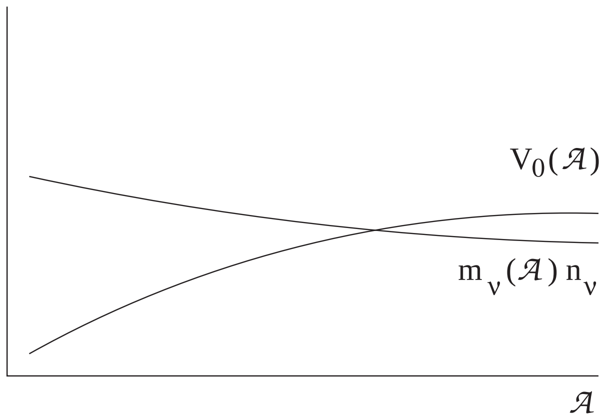
The effective potential of the system is

$$V(m_\nu) = m_\nu n_\nu + V_0(m_\nu).$$

Thermal background and scalar potential V_0 acts in opposite direction and the minimum of the potential satisfies

$$V'(m_\nu) = n_\nu + V_0'(m_\nu) = 0.$$


V0(mν)
V'(mν) = 0
mν nν
mν


V0(𝒜)
mν (𝒜) nν
𝒜

For a simple equation of state $p(t) = \omega\rho(t)$.

the density of matter varies with the scale factor as $\rho \propto R^{-3(1+\omega)}$.

$\omega(t)$ **depends on contribution of** $V_0(m_\nu)$, $\Omega_{DE} = \rho_{DE}/\rho_c$;

contributions of mass varying neutrinos $\Omega_\nu = m_\nu n_\nu/\rho_c$;

and is given by

$$\omega = -1 - \frac{V'(m_\nu)}{m_\nu V} = \frac{\Omega_\nu}{\Omega_\nu + \Omega_{DE}} - 1,$$

Observed value $\omega \sim -1$ **at present time implies** $\Omega(DE) \gg \Omega(\nu)$.

For small $d\omega/dn_\nu$, $m_\nu \propto n_\nu^\omega$.

We consider the scalar potential to be of the form

$$V_0(m_\nu) = \Lambda^4 \log\left(1 + \left|\frac{\mu}{m_\nu(\mathcal{A})}\right|\right).$$

This leads to an effective potential

$$-\mathcal{L}_{eff} = m_{\nu ij}(\mathcal{A})\ \nu_i \nu_j + H.c. + \Lambda^4 \log(1 + |M_1(\mathcal{A})/\mu|).$$

Thus the minima corresponds to $m_\nu \propto n_\nu^{-1}$,

implying $\omega = -1$.

Models with right-handed neutrinos:

In a model with right-handed neutrinos, start with Lagrangian

$$\mathrm{L} = \mathrm{m_{lr}}\nu_\mathrm{l}\nu_\mathrm{r} + \mathrm{M}(\mathcal{A})\nu_\mathrm{l}\nu_\mathrm{r} + \mathrm{h.c.} + \Lambda^4 \log(1 + |\mathrm{M}(\mathcal{A})/\mu|),$$

with $M(\mathcal{A})/mu \gg 1$, so that low-energy effective Lagrangian is

$$\mathrm{L} = \frac{\mathrm{m_{lr}^2}}{\mathrm{M}(\mathcal{A})}\nu_\mathrm{l}\nu_\mathrm{l} + \mathrm{h.c.} + \Lambda^4 \log(|\mathrm{M}(\mathcal{A})/\mu|)$$

The dynamics will depend on the form of $M(\mathcal{A})$, but equation of state will be same. Neutrino mass will depend on neutrino density.

Forms of $M(\mathcal{A})$ considered are:

$$\mathrm{M}(\mathcal{A}) = \lambda\mathcal{A} \quad \mathrm{rmand} \quad \mathrm{M}(\mathcal{A}) = \mathrm{M}e^{\mathcal{A}^2/\mathrm{f}^2}.$$

The main drawback is $m_{lr} \sim 1$ eV from naturalness.

Models with Triplet Higgs and Dark Energy:

Consider a triplet Higgs model for neutrino masses:

$$\mathcal{L} = \mathrm{f_{ij}}\ \xi \ell_\mathrm{i} \ell_\mathrm{j} + \mathrm{h}\ \mathcal{A} \xi^\dagger \phi \phi + \mathrm{m}_\xi^2 \xi^\dagger \xi$$

Integrating out ξ below the electroweak scale gives:

$$\mathcal{L} = \mathrm{f_{ij}}\ \frac{\langle \phi \rangle^2}{\mathrm{m}_\xi^2}\ \mathcal{A}\ \nu_\mathrm{l} \nu_\mathrm{l}$$

The natural scale for m_ξ is between 80–500 GeV

Triplet Higgs must be observed at LHC or ILC.
Acceleron production should have consequences.
Majoron coupling with charged fermions is small.
Varying neutrino mass leads to new phenomenology.

Neutrino Masses and Pseudo Nambu-Goldstone Bosons

C.T. Hill, I. Mocioiu, E.A. Paschos and US

Right-handed neutrino masses originate from a global symmetry breaking, but the corresponding Goldstone boson picks up tiny mass because of soft breaking of the global symmetry at low energies.

Consider Φ acquiring vacuum expectation values: $\langle\Phi\rangle = \sigma$

One may write the field as: $\Phi \to (\Phi^{phys} + \sigma)e^{i\phi/\sigma}$
Φ^{phys}: real physical field and $i\phi$: Nambu-Goldstone boson

The Lagrangian now becomes

$$\begin{aligned} \mathcal{L}_\Phi &= \frac{1}{2}M_\Phi^2\Phi^\dagger\Phi + \frac{1}{4}\lambda(\Phi^\dagger\Phi)^2 + \partial_\mu\Phi^\dagger\partial^\mu\Phi \\ &= \frac{\sigma^2}{f^2}\partial_\mu\phi\partial^\mu\phi + f(\Phi^\dagger\Phi) \qquad (1) \end{aligned}$$

where $f(\Phi^\dagger\Phi) = f[(\Phi^{phys} + \sigma)^2]$ represents terms excluding ϕ.

There are no mass terms for the Nambu-Goldstone bosons.

If the global symmetry is explicitly broken, Nambu-Goldstone bosons become massive.

For soft breaking, the theory remains renormalizable

$\Rightarrow$ light pseudo Nambu-Goldstone Boson (PNGB)

Nambu-Goldstone Bosons receive small mass radiatively.

The Model:

Extend the standard model with two singlet neutrinos $N_i, i = 1, 2$ and

two singlet scalars $\Phi_i, i = 1, 2$ with interactions

$$\mathcal{L}_M = \frac{1}{2}\alpha_1 \bar{N}_1 N_1^c \Phi_1 + \frac{1}{2}\alpha_2 \bar{N}_2 N_2^c \Phi_2.$$

It possess an $U(1)_A \times U(1)_B$ global symmetry, under which

$$N_1 \equiv (1, 0); \quad N_2 \equiv (0, 1); \quad \Phi_1 \equiv (2, 0); \quad \Phi_2 \equiv (0, 2)$$

Vacuum expectation values (vevs) $\langle \Phi_i \rangle = \sigma_i$

give Majorana masses $M_i = \alpha_i \sigma_i$ to N_i

and break global symmetries spontaneously

leading to Nambu-Goldstone bosons.

Writing $\Phi_i \to \sigma_i e^{2i\phi_i/f}$, it is possible to absorb the Nambu-Goldstone modes ϕ_i by shifting $N_i \to e^{i\phi_i} N_i$.

The global symmetries will prevent Yukawa couplings
$f_{\alpha i} N_\alpha \ell_i H$

If Yukawa couplings are included, 3-loop divergent diagrams will make the PNGB heavy.

Add soft terms $m_{\alpha i} \bar{N}_\alpha \ell_i$ that break $U(1)_{(A-B)}$ explicitly,

leading to one pseudo Nambu-Goldstone boson (PNGB):
$\phi = \phi_1 - \phi_2$.

Replacing fields $\Phi_i \to \sigma_i e^{i\phi_i}$ and including soft terms with phases, the Lagrangian becomes

$$\begin{aligned}\mathcal{L}_{mass} &= \frac{1}{2} M_1 \bar{N}_1 N_1^c e^{2i\phi_1/f} + \frac{1}{2} M_2 \bar{N}_2 N_2^c e^{2i\phi_2/f} + m e^{i\alpha} \bar{N}_1 \nu_1 \\ &+ m\epsilon e^{i\beta} \bar{N}_1 \nu_2 + \lambda m \epsilon' e^{i\gamma} \bar{N}_2 \nu_1 + \lambda m e^{i\xi} \bar{N}_2 \nu_2.\end{aligned}$$

Rephasing of the fields:

$$N_i \longrightarrow e^{i\phi_2/f} N_i \qquad \nu_i \longrightarrow e^{i\phi_2/f} \nu_i$$

leads to (with $2\eta = \gamma - \alpha + \beta - \xi$)

$$\begin{aligned}\mathcal{L}_\mu \quad = \quad & \frac{1}{2} M_1 \bar{N}_1 N_1^c e^{2i\phi/f} + \frac{1}{2} M_2 \bar{N}_2 N_2^c + m \bar{N}_1 \nu_1 + m\epsilon e^{i\eta} \bar{N}_1 \nu_2 \\ & + \lambda m \epsilon' e^{i\eta} \bar{N}_2 \nu_1 + \lambda m \bar{N}_2 \nu_2 + H.c.\end{aligned}$$

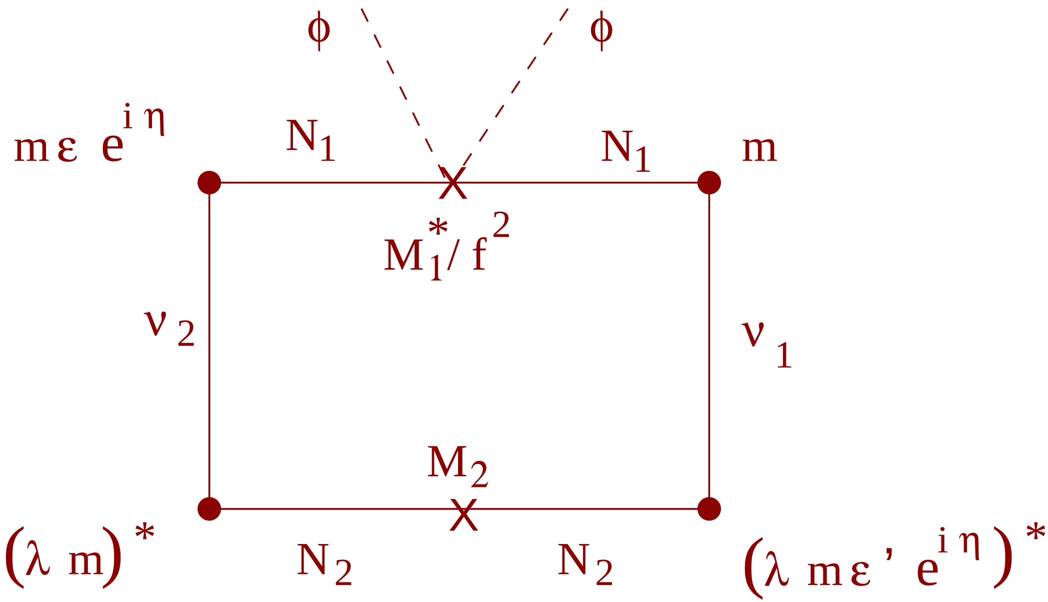
The field $\phi = \phi_1 - \phi_2$ cannot be rotated away.

No general phase transformations

$$N_i \longrightarrow e^{ip_i} N_i \qquad \text{and} \qquad \nu_i \longrightarrow e^{iq_i}$$

can absorb ϕ and η.

The radiative diagram (which is invariant under any rephasing) gives mass to the PNGB $i\phi = \mathcal{A}$.



Coleman-Weinberg effective potential

$$V_{eff}(\phi^2) = -\frac{m^4\lambda^2\epsilon\epsilon'}{4\pi^2}\frac{M_1M_2}{M_1^2-M_2^2}\cos\left(\frac{2\phi}{f}\right).$$

$$= -\frac{m^4\lambda^2\epsilon\epsilon'}{4\pi^2}\frac{M_1M_2}{M_1^2-M_2^2}\left(1+2\frac{\mathcal{A}^2}{f^2}+O(\frac{\mathcal{A}^4}{f^4})+\cdots\right).$$

implies small finite mass to the PNGB:

$$m_\phi = \frac{m^2\lambda\sqrt{\epsilon\epsilon'}}{\pi f}\frac{M_1M_2}{M_1^2-M_2^2}.$$

PNGB ϕ is now like an axion.

Origin of Soft terms

Introduce three doublet Higgs scalars H_0, H_1, H_2 so that the Yukawa couplings

$$\mathcal{L}_{mass} = f_{11}\bar{N}_1\ell_1 H_0 + f_{12}\bar{N}_1\ell_2 H_1 + f_{21}\bar{N}_2\ell_1 H_2 + f_{22}\bar{N}_2\ell_2 H_0.$$

do not break the global symmetries.

However, after electroweak symmetry breaking, soft terms breaking the global symmetry will be allowed.

After proper rephasing the final mass matrix becomes

$$-\mathcal{L}_{mass} = \frac{1}{2}M_1\bar{N}_1 N_1^c e^{2i\phi_1/f} + \frac{1}{2}M_2\bar{N}_2 N_2^c e^{2i\phi_2/f} + me^{i\alpha}\bar{N}_1\nu_1 + m\epsilon e^{i\beta}\bar{N}_1\nu_2$$
$$+\lambda m\epsilon' e^{i\gamma}\bar{N}_2\nu_1 + \lambda m e^{i\xi}\bar{N}_2\nu_2.$$

The PNGB then becomes the acceleron field.

Origin of Dark Energy:

Neutrino masses (M_1 and hence m^2/M) varies as a function of PNGB $\mathcal{A}$ (the acceleron)

$$\mathcal{L}_\mu = \frac{1}{2}M_1(\mathcal{A})\bar{N}_1^c N_1 + \frac{1}{2}M_2\bar{N}_2^c N_2 + m\bar{N}_1\nu_1 + m\epsilon\bar{N}_1\nu_2 + \lambda m\epsilon'\bar{N}_2\nu_1 + \lambda m\bar{N}_2\nu_2 + H.c.$$

where $M_1(\mathcal{A})$ varies with the acceleron field $\mathcal{A}$.

This can then explain why the scale of dark energy is close to the neutrino mass scale.

See-saw Neutrino Masses:

Assume $M_i \gg m_{ij}$**.**

The PNGB have a mass comparable to the neutrino masses and hence will show up long range force.

The time-development of the light states is determined by the matrix

$$-\mathcal{L}_{eff} = m_{ij}^T M_i^{-1} m_{ij} \ \nu_i \nu_j$$

$$= \frac{m^2}{M} \left[\begin{pmatrix} \nu_1 & \nu_2 \end{pmatrix} \begin{pmatrix} 1 + (\lambda\epsilon')^2 e^{2i\eta} & ei\eta(\epsilon + \lambda\epsilon') \\ ei\eta(\epsilon + \lambda\epsilon') & \lambda^2 + \epsilon^2 e^{2i\eta} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \right].$$

with mixing angle $\tan 2\theta = 2 \frac{\epsilon + \lambda\epsilon'}{(1-\lambda^2)e - i\eta + (\lambda^2\epsilon'^2 - \epsilon^2)ei\eta}$

which gives large mixing angle for $\epsilon, \epsilon' \ll 1$ **and** $\lambda \approx 1$**.**

This mass matrix can be diagonalized to

$$M^{diag} = \begin{pmatrix} m_1 e^{i\theta_1} & 0 \\ 0 & m_2 e^{i\theta_2} \end{pmatrix}$$ **with**

$$m_{1,2} = \frac{m^2}{M}\left[1 \pm (\epsilon + \lambda\epsilon') + O(\epsilon^2)\right], \quad \text{and} \quad \tan\theta_{1,2} = \frac{1 \pm (\epsilon + \lambda\epsilon')\cos\eta}{\pm(\epsilon + \lambda\epsilon')\sin\eta},$$

Including interactions of ϕ with neutrinos, the final mass and interaction Lagrangian is given by

$$\mathcal{L} = \frac{m^2}{2M}\,\overline{\Psi^c}\left\{\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} - \frac{\phi}{f}e^{i\eta}\begin{pmatrix} (\epsilon - \lambda\epsilon')e^{-i\theta_1} & 0 \\ 0 & (\lambda\epsilon' - \epsilon)e^{-i\theta_2} \end{pmatrix}\right\}\Psi + H.C.$$

The model thus predicts long-range force that can be detected in neutrino oscillation experiments.

Leptogenesis is possible in this model.

DARK MATTER:

It is possible to explain neutrino masses radiatively, that requires a second Higgs doublet.

The new Higgs doublet does not couple to usual fermions and can be a cold dark matter candidate with mass of around 50 GeV.

It is also possible to have right-handed neutrinos as cold dark matter with mass around 50 GeV with suppressed CC interactions.

SUMMARY:

Neutrinos might have played an extremely important role in the early universe.

Three major problems of the early universe: Baryon asymmetry of the universe, Dark matter and Dark energy, may have solutions hidden in the neutrino physics

Some of the models of neutrino physics have direct predictions that could be verified in the near future.