Neutrino Oscillation Phenomenology - I

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- Can be explained if neutrinos had mass and if they were also “mixed”

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- In other words in a simple 2 generation picture we can write

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- The states ν_1 and ν_2 are "mass eigenstates"
  The states ν_e and ν_μ are "flavor eigenstates"

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- After time t

$$\text{"}\nu_e(t)\text{''} = \cos\theta \; e^{-iE_1 t} \; \nu_1 + \sin\theta \; e^{-iE_2 t} \; \nu_2$$

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- "Survival Probability"

$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad (\Delta m^2 = m_2^2 - m_1^2)$$

Atmospheric Neutrinos

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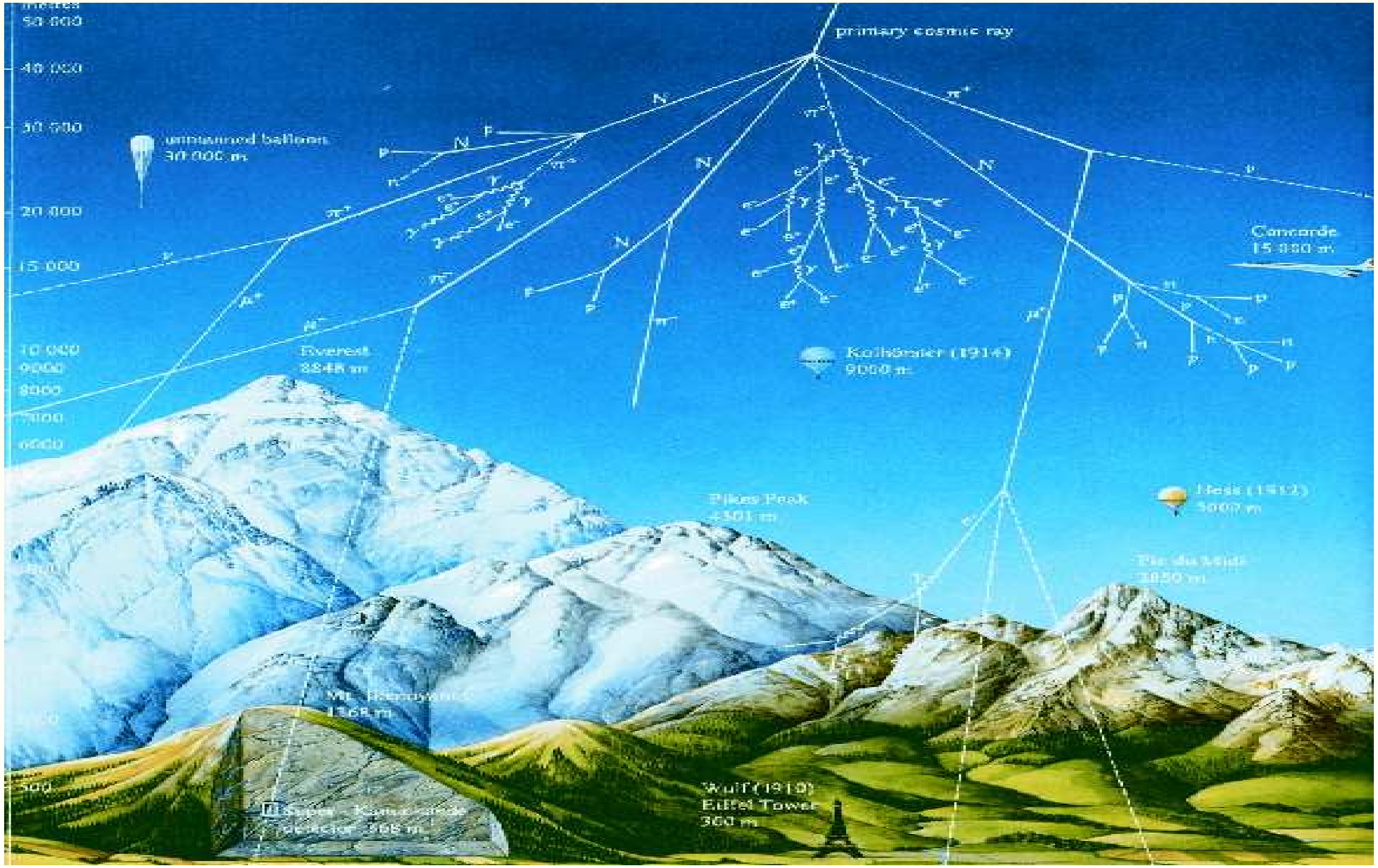
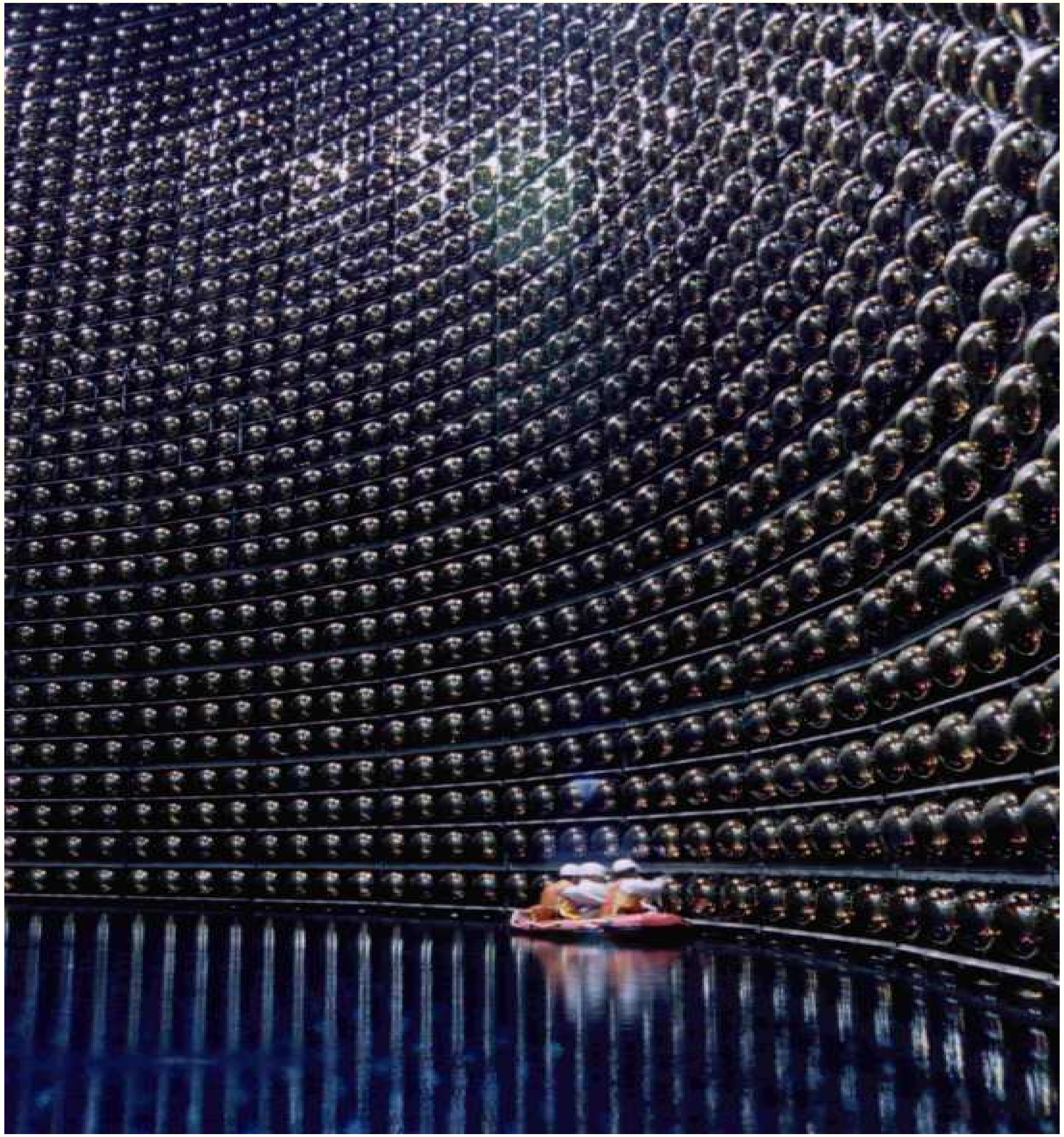


$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_{e}(\bar{\nu}_{e}) + \bar{\nu}_{\mu}(\nu_{\mu})$$

Atmospheric Neutrinos



Detected in SK via:
$\nu_l + N \rightarrow l + N'$

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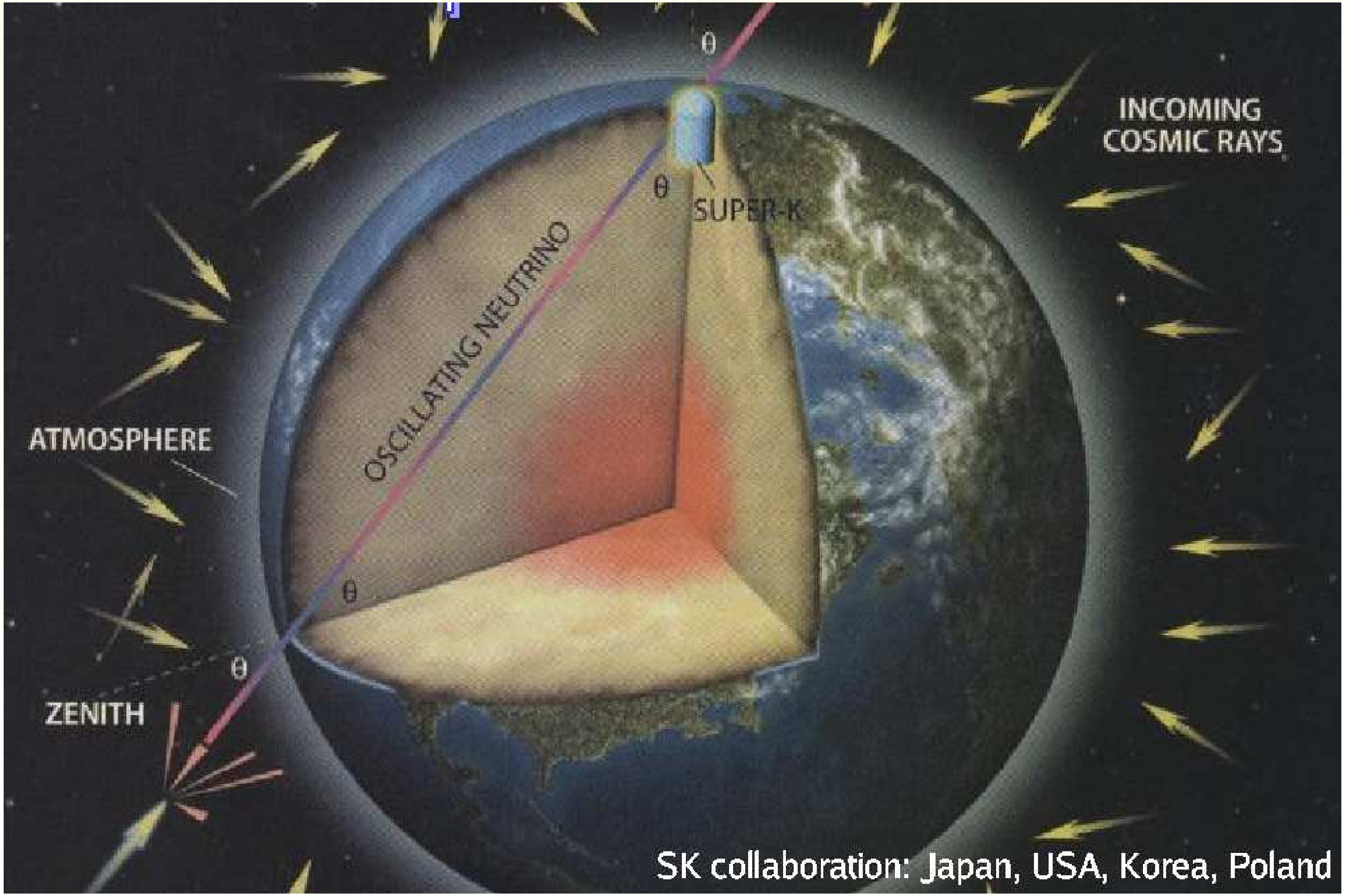
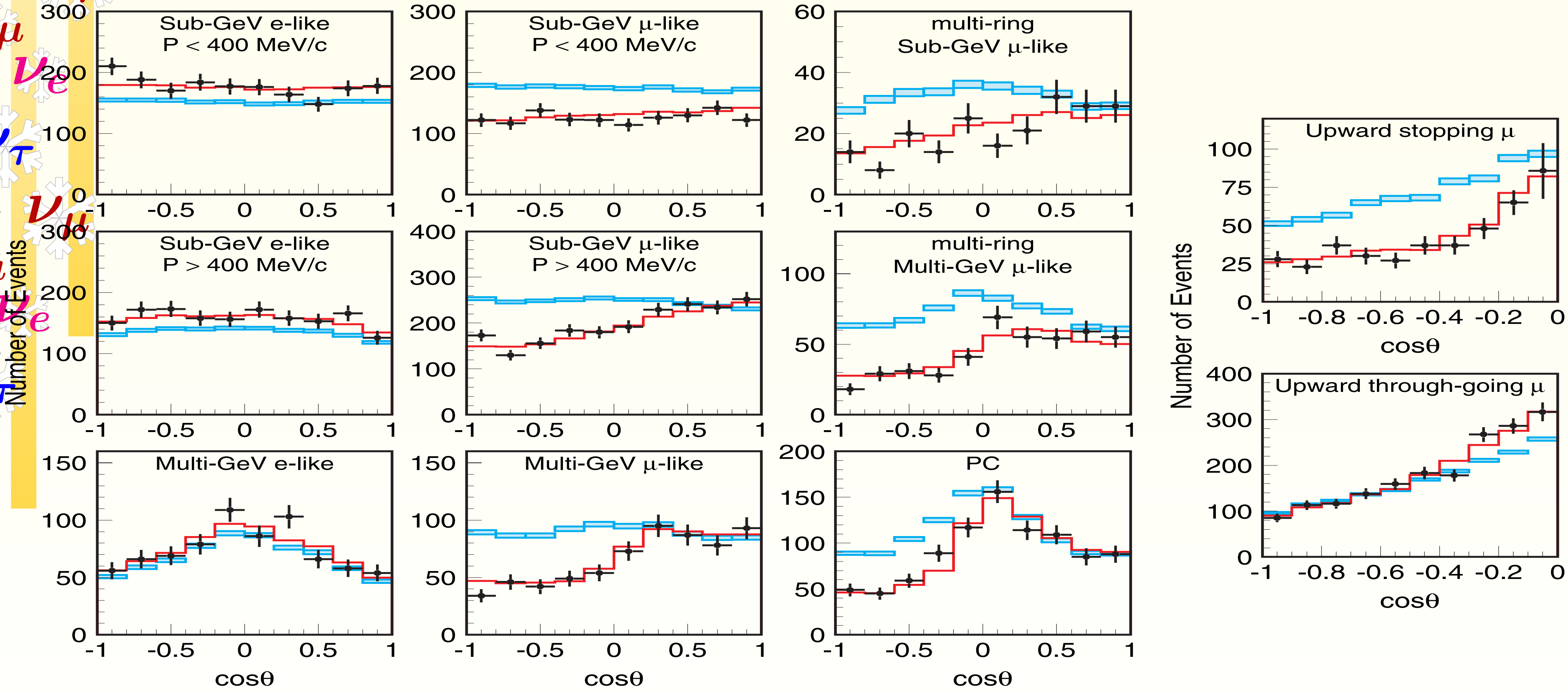


Atmospheric Neutrinos

Zenith Angle Distributions



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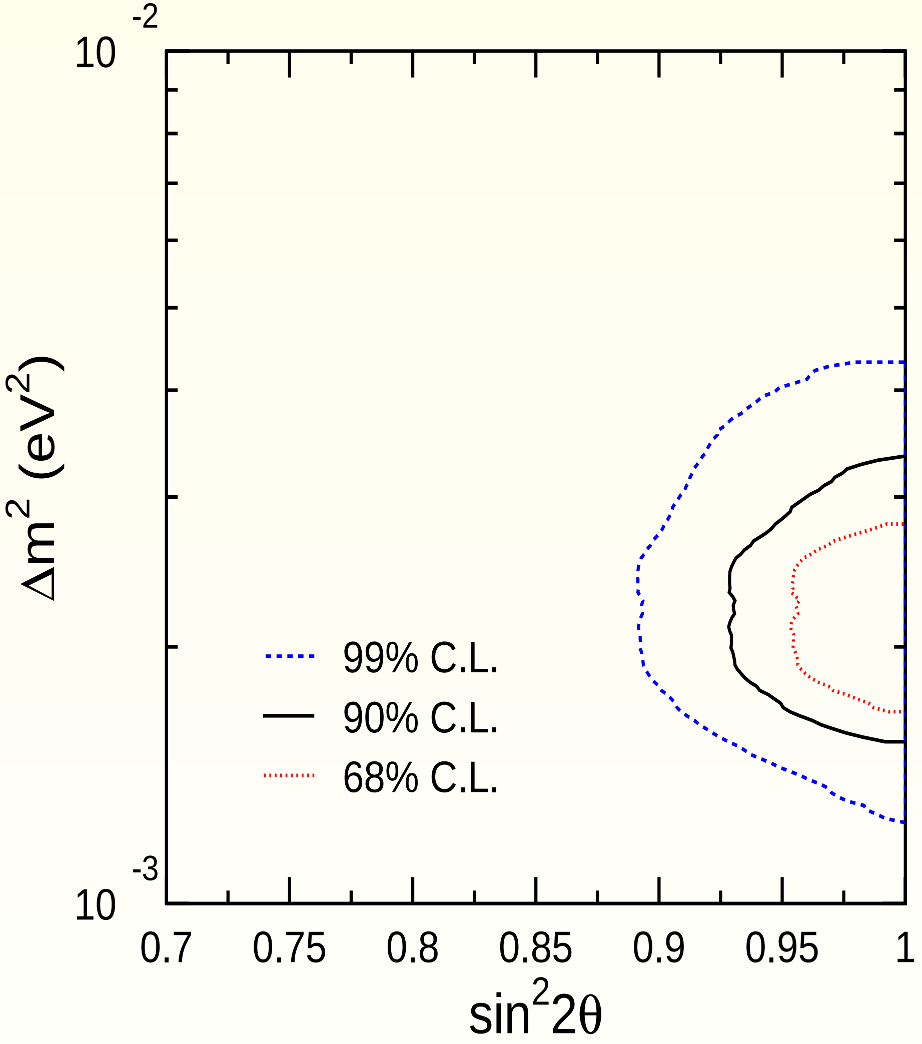


Data best explained by

$\nu_\mu \to \nu_\tau$ vacuum oscillations:

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Best-Fit:

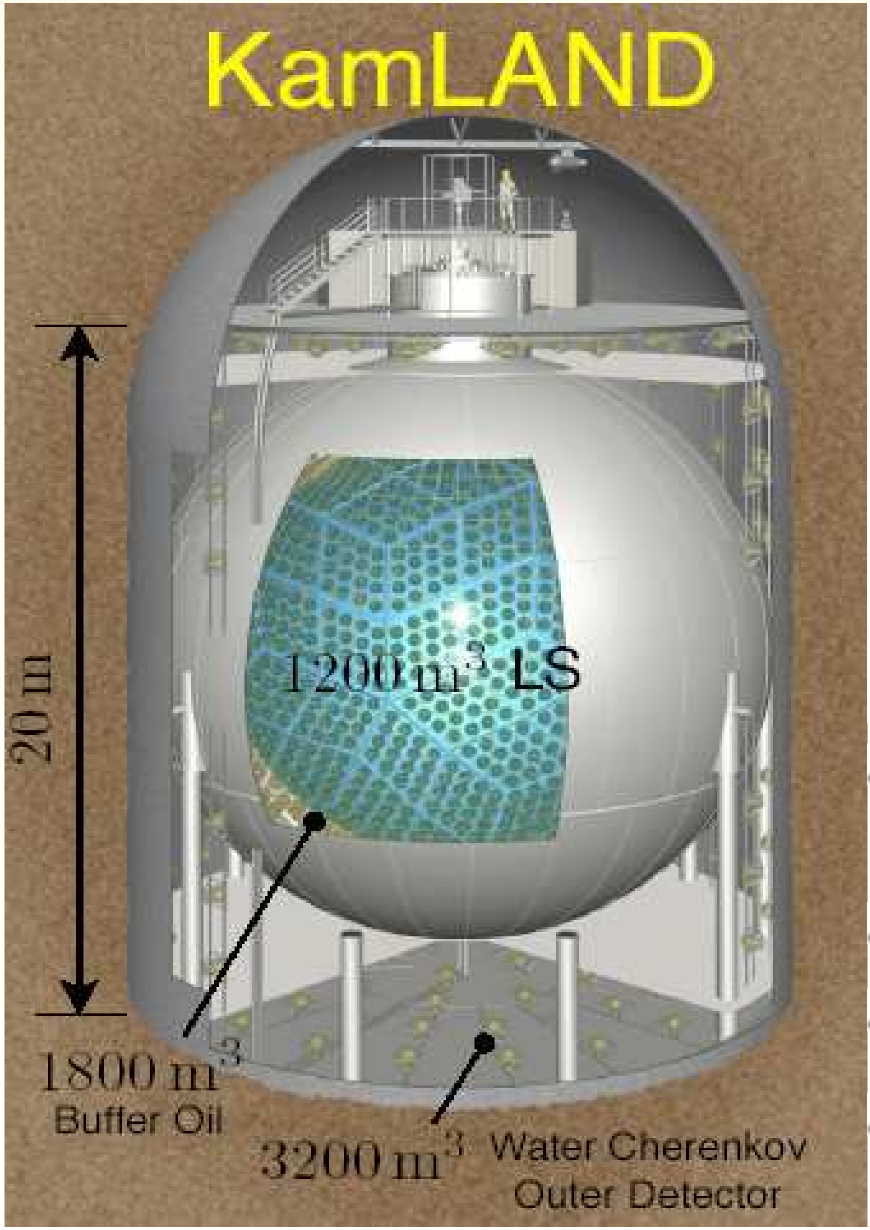
$\Delta m^2_{31} = 2.1 \times 10^{-3}$ eV^2

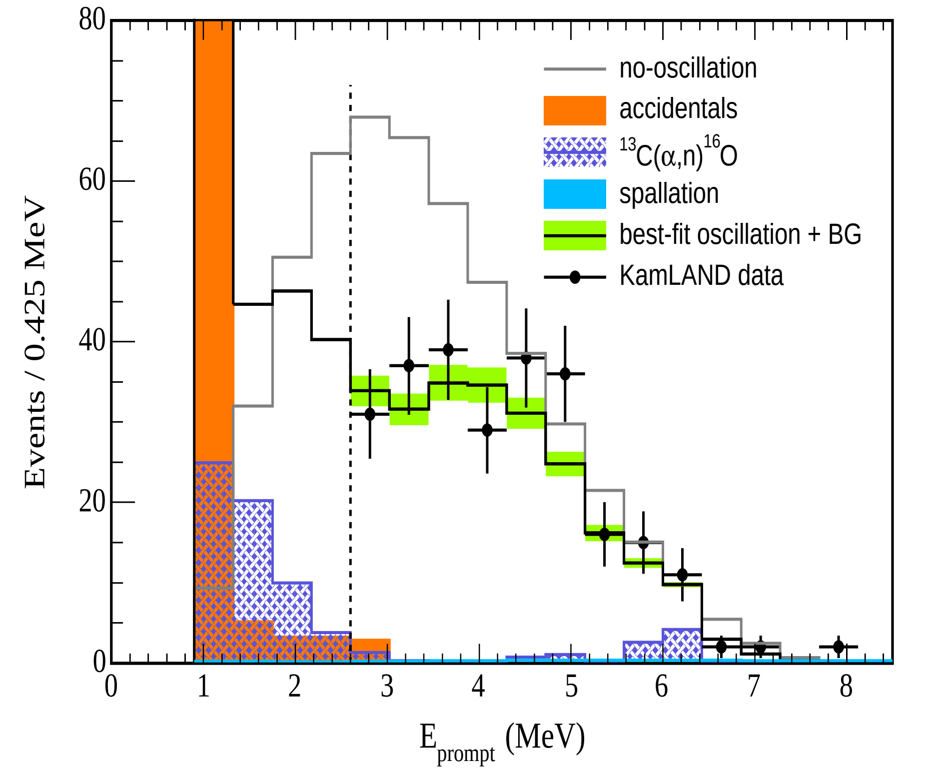
$\sin^2 2\theta_{23} = 1.0$

SK Collaboration, hep-ph/0501064

- Results confirmed by K2K (Japan) and MINOS (US)

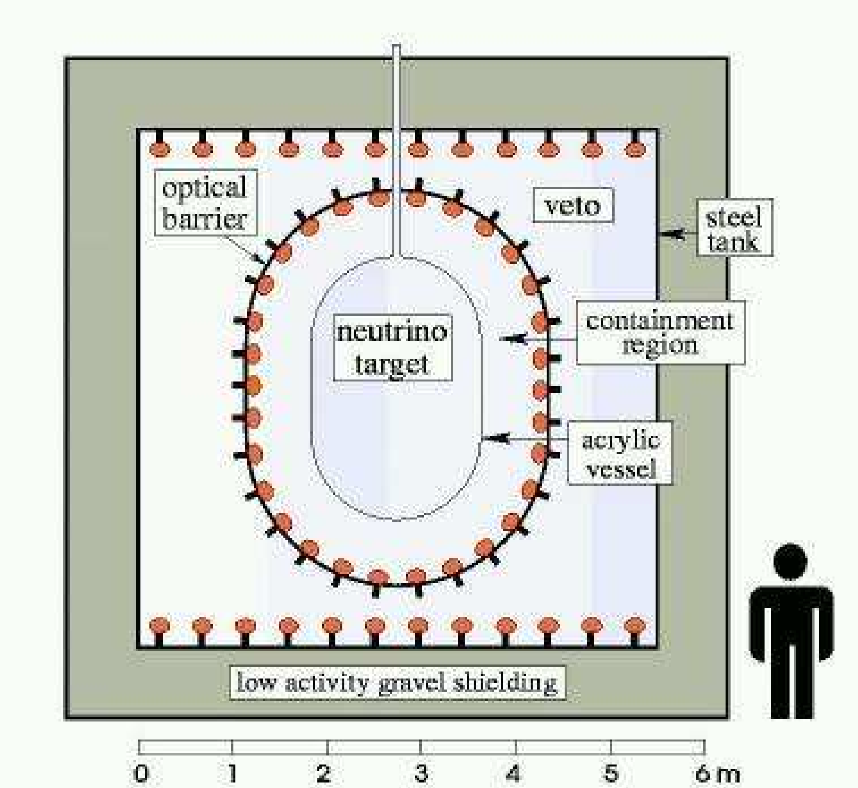
KamLAND





- Reactor antineutrinos detected in liquid scintillator
- $p + \bar{\nu}_e \rightarrow n + e^+$
- $\frac{Observed}{Expected} = 0.686 \pm 0.063$
- Δm^2_{21}=8×10^{-5} eV^2,$\sin^2 \theta_{12} = 0.3$

CHOOZ



- Detects $\bar{\nu}_e$ through

$$\bar{\nu}_e + p \to e^+ + n$$

- $L \approx 1$ km
- $E \sim 4$ MeV
- $L/E \sim 250$ km/MeV

$$\lambda_{21} \sim 124\text{km}; \;\; \lambda_{31} \sim 4\text{km}$$

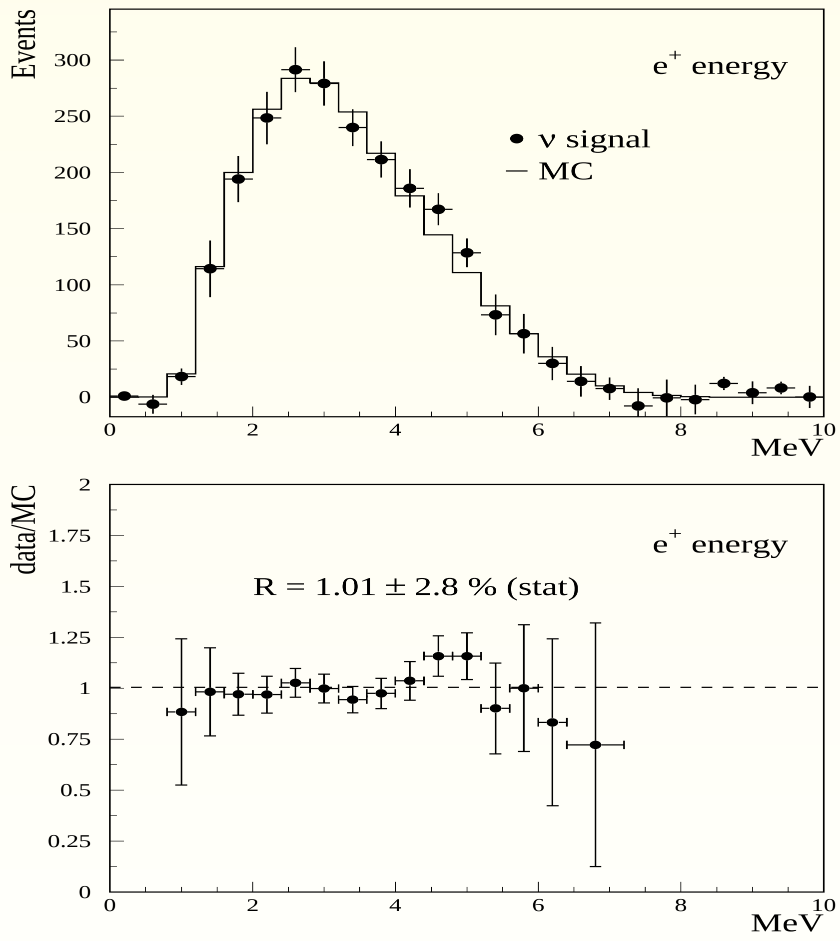
$$L << \lambda_{21} \Rightarrow \sin^2 \frac{\Delta m^2_{21} L}{4E} \to 0$$

- Δm^2_{31} driven oscillations will be relevant

CHOOZ

$$
\begin{aligned}
P_{ee} &= 1 - 4\sum_{j>i} |U_{ei}|^2|U_{ej}|^2 \sin^2\frac{\Delta m^2_{ji}L}{4E} \\
&= 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\frac{\Delta m^2_{21}L}{4E} \\
&\quad -4|U_{e1}|^2|U_{e3}|^2 \sin^2\frac{\Delta m^2_{31}L}{4E} - 4|U_{e2}|^2|U_{e3}|^2 \sin^2\frac{\Delta m^2_{32}L}{4E} \\
&\approx 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\frac{\Delta m^2_{21}L}{4E} \\
&\quad -4|U_{e3}|^2(|U_{e1}|^2 + |U_{e2}|^2) \sin^2\frac{\Delta m^2_{31}L}{4E} \\
&= 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\frac{\Delta m^2_{21}L}{4E} - 4|U_{e3}|^2(1 - |U_{e3}|^2) \sin^2\frac{\Delta m^2_{31}L}{4E} \\
&\approx 1 - 4|U_{e3}|^2(1 - |U_{e3}|^2) \sin^2\frac{\Delta m^2_{31}L}{4E} = 1 - \sin^2 2\theta_{13} \sin^2\frac{\Delta m^2_{31}L}{4E}
\end{aligned}
$$

CHOOZ



- No depletion of events observed
- $P_{ee}(observed) \approx 1$
- $P_{ee}(Th) = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m^2_{31} L}{4E}$
- We know that Δm^2_{31} was relevant for oscillations
- Therefore $\theta_{13} \to$ small

At 3σ, $\sin^2\theta_{13} < 0.04$

- Limit on θ_{13} will come from a χ^2 analysis of the data and will be determined mainly by the error involved.

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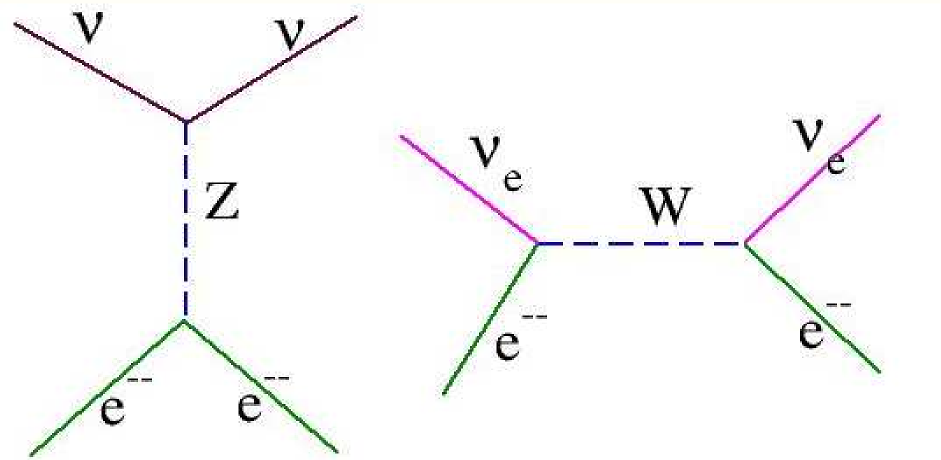
$$\begin{aligned} \nu_e + e^- &\rightarrow \nu_e + e^- \ (\mathrm{CC} + \mathrm{NC}) \\ \bar{\nu}_e + e^- &\rightarrow \bar{\nu}_e + e^- \ (\mathrm{CC} + \mathrm{NC}) \\ \nu_x + e, p, n &\rightarrow \nu_x + e, p, n \ (\mathrm{NC}) \end{aligned}$$



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- $V_{CC} = \sqrt{2}G_F N_e$

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- One can similarly get for the neutral current

$$V_{NC} = \sqrt{2} G_F \sum_f N_f [I_{3L}^{(f)} - 2\sin^2\theta_W Q^{(f)}]$$

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- Evolution equation in the flavor eigenbasis:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} = \frac{1}{2E}\mathcal{M}_F^m\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix}$$

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$$\mathcal{M}_F^m = U\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}U^\dagger + \begin{pmatrix}A+A_{nc} & 0\\ 0 & A_{nc}\end{pmatrix}$$

- $A = \pm 2\sqrt{2}G_F n_e E$ ($+ \Rightarrow$ neutrinos) ($- \Rightarrow$ antineutrinos)
- $A_{nc} = \mp\sqrt{2}G_F n_n E$ ($- \Rightarrow$ neutrinos) ($+ \Rightarrow$ antineutrinos)
- A has dimensions of eV^2
- $A = 1.5 \times 10^{-5}$ eV^2 $\left(\frac{E}{MeV}\right)$ [core of sun]
- $A = 1.7 \times 10^{-3}$ eV^2 $\left(\frac{\rho}{4.5\ gm/cc}\right)$ $\left(\frac{E}{5\ GeV}\right)$ [earth]

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- Evolution equation in the flavor eigenbasis:

$$i\frac{d}{dt}\begin{pmatrix}a_{ee}(t)\\ a_{e\mu}(t)\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}A - \Delta m^2\cos 2\theta & \Delta m^2 \sin 2\theta\\ \Delta m^2\sin 2\theta & -A + \Delta m^2\cos 2\theta\end{pmatrix}\begin{pmatrix}a_{ee}(t)\\ a_{e\mu}(t)\end{pmatrix}$$

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- Solve analytically the coupled differential equation of motion to get a_{ee} and then $P_{ee} = |a_{ee}|^2$.

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- Diagonalize the mass matrix in matter $\mathcal{M}_F^m$

$$U_m^\dagger \mathcal{M}_F^m U_m = \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix}$$

$$\nu_\alpha = \sum_k U_{\alpha k}^m \nu_k^m$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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- Eigenvalues of this mass matrix are:

$$M_{2,1}^2 = \pm\frac{1}{2}\left[(-A + \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2\right]^{1/2}$$

$$\Delta m_m^2 = \left[(-A + \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2\right]^{1/2}$$

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- Eigenvectors of this mass matrix give:

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{-A + \Delta m^2 \cos 2\theta}$$

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- If $N_e \to 0$, $A << \Delta m^2$

$$\begin{aligned} \Delta m_m^2 &\to \Delta m^2 \\ \theta_m &\to \theta \end{aligned}$$

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$$\theta_m \rightarrow \theta$$

- If $N_e \to$ very large, $A >> \Delta m^2$

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Oscillation Probabilities in Constant Matter

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MSW Resonance in Matter

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- We see that ν_1^m and ν_2^m themselves get mixed in matter with varying density. $\Rightarrow$ Not stationary states.
- However, if the condition

$$\left|\frac{d\theta_m}{dx}\right| \ll \left|M_2^2 - M_1^2\right|$$

is satisfied, then we can neglect the off-diagonal terms in the mass matrix above and then ν_1^m and ν_2^m go unchanged in matter.

Oscillation Probabilities in Varying Matter

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- This condition will be easily satisfied when:
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  - √ $A \ll \Delta m^2 \cos 2\theta$

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- In fact, it will have maximal violation when:
  - $\star\ A = \Delta m^2 \cos 2\theta \Rightarrow$ at the resonance

Oscillation Probabilities in Varying Matter

- Define an adiabaticity parameter

$$\gamma = \left| \frac{(M_2^2 - M_1^2)/2E}{d\theta_m/dx} \right|_{x=x_{res}}$$

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- $\gamma \gg 1 \Rightarrow$ Adiabatic Transition
- $\gamma \sim 1 \Rightarrow$ Non-Adiabatic Transition
- $\gamma \ll 1 \Rightarrow$ Extreme Non-Adiabatic Transition

Adiabatic Transition in Matter – MSW Effect

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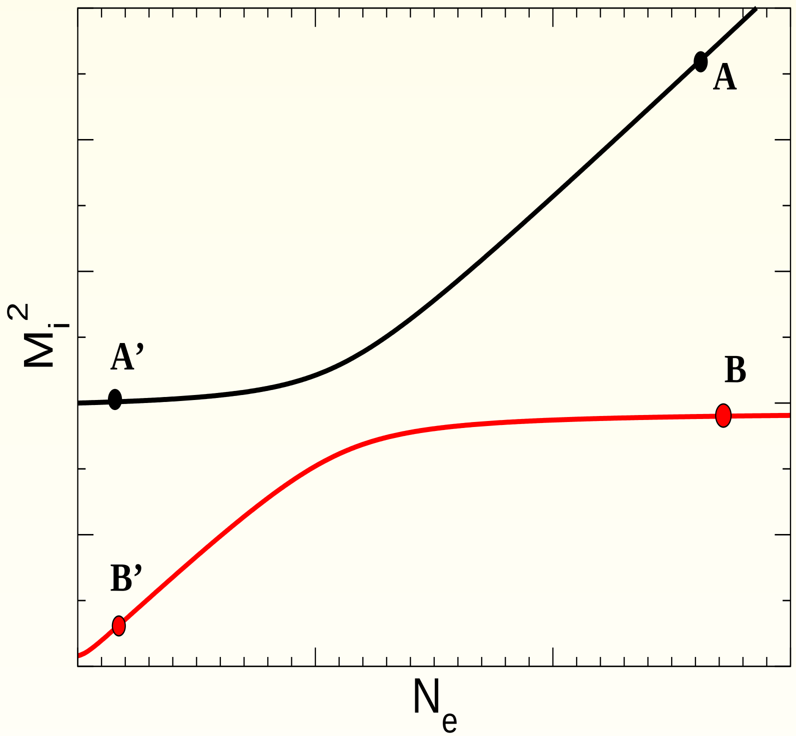

- The adiabaticity condition is satisfied so the mass eigenstates (created at A or B) evolve independently in matter and emerge in vacuum at A' and B'

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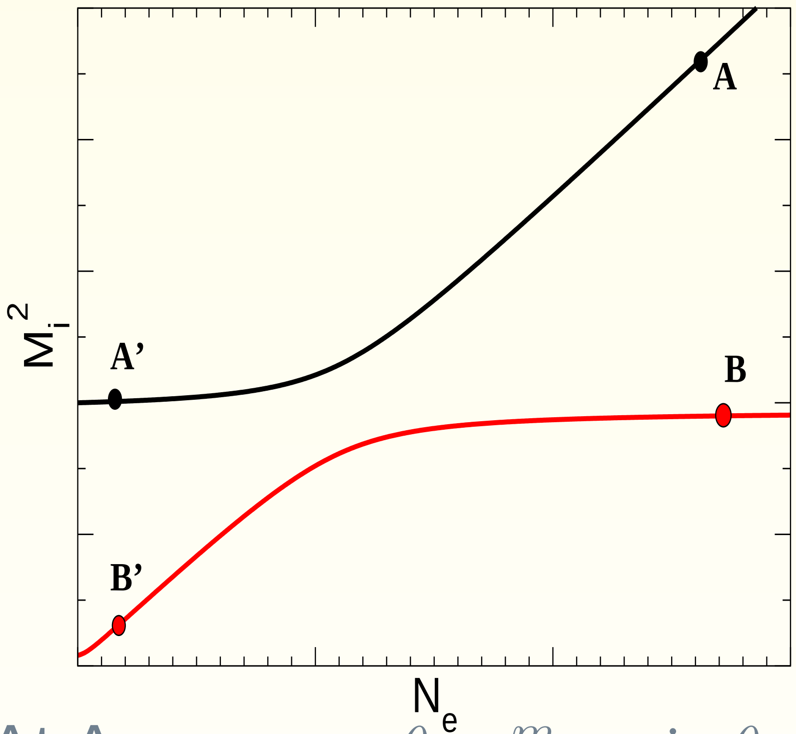

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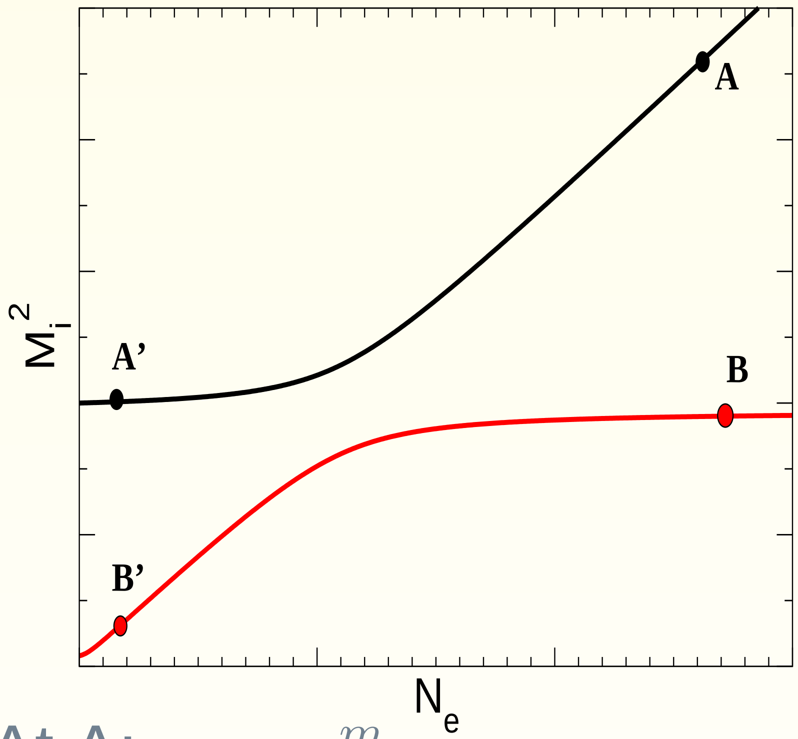

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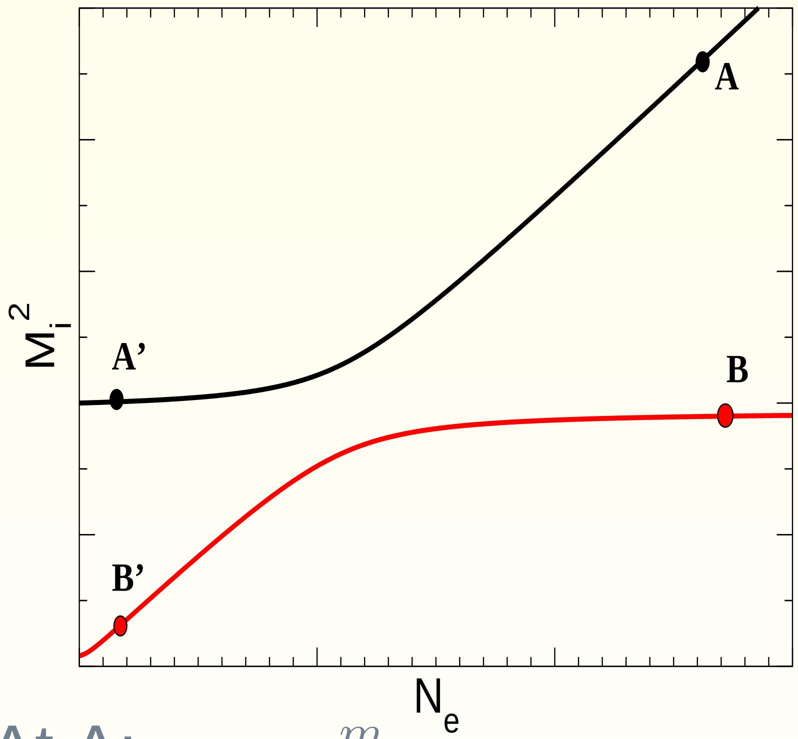

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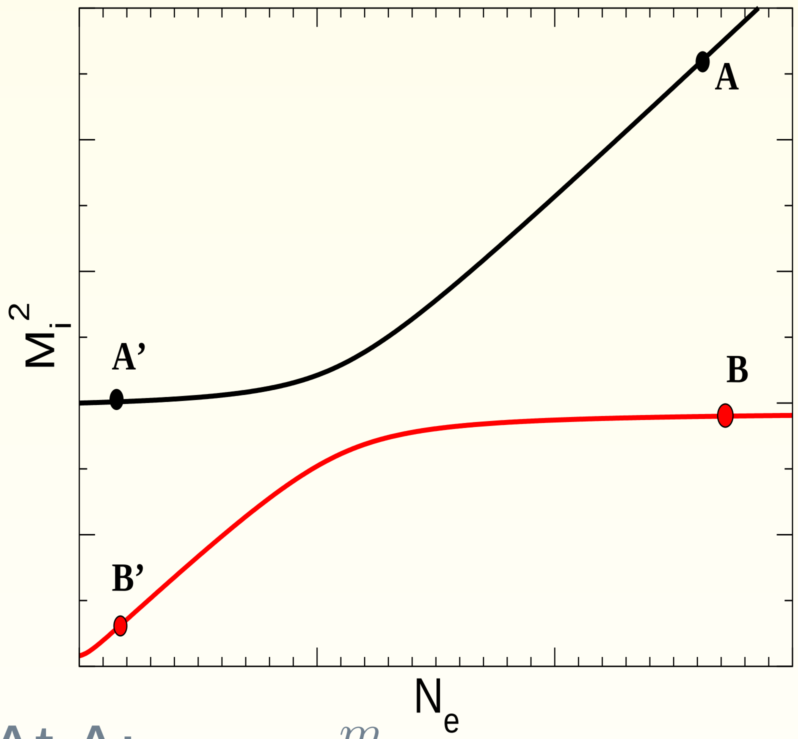

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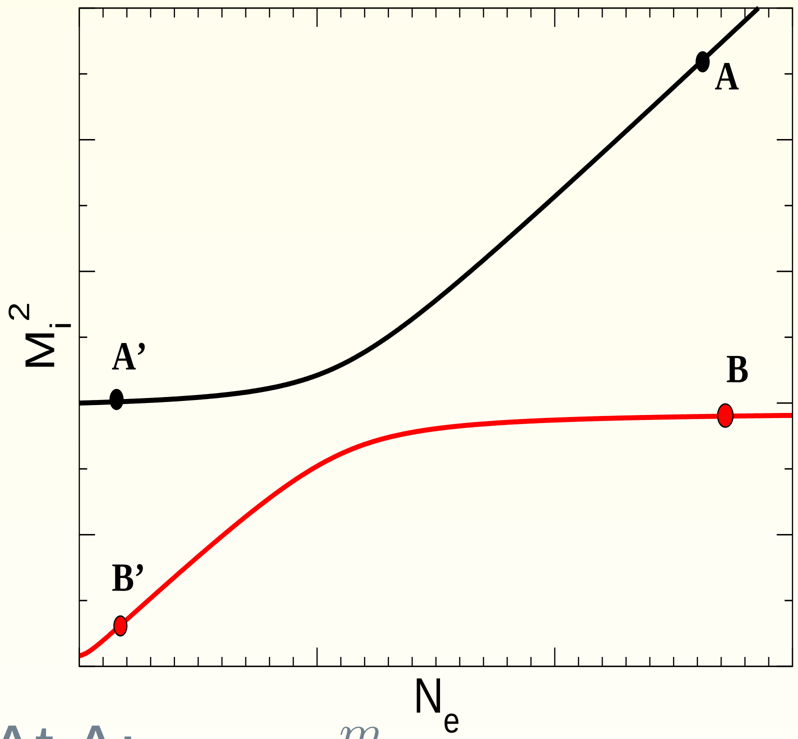

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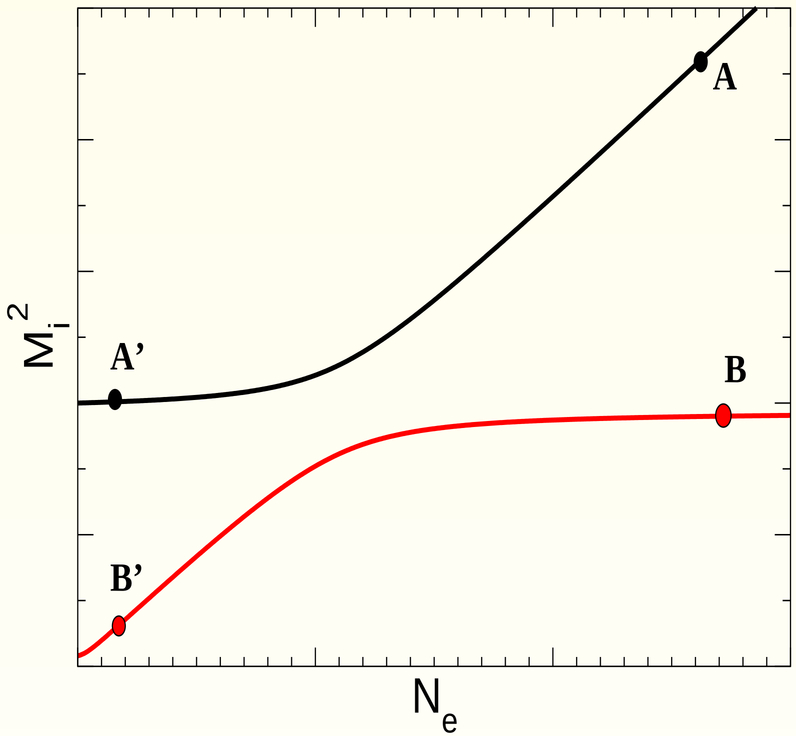
- $\boxed{P_{ee} = \sin^2\theta}$ (MSW Effect)

Adiabatic Transition in Matter – MSW Effect



- The adiabaticity condition is satisfied so the mass eigenstates (created at A or B) evolve independently in matter and emerge in vacuum at A’ and B’

- For general adiabatic case where θ_m is not = $\pi/2$

$$P_{ee} = \frac{1}{2} + \frac{1}{2}\cos 2\theta \cos 2\theta_m$$

Non-Adiabatic Transition in Matter

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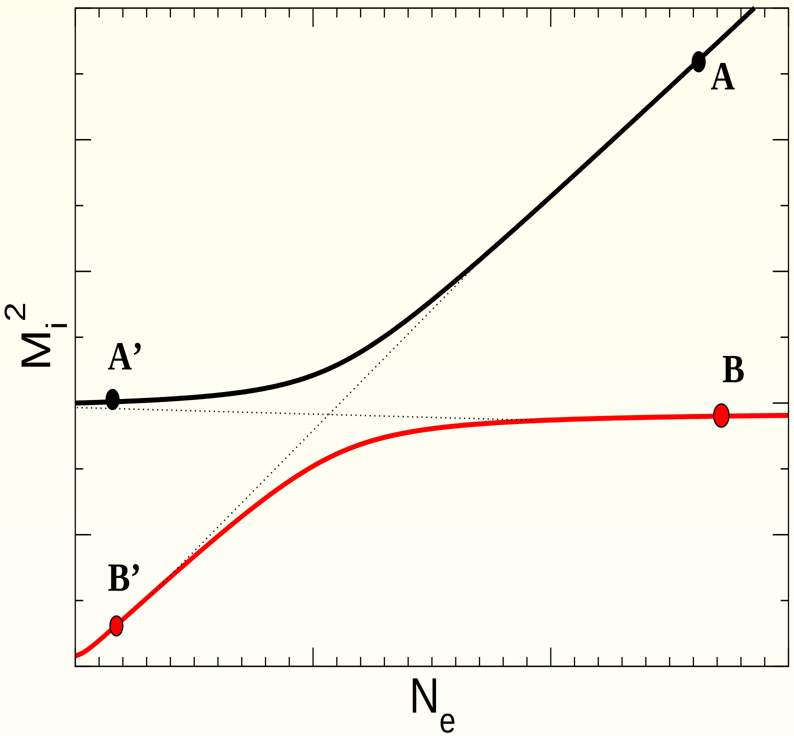

- The adiabaticity condition is broken so there is a finite probability that the mass eigenstates (created at A or B) cross-over to each other at the resonance

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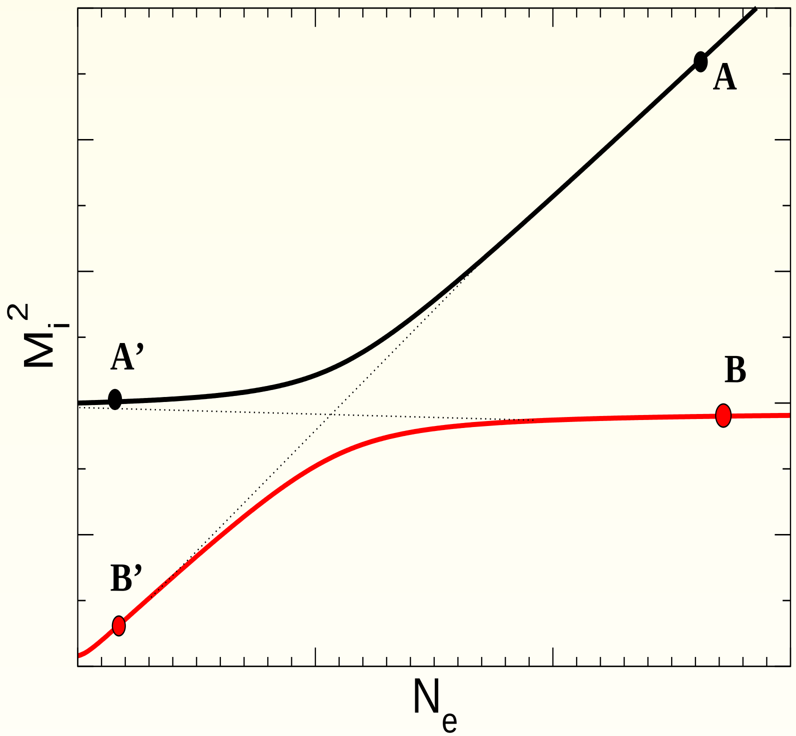


- The adiabaticity condition is broken so there is a finite probability that the mass eigenstates (created at A or B) cross-over to each other at the resonance

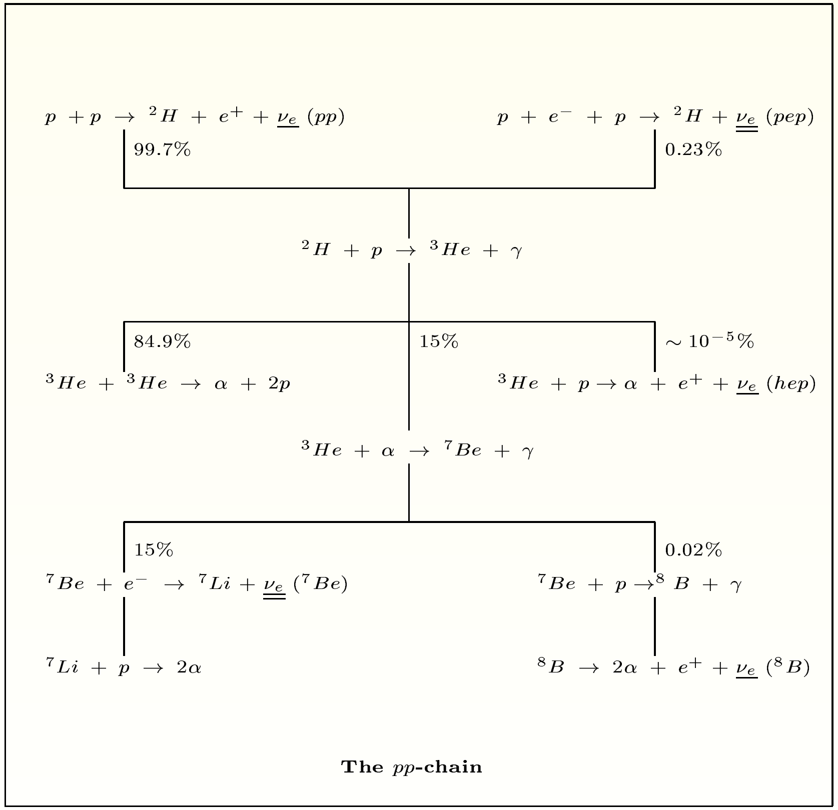
- The survival probability is

$$P_{ee} = \frac{1}{2} + \left(\frac{1}{2} - P_J\right) \cos 2\theta \cos 2\theta_m$$

$$P_J \equiv |\langle \nu_1^m(x_+)|\nu_2^m(x_-)\rangle|^2$$

Solar Neutrinos

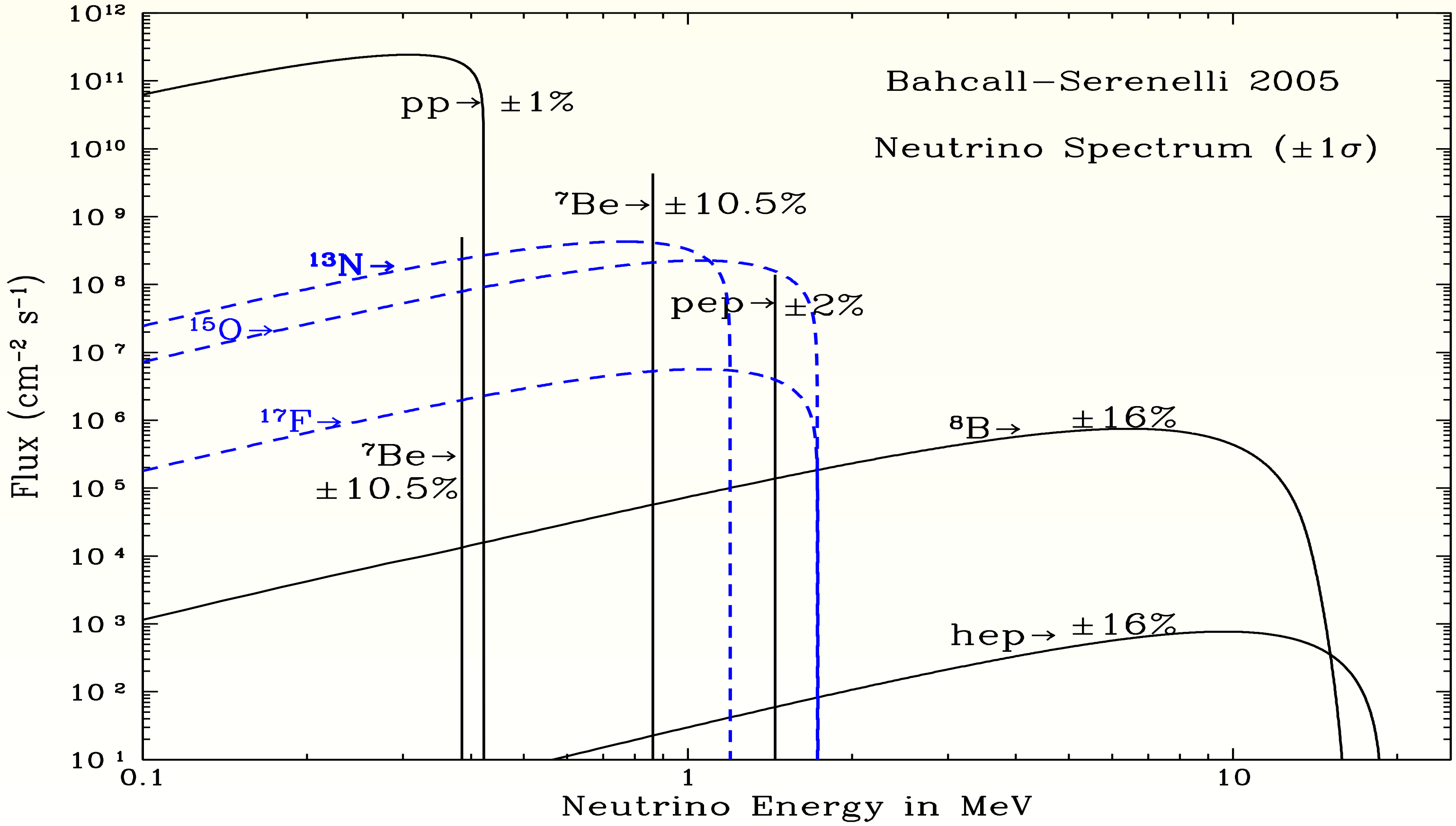
Solar Neutrino Flux



Solar Neutrino Flux

$$4p \rightarrow ^4He + 2e^+ + 2\nu_e + 28\text{MeV} \quad \text{ONLY } \nu_e$$

- Standard Solar Model gives the solar neutrino fluxes:



Solar Neutrino Experiments

Solar Neutrino Experiments

- Chlorine Experiment $E_{th} = 0.81 MeV$
  - $\star$ $^{37}Cl + \nu_e \rightarrow ^{37} Ar + e^-$ $^{7}Be, ^{8}B$

$$Observed/SSM(BP2004) = 0.301 \pm 0.027$$

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$$Observed/SSM(BP2004) = 0.301 \pm 0.027$$

- Gallium Experiment $E_{th} = 0.23 MeV$
  - $\star\ ^{71}Ga + \nu_e \rightarrow ^{71}Ge + e^-$ $pp, ^7Be, ^8B$

$$Observed/SSM(BP2004) = 0.52 \pm 0.029$$

Solar Neutrino Experiments

- Chlorine Experiment $E_{th} = 0.81 MeV$
  - $\star\ ^{37}Cl + \nu_e \rightarrow ^{37}Ar + e^-$ $^7Be, ^8B$

$$Observed/SSM(BP2004) = 0.301 \pm 0.027$$

- Gallium Experiment $E_{th} = 0.23 MeV$
  - $\star\ ^{71}Ga + \nu_e \rightarrow ^{71}Ge + e^-$ $pp, ^7Be, ^8B$

$$Observed/SSM(BP2004) = 0.52 \pm 0.029$$

- Super-Kamiokande $E_{e,th} = 5.0 MeV$
  - $\star\ e^- + \nu_x \rightarrow e^- + \nu_x,$ ^8B,hep

$$Observed/SSM(BP2004) = 0.406 \pm 0.014$$

Solar Neutrino Experiments

- SNO $T_{e,th} = 5.0 MeV$
  - ⋆ $d + \nu_e \rightarrow p + p + e^-$ (CC) ^8B,hep

    $$Observed/SSM(BP2004) = 0.290 \pm 0.018$$

  - ⋆ $d + \nu_x \rightarrow p + n + \nu_x$ (NC)

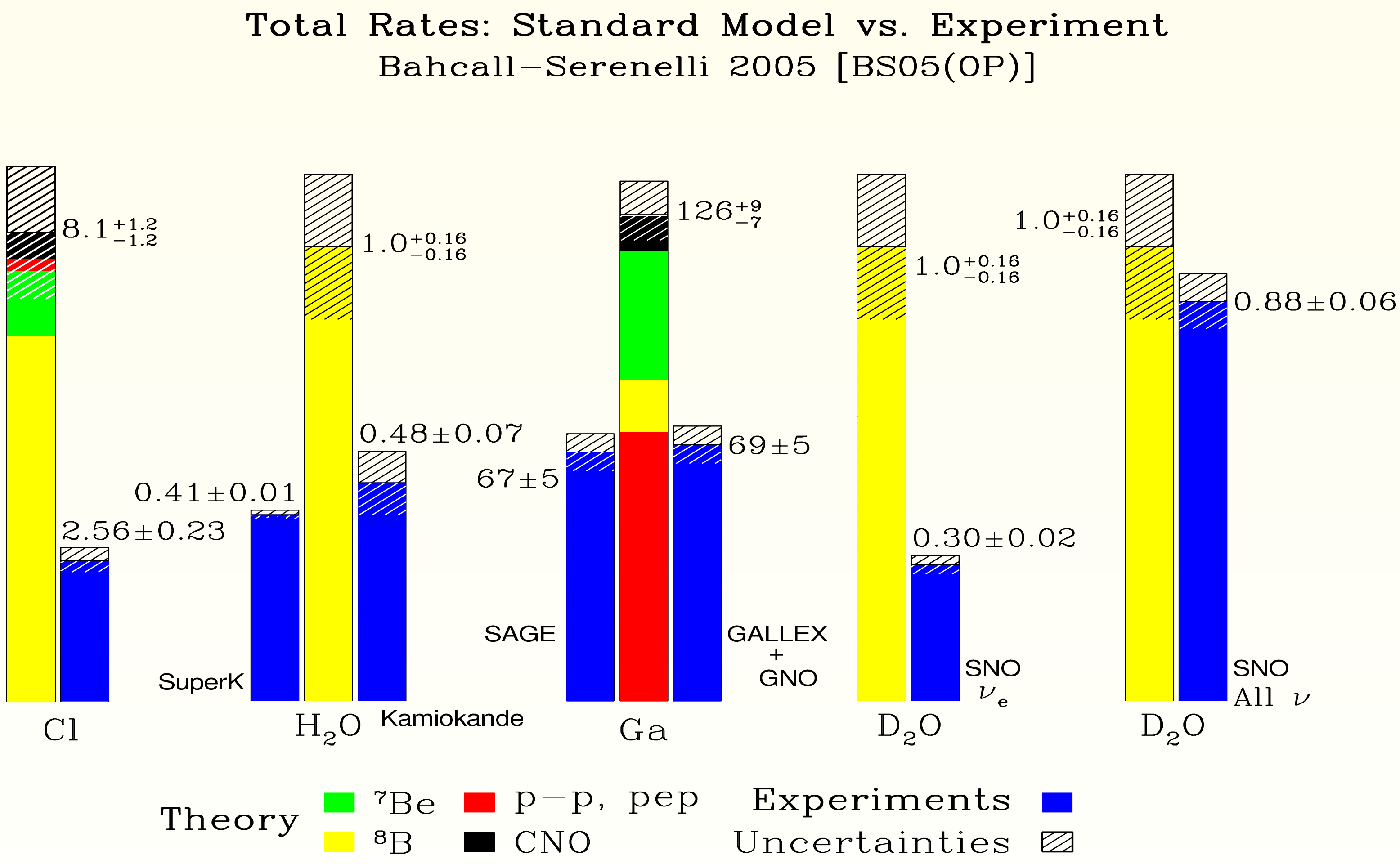
    $$Observed/SSM(BP2004) = 0.853 \pm 0.072$$

  - ⋆ $e^- + \nu_x \rightarrow e^- + \nu_x$ (ES)

    $$Observed/SSM(BP2004) = 0.406 \pm 0.046$$

Features of Solar Neutrino Data

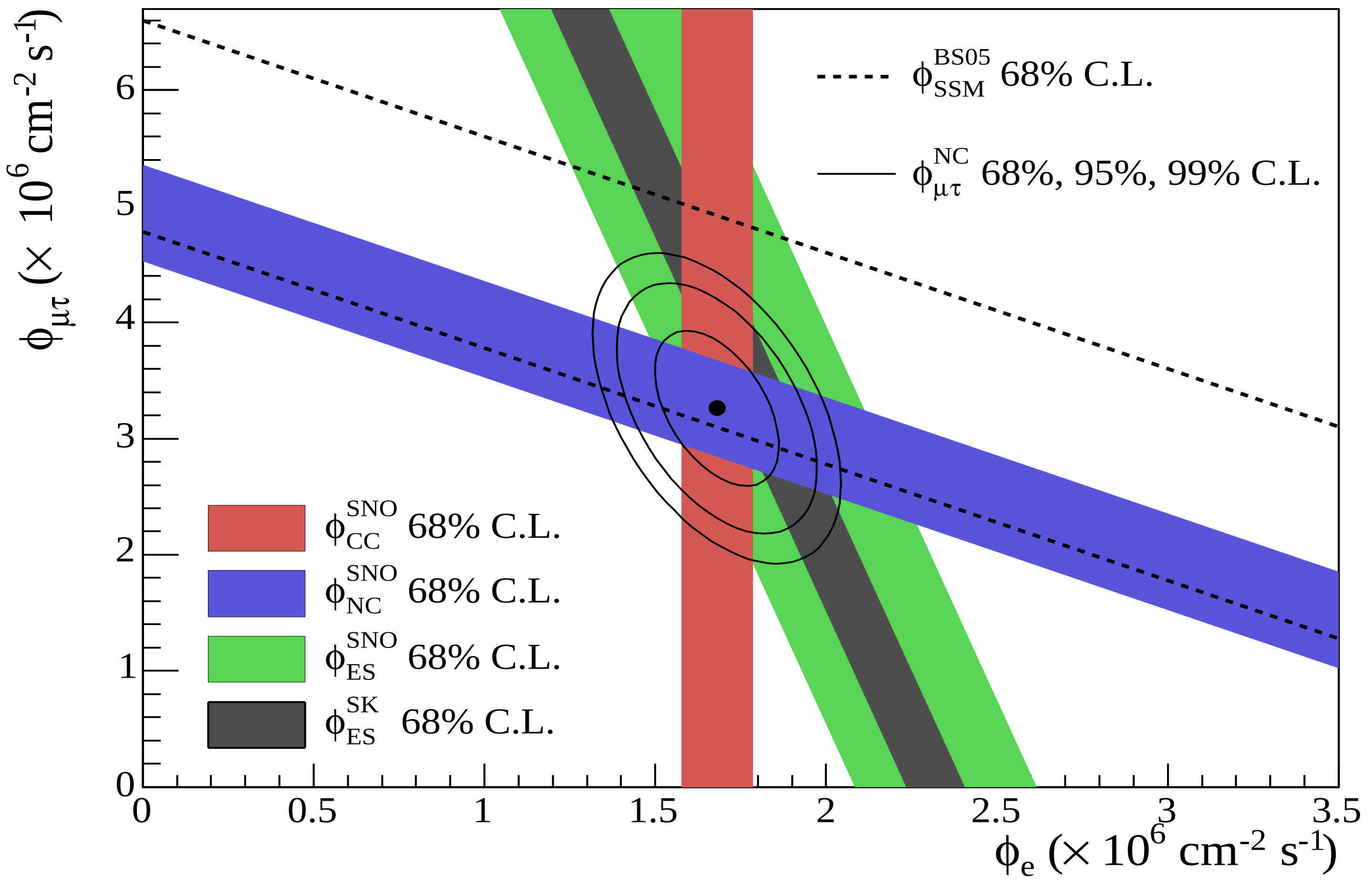
Features of Solar Neutrino Data



J.N. Bahcall, *http://www.sns.ias.edu/ jnb/*

- Neutrinos are definitely oscillating

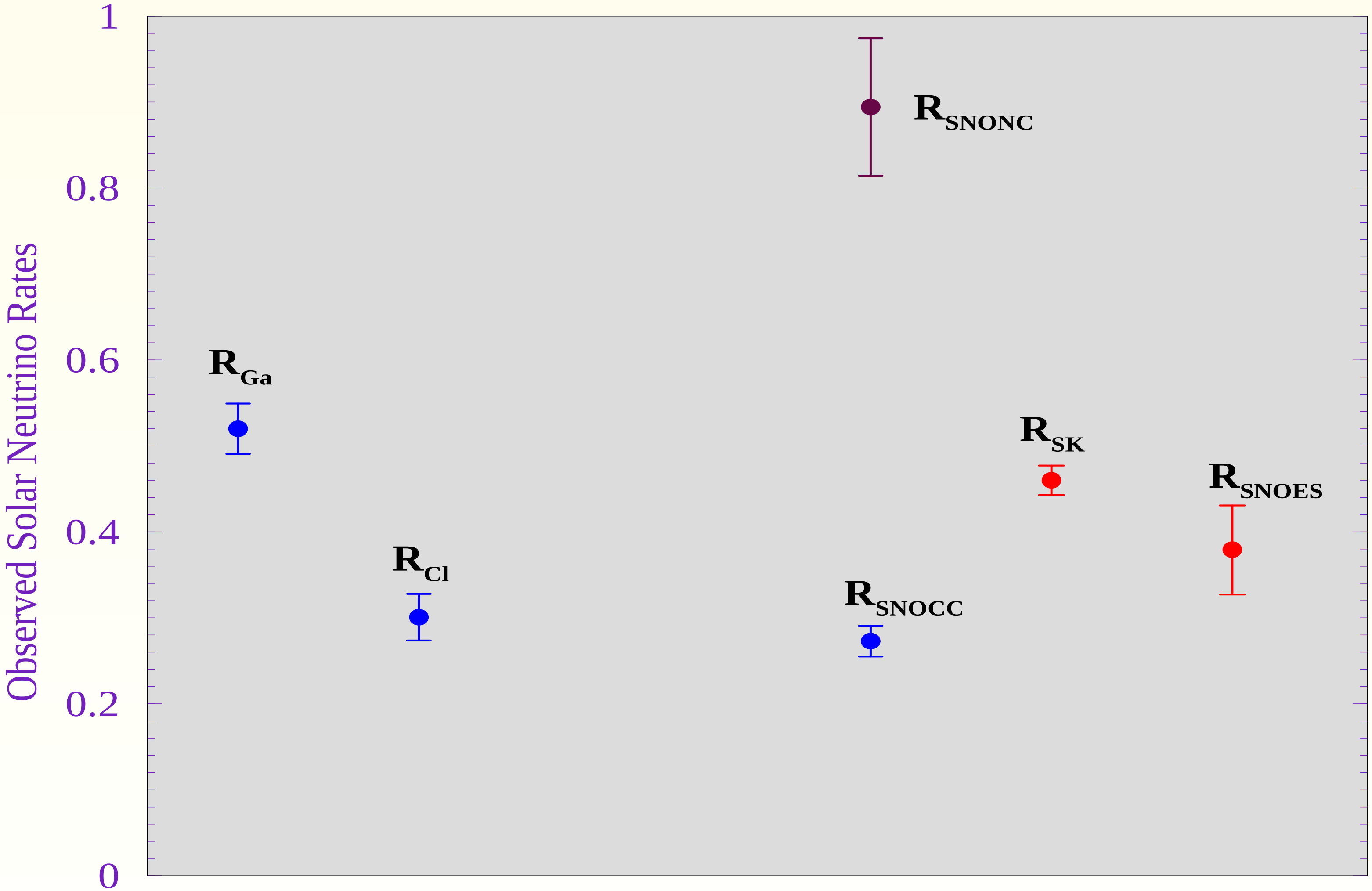
Features of Solar Neutrino Data



SNO Collaboration, hep-ph/050202

- No doubt about the presence of ν_μ and/or ν_τ.

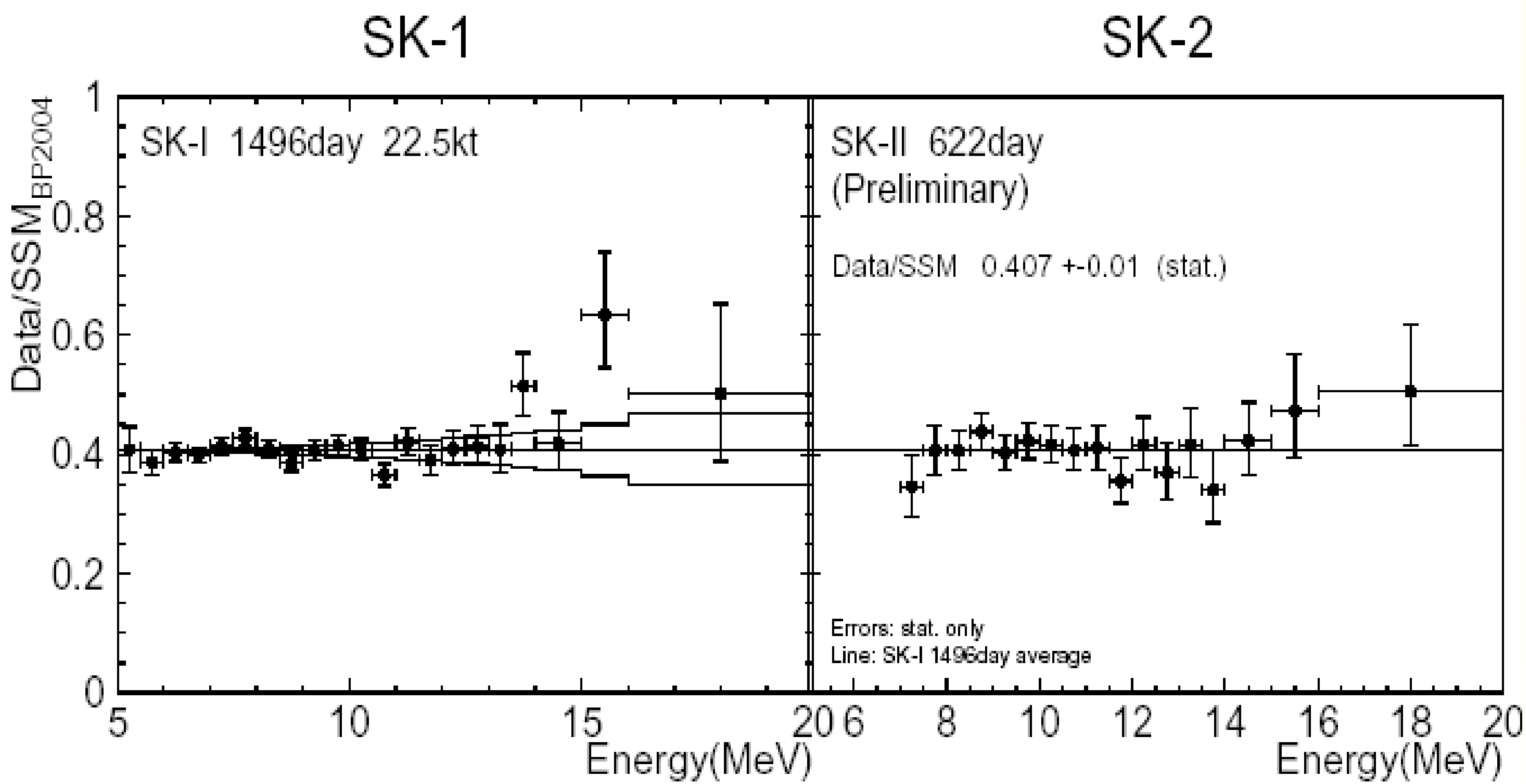
Features of Solar Neutrino Data



- There is a marked energy dependence in the suppression.

Features of Solar Neutrino Data

- Suppression is energy independent above $E = 5$ MeV



SK Collaboration

Features of Solar Neutrino Data

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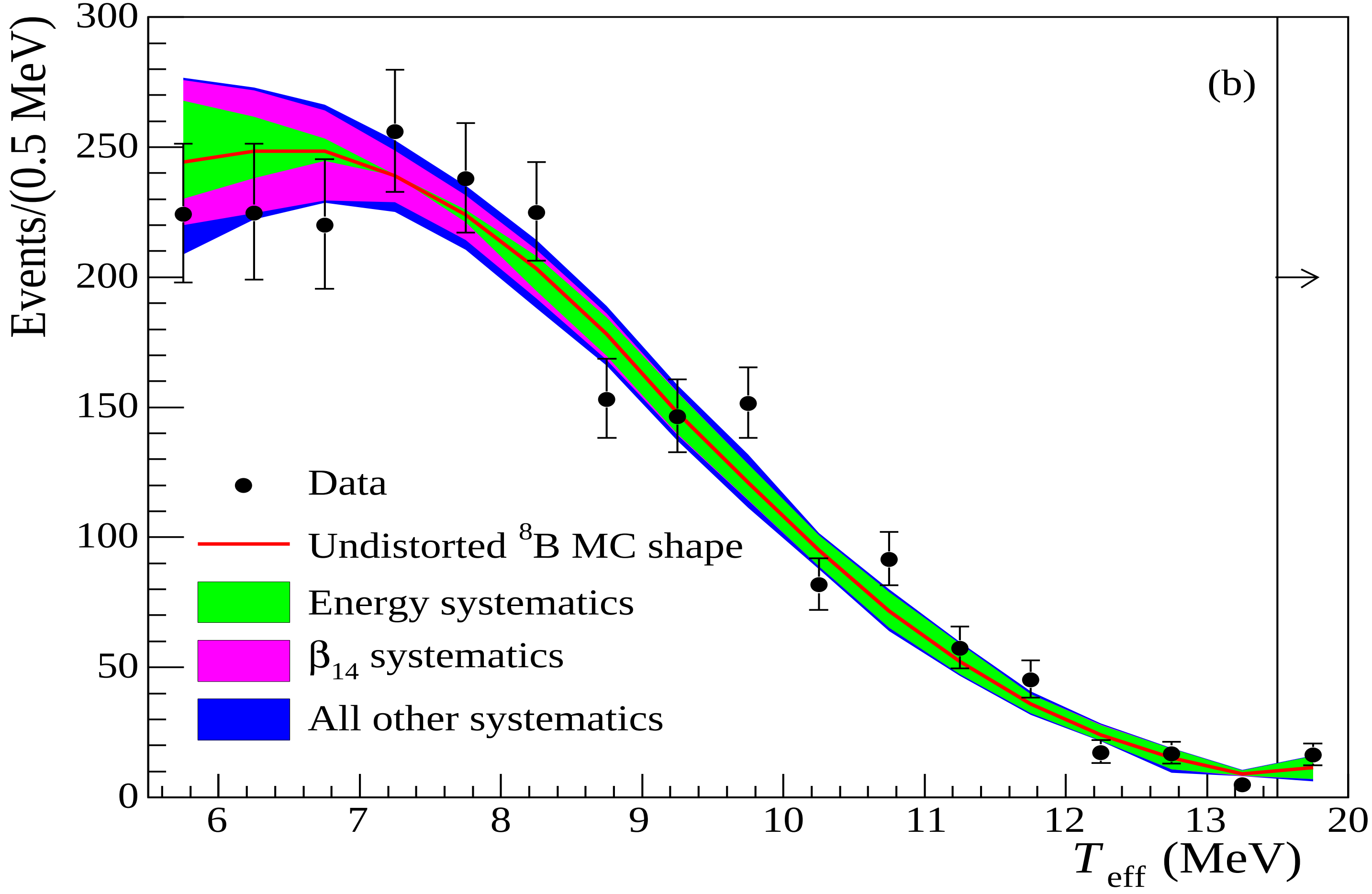
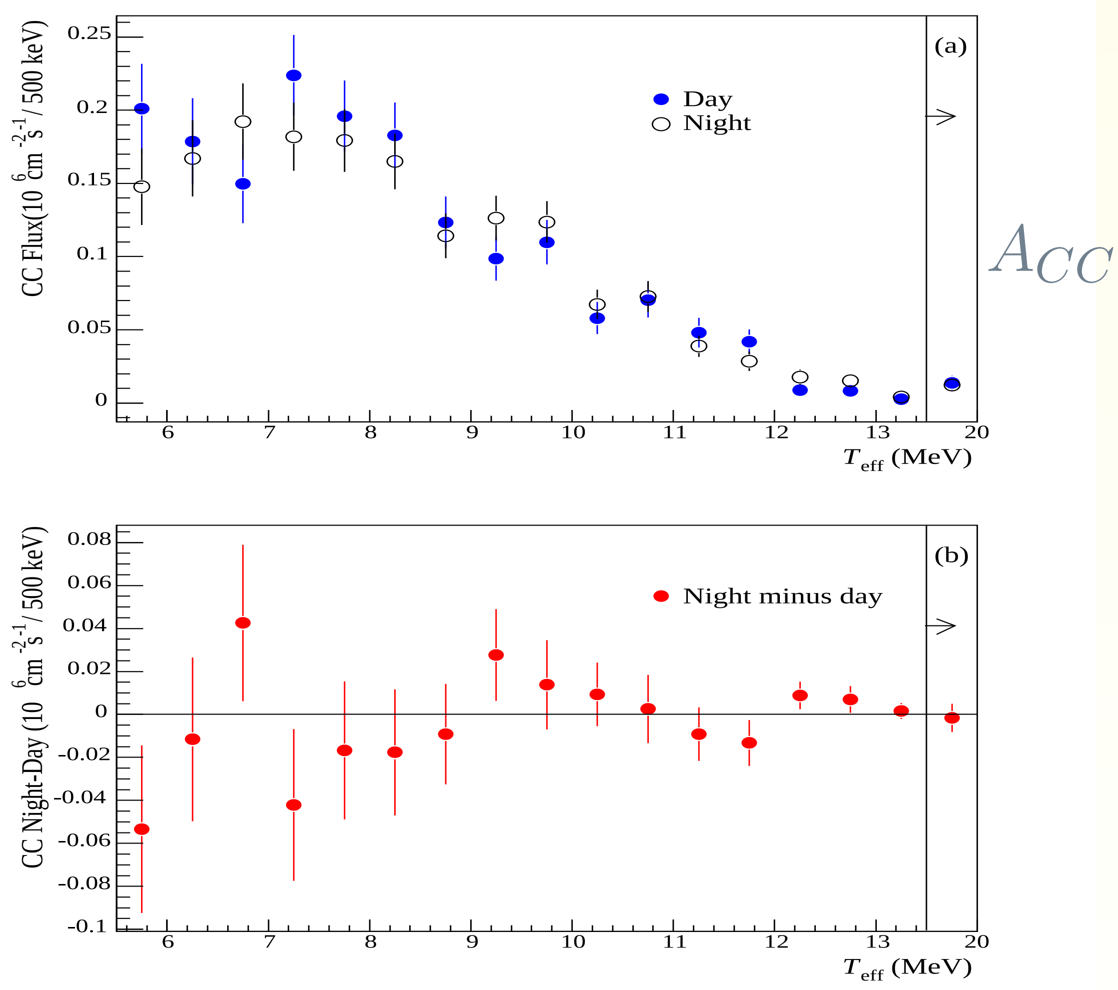


SNO Collaboration, hep-ph/050202

Features of Solar Neutrino Data

- Small difference between the rates at day and night



$$
\begin{aligned}
A_{CC} &= 2\frac{\phi_N - \phi_D}{\phi_N + \phi_D} \\
&= -0.21 \pm 0.063 \pm 0.035
\end{aligned}
$$

SNO Collaboration, hep-ph/050202

- There are similar results from SK.

Neutrino Oscillations can explain ALL features of the solar neutrino data

Analysis of Solar Neutrino Data

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- Need to do a χ^2 analysis to pin down the parameters

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- The Covariance Approach

$$\chi^2 = \sum_{i,j=1}^{N} (R_i^{data} - R_i^{theory})(\sigma_{ij}^2)^{-1}(R_j^{data} - R_j^{theory}) \ ,$$

$$\sigma_{ij}^2 = \delta_{ij}\sigma_i\sigma_j + \sum_{k=1}^{K} \sigma_i^k \sigma_j^k \rho_{ij}, \ N \rightarrow \text{bins}, \ K \rightarrow \text{theory errors}$$

Analysis of Solar Neutrino Data

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- The "Pull" Approach

$$\chi^2 = \min_{\xi_k} \left[\sum_{i=1}^{N} \left(\frac{R_i^{data} - (R_i^{theory} + \sum_{k=1}^{K} \xi_k \sigma_k)}{\sigma_i^{uncorr}} \right)^2 + \sum_{k=1}^{K} \xi_k^2 \right]$$

$\xi_k \rightarrow$ free parameters,

$N \rightarrow$ no. of bins, $K \rightarrow$ no. of theory errors

Predicted Rates from Theory

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- Expected event rate in radiochemical expts Cl and Ga is

$$R_i^{th} = \sum_{k=1}^{8} \int_{E_\nu^{th}} \phi_k(E_\nu)\sigma_i(E_\nu)\langle P_{ee}(E_\nu)\rangle dE_\nu$$

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Gallium Experiments (SAGE, GALLEX, GNO)

- Detection Channel: $^{71}Ga + \nu_e \rightarrow ^{71}Ge + e^-$
- This is a **charged current** reaction: only ν_e detected
- Energy Threshold: $E_\nu > 0.23 MeV$
- Solar Neutrinos Detected: pp, ^7Be, ^8B, hep and CNO ν's

$$Observed/SSM(BP2004) = 0.520 \pm 0.029$$

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Chlorine Experiment (Homestake)

- Detection Channel: $^{37}Cl + \nu_e \rightarrow ^{37}Ar + e^-$
- This is a **charged current** reaction: only ν_e detected
- Energy Threshold: $E_\nu > 0.81 MeV$
- Solar Neutrinos Detected: ^7Be, ^8B, hep and CNO ν's

$$Observed/SSM(BP2004) = 0.301 \pm 0.027$$

Predicted Rates from Theory

- Expected event rate in Super-Kamiokande is

$$R_{SK}^{th} = \int_{E_A^{th}} dE_A \int dE_T R(E_A, E_T) \int dE_\nu \lambda_{\nu_e}(E_\nu)$$

$$\times \left[\frac{d\sigma_{\nu_e}}{dE_T}\langle P_{ee}(E_\nu)\rangle + \frac{d\sigma_{\nu_x}}{dE_T}\langle P_{ex}(E_\nu)\rangle\right]$$

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$$R(E_A, E_T) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(E_T - E_A)^2}{2\sigma^2} \right)$$

$$\sigma = 1.6 \sqrt{\frac{E_T}{10\mathrm{MeV}}}$$

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Chlorine Experiment

Detects only ν_e

Solar Neutrinos Detected:

^7Be, ^8B, hep and CNO ν's

$R_{Cl} = 0.301 \pm 0.027$

SK Experiment

Detects ν_e ($\approx 83\%$) and ν_x

Solar Neutrinos Detected:

^8B, hep and CNO ν's

$R_{SK} = 0.406 \pm 0.014$

- Even before SNO, Astrophysical Solutions could not solve this

Predicted Rates from Theory

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Predicted Rates from Theory

- Expected CC event rate in SNO is

$$R_{CC}^{th} = \frac{\int dE_\nu \lambda_{\nu_e}(E_\nu)\sigma_{CC}(E_\nu)\langle P_{ee}(E_\nu)\rangle}{\int dE_\nu \lambda_{\nu_e}(E_\nu)\sigma_{CC}(E_\nu)}$$

$$\sigma_{CC} = \int_{E_A^{th}} dE_A \int_0^\infty dE_T R(E_A, E_T) \frac{d\sigma_{\nu_e d}(E_T, E_\nu)}{dE_T}$$

$$\sigma_{SNO} = (-0.131 + 0.383\sqrt{(E_T - m_e)} + 0.03731(E_T - m_e))$$

- Expected NC event rate in SNO is

$$R_{NC}^{th} = \int_0^\infty \int_0^\infty \int_{5.5}^\infty \lambda_\nu(E_\nu) \frac{d\sigma}{dE_T} R(E_A, E_T) dE_\nu dE_A dE_T$$

- Expression for ES event rate is similar to SK

Neutrino Oscillation Solution

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- The answers to these questions will also tell us why LMA is the favored solution.

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- The answers to these questions will also tell us why LMA is the favored solution.
- To understand that we will have to look at the solar neutrino oscillation survival probability.

Survival Probability of $\nu_{e\odot}$

Since solar neutrinos travel through
(1) solar matter
(2) vacuum between sun and earth
(3) earth matter
The amplitude of a ν_e to remain a ν_e is

$$A_{ee} = A^{\odot}_{e1} A^{vac}_{11} A^{\oplus}_{1e} + A^{\odot}_{e2} A^{vac}_{22} A^{\oplus}_{2e}$$

- $A^{\odot}_{ek} (k = 1, 2)$ gives the probability amplitude of $\nu_e \to \nu_k$ transition at the solar surface,
- A^{vac}_{kk} is the survival amplitude from the solar surface to the surface of the Earth,
- $A^{\oplus}_{ke}$ is the $\nu_k \to \nu_e$ transition amplitudes inside Earth.

Survival Probability of $\nu_{e\odot}$

$$A_{ek}^{\odot} = a_{ek}^{\odot} e^{-i\phi_k^{\odot}}$$

where $\phi_k^{\odot}$ is the phase picked up by the neutrinos on their way from the production point in the central regions to the surface of the Sun and

$${a_{e1}^{\odot}}^2 = \frac{1}{2} + (\frac{1}{2} - P_J) \cos 2\theta_m$$

θ_m is the mixing angle at the production point of the neutrino, P_J is the non-adiabatic jump probability

$$A_{kk}^{vac} = e^{-iE_k(L-R_{\odot})}$$

$A_{ke}^{\oplus}$ will depend on whether neutrinos cross the earth of not. For no earth effect

$$A_{1e}^{\oplus} = \cos\theta; \;\; A_{2e}^{\oplus} = \sin\theta$$

Survival Probability of $\nu_{e\odot}$

$$A_{ee} = A_{e1}^{\odot} A_{11}^{vac} A_{1e}^{\oplus} + A_{e2}^{\odot} A_{22}^{vac} A_{2e}^{\oplus}$$

$$\begin{aligned} P_{ee} &= |A_{ee}|^2 \\ &= {a_{e1}^{\odot}}^2 |A_{1e}^{\oplus}|^2 + {a_{e2}^{\odot}}^2 |A_{2e}^{\oplus}|^2 \\ &\quad + 2 a_{e1}^{\odot} a_{e2}^{\odot} Re[A_{1e}^{\oplus} {A_{2e}^{\oplus}}^* e^{i(E_2 - E_1)(L - R_\odot)} e^{i(\phi_2^{\odot} - \phi_1^{\odot})}] \end{aligned}$$

Identifying $P_\odot = {a_{e1}^{\odot}}^2$ and $P_\oplus = |A_{1e}^{\oplus}|^2$

$$\begin{aligned} P_{ee} &= P_\odot P_\oplus + (1 - P_\odot)(1 - P_\oplus) \\ &\quad + 2\sqrt{P_\odot (1 - P_\odot) P_\oplus (1 - P_\oplus)} \cos\xi \end{aligned}$$

- ξ contains all the phases collected in Sun and Vacuum.

Survival Probability of $\nu_{e\odot}$

$$
\begin{aligned}
P_{ee} &= P_\odot P_\oplus + (1-P_\odot)(1-P_\oplus) \\
&\quad +2\sqrt{P_\odot(1-P_\odot)P_\oplus(1-P_\oplus)}\cos(\Delta m^2(L-R_\odot)/2E)
\end{aligned}
$$

- $\Delta m^2/E \lesssim 5\times 10^{-10}$ eV^2/MeV $\Rightarrow$ extremely non-adiabatic
- $P_\odot = \frac{1}{2} + (\frac{1}{2} - P_J)\cos 2\theta_m$
- So, $\theta_m \approx \pi/2$, $P_\odot \approx P_J \approx \cos^2\theta$ and $P_\oplus = \cos^2\theta$

$$
\begin{aligned}
P_{ee}^{vac} &= \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta\cos(\Delta m^2(L-R_\odot)/2E) \\
&= 1 - \sin^2 2\theta \sin^2(\Delta m^2(L-R_\odot)/4E)
\end{aligned}
$$

Vacuum Oscillations Regime

Survival Probability of $\nu_{e\odot}$

$$\begin{aligned} P_{ee} &= P_\odot P_\oplus + (1-P_\odot)(1-P_\oplus) \\ &+2\sqrt{P_\odot(1-P_\odot)P_\oplus(1-P_\oplus)}\cos(\Delta m^2(L-R_\odot)/2E) \end{aligned}$$

For $\Delta m^2/E \gtrsim 10^{-8}$ eV^2/MeV, the $\cos(\Delta m^2(L-R_\odot)/2E)$ term averages out to zero.

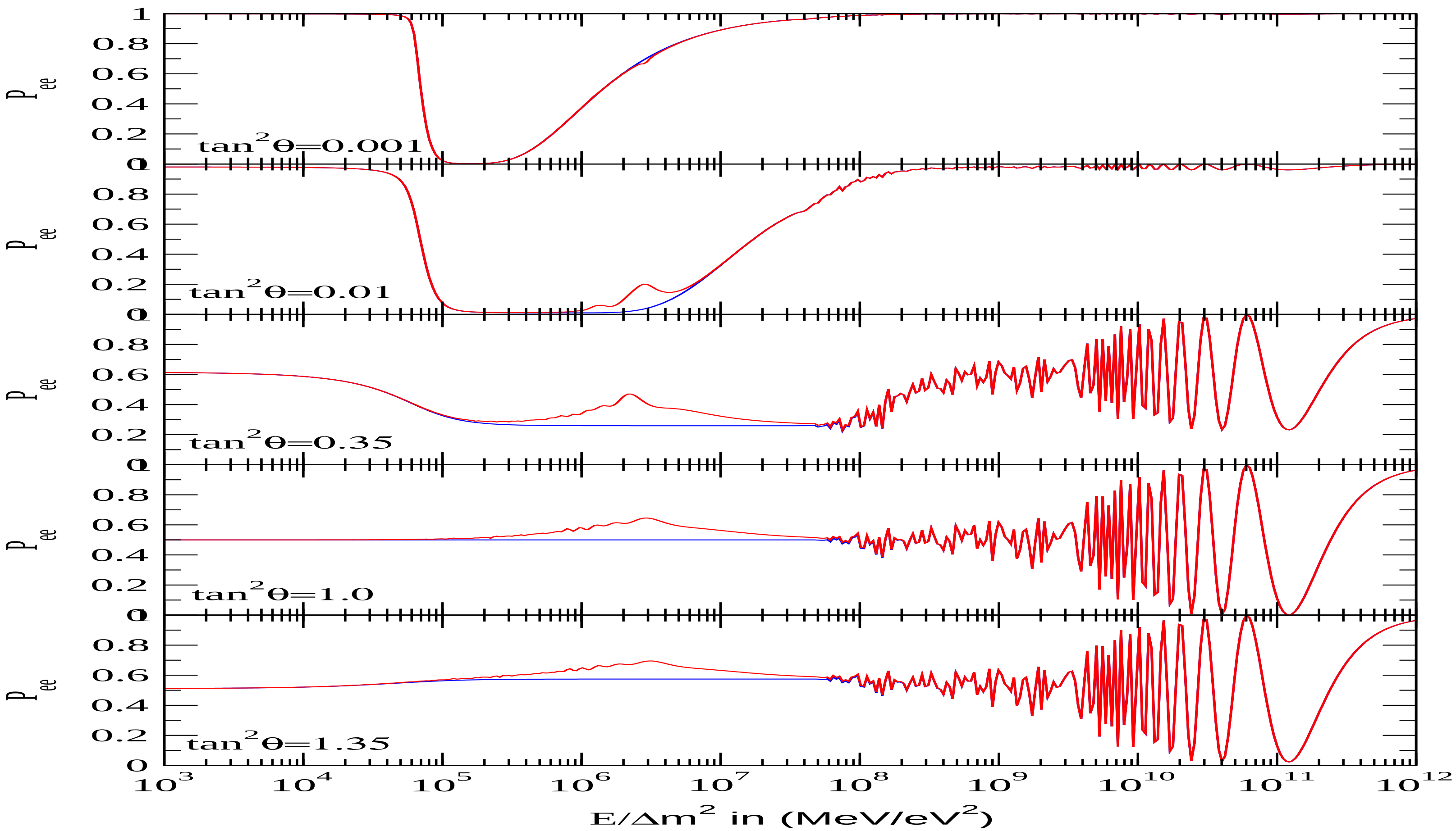
$$P_\odot = \frac{1}{2} + (\frac{1}{2} - P_J)\cos 2\theta_m,\ 1 - P_\odot = \frac{1}{2} - (\frac{1}{2} - P_J)\cos 2\theta_m$$

$$\begin{aligned} P_{ee}^{MSW} &= \left(\frac{1}{2} + (\frac{1}{2} - P_J)\cos 2\theta_m\right)\cos^2\theta \\ &+ \left(\frac{1}{2} - (\frac{1}{2} - P_J)\cos 2\theta_m\right)\sin^2\theta \end{aligned}$$

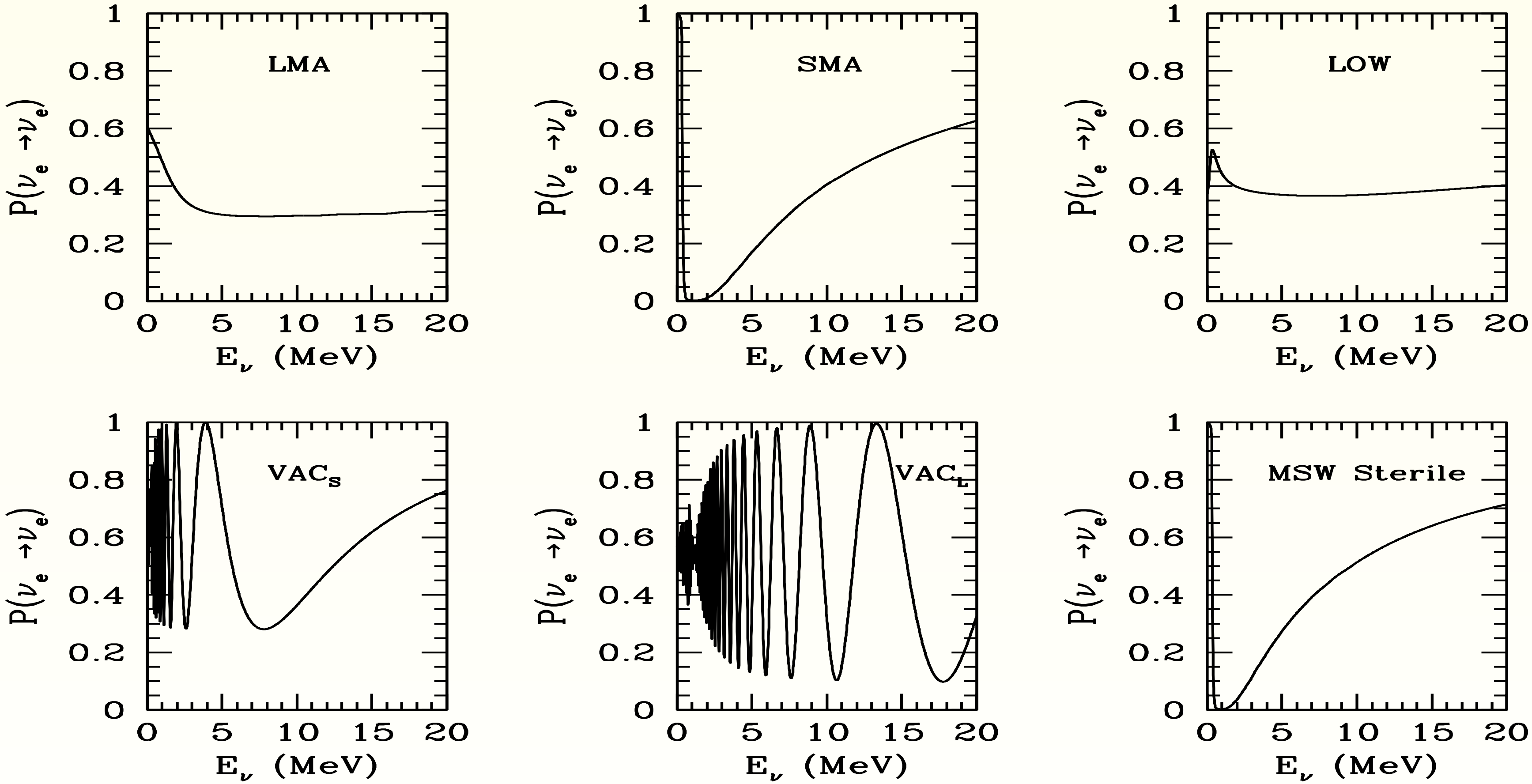
$$P_{ee}^{MSW} = \frac{1}{2} + (\frac{1}{2} - P_J)\cos 2\theta_m \cos 2\theta$$

MSW Regime – LMA, SMA, LOW

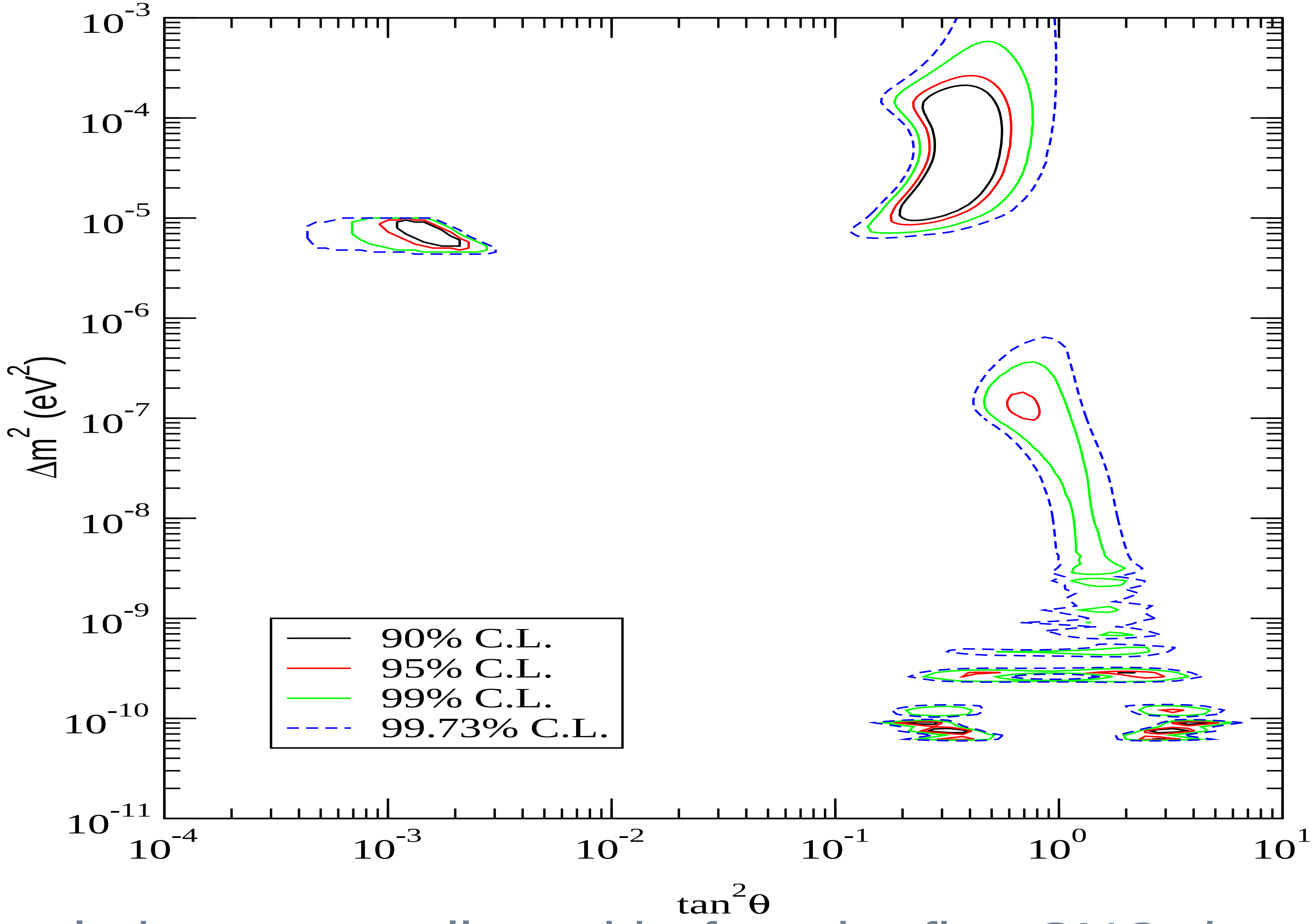
Survival Probability of $\nu_{e\odot}$



Survival Probability of $\nu_{e\odot}$



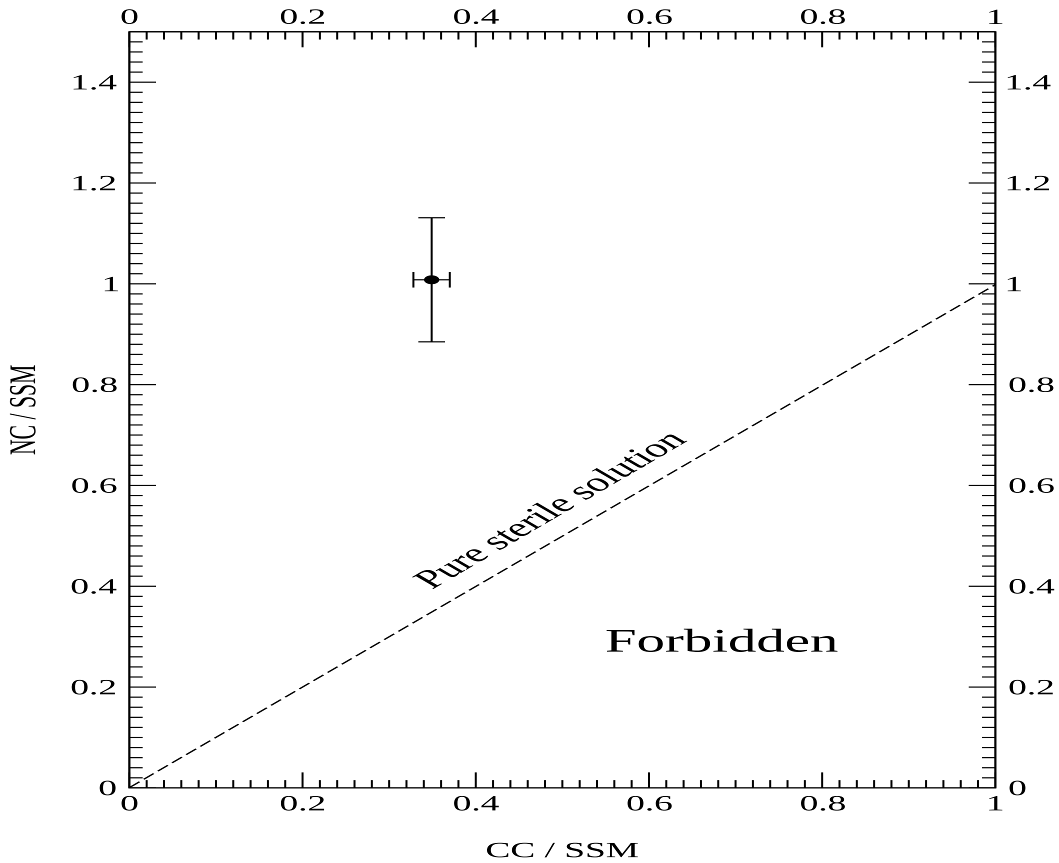
- Before the SNO, all six were allowed!
- LMA was not even the favored solution!!



All solutions were allowed before the first SNO data.

Solutions of Solar Neutrino Problem

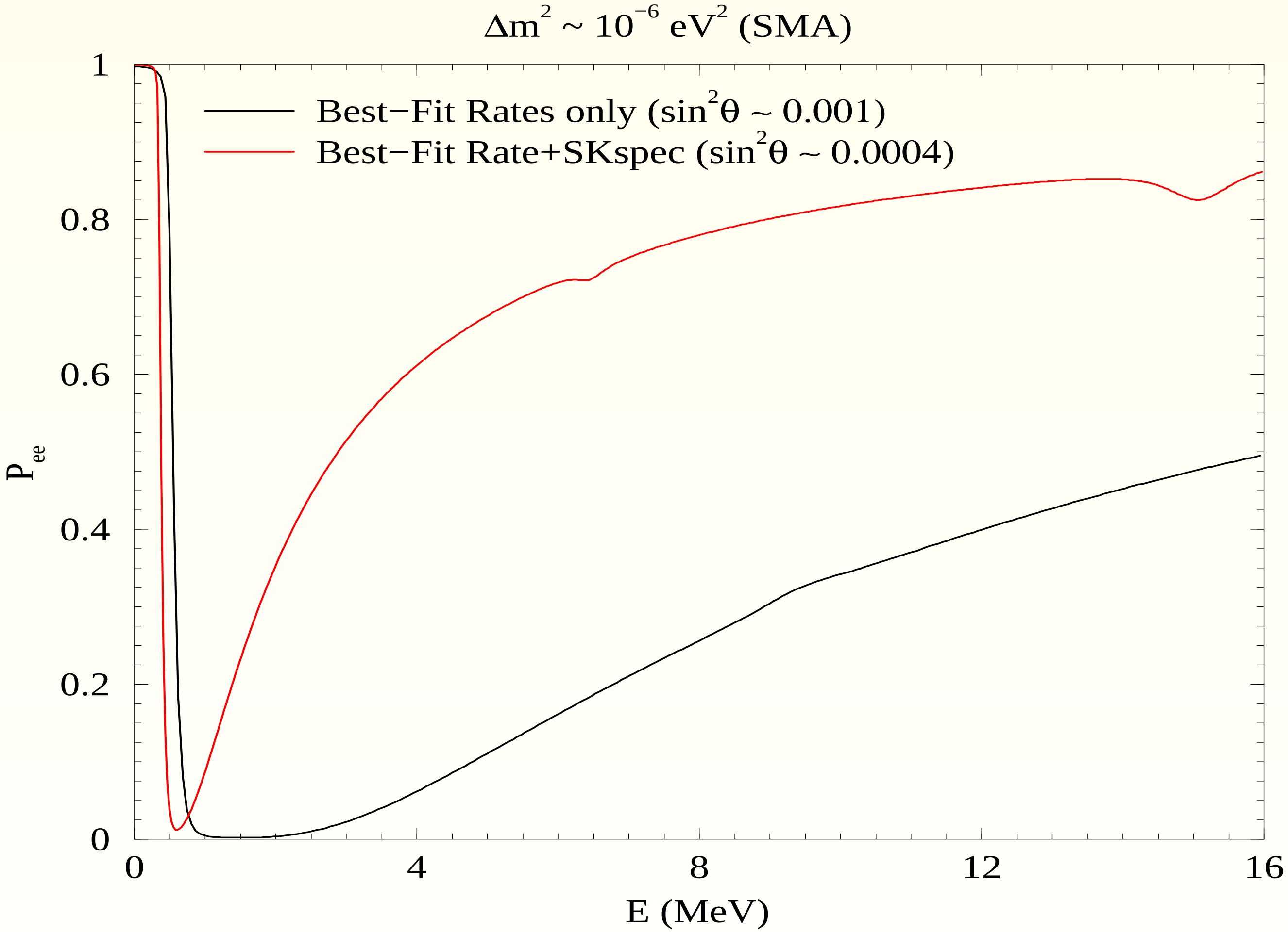
- The sterile solution is ruled out from SNO CC/NC data



- Now increased to 7.8σ with latest SNO data

Solutions of Solar Neutrino Problem

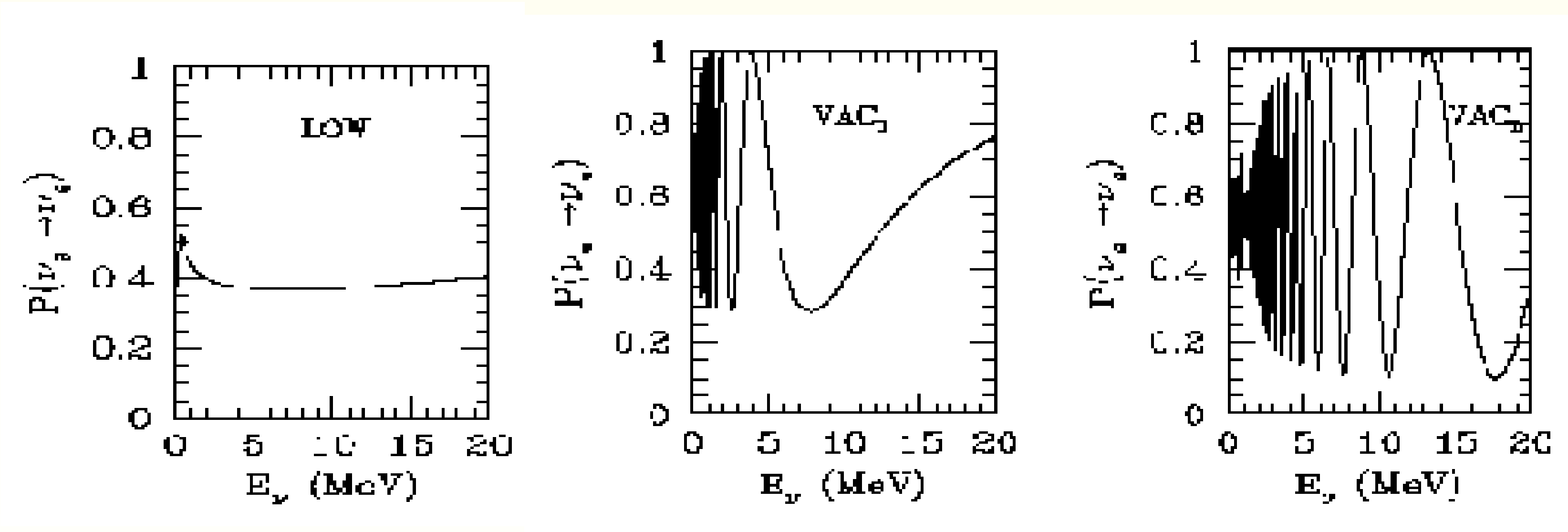
- SMA is ruled out from SNO CC and SK spectrum



- $\sin^2\theta \sim 10^{-3}$ soln cannot explain the SK spectrum
- $\sin^2\theta \sim 10^{-4}$ soln cannot explain the SK and CC rates

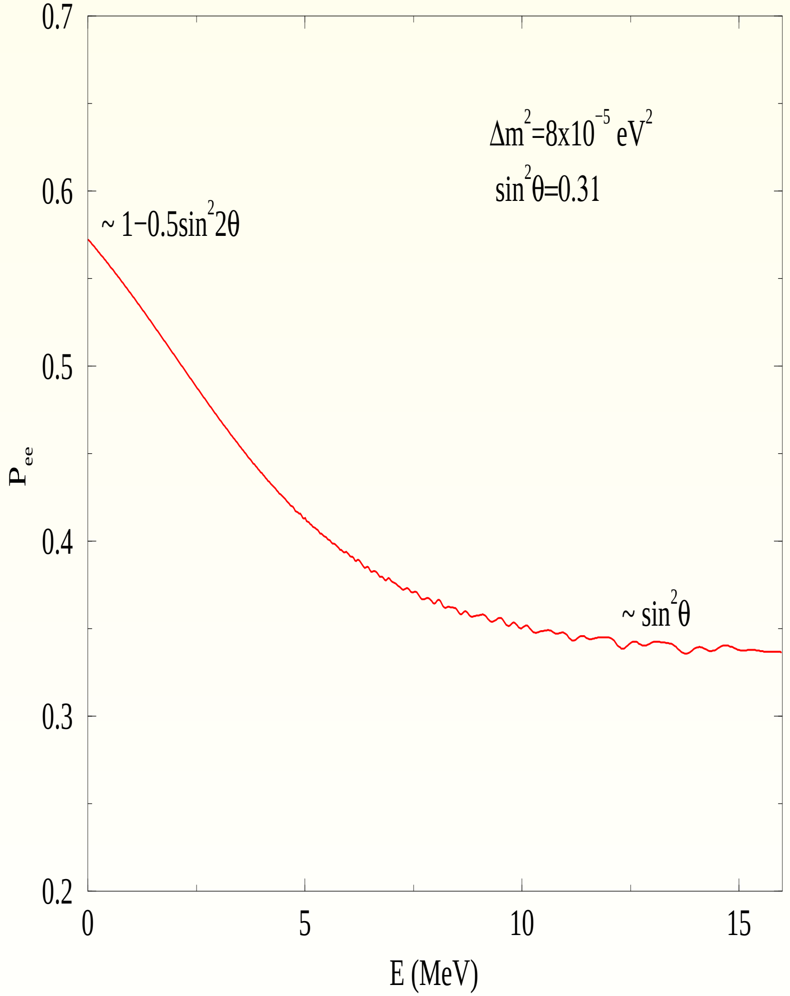
Solutions of Solar Neutrino Problem

- LOW, and Vacuum Oscillation solutions and Maximal Mixing are ruled out by the SNO salt phase NC data



Solutions of Solar Neutrino Problem

- LMA explains all features of the data:



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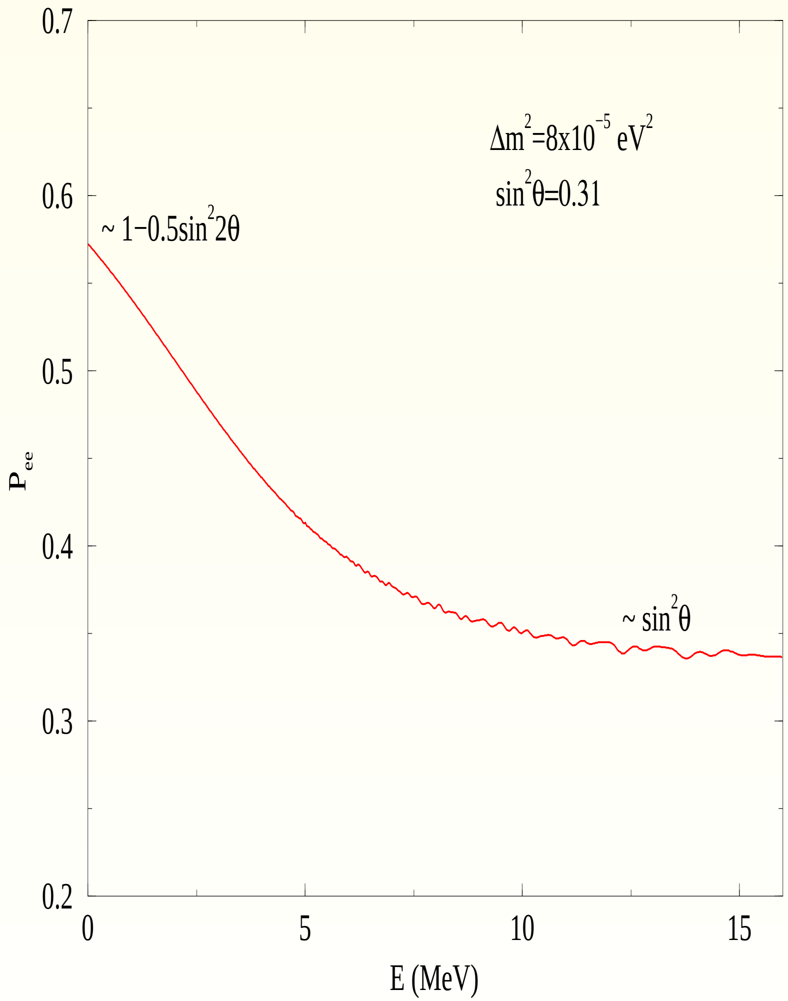

- Low energy pp neutrinos:

$$P_{ee} \approx 1 - \frac{1}{2}\sin^2 2\theta$$
$$\approx 0.57$$

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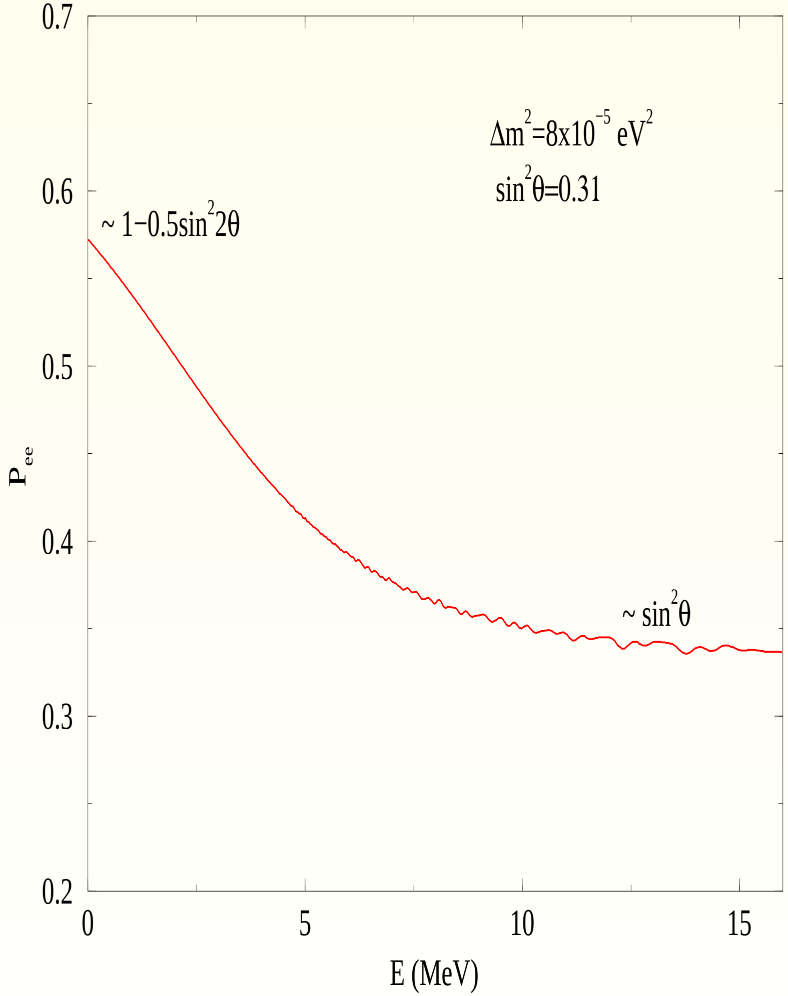

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- High energy ^8B neutrinos:

$$P_{ee} \approx \sin^2\theta \approx 0.31$$

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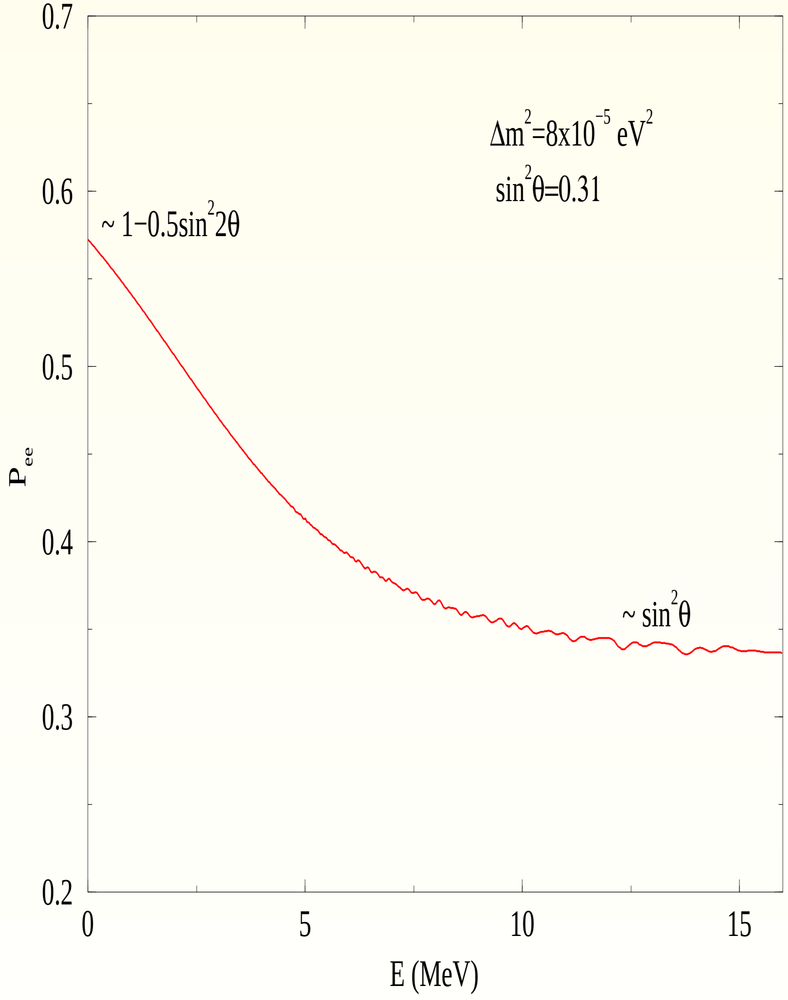

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- Energy spectrum is almost flat above 5 MeV.

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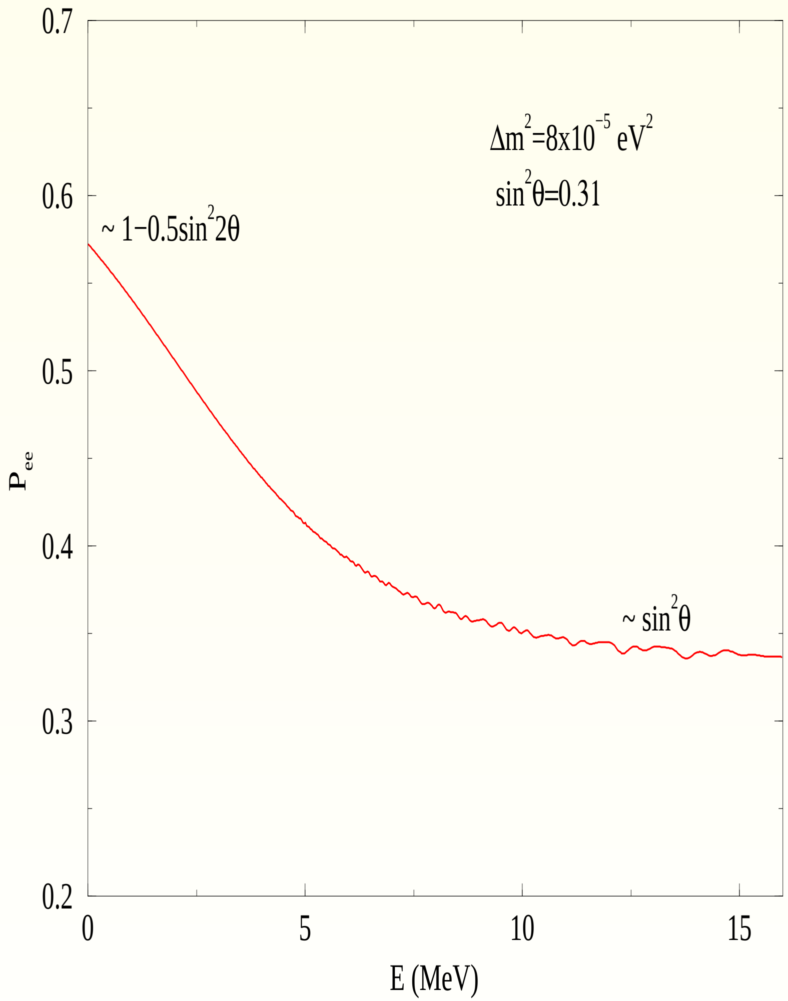

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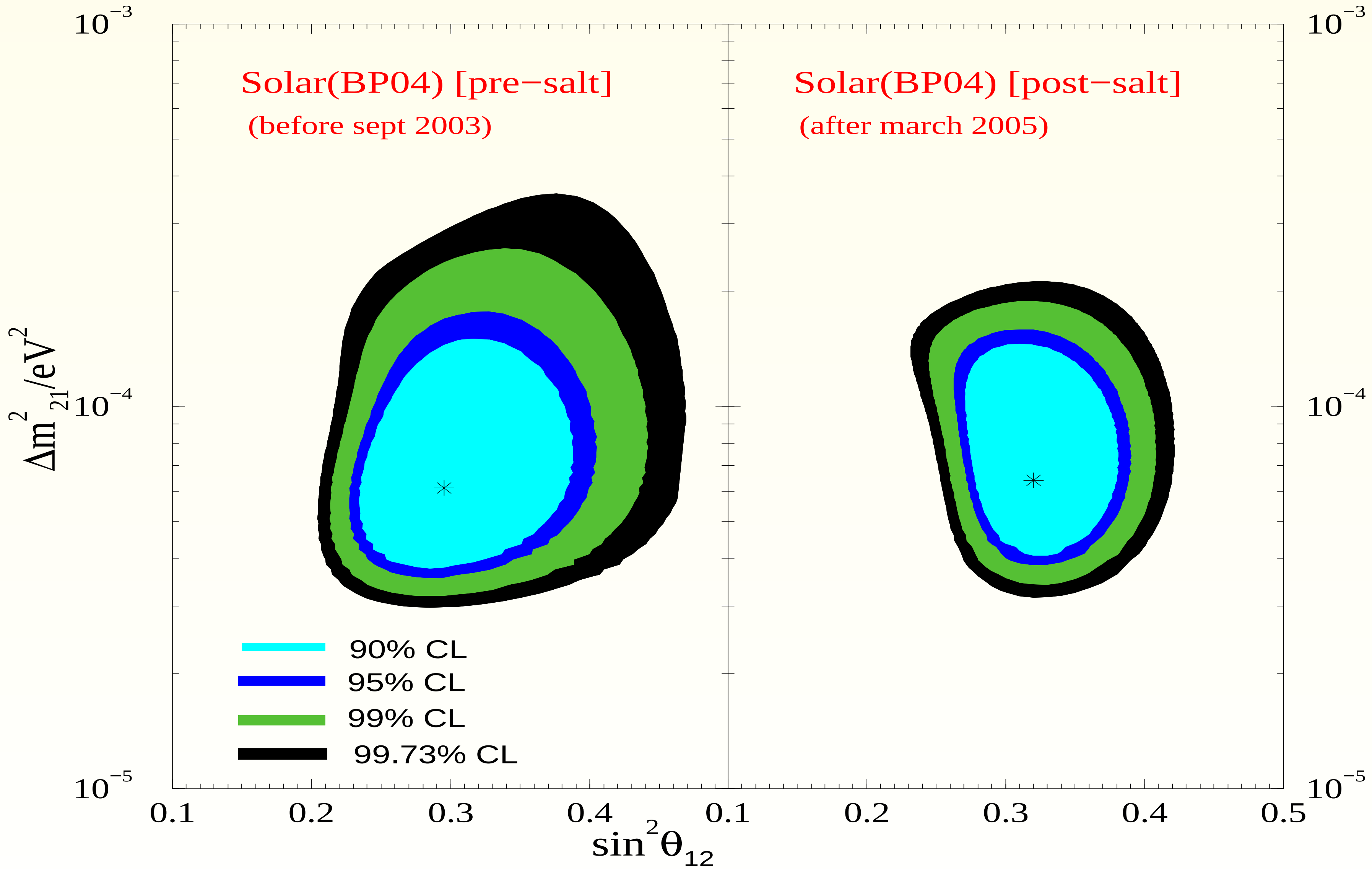
- Energy spectrum is almost flat above 5 MeV.

- Very small D/N effect

LMA is The Solution



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