Neutrino Oscillation Phenomenology - II

Sandhya Choubey

Harish-Chandra Research Institute, Allahabad, India

JIGSAW 2007

Joint Indo-German School And Workshop 2007

TIFR, Mumbai, 12-23 February 2007

Neutrino Phenomenology – Future Directions

Neutrino Masses and Mixings

Neutrino Masses and Mixings

- Assume that there are only three light neutrinos

Neutrino Masses and Mixings

- Assume that there are only three light neutrinos
- 9 free parameters in the light neutrino mass matrix
  - ⋆ 3 neutrino masses: $m_1,\ m_2$ and m_3
  - ⋆ 3 mixing angles: $\theta_{12},\ \theta_{23}$ and θ_{13}
  - ⋆ 3 phases: δ, α and β

Neutrino Masses and Mixings

- Assume that there are only three light neutrinos
- 9 free parameters in the light neutrino mass matrix
  - ⋆ 3 neutrino masses: $m_1, \ m_2$ and m_3
  - ⋆ 3 mixing angles: $\theta_{12}, \ \theta_{23}$ and θ_{13}
  - ⋆ 3 phases: δ, α and β
- Neutrino oscillation experiments sensitive to
  - ⋆ $m_2^2 - m_1^2 \equiv \Delta m_{21}^2$ and $m_3^2 - m_2^2 \equiv \Delta m_{32}^2$
  - ⋆ all the three mixing angles
  - ⋆ one CP phase: δ (Dirac phase)

Neutrino Masses and Mixings

- Assume that there are only three light neutrinos
- 9 free parameters in the light neutrino mass matrix
  - ⋆ 3 neutrino masses: $m_1,\ m_2$ and m_3
  - ⋆ 3 mixing angles: $\theta_{12},\ \theta_{23}$ and θ_{13}
  - ⋆ 3 phases: δ, α and β
- Neutrino oscillation experiments sensitive to
  - ⋆ $m_2^2 - m_1^2 \equiv \Delta m_{21}^2$ and $m_3^2 - m_2^2 \equiv \Delta m_{32}^2$
  - ⋆ all the three mixing angles
  - ⋆ one CP phase: δ (Dirac phase)
- The CP phases α and β are observable in $\Delta L = 2$ process (Majorana phases)

Neutrino Masses and Mixings

- Assume that there are only three light neutrinos
- 9 free parameters in the light neutrino mass matrix
  - ⋆ 3 neutrino masses: $m_1, \; m_2$ and m_3
  - ⋆ 3 mixing angles: $\theta_{12}, \; \theta_{23}$ and θ_{13}
  - ⋆ 3 phases: δ, α and β
- Neutrino oscillation experiments sensitive to
  - ⋆ $m_2^2 - m_1^2 \equiv \Delta m_{21}^2$ and $m_3^2 - m_2^2 \equiv \Delta m_{32}^2$
  - ⋆ all the three mixing angles
  - ⋆ one CP phase: δ (Dirac phase)
- The CP phases α and β are observable in $\Delta L = 2$ process (Majorana phases)
- Absolute neutrino mass scale can be accessible in
  - ⋆ Tritium beta decay experiments
  - ⋆ Neutrinoless double beta decay experiments
  - ⋆ Cosmology

What we know about the parameters

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$
- $\sin^2\theta_{12} \sim 0.31$ $\star$

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$
- $\sin^2\theta_{12} \sim 0.31$ $\star$
- $\Delta m^2_{21} > 0$ $\star$

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$
- $\sin^2 \theta_{12} \sim 0.31$ $\star$
- $\Delta m^2_{21} > 0$ $\star$
- $\sin^2 \theta_{12}$ is not maximal $\star$

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$
- $\sin^2\theta_{12} \sim 0.31$ $\star$
- $\Delta m^2_{21} > 0$ $\star$
- $\sin^2\theta_{12}$ is not maximal $\star$
- $|\Delta m^2_{32}| \sim 2 - 3 \times 10^{-3}$ eV^2 $\star$

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$
- $\sin^2\theta_{12} \sim 0.31$ $\star$
- $\Delta m^2_{21} > 0$ $\star$
- $\sin^2\theta_{12}$ is not maximal $\star$
- $|\Delta m^2_{32}| \sim 2 - 3 \times 10^{-3}$ eV^2 $\star$
- $\sin^2 2\theta_{23} \sim 1$ $\star$

What we know about the parameters

- $\Delta m^2_{21} \sim 8 \times 10^{-5}$ eV^2 $\star$
- $\sin^2\theta_{12} \sim 0.31$ $\star$
- $\Delta m^2_{21} > 0$ $\star$
- $\sin^2\theta_{12}$ is not maximal $\star$
- $|\Delta m^2_{32}| \sim 2 - 3 \times 10^{-3}$ eV^2 $\star$
- $\sin^2 2\theta_{23} \sim 1$ $\star$
- $\sin^2 2\theta_{13} < O(0.1)$ $\star$

$\star$ ATM + K2K + MINOS
$\star$ Solar Neutrino Experiments + KamLAND
$\star$ CHOOZ and Palo Verde

What we still don't know about the parameters

- Are 3 flavor oscillations enough?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?
- Is $\sin^2 2\theta_{23}$ exactly maximal $(= 1)$?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?
- Is $\sin^2 2\theta_{23}$ exactly maximal $(= 1)$?
- If $\sin^2 2\theta_{23} \neq 1$, what is its octant?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?
- Is $\sin^2 2\theta_{23}$ exactly maximal $(= 1)$?
- If $\sin^2 2\theta_{23} \neq 1$, what is its octant?
- How small is $\sin^2 2\theta_{13}$? Is it zero?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?
- Is $\sin^2 2\theta_{23}$ exactly maximal $(= 1)$?
- If $\sin^2 2\theta_{23} \neq 1$, what is its octant?
- How small is $\sin^2 2\theta_{13}$? Is it zero?
- Is the CP phase non-zero?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?
- Is $\sin^2 2\theta_{23}$ exactly maximal ($=1$)?
- If $\sin^2 2\theta_{23} \neq 1$, what is its octant?
- How small is $\sin^2 2\theta_{13}$? Is it zero?
- Is the CP phase non-zero?
- Are the neutrinos really Majorana particles?

What we still don't know about the parameters

- Are 3 flavor oscillations enough?
- What are the precise values of Δm^2_{21} and $\sin^2\theta_{12}$?
- What is the precise value of $|\Delta m^2_{32}|$?
- What is the sign of Δm^2_{32}
- Is the neutrino spectrum hierarchical or not?
- Is $\sin^2 2\theta_{23}$ exactly maximal ($= 1$)?
- If $\sin^2 2\theta_{23} \neq 1$, what is its octant?
- How small is $\sin^2 2\theta_{13}$? Is it zero?
- Is the CP phase non-zero?
- Are the neutrinos really Majorana particles?
- Is there a link between low energy CPV in the lepton sector and can we link it to the baryon asymmetry?

Next Generation Experiments

- Future Solar Neutrino Experiments
- Reactor Experiments
- Future Atmospheric Neutrino Experiments
- Long Baseline Experiments with ConventionalBeam
- Long Baseline Experiments with Super Beams
- Long Baseline Experiments with Beta Beams
- Long Baseline Experiments with Neutrino Factories
- Neutrinoless Double Beta Decay Experiments

Next Generation Experiments

- Future Solar Neutrino Experiments
- Reactor Experiments
- Future Atmospheric Neutrino Experiments
- Long Baseline Experiments with ConventionalBeam
- Long Baseline Experiments with Super Beams
- Long Baseline Experiments with Beta Beams
- Long Baseline Experiments with Neutrino Factories
- Neutrinoless Double Beta Decay Experiments

Future Solar Neutrino Experiments

Future Solar Neutrino Experiments

- Why do them?

- To measure Δm^2_{21} and θ_{12} more precisely.

- To measure the solar neutrino fluxes more precisely.

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

SNO:

- Measures the ν_e flux thru CC interactions

$$R_{CC} = \phi_B P_{ee}^{sno}$$

- Measures the total active ν flux thru NC interactions

$$R_{NC} = \phi_B$$

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

SNO:

- Measures the ν_e flux thru CC interactions

$$R_{CC} = \phi_B P_{ee}^{sno}$$

- Measures the total active ν flux thru NC interactions

$$R_{NC} = \phi_B$$

$\Rightarrow$

$$\frac{R_{CC}}{R_{NC}} = P_{ee}^{sno} \simeq \sin^2\theta_{12}$$

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

SNO:

- Measures the ν_e flux thru CC interactions

$$R_{CC} = \phi_B P_{ee}^{sno}$$

- Measures the total active ν flux thru NC interactions

$$R_{NC} = \phi_B$$

$\Rightarrow$

$$\frac{R_{CC}}{R_{NC}} = P_{ee}^{sno} \simeq \sin^2\theta_{12}$$

- SNO phase-III results which will have lower uncertainties on R_{NC} will improve $\sin^2\theta_{12}$ accuracy

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO
- A bit like SK – but still different

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO
- A bit like SK – but still different
- Ratio of CC to NC in $\nu - e$ scattering depends on E

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO
- A bit like SK – but still different
- Ratio of CC to NC in $\nu - e$ scattering depends on E
- For SK $r_{sk} \simeq 0.17$

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO
- A bit like SK – but still different
- Ratio of CC to NC in $\nu - e$ scattering depends on E
- For SK $r_{sk} \simeq 0.17$
- For pp $r_{pp} \simeq 0.3$

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO
- A bit like SK – but still different
- Ratio of CC to NC in $\nu - e$ scattering depends on E
- For SK $r_{sk} \simeq 0.17$
-
- For pp $r_{pp} \simeq 0.3$

$$P^{pp}_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

$$R_{pp} = P^{pp}_{ee} + r_{pp} * (1 - P^{pp}_{ee})$$

Measuring Δm^2_{21} and $\sin^2\theta_{12}$ precisely – Solar

pp:

- Will have energy threshold $\sim$ keV
- Observe low energy pp νs through $\nu - e$ scatt
- Not a pure measurement of P_{ee} like SNO
- A bit like SK – but still different
- Ratio of CC to NC in $\nu - e$ scattering depends on E
- For SK $r_{sk} \simeq 0.17$
-
- For pp $r_{pp} \simeq 0.3$

$$P^{pp}_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}$$

$$R_{pp} = P^{pp}_{ee} + r_{pp} * (1 - P^{pp}_{ee})$$

- These experiments should have error $\sim$ 1%

Comparing Experiments

Comparing Experiments

- SNO: $P_{ee} \simeq \sin^2\theta_{12}$
- pp: $P_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}$
- Reactor: $P_{ee} \simeq 1 - \sin^2 2\theta_{12}\sin^2(1.27\Delta m^2_{21}L/E)$
- SPMIN: $P_{ee} \simeq 1 - \sin^2 2\theta_{12}$

Comparing Experiments

- SNO: $P_{ee} \simeq \sin^2\theta_{12}$
- pp: $P_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}$
- Reactor: $P_{ee} \simeq 1 - \sin^2 2\theta_{12}\sin^2(1.27\Delta m^2_{21}L/E)$
- SPMIN: $P_{ee} \simeq 1 - \sin^2 2\theta_{12}$

- The $\sin^2\theta_{12}$ uncertainty in the three cases are:
  - $\Delta(\sin^2\theta_{12}) \simeq \Delta P_{ee}$
  - $\Delta(\sin^2\theta_{12}) \simeq \frac{\Delta P_{ee}}{2\cos 2\theta_{12}}$
  - $\Delta(\sin^2\theta_{12}) \simeq \frac{\Delta P_{ee}}{4\cos 2\theta_{12} < \sin^2 \frac{\Delta m^2_{21} L}{4E} >}$
  - $\Delta(\sin^2\theta_{12}) \simeq \frac{\Delta P_{ee}}{4\cos 2\theta_{12}}$

Comparing Experiments

- SNO: $P_{ee} \simeq \sin^2\theta_{12}$
- pp: $P_{ee} \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}$
- Reactor: $P_{ee} \simeq 1 - \sin^2 2\theta_{12} \sin^2(1.27\Delta m^2_{21} L/E)$
- SPMIN: $P_{ee} \simeq 1 - \sin^2 2\theta_{12}$

- The $\sin^2\theta_{12}$ uncertainty in the three cases are:
  - $\Delta(\sin^2\theta_{12}) \simeq \Delta P_{ee}$
  - $\Delta(\sin^2\theta_{12}) \simeq \frac{\Delta P_{ee}}{2\cos 2\theta_{12}}$
  - $\Delta(\sin^2\theta_{12}) \simeq \frac{\Delta P_{ee}}{4\cos 2\theta_{12} < \sin^2 \frac{\Delta m^2_{21} L}{4E} >}$
  - $\Delta(\sin^2\theta_{12}) \simeq \frac{\Delta P_{ee}}{4\cos 2\theta_{12}}$
- Solar experiments DO NOT have good Δm^2_{21} sensitivity

Measuring the Solar Neutrino Parameters

Experiment	$\|\Delta m^2_{21}\|$	$\sin^2\theta_{12}$
Solar + KL	12%	22%

Bandyopadhyay, S.C., Goswami (2005)

Measuring the Solar Neutrino Parameters

Experiment	$\lvert\Delta m^2_{21}\rvert$	$\sin^2\theta_{12}$
Solar + KL	12%	22%
Solar + KL + SNO3	12%	18%

Bandyopadhyay, S.C., Goswami (2005)

Measuring the Solar Neutrino Parameters

Experiment	$\lvert\Delta m^2_{21}\rvert$	$\sin^2\theta_{12}$
Solar + KL	12%	22%
Solar + KL + SNO3	12%	18%
Solar + SNO3 + 3kTy KL	6%	16%

Bandyopadhyay, S.C., Goswami (2005)

Measuring the Solar Neutrino Parameters

Experiment	$\lvert\Delta m^2_{21}\rvert$	$\sin^2\theta_{12}$
Solar + KL	12%	22%
Solar + KL + SNO3	12%	18%
Solar + SNO3 + 3kTy KL	6%	16%
Solar + SNO3 + 3kTy KL + pp	6%	12%

Bandyopadhyay, S.C., Goswami, Petcov hep-ph/0410283

Measuring the Solar Neutrino Parameters

Experiment	$\lvert\Delta m^2_{21}\rvert$	$\sin^2\theta_{12}$
Solar + KL	12%	22%
Solar + KL + SNO3	12%	18%
Solar + SNO3 + 3kTy KL	6%	16%
Solar + SNO3 + 3kTy KL + pp	6%	12%
SKGd 5 years	2.4%	18%

Beacom and Vagins, hep-ph/0309300

S.C. and Petcov, hep-ph/0404103

Measuring the Solar Neutrino Parameters

Experiment	$\|\Delta m^2_{21}\|$	$\sin^2\theta_{12}$
Solar + KL	12%	22%
Solar + KL + SNO3	12%	18%
Solar + SNO3 + 3kTy KL	6%	16%
Solar + SNO3 + 3kTy KL + pp	6%	12%
SKGd 5 years	2.4%	18%
SPMIN	5%	6%

Bandyopadhyay, S.C., Goswami, hep-ph/0302243

Minakata *et al.*, hep-ph/0407326

Bandyopadhyay, S.C., Goswami, Petcov, hep-ph/0410283

Probing the Sun through Neutrinos

Probing the Sun through Neutrinos

- Solar neutrino fluxes depend directly on solar model parameters such as:
  - ⋆ The S-factors: $S_{11}, S_{33}, S_{34}, S_{1,14}, S_{17}, S_{e-7}$
  - ⋆ Solar luminosity $L_\odot$
  - ⋆ Solar metallicity Z/X
  - ⋆ Solar opacity $O_\odot$
  - ⋆ Solar age $\tau_\odot$
  - ⋆ Solar diffusion $D_\odot$

John Bahcall

Probing the Sun through Neutrinos

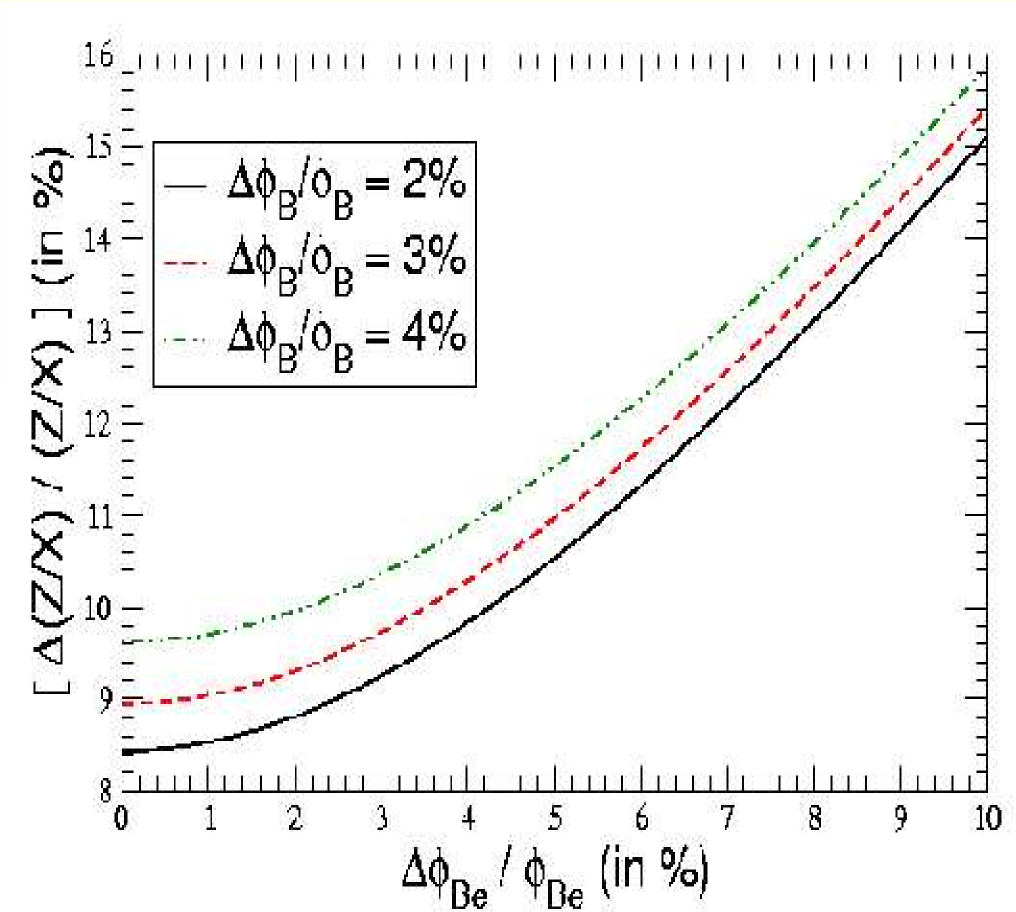
- Solar neutrino fluxes depend directly on solar model parameters such as:
  - ⋆ The S-factors: S_{11}, S_{33}, S_{34}, $S_{1,14}$, S_{17}, S_{e-7}
  - ⋆ Solar luminosity $L_\odot$
  - ⋆ Solar metallicity Z/X
  - ⋆ Solar opacity $O_\odot$
  - ⋆ Solar age $\tau_\odot$
  - ⋆ Solar diffusion $D_\odot$
- For instance:

$$\begin{aligned}
\phi_B &= C_B\,(S_{11})^{-2.59}(S_{33})^{-0.40}(S_{34})^{+0.81}(S_{1,14})^{+0.01} \\
&\times (S_{17})^{+1.0}(S_{e-7})^{-1.0}(L_\odot)^{+6.76}(\tau_\odot)^{+1.28}(O_\odot)^{-2.93} \\
&\times (D_\odot)^{-2.20}(Z/X)^{+1.36}
\end{aligned}$$

John Bahcall

Probing the Sun through Neutrinos

- Direct measurement of solar neutrino fluxes ^8B, $7Be$ and pp, can be turned around to give information on these parameters

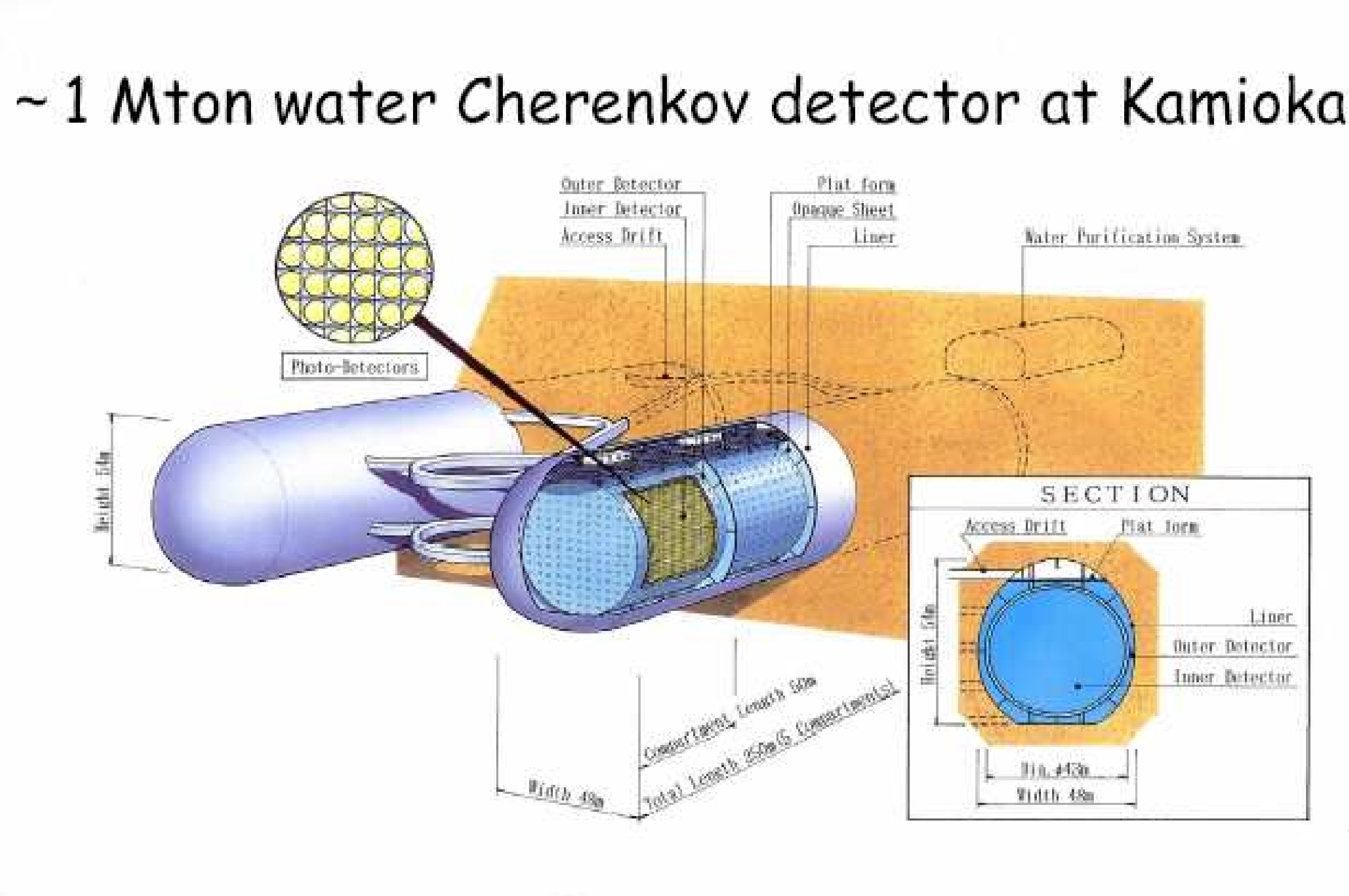


Bandyopadhyay, SC, Goswami, Petcov, hep-ph/0608323

What More Can We Learn From ATM Neutrinos

What More Can We Learn From ATM Neutrinos

- Using Megaton Water Cerenkov Detectors

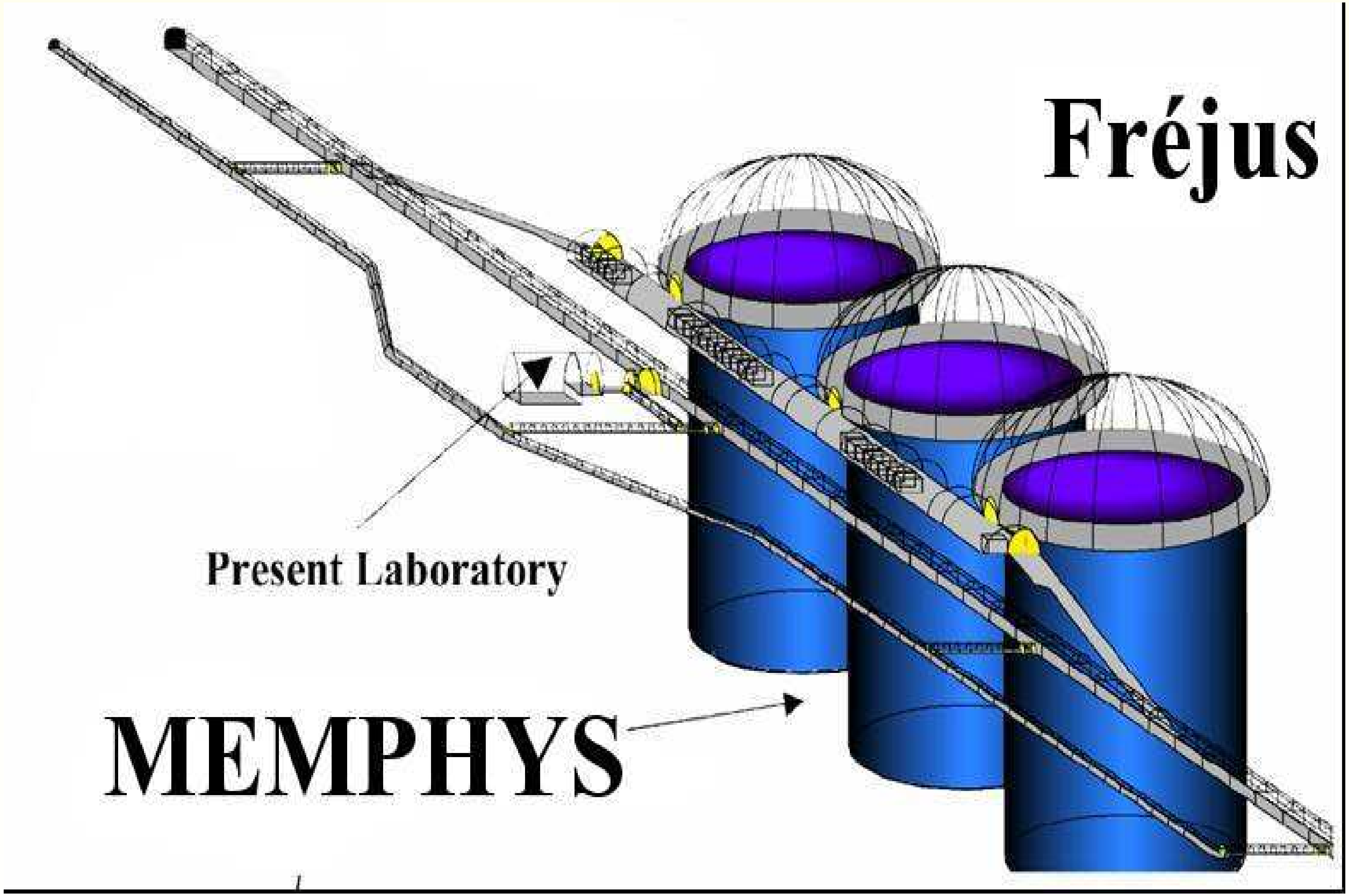


What More Can We Learn From ATM Neutrinos

- Using Megaton Water Cerenkov Detectors

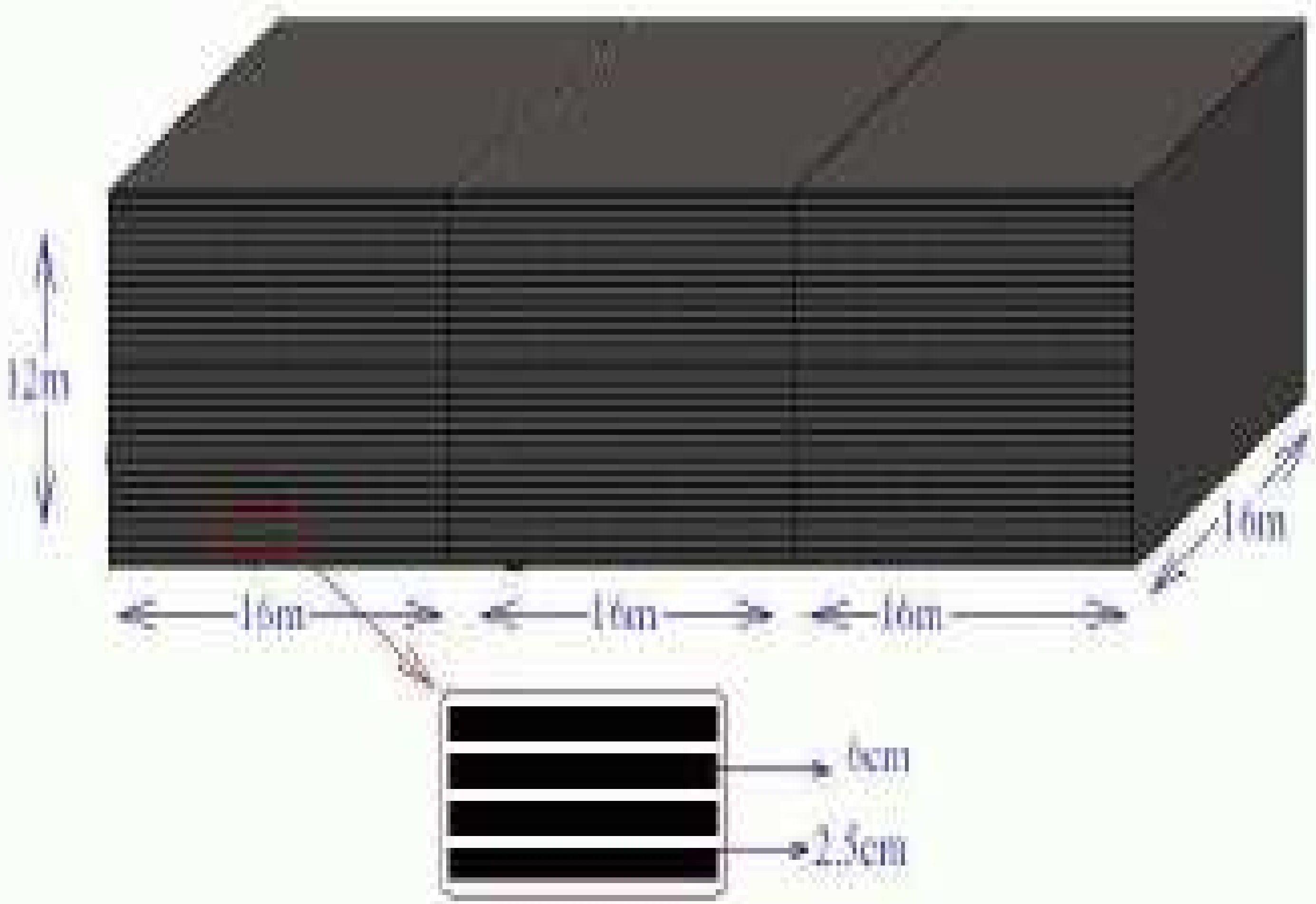
What More Can We Learn From ATM Neutrinos

- Using Megaton Water Cerenkov Detectors



What More Can We Learn From ATM Neutrinos

- Using Large Magnetized Iron Calorimeters (INO-ICAL)



What More Can We Learn From ATM Neutrinos

- Advantages of Large Magnetized Iron Detectors:

- Charge discrimination capacity

$\Rightarrow$ Can distinguish neutrino from antineutrinos

- Very good E and L resolution

$\Rightarrow$ Fine binning of data possible

What More Can We Learn From ATM Neutrinos

- Advantages of Large Magnetized Iron Detectors:
- Charge discrimination capacity

$\Rightarrow$ Can distinguish neutrino from antineutrinos

- Very good E and L resolution

$\Rightarrow$ Fine binning of data possible

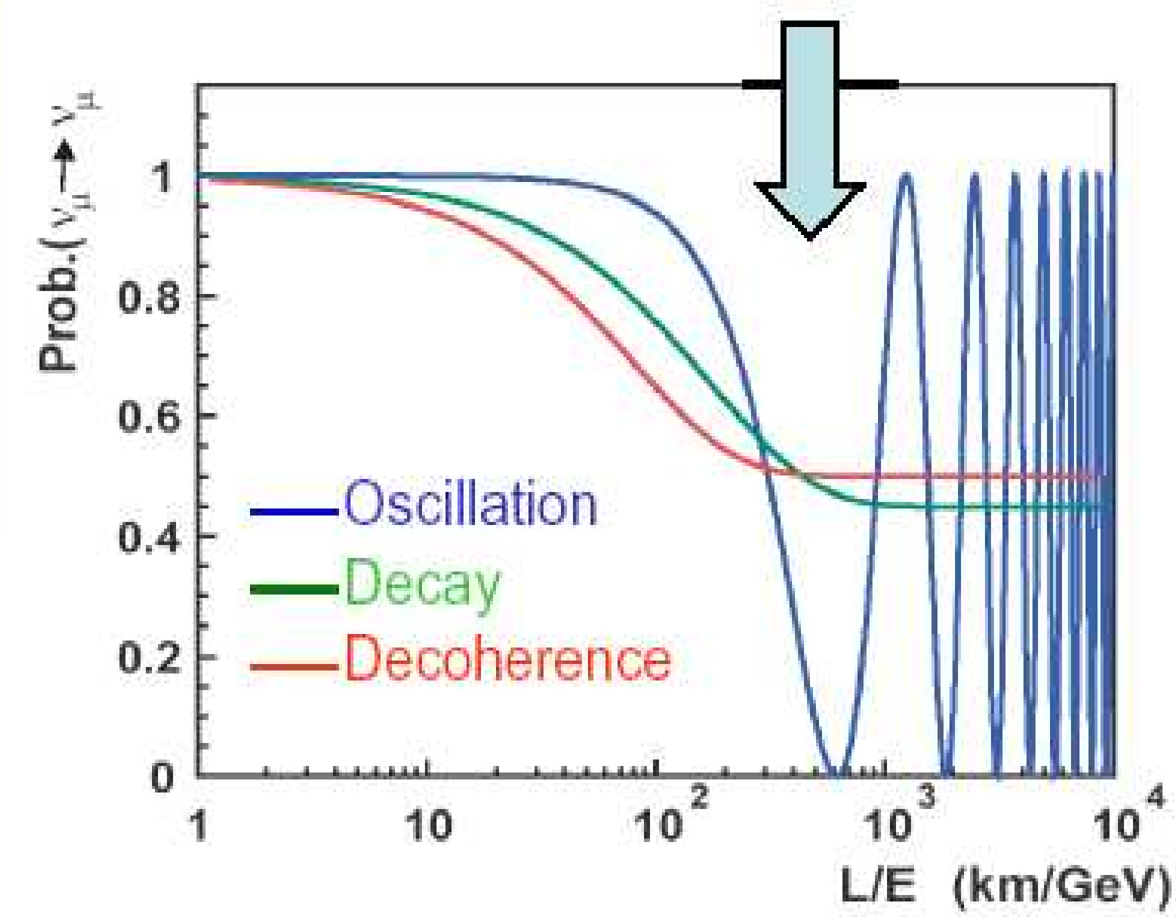
- Advantages of Water Cerenkov Detectors:
- Can detect electrons
- Low threshold

$\Rightarrow$ Can detect lower E neutrinos

Confirmation of Oscillations

Confirmation of Oscillations

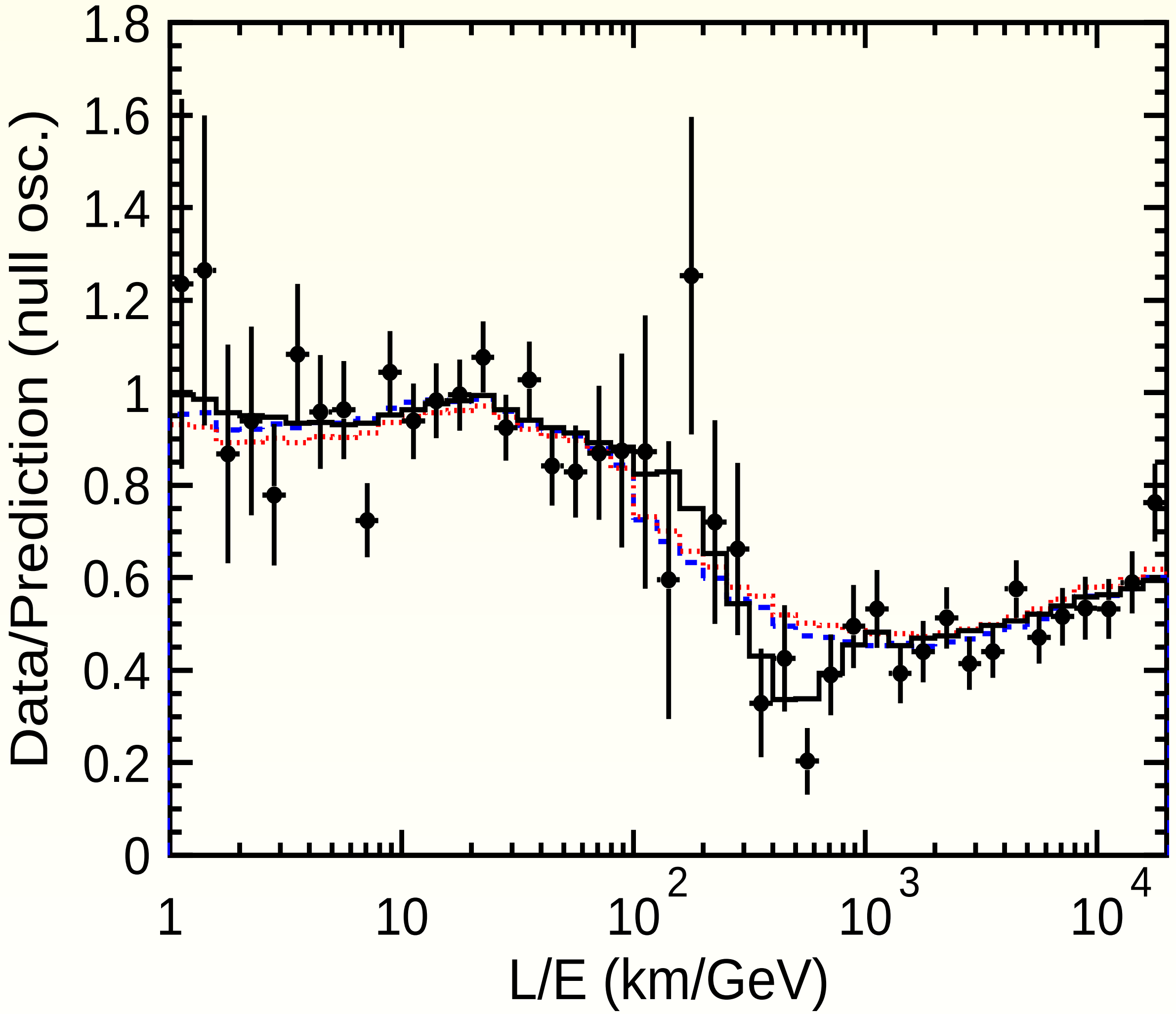
- Smoking Gun Signal for $\nu_\mu - \nu_\tau$ Oscillations



SK collab, Koshio talk, NANP '05

- Its important to observe the characteristic "dip" in L/E

Confirmation of Oscillations

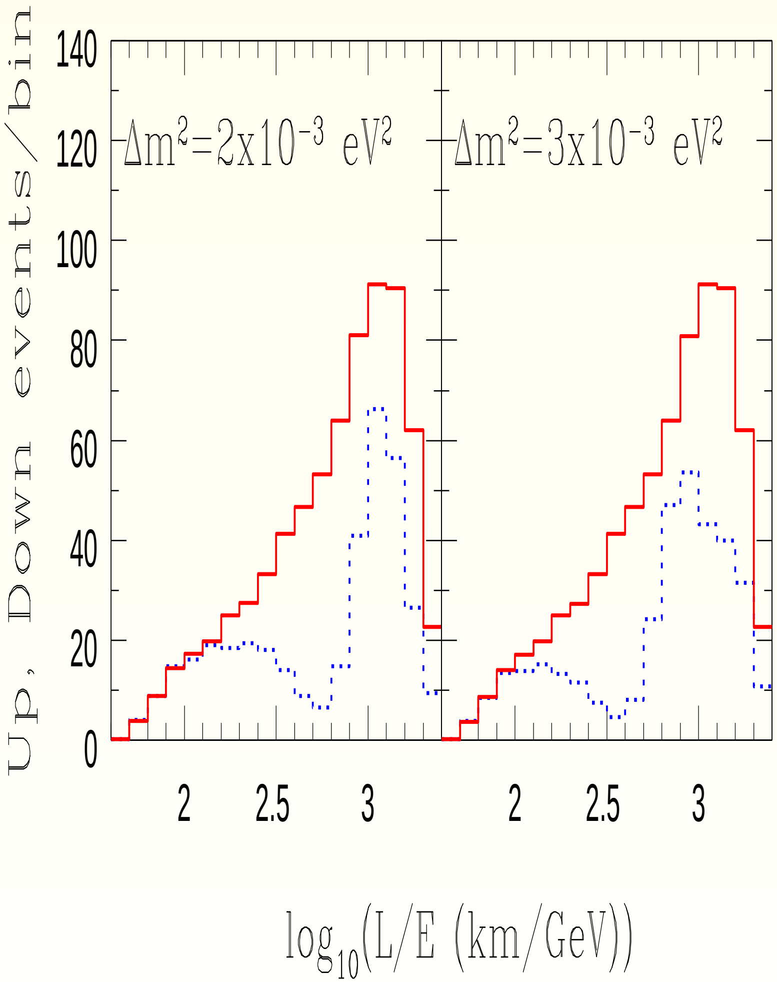


- There is an indication from the SK L/E data

SK Collaboration, hep-ex/04xxxxx

Confirmation of Oscillations

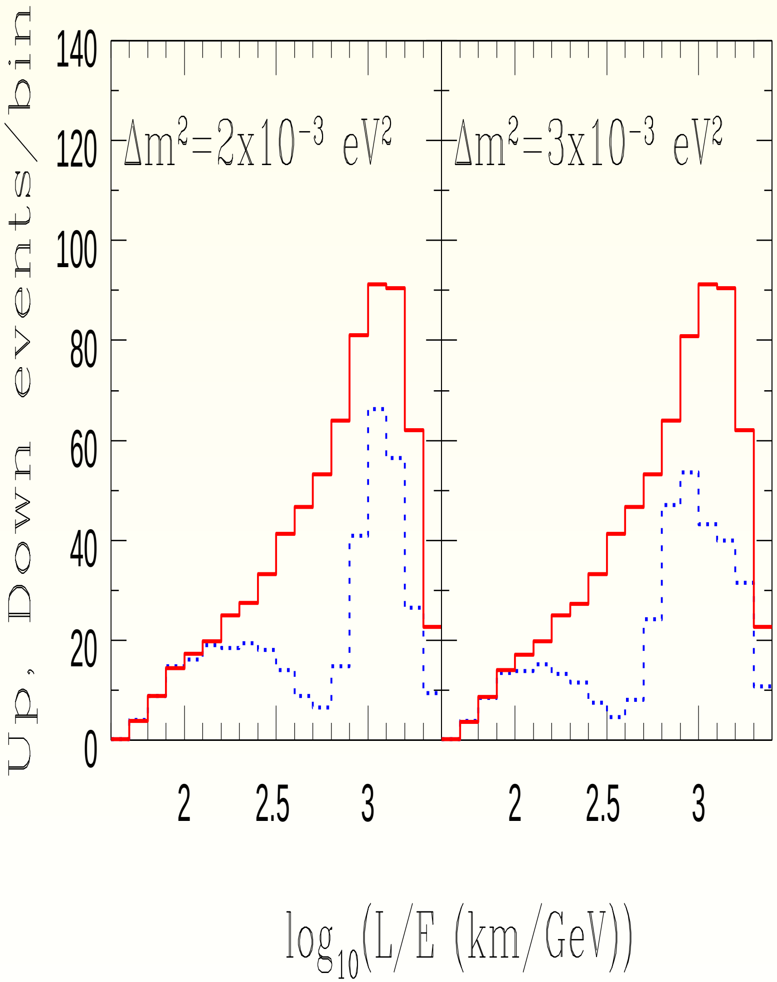
- In INO-ICAL

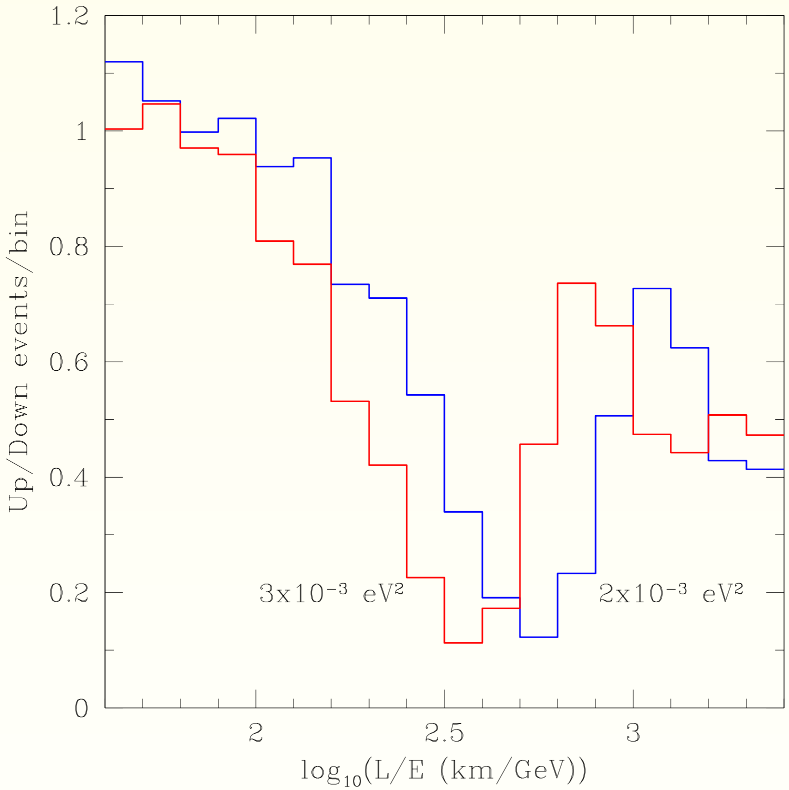


INO collaboration

Confirmation of Oscillations

- In INO-ICAL





INO collaboration

- The first oscillation dip should be clearly observable

Precision Measurement of Δm^2_{31} and θ_{23}

Precision Measurement of Δm^2_{31} and θ_{23}

Experiment	$\|\Delta m^2_{31}\|$	$\sin^2\theta_{23}$
Current	23%	33%

Precision Measurement of Δm^2_{31} and θ_{23}

Experiment	$\|\Delta m^2_{31}\|$	$\sin^2\theta_{23}$
Current	23%	33%
MINOS+CNGS	13%	38%
T2K (5 yrs)	6%	22%
NOνA (5 yrs)	13%	42%
Combination	4.5%	20%

Huber et al hep-ph/0403068

Precision Measurement of Δm^2_{31} and θ_{23}

Experiment	$\lvert\Delta m^2_{31}\rvert$	$\sin^2\theta_{23}$
Current	23%	33%
MINOS+CNGS	13%	38%
T2K (5 yrs)	6%	22%
NOνA (5 yrs)	13%	42%
Combination	4.5%	20%
SK20 (1.84 MTy)	17%	24%

Huber et al hep-ph/0403068

Gonzalez-Garcia et al, hep-ph/0408170

Precision Measurement of Δm^2_{31} and θ_{23}

Experiment	$\|\Delta m^2_{31}\|$	$\sin^2\theta_{23}$
Current	23%	33%
MINOS+CNGS	13%	38%
T2K (5 yrs)	6%	22%
NOνA (5 yrs)	13%	42%
Combination	4.5%	20%
SK20 (1.84 MTy)	17%	24%
INO (250 kTy)	10%	30%

Huber et al hep-ph/0403068

Gonzalez-Garcia et al, hep-ph/0408170

INO Collaboration

Three Flavor Oscillations

Three Flavor Oscillations in Vacuum

- Flavor Eigenstates $\neq$ Mass Eigenstates

Three Flavor Oscillations in Vacuum

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$

Three Flavor Oscillations in Vacuum

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Three Flavor Oscillations in Vacuum

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$P_{\beta\gamma}(L) = \delta_{\beta\gamma} \quad -4\sum_{j>1} Re\left(U_{\beta i}U^\star_{\gamma i}U^\star_{\beta j}U_{\gamma j}\right)\frac{\sin^2 \Delta m^2_{ij}L}{4E}$$

$$\pm 2\sum_{j>1} Im\left(U_{\beta i}U^\star_{\gamma i}U^\star_{\beta j}U_{\gamma j}\right)\frac{\sin \Delta m^2_{ij}L}{2E}.$$

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i} |\nu_i^m\rangle$

$$U^m = \begin{pmatrix} c^m_{12}c^m_{13} & s^m_{12}c^m_{13} & s^m_{13}e^{-i\delta^m} \\ -s^m_{12}c^m_{23} - c^m_{12}s^m_{23}s^m_{13}e^{i\delta^m} & c^m_{12}c^m_{23} - s^m_{12}s^m_{23}s^m_{13}e^{i\delta^m} & s^m_{23}c^m_{13} \\ s^m_{12}s^m_{23} - c^m_{12}c^m_{23}s^m_{13}e^{i\delta^m} & -c^m_{12}s^m_{23} - s^m_{12}c^m_{23}s^m_{13}e^{i\delta^m} & c^m_{23}c^m_{13} \end{pmatrix}$$

$$P^m_{\beta\gamma}(L) = \delta_{\beta\gamma} \quad -4\sum_{j>1} Re\left(U^m_{\beta i}U^{m\star}_{\gamma i}U^{m\star}_{\beta j}U^m_{\gamma j}\right)\frac{\sin^2\left(\Delta m^2_{ij}\right)^m L}{4E}$$

$$\pm 2\sum_{j>1} Im\left(U^m_{\beta i}U^{m\star}_{\gamma i}U^{m\star}_{\beta j}U^m_{\gamma j}\right)\frac{\sin\left(\Delta m^2_{ij}\right)^m L}{2E}.$$

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i} |\nu^m_i\rangle$

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i}|\nu^m_i\rangle$

$$M_F = U M_D U^\dagger \quad (Vacuum)$$

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i} |\nu^m_i\rangle$

$$M^m_F = U^m M^m_D U^{m\dagger} \quad (Matter)$$

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i} |\nu^m_i\rangle$

$$M^m_F = U^m M^m_D U^{m\dagger} \quad (Matter)$$

$$M^m_F = U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- $A = \pm 2\sqrt{2} G_F n_e E$ ($+ \Rightarrow$ neutrinos) ($- \Rightarrow$ antineutrinos)

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i}|\nu^m_i\rangle$

$$M_F^m = U^m M_D^m U^{m\dagger} \quad (Matter)$$

$$M_F^m = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- $A = \pm 2\sqrt{2} G_F n_e E$ ($+ \Rightarrow$ neutrinos) ($- \Rightarrow$ antineutrinos)

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i}|\nu^m_i\rangle$

$$M^m_F = U^m M^m_D U^{m\dagger} \quad (Matter)$$

$$M^m_F = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- $A = \pm 2\sqrt{2} G_F n_e E$ ($+ \Rightarrow$ neutrinos) ($- \Rightarrow$ antineutrinos)
- $A = \pm 7.56 \times 10^{-5} \rho(\mathrm{gm/cc}) E(\mathrm{GeV})$ eV^2

Three Flavor Oscillations in Matter

- Flavor Eigenstates $\neq$ Mass Eigenstates
- $|\nu_\alpha\rangle = \sum_i U^m_{\alpha i} |\nu^m_i\rangle$

$$M^m_F = U^m M^m_D U^{m\dagger} \quad (Matter)$$

$$M^m_F = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- $A = \pm 2\sqrt{2} G_F n_e E$ (+ $\Rightarrow$ neutrinos) ($-$ $\Rightarrow$ antineutrinos)
- $A = \pm 7.56 \times 10^{-5} \rho(\mathrm{gm/cc}) E(\mathrm{GeV})$ eV^2
- $|A| \sim 2 \times 10^{-3} \frac{\rho}{4.5(\mathrm{gm/cc})} \frac{E}{5.0(\mathrm{GeV})}$ eV^2 $\sim |\Delta m^2_{31}|$

Three Flavor Oscillations in Matter

- Mass squared difference in matter changes to:

$$(\Delta m^2_{31})^m = \sqrt{(\Delta m^2_{31}\cos 2\theta_{13} - A)^2 + (\Delta m^2_{31}\sin 2\theta_{13})^2}$$

Three Flavor Oscillations in Matter

- Mass squared difference in matter changes to:

$$(\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}$$

- Mixing angle in matter changes to:

$$\sin 2\theta_{13}^m = \sin 2\theta_{13} \frac{\Delta m_{31}^2}{(\Delta m_{31}^2)^m}$$

Three Flavor Oscillations in Matter

- Mass squared difference in matter changes to:

$$(\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}$$

- Mixing angle in matter changes to:

$$\sin 2\theta_{13}^m = \sin 2\theta_{13} \, \frac{\Delta m_{31}^2}{(\Delta m_{31}^2)^m}$$

- Both of these depend on the sign of Δm_{31}^2

Three Flavor Oscillations in Matter

- Mass squared difference in matter changes to:

$$(\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}$$

- Mixing angle in matter changes to:

$$\sin 2\theta_{13}^m = \sin 2\theta_{13} \frac{\Delta m_{31}^2}{(\Delta m_{31}^2)^m}$$

- Both of these depend on the sign of Δm_{31}^2
- Effect is opposite for neutrinos and antineutrinos

Three Flavor Oscillations in Matter

- Mass squared difference in matter changes to:

$$(\Delta m^2_{31})^m = \sqrt{(\Delta m^2_{31}\cos 2\theta_{13} - A)^2 + (\Delta m^2_{31}\sin 2\theta_{13})^2}$$

- Mixing angle in matter changes to:

$$\sin 2\theta^m_{13} = \sin 2\theta_{13}\,\frac{\Delta m^2_{31}}{(\Delta m^2_{31})^m}$$

- Both of these depend on the sign of Δm^2_{31}
- Effect is opposite for neutrinos and antineutrinos
- When

$$A = \Delta m^2_{31}\cos 2\theta_{13}$$

$$\sin 2\theta^m_{13} = 1$$

Three Flavor Oscillations in Matter

- Mass squared difference in matter changes to:

$$(\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}$$

- Mixing angle in matter changes to:

$$\sin 2\theta_{13}^m = \sin 2\theta_{13} \frac{\Delta m_{31}^2}{(\Delta m_{31}^2)^m}$$

- Both of these depend on the sign of Δm_{31}^2
- Effect is opposite for neutrinos and antineutrinos
- When

$$A = \Delta m_{31}^2 \cos 2\theta_{13}$$

$$\sin 2\theta_{13}^m = 1$$

Matter Enhanced (MSW) Resonance

Wolfenstein 1978, Mikheyev and Smirnov 1985-6

Three Generation Oscillation Probabilities

Electron ν Transition/Survival Probability

Electron ν Transition/Survival Probability

$$\lim_{\Delta m^2_{21} \to 0} P_{\mu e}(L,E) = \sin^2\theta_{23} \sin^2 2\theta^m_{13} \sin^2 \frac{(\Delta m^2_{31})^m L}{4E}$$

$$\lim_{\Delta m^2_{21} \to 0} P_{\tau e}(L,E) = \cos^2\theta_{23} \sin^2 2\theta^m_{13} \sin^2 \frac{(\Delta m^2_{31})^m L}{4E}$$

$$\lim_{\Delta m^2_{21} \to 0} P_{ee}(L,E) = 1 - \sin^2 2\theta^m_{13} \sin^2 \frac{(\Delta m^2_{31})^m L}{4E}$$

Electron ν Transition/Survival Probability

$$\lim_{\Delta m^2_{21} \to 0} P_{\mu e}(L,E) = \sin^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$

$$\lim_{\Delta m^2_{21} \to 0} P_{\tau e}(L,E) = \cos^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$

$$\lim_{\Delta m^2_{21} \to 0} P_{ee}(L,E) = 1 - \sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$

- Matter effects will be maximal when:
  - $\star$ $\sin^2 2\theta^m_{13} = 1$
  - $\star$ $\sin^2\frac{(\Delta m^2_{31})^m L}{4E} = 1$ Ghoshal,Gandhi,Goswami,Mehta,UmaSankar (2004)

are satisfied simultaneously

Electron ν Transition/Survival Probability

$$\lim_{\Delta m^2_{21}\to 0} P_{\mu e}(L,E) = \sin^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$

$$\lim_{\Delta m^2_{21}\to 0} P_{\tau e}(L,E) = \cos^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$

$$\lim_{\Delta m^2_{21}\to 0} P_{ee}(L,E) = 1-\sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$

- Matter effects will be maximal when:
  - $\star$ $\sin^2 2\theta^m_{13} = 1$
  - $\star$ $\sin^2\frac{(\Delta m^2_{31})^m L}{4E} = 1$ Ghoshal,Gandhi,Goswami,Mehta,UmaSankar (2004)

  are satisfied simultaneously
- This condition of maximal matter effects is

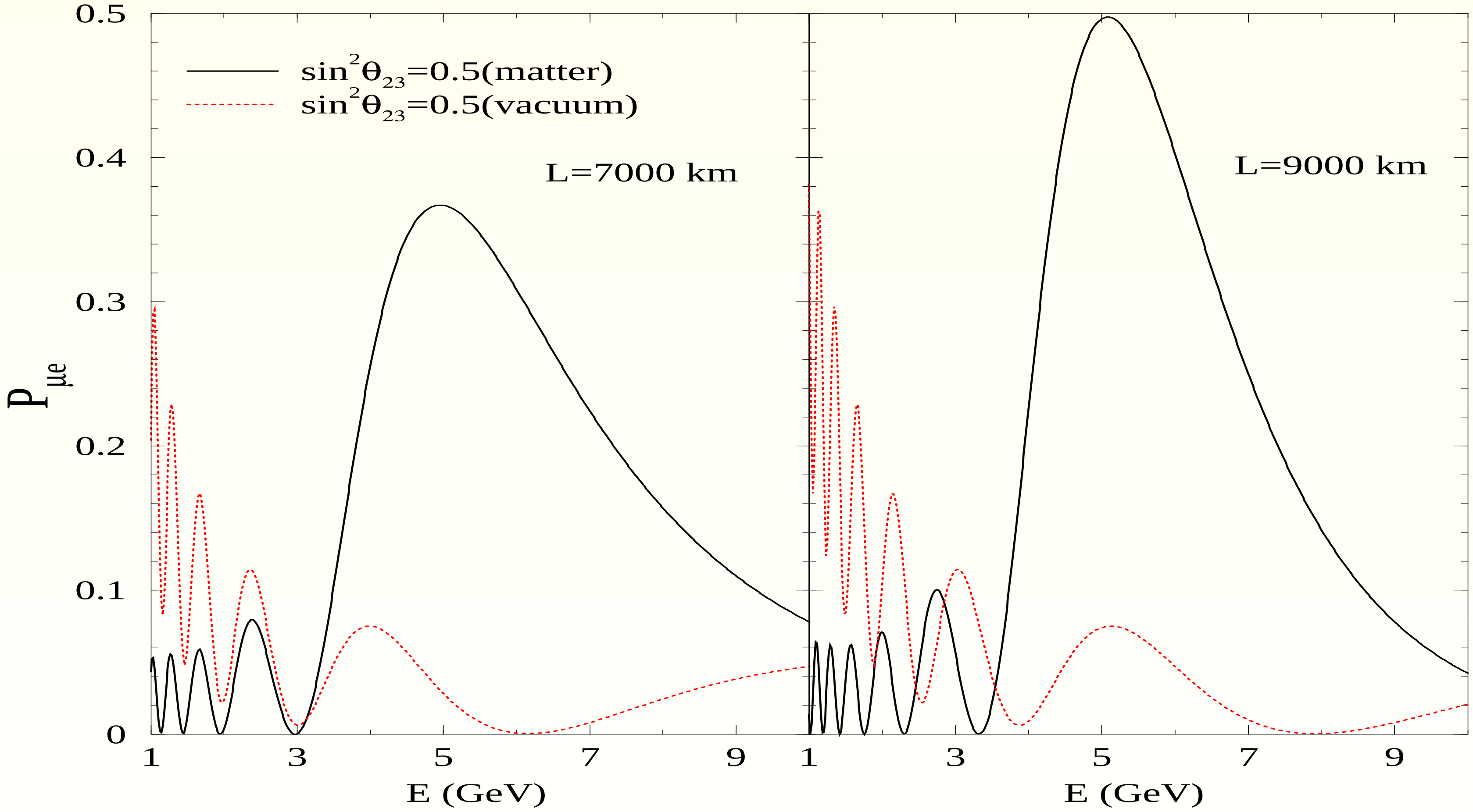
$$(\rho L)^{max} \simeq \frac{(2p+1)\pi 5.18\times 10^3}{\tan 2\theta_{13}}\ \mathrm{km\ gm/cc}$$

Electron ν Transition/Survival Probability

$$\lim_{\Delta m^2_{21} \to 0} P_{\mu e}(L,E) = \sin^2\theta_{23} \sin^2 2\theta^m_{13} \sin^2 \frac{(\Delta m^2_{31})^m L}{4E}$$

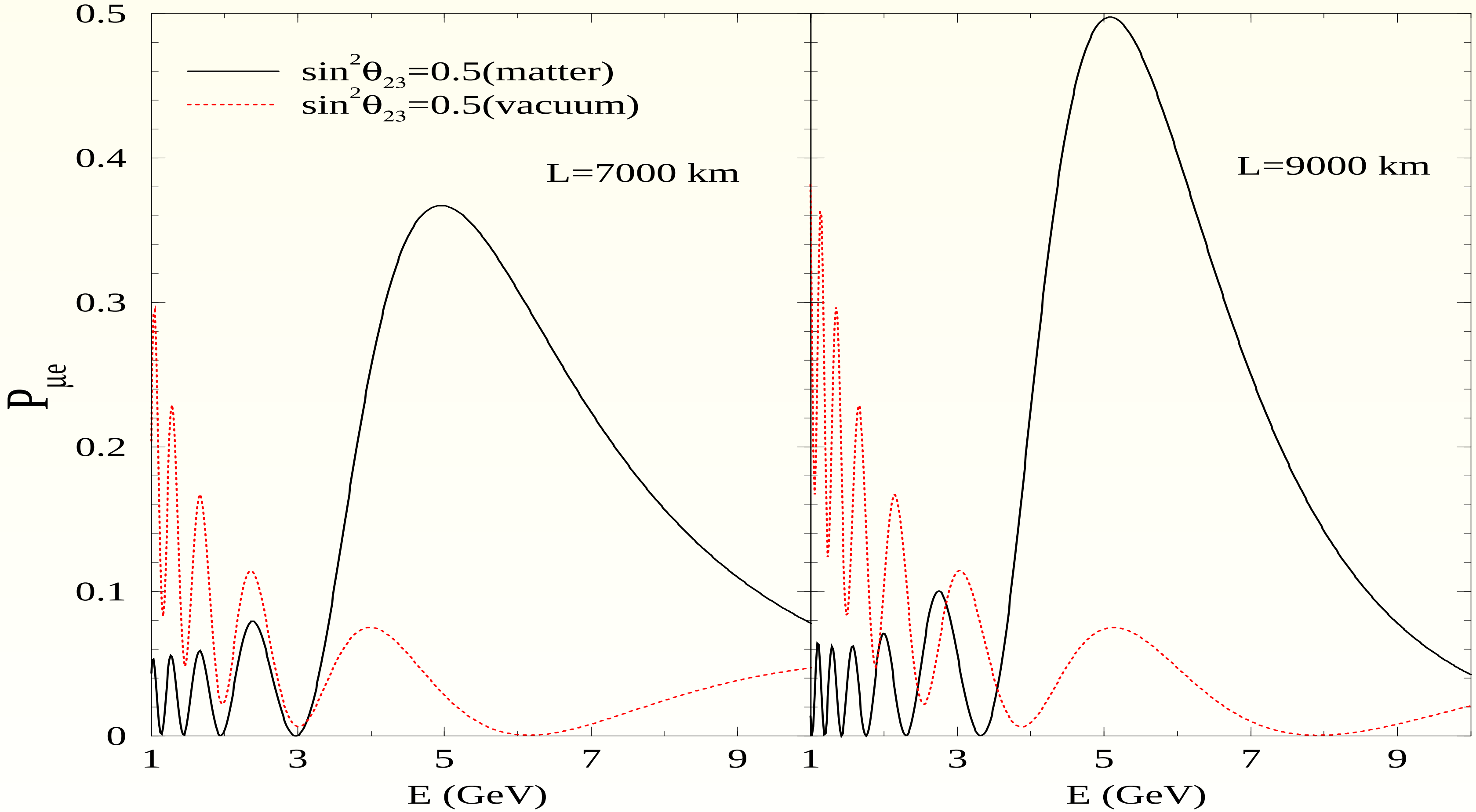
Electron ν Transition/Survival Probability

$$\lim_{\Delta m^2_{21}\to 0} P_{\mu e}(L,E) = \sin^2\theta_{23}\sin^2 2\theta^m_{13}\sin^2\frac{(\Delta m^2_{31})^m L}{4E}$$



Electron ν Transition/Survival Probability

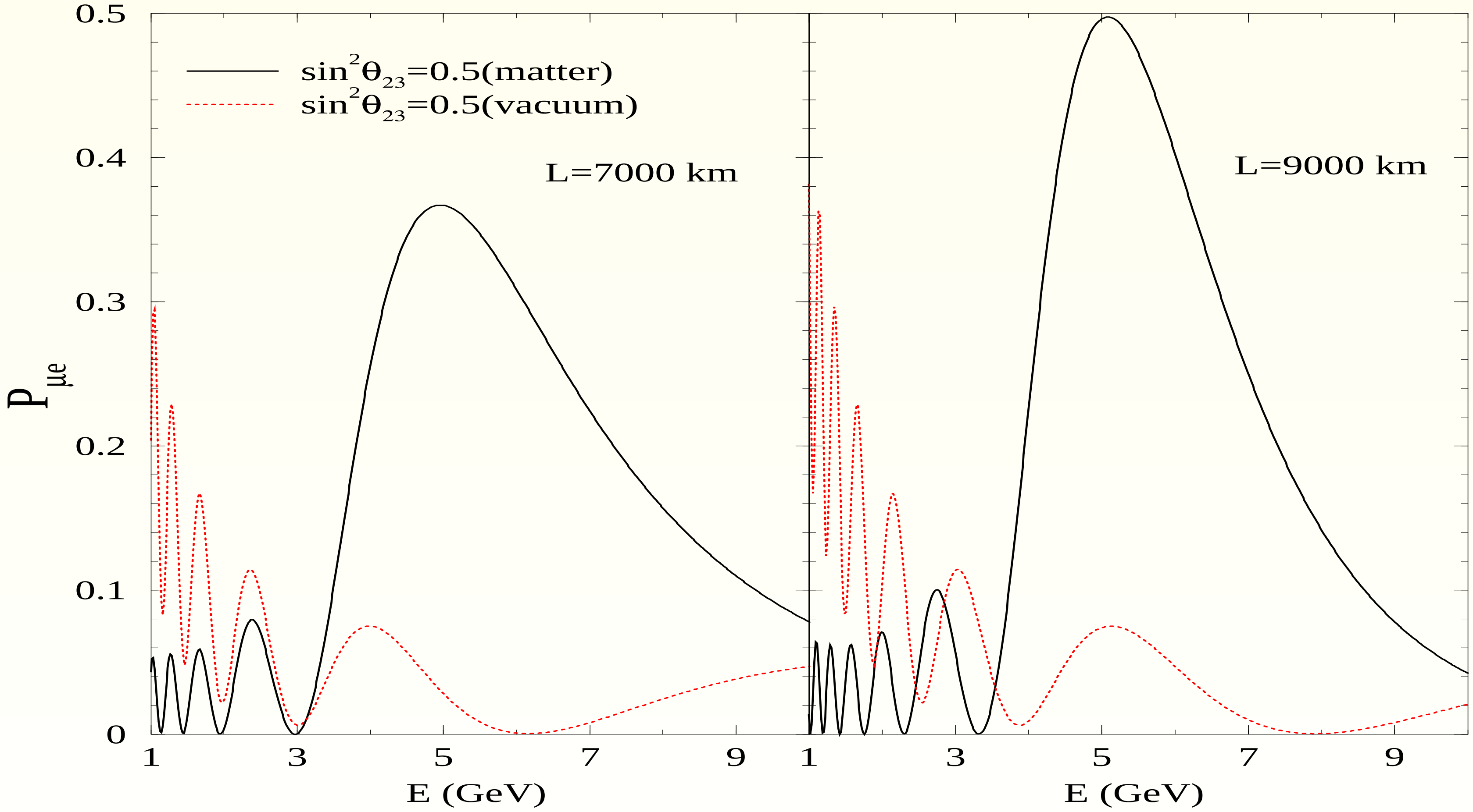
$$\lim_{\Delta m^2_{21} \to 0} P_{\mu e}(L,E) = \sin^2\theta_{23} \sin^2 2\theta^m_{13} \sin^2 \frac{(\Delta m^2_{31})^m L}{4E}$$



- Above ~ 3 GeV, matter effects increase for all E and L

Electron ν Transition/Survival Probability

$$\lim_{\Delta m^2_{21} \to 0} P_{\mu e}(L,E) = \sin^2\theta_{23} \sin^2 2\theta^m_{13} \sin^2 \frac{(\Delta m^2_{31})^m L}{4E}$$



- Sub-dominant Δm^2_{21} oscillations are also very crucial

Muon Neutrino Survival Probability

$$\lim_{\Delta m^2_{21} \to 0} P_{\mu\mu}(L,E) = 1 - P^1_{\mu\mu}(L,E) - P^2_{\mu\mu}(L,E) - P^3_{\mu\mu}(L,E)$$

$$P^1_{\mu\mu}(L,E) = \sin^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2\frac{(A+\Delta m^2_{31}) - (\Delta m^2_{31})^m}{8E}L$$

$$P^2_{\mu\mu}(L,E) = \cos^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2\frac{(A+\Delta m^2_{31}) + (\Delta m^2_{31})^m}{8E}L$$

$$P^3_{\mu\mu}(L,E) = \sin^2 2\theta^m_{13}\sin^4\theta_{23}\sin^2\frac{(\Delta m^2_{31})^m}{4E}L$$

Muon Neutrino Survival Probability

$$\lim_{\Delta m^2_{21} \to 0} P_{\mu\mu}(L,E) = 1 - P^1_{\mu\mu}(L,E) - P^2_{\mu\mu}(L,E) - P^3_{\mu\mu}(L,E)$$

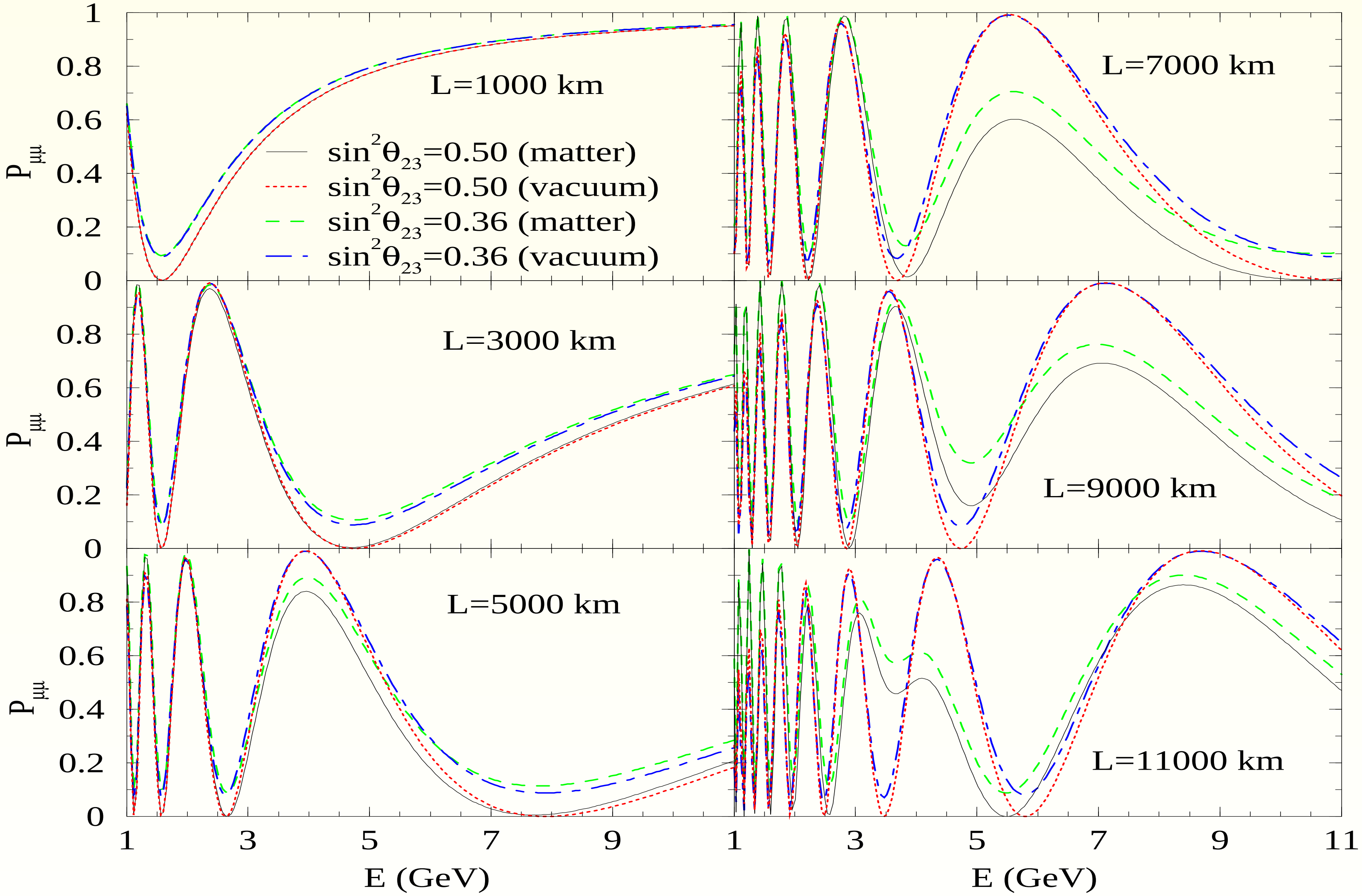
$$P^1_{\mu\mu}(L,E) = \sin^2\theta^m_{13} \sin^2 2\theta_{23} \sin^2 \frac{(A+\Delta m^2_{31}) - (\Delta m^2_{31})^m}{8E} L$$

$$P^2_{\mu\mu}(L,E) = \cos^2\theta^m_{13} \sin^2 2\theta_{23} \sin^2 \frac{(A+\Delta m^2_{31}) + (\Delta m^2_{31})^m}{8E} L$$

$$P^3_{\mu\mu}(L,E) = \sin^2 2\theta^m_{13} \sin^4\theta_{23} \sin^2 \frac{(\Delta m^2_{31})^m}{4E} L$$

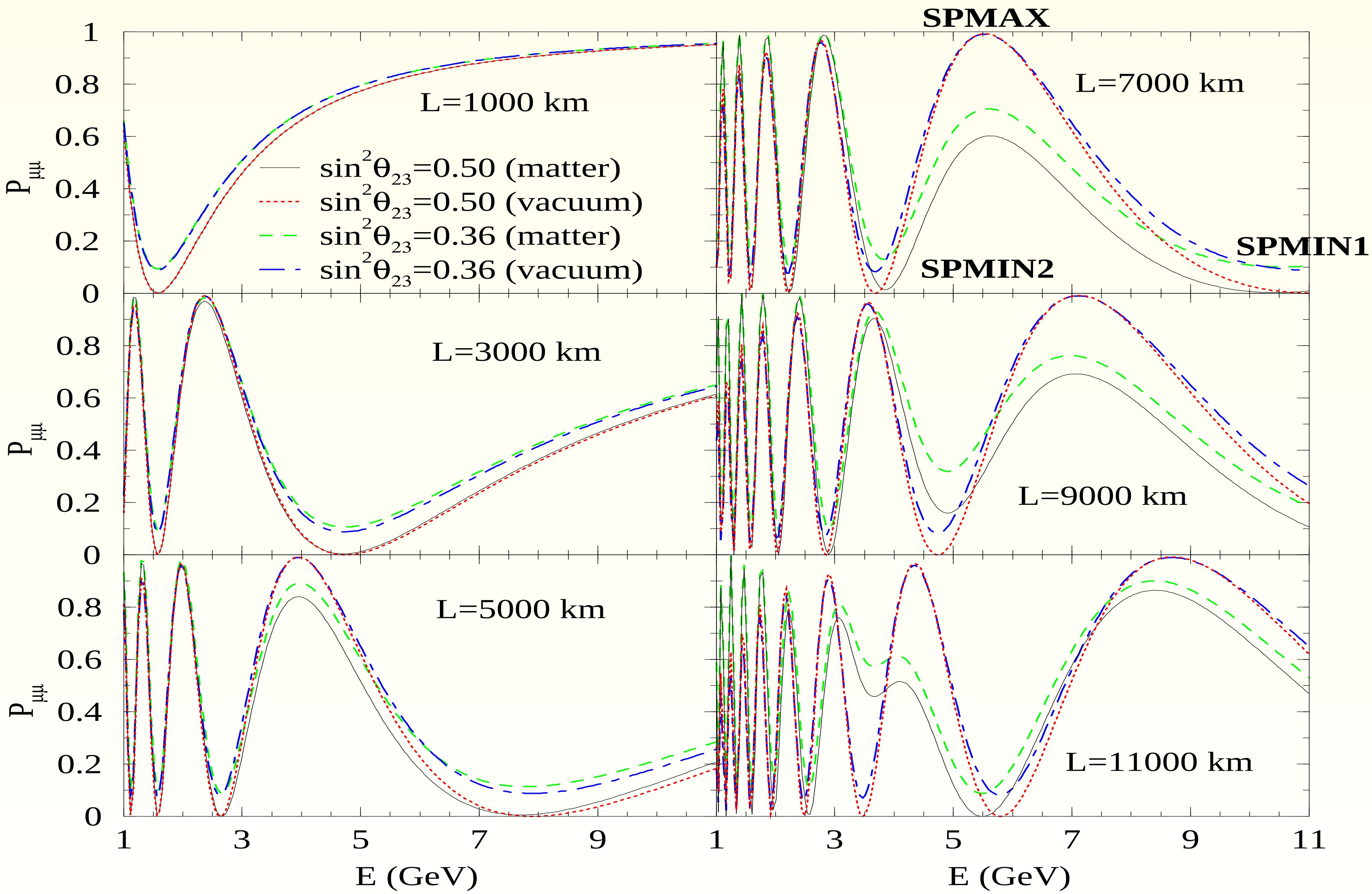
- Dependence on θ_{23} in the form $\sin^4\theta_{23}$
- Octant sensitivity is expected to be good

Large Matter Effects in ν_μ Survival Probability



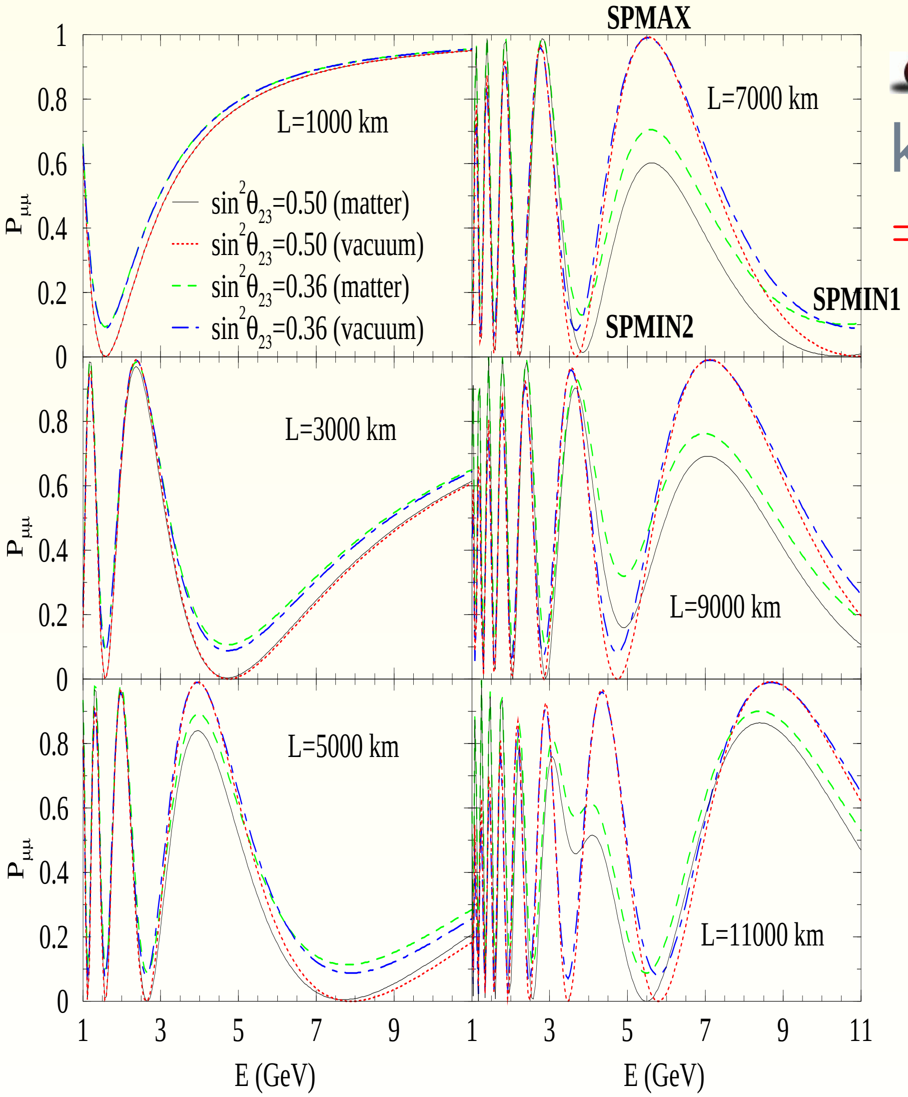
S.C. and P. Roy, hep-ph/0509197

Large Matter Effects in ν_μ Survival Probability



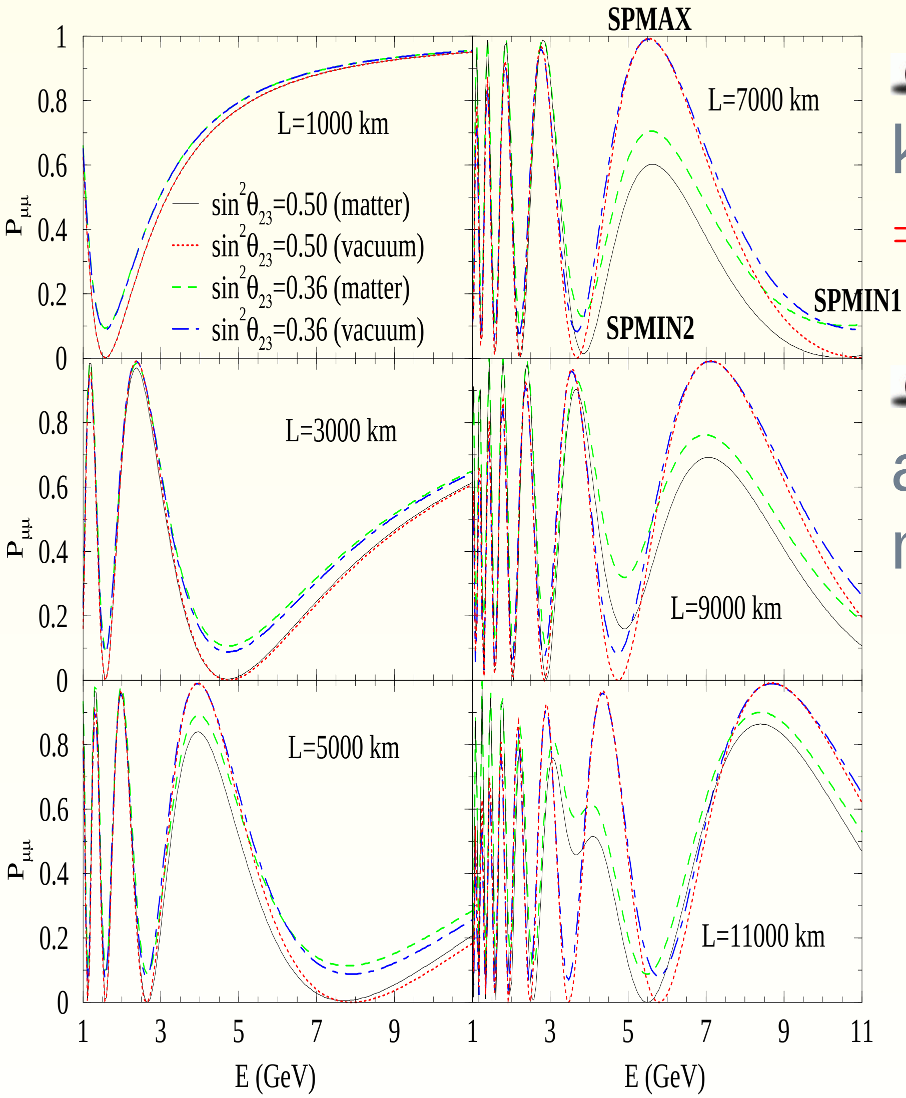
S.C. and P. Roy, hep-ph/0509197

Large Matter Effects in ν_μ Survival Probability



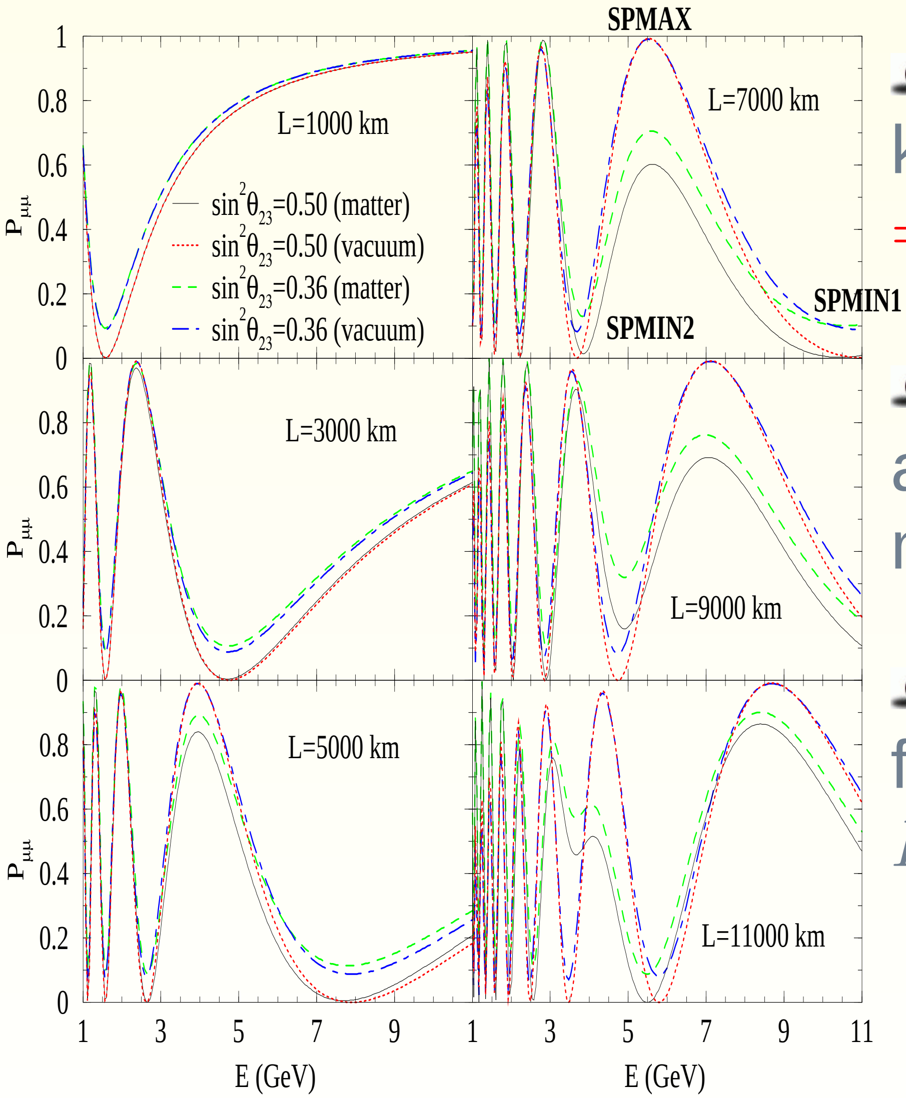
- Max effect for $L \simeq 7000$ km and $E \simeq 5$ GeV $\Rightarrow (E_{\mathrm{SPMAX}} \simeq E_{\mathrm{res}})$

Large Matter Effects in ν_μ Survival Probability



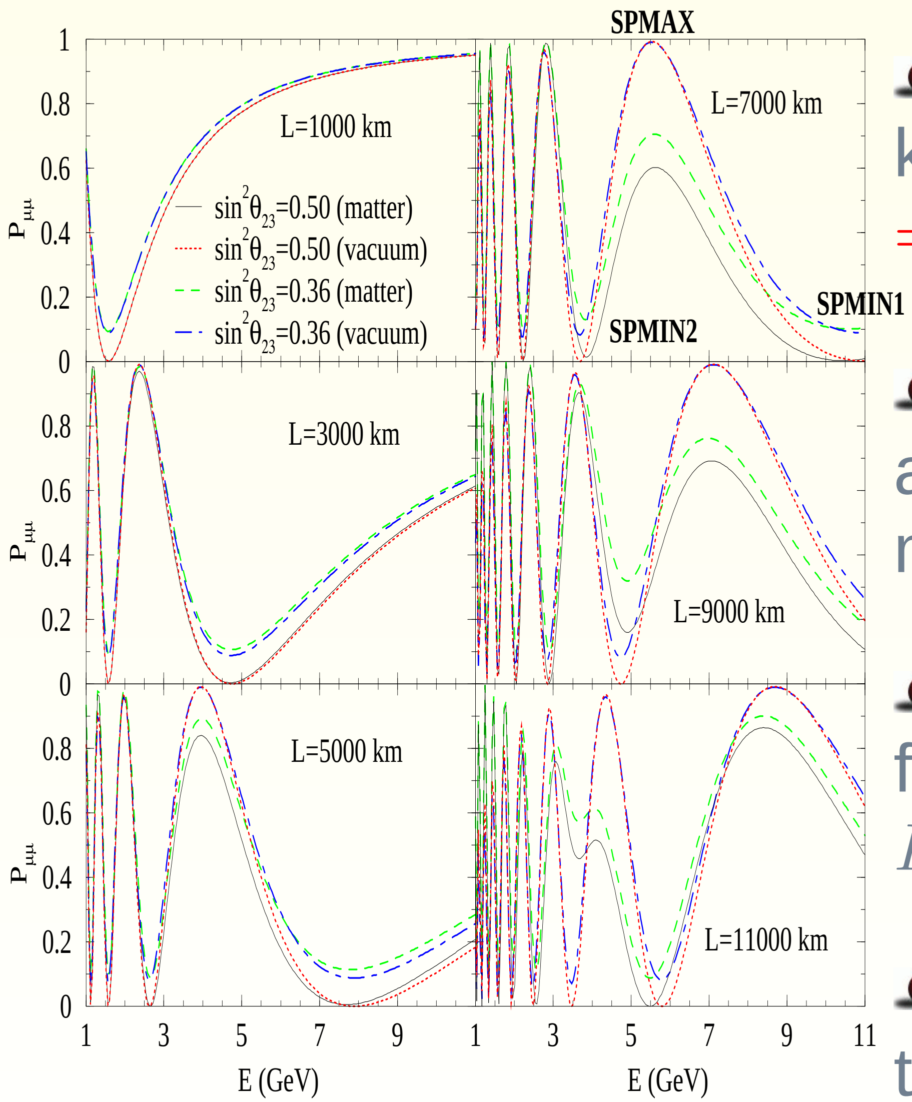
- Max effect for $L \simeq 7000$ km and $E \simeq 5$ GeV $\Rightarrow$ $(E_{\rm SPMAX} \simeq E_{\rm res})$
- $P_{\mu\mu}$ decreases (increases) at SPMAX (SPMIN) due to matter effects

Large Matter Effects in ν_μ Survival Probability



- Max effect for $L \simeq 7000$ km and $E \simeq 5$ GeV $\Rightarrow$ $(E_{\rm SPMAX} \simeq E_{\rm res})$
- $P_{\mu\mu}$ decreases (increases) at SPMAX (SPMIN) due to matter effects
- Sign of the earth matter effects depends on both E and L

Large Matter Effects in ν_μ Survival Probability



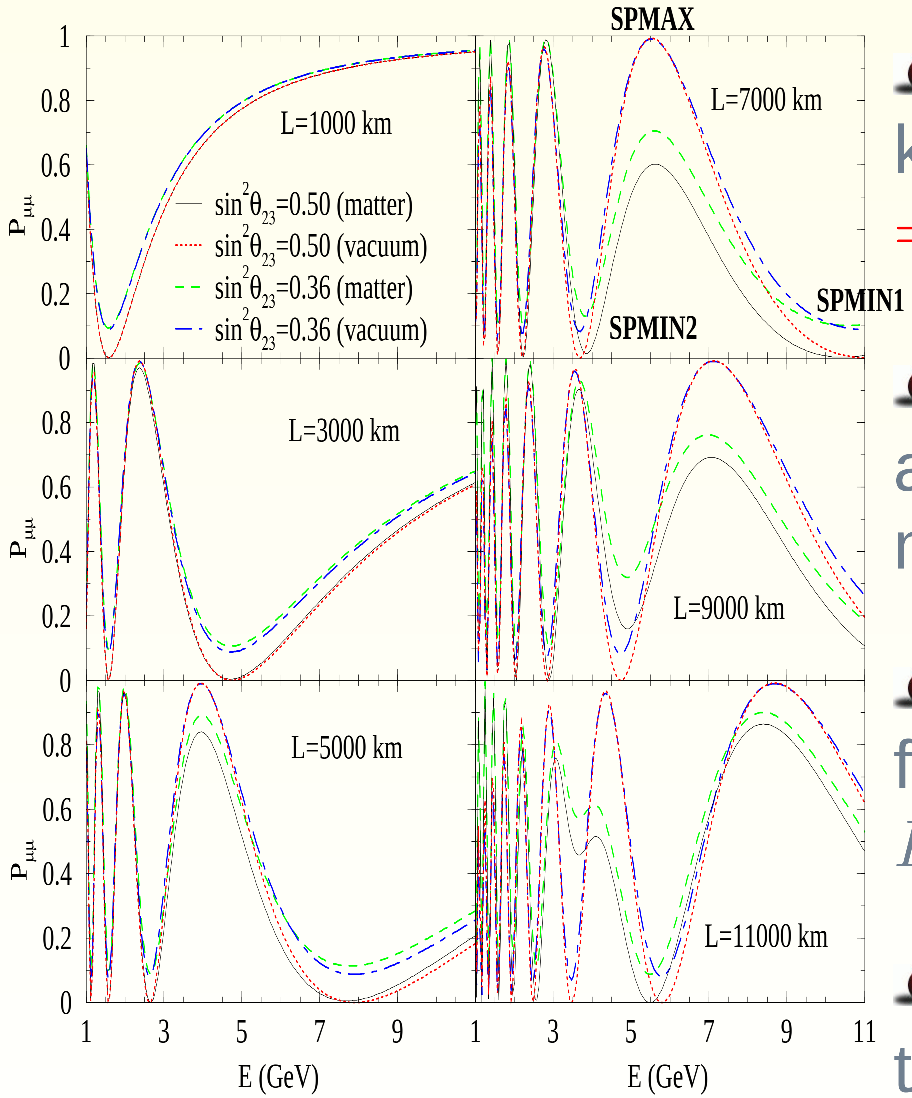
- Max effect for $L \simeq 7000$ km and $E \simeq 5$ GeV $\Rightarrow$ $(E_{\rm SPMAX} \simeq E_{\rm res})$

- $P_{\mu\mu}$ decreases (increases) at SPMAX (SPMIN) due to matter effects

- Sign of the earth matter effects depends on both E and L

- Matter effects depend on the value of $\sin^2\theta_{23}$

Large Matter Effects in ν_μ Survival Probability



- Max effect for $L \simeq 7000$ km and $E \simeq 5$ GeV $\Rightarrow$ $(E_{\rm SPMAX} \simeq E_{\rm res})$
- $P_{\mu\mu}$ decreases (increases) at SPMAX (SPMIN) due to matter effects
- Sign of the earth matter effects depends on both E and L
- Matter effects depend on the value of $\sin^2\theta_{23}$
- Most important to choose the bins properly in E and L

Atmospheric Neutrino Events

Atmospheric Neutrino Events

Change in number of muon events:

$$N_\mu = N_\mu^0 P_{\mu\mu} + N_e^0 P_{e\mu}$$

$$= N_\mu^0 \left[P_{\mu\mu} + \frac{1}{r} P_{e\mu} \right]; \quad (\text{where } r = \frac{N_\mu^0}{N_e^0})$$

$$1 - \frac{N_\mu}{N_\mu^0} \simeq (P_{\mu\mu}^1 + P_{\mu\mu}^2) + (P_{\mu\mu}^3)' s_{23}^2 (s_{23}^2 - \frac{1}{r})$$

$$(P_{\mu\mu}^3)' = \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m}{4E} L$$

- Can be used to study maximality and octant of θ_{23}
- Can be used to study the neutrino mass hierarchy
- Δm_{21}^2 and δ_{CP} bring in small effects

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$

$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$

$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$

$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$

$$+ \sin\theta_{23}\cos\theta_{23}\, r\, Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

- Δm_{21}^2-driven oscillation effect

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$

$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$

$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

- Δm_{21}^2-driven oscillation effect
- Brings an excess (depletion) in the sub-GeV electron event sample for $\sin^2\theta_{23} < 0.5 (> 0.5)$

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m^2_{21})^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m^2_{31})^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

- Δm^2_{21}-driven oscillation effect
- Brings an excess (depletion) in the sub-GeV electron event sample for $\sin^2\theta_{23} < 0.5 (> 0.5)$
- Can be used for studying maximality and octant of θ_{23}

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$

$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$

$$+ \sin\theta_{23}\cos\theta_{23}\ r\ Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

- θ_{13}-driven oscillation effect

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$

$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$

$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

- θ_{13}-driven oscillation effect
- Reduces the excess (depletion) in the sub-GeV electron event sample for $\sin^2\theta_{23} < 0.5 (> 0.5)$ – bad!!

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\ r\ Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

- θ_{13}-driven oscillation effect
- Reduces the excess (depletion) in the sub-GeV electron event sample for $\sin^2\theta_{23} < 0.5 (> 0.5)$ – bad!!
- Big earth matter effects $\Rightarrow$ big change in multi-GeV e sample, the change being $\sin^2\theta_{23}$ dependent – excellent!!

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\, r\, Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

- θ_{13}-driven oscillation effect
- Reduces the excess (depletion) in the sub-GeV electron event sample for $\sin^2\theta_{23} < 0.5 (> 0.5)$ – bad!!
- Big earth matter effects $\Rightarrow$ big change in multi-GeV e sample, the change being $\sin^2\theta_{23}$ dependent – excellent!!
- Can be used for studying maximality and octant of θ_{23}

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\ r\ Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

- θ_{13}-driven oscillation effect
- Reduces the excess (depletion) in the sub-GeV electron event sample for $\sin^2\theta_{23} < 0.5 (> 0.5)$ – bad!!
- Big earth matter effects $\Rightarrow$ big change in multi-GeV e sample, the change being $\sin^2\theta_{23}$ dependent – excellent!!
- Can be used for studying maximality and octant of θ_{23}
- Can be used for studying the neutrino mass hierarchy

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$

$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$

$$+ \sin\theta_{23}\cos\theta_{23}\ r\ Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12} \exp(-i\delta_{CP})\right]$$

- "interference" term which depends on δ_{CP}

Atmospheric Neutrino Events

Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\; r\; Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

- “interference” term which depends on δ_{CP}

- It might cancel the effect of the Δm_{21}^2 and θ_{13} terms depending on the value of δ_{CP} – bad bad bad...

Atmospheric Neutrino Events

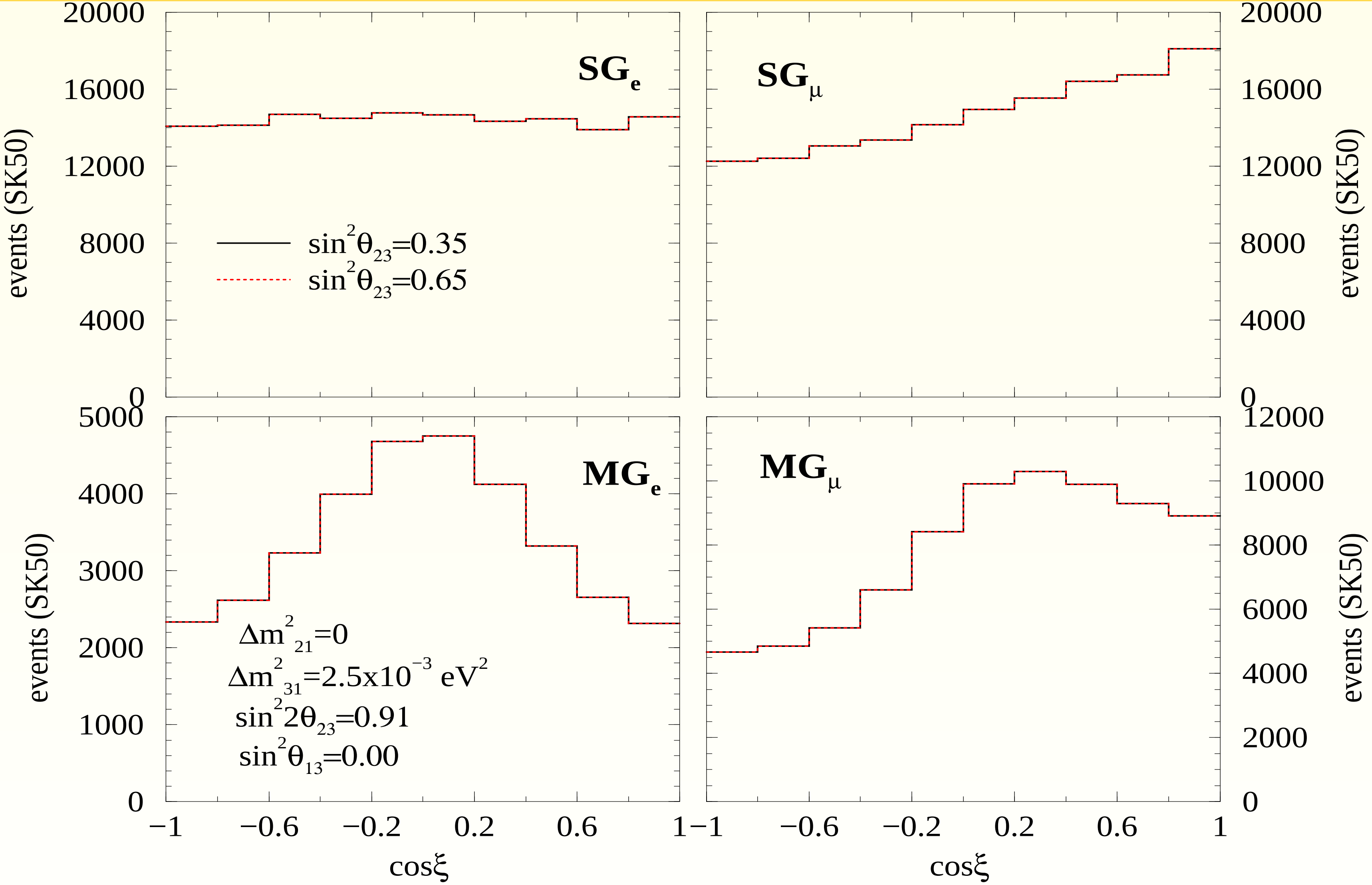
Change in number of electron events:

$$\frac{N_e}{N_e^0} - 1 \simeq \sin^2 2\theta_{12}^m \sin^2\left(\frac{(\Delta m_{21}^2)^m L}{4E}\right) \times (r\cos^2\theta_{23} - 1)$$
$$+ \sin^2 2\theta_{13}^m \sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \times (r\sin^2\theta_{23} - 1)$$
$$+ \sin\theta_{23}\cos\theta_{23}\, r\, Re\left[A_{13}^* A_{12}\exp(-i\delta_{CP})\right]$$

- "interference" term which depends on δ_{CP}
- It might cancel the effect of the Δm_{21}^2 and θ_{13} terms depending on the value of δ_{CP} – bad bad bad...
- But it might tell us if $\delta_{CP} = 0$ or π (Fogli et al. hep-ph/0506083)

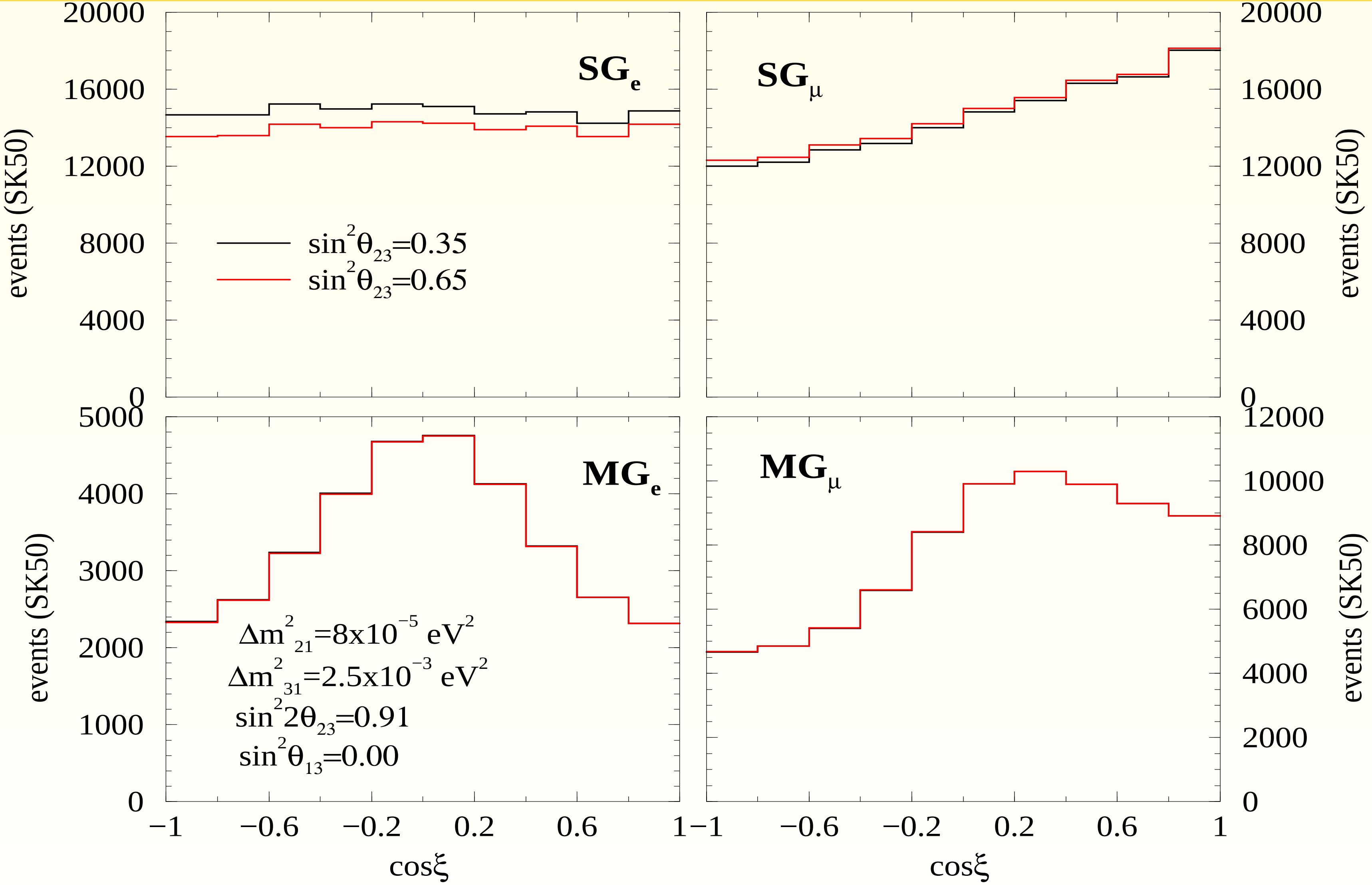
Atmospheric ν Events in MTon Water Detector

Atmospheric ν Events in MTon Water Detector



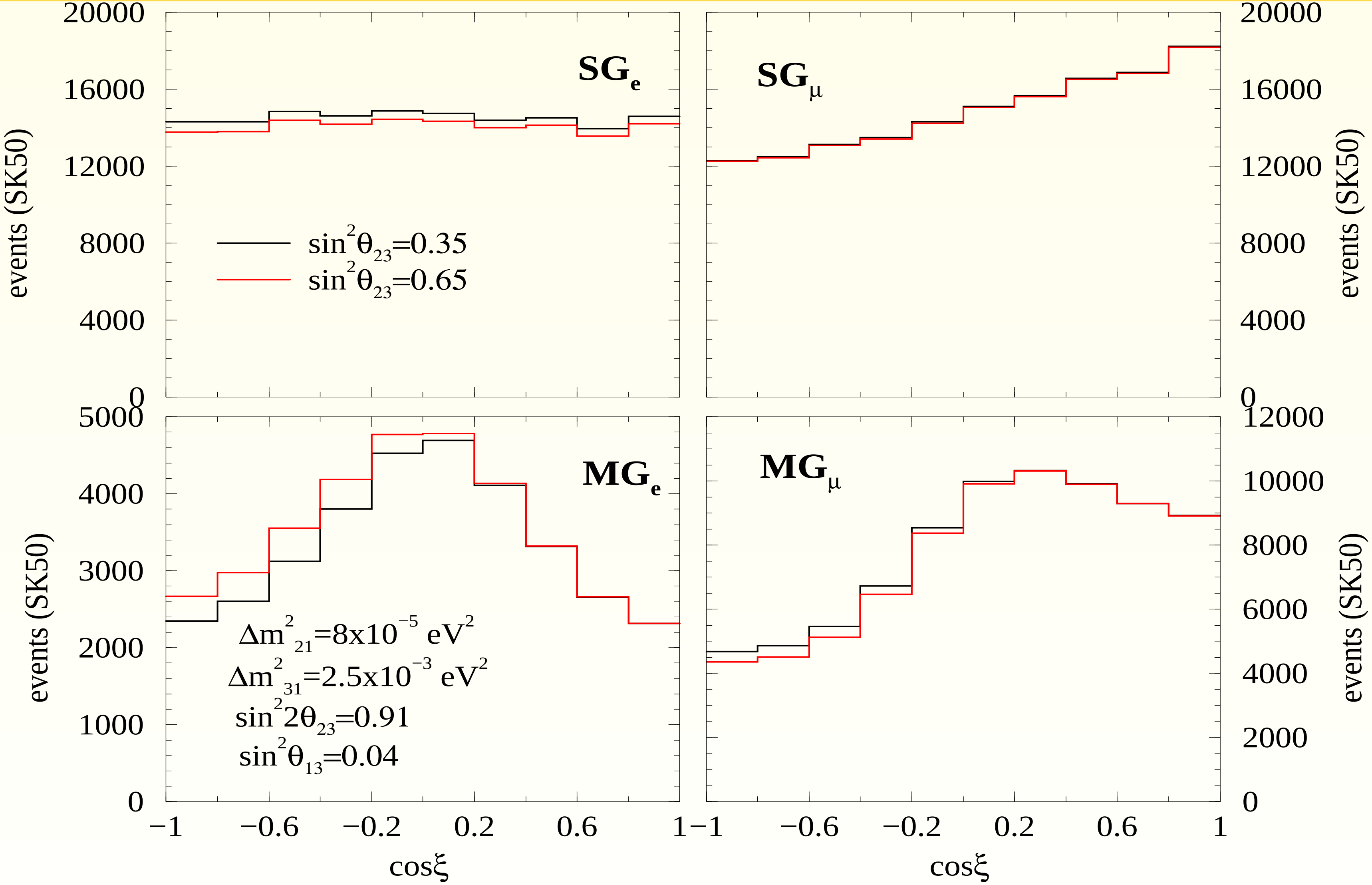
- These are just 2-gen $\nu_\mu \rightarrow \nu_\tau$ oscillations

Atmospheric ν Events in MTon Water Detector



- Δm^2_{21}-driven oscillations bring in octant sensitivity in SGe events

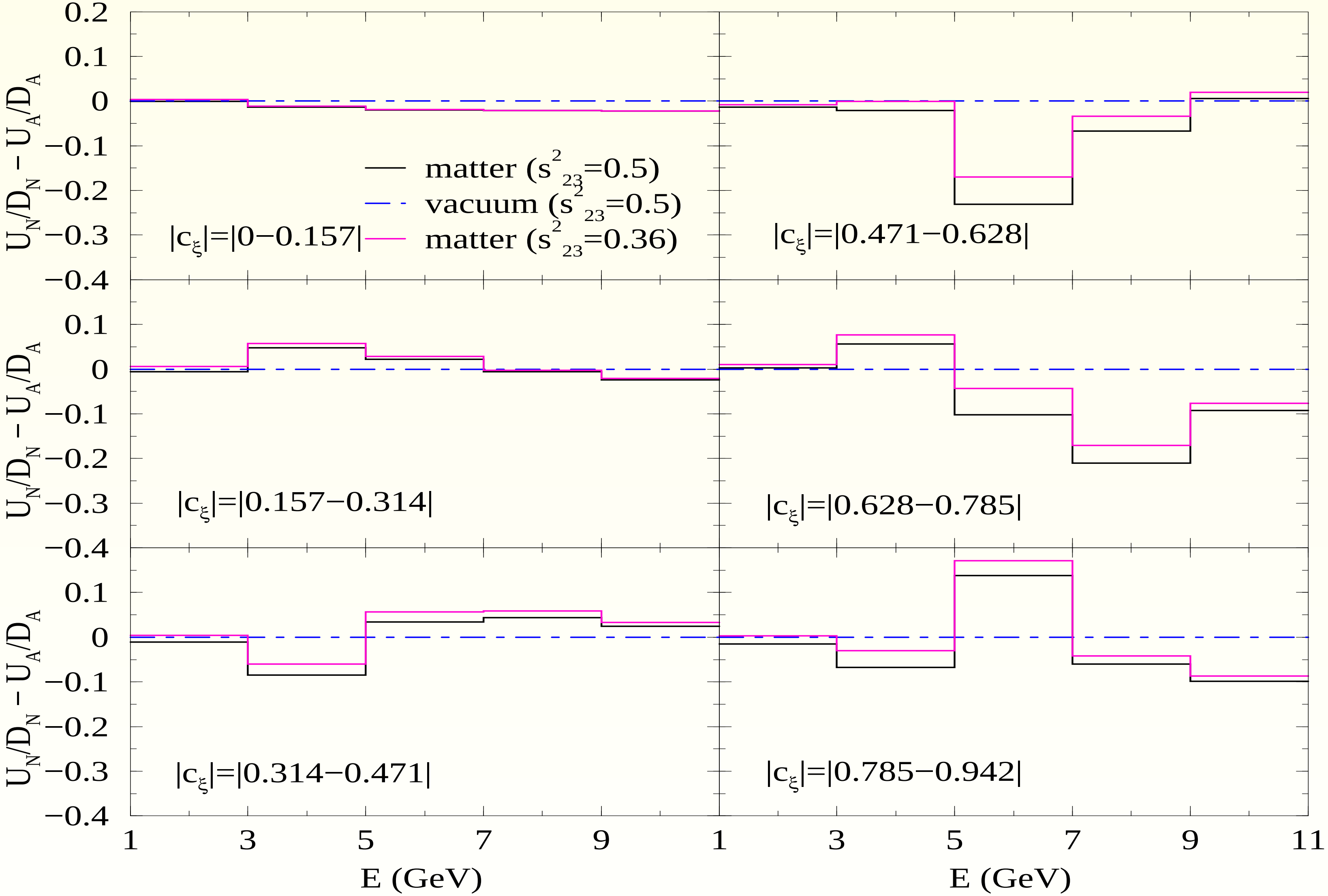
Atmospheric ν Events in MTon Water Detector



- θ_{13} brings in more octant sensitivity through matter effects

Atmospheric ν Events in INO-ICAL

Atmospheric Neutrino Events in INO-ICAL



S.C and P. Roy hep-ph/0509197

Physics from Sub-Dominant Oscillations

Determining Octant of θ_{23}

- Δm^2_{21} driven oscillations give an excess of events in the sub-GeV electron sample. The excess depends on θ_{23}. This can be used in megaton water Cerenkov detectors to study octant. Octant sensitivity is possible even if $\theta_{13} = 0$ if $\sin^2\theta_{23}$ is sufficiently far from maximal.

Determining Octant of θ_{23}

- θ_{13} driven matter effects appear in the multi-GeV electron sample. This effect is θ_{23} dependent. This can be used in megaton water Cerenkov detectors to study octant. Both θ_{13} should be sufficiently large and $\sin^2\theta_{23}$ away from maximal for this.

Determining Octant of θ_{23}

- θ_{13} driven matter effects appear in the multi-GeV electron sample. This effect is θ_{23} dependent. This can be used in megaton water Cerenkov detectors to study octant. Both θ_{13} should be sufficiently large and $\sin^2\theta_{23}$ away from maximal for this.
- θ_{13} driven matter effects appear in the multi-GeV muon sample. This excess is θ_{23} dependent. This can be used in large magnetized iron detectors to study octant. Both θ_{13} should be sufficiently large and $\sin^2\theta_{23}$ away from maximal for this.

Determining Octant of θ_{23}

- θ_{13} driven matter effects appear in the multi-GeV electron sample. This effect is θ_{23} dependent. This can be used in megaton water Cerenkov detectors to study octant. Both θ_{13} should be sufficiently large and $\sin^2\theta_{23}$ away from maximal for this.
- θ_{13} driven matter effects appear in the multi-GeV muon sample. This excess is θ_{23} dependent. This can be used in large magnetized iron detectors to study octant. Both θ_{13} should be sufficiently large and $\sin^2\theta_{23}$ away from maximal for this.
- Matter effects in general improve $\sin^2\theta_{23}$ sensitivity $\Rightarrow$ Maximality sensitivity improves if θ_{13} is large.

Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.

Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.
- θ_{13} driven matter effects in multi-GeV muons can be used in large magnetized iron detectors to study hierarchy.

Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.
- θ_{13} driven matter effects in multi-GeV muons can be used in large magnetized iron detectors to study hierarchy.
- For magnetized iron, sensitivity increases as the E and L resolution improves

Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.
- θ_{13} driven matter effects in multi-GeV muons can be used in large magnetized iron detectors to study hierarchy.
- For magnetized iron, sensitivity increases as the E and L resolution improves
- Sensitivity increases with true value of $\sin^2\theta_{13}$.

Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.
- θ_{13} driven matter effects in multi-GeV muons can be used in large magnetized iron detectors to study hierarchy.
- For magnetized iron, sensitivity increases as the E and L resolution improves
- Sensitivity increases with true value of $\sin^2\theta_{13}$.
- Sensitivity increases with true value of $\sin^2\theta_{23}$.

Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.
- θ_{13} driven matter effects in multi-GeV muons can be used in large magnetized iron detectors to study hierarchy.
- For magnetized iron, sensitivity increases as the E and L resolution improves
- Sensitivity increases with true value of $\sin^2\theta_{13}$.
- Sensitivity increases with true value of $\sin^2\theta_{23}$.
- Increase in sensitivity with $\sin^2\theta_{23}$ is more for electron sample than muon sample.

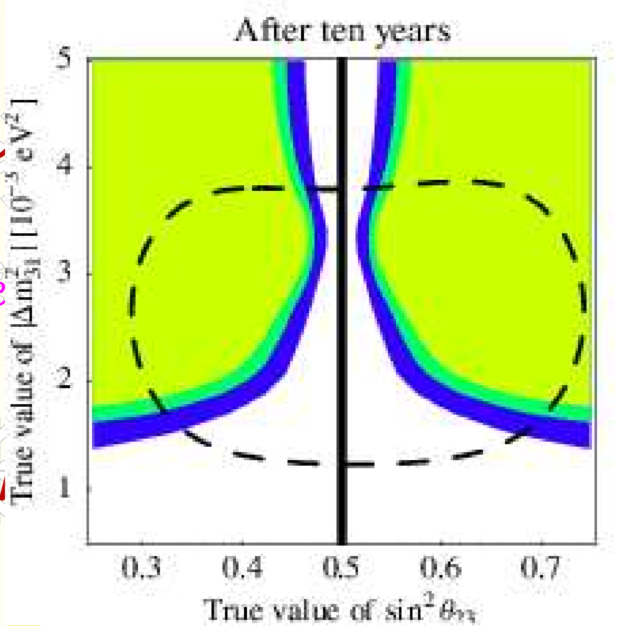
Determining $sgn(\Delta m^2_{31})$

- θ_{13} driven matter effects in multi-GeV electrons can be used in megaton water Cerenkov detectors to study hierarchy.
- θ_{13} driven matter effects in multi-GeV muons can be used in large magnetized iron detectors to study hierarchy.
- For magnetized iron, sensitivity increases as the E and L resolution improves
- Sensitivity increases with true value of $\sin^2\theta_{13}$.
- Sensitivity increases with true value of $\sin^2\theta_{23}$.
- Increase in sensitivity with $\sin^2\theta_{23}$ is more for electron sample than muon sample.
- Sens of the electron sample gets affected by δ_{CP}.

Backup Slides

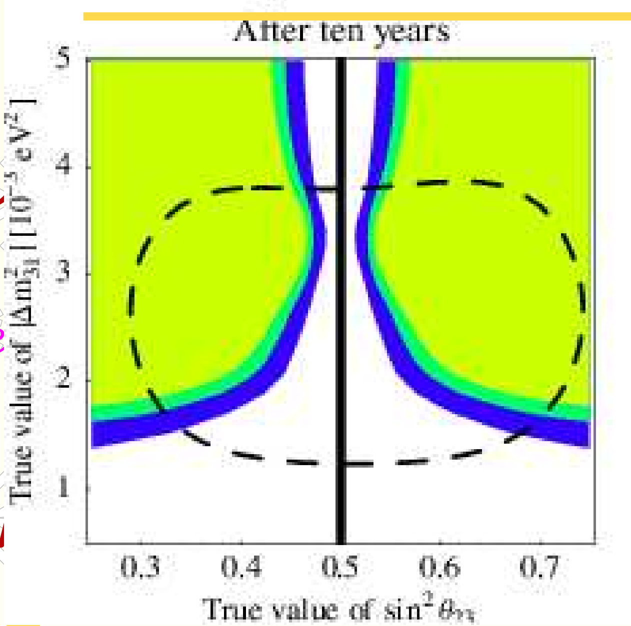
Testing Maximality of θ_{23}

Testing maximality of θ_{23}



$$|D| \equiv |(\sin^2\theta_{23} - 0.5)|$$

Testing maximality of θ_{23}



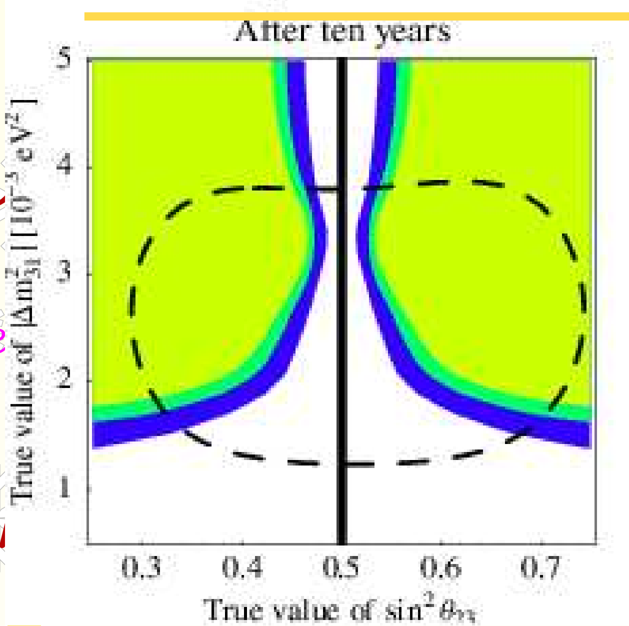
$|D|$ within 14%

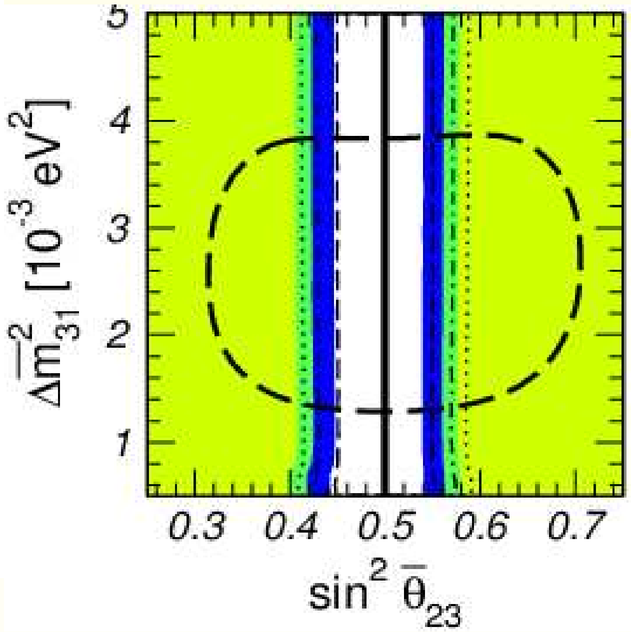
LBL combined

Antusch, et al,

hep-ph/0404268

Testing maximality of θ_{23}





$|D|$ within 14%

LBL combined

Antusch, et al,

hep-ph/0404268

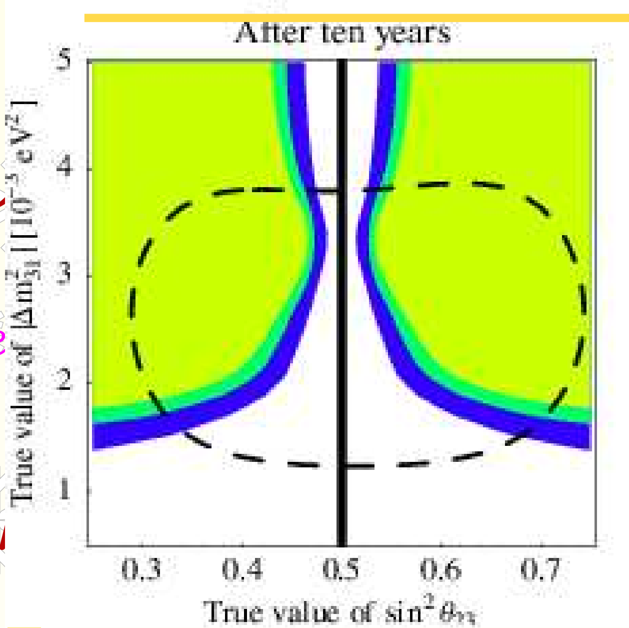
$|D|$ within 19%

SK50

Gonzalez-Garcia, et al,

hep-ph/0408170

Testing maximality of θ_{23}

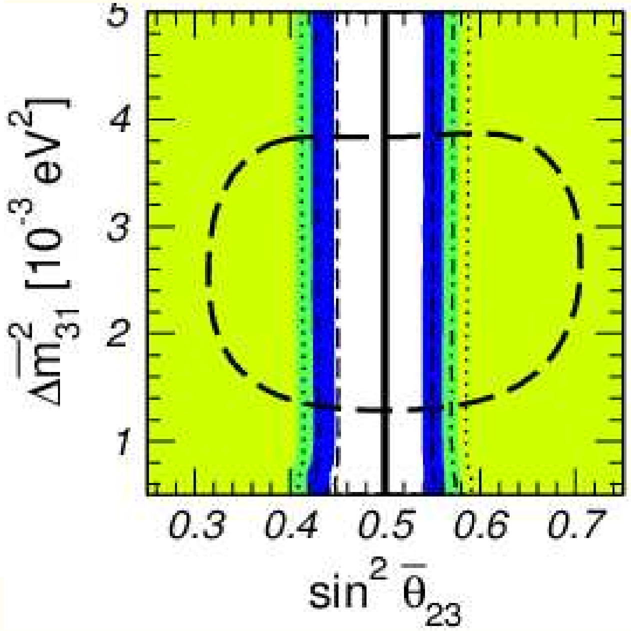


$|D|$ within 14%

LBL combined

Antusch, et al,

hep-ph/0404268

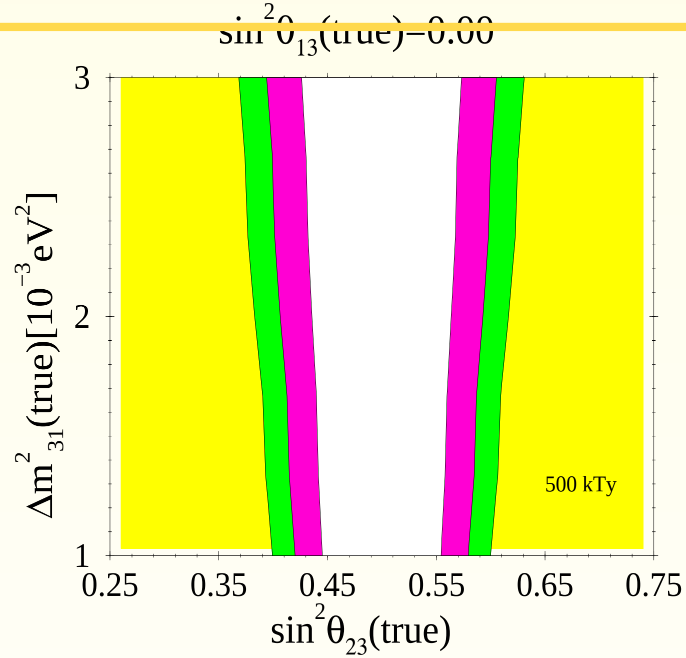


$|D|$ within 19%

SK50

Gonzalez-Garcia, et al,

hep-ph/0408170



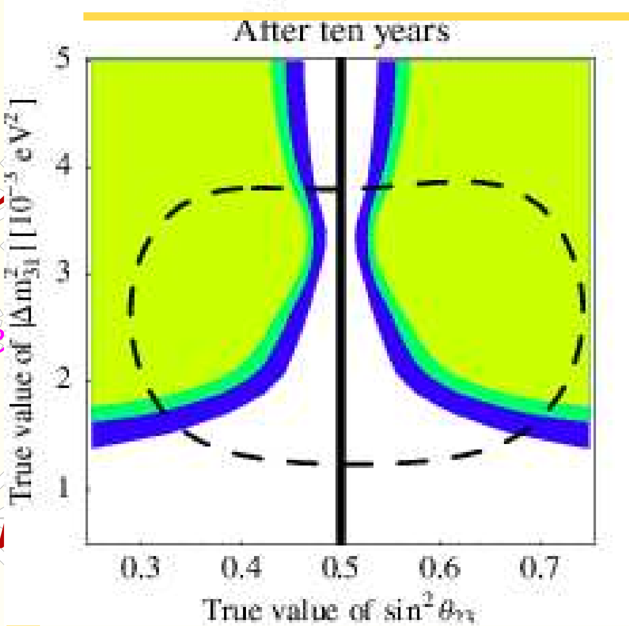
$|D|$ within 25%

INO-ICAL 500 kTy

S.C. and P. Roy,

hep-ph/0509197

Testing maximality of θ_{23}

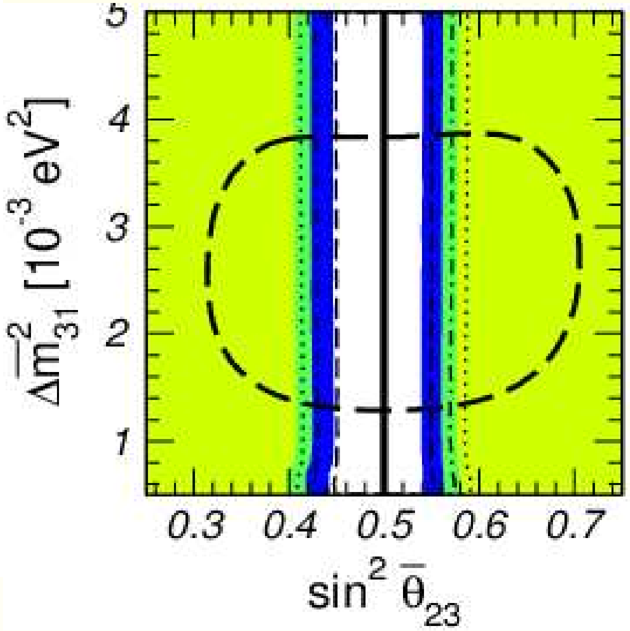


$|D|$ within 14%

LBL combined

Antusch, et al,

hep-ph/0404268

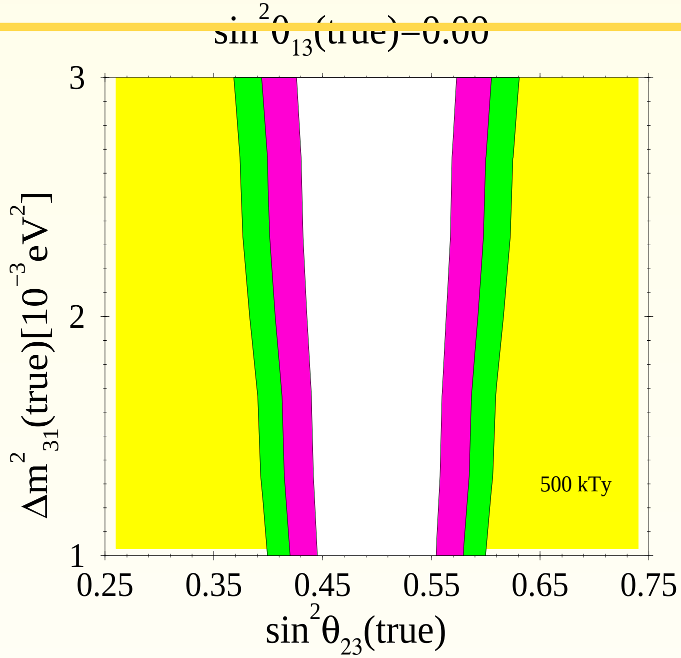


$|D|$ within 19%

SK50

Gonzalez-Garcia, et al,

hep-ph/0408170



$|D|$ within 25%

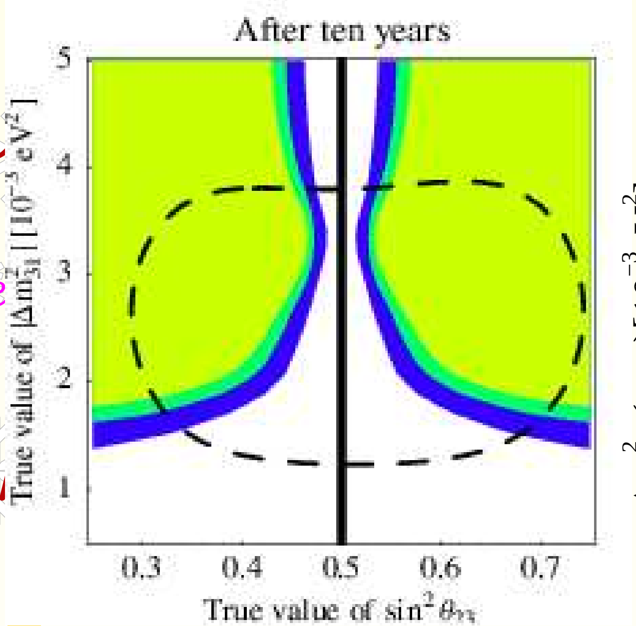
INO-ICAL 500 kTy

S.C. and P. Roy,

hep-ph/0509197

- Sensitivity to $|D| \equiv |(\sin^2\theta_{23} - 0.5)|$ comparable

Testing Maximality of θ_{23}

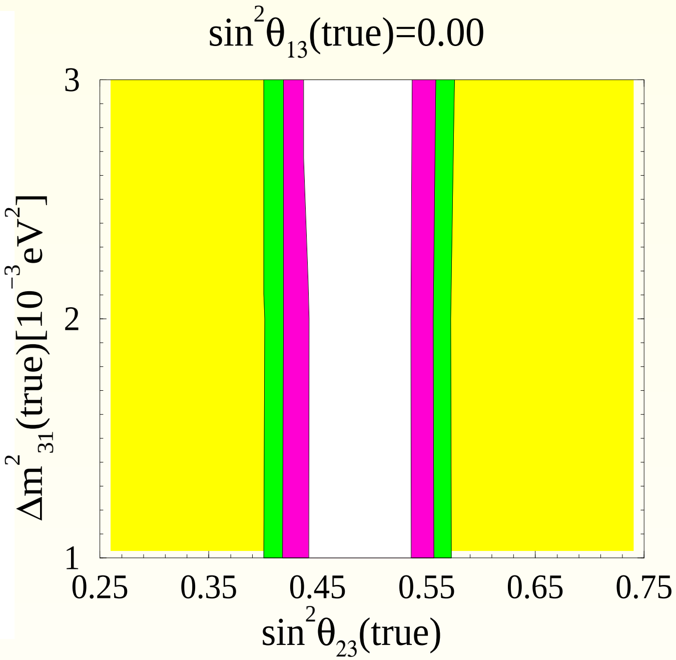


$|D|$ within 14%

LBL combined

Antusch, et al,

hep-ph/0404268

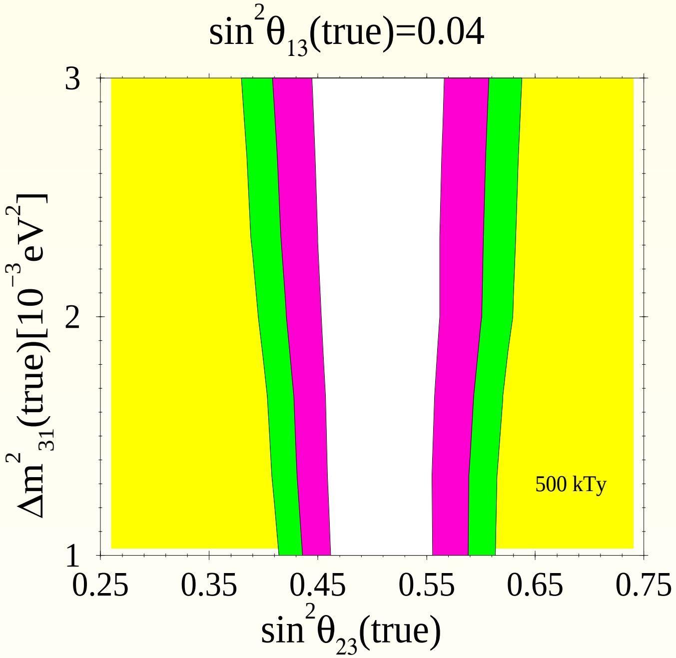


$|D|$ within 20%

SK50

preliminary

.



$|D|$ within 23%

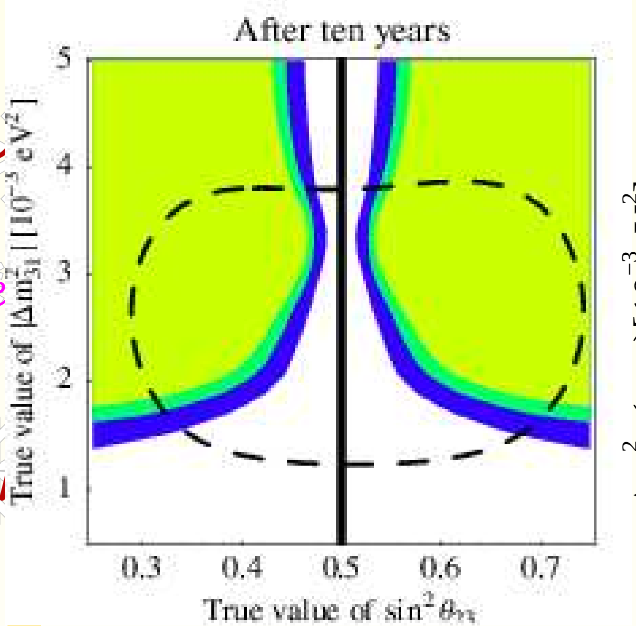
INO-ICAL 500 kTy

S.C. and P. Roy,

hep-ph/0509197

- Sensitivity to $|D|$ in ICAL improves marginally with θ_{13}

Testing Maximality of θ_{23}

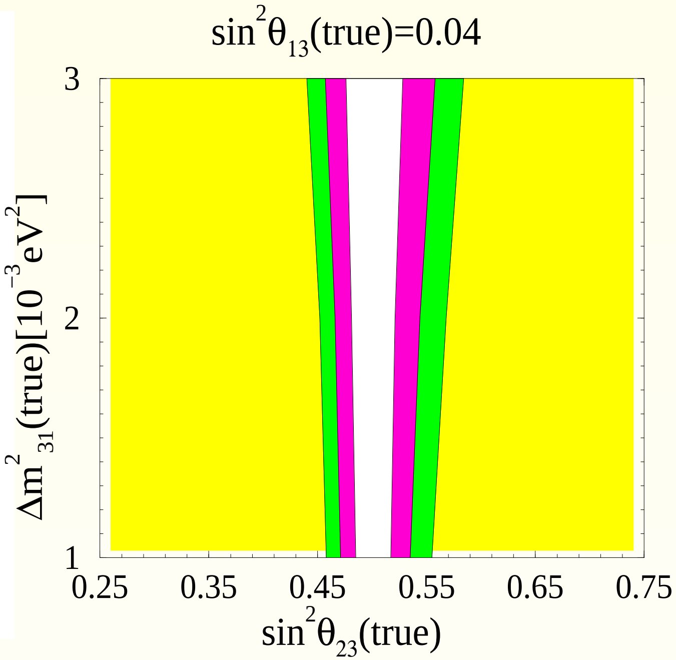


$|D|$ within 14%

LBL combined

Antusch, et al,

hep-ph/0404268

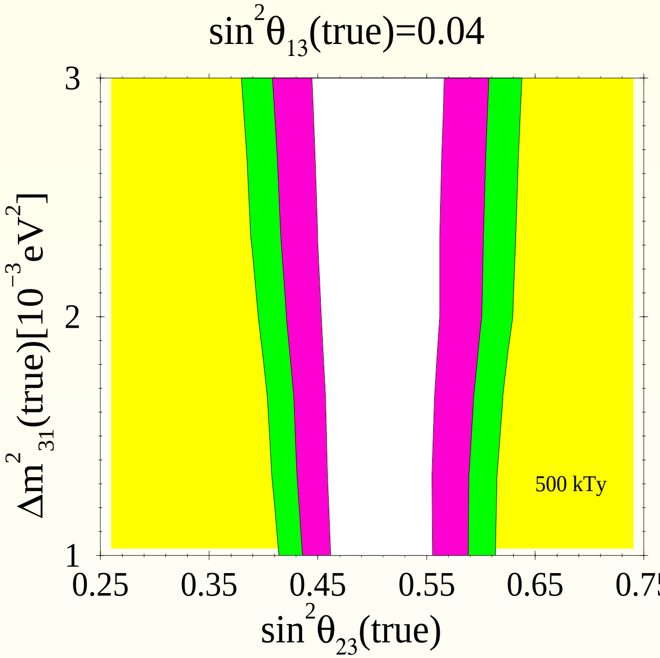


$|D|$ within 11%

SK50

preliminary

.



$|D|$ within 23%

INO-ICAL 500 kTy

S.C. and P. Roy,

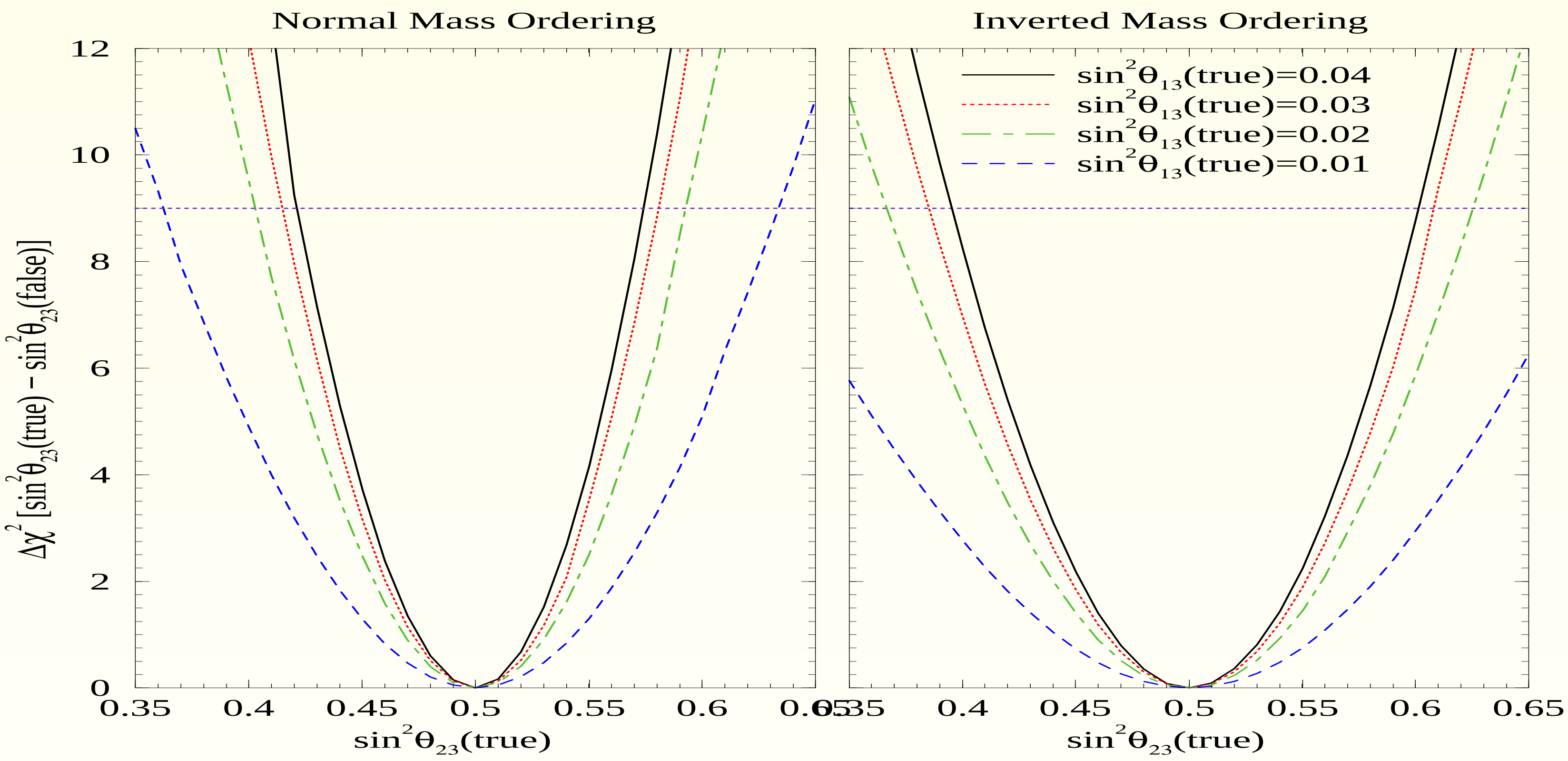
hep-ph/0509197

- Sensitivity to $|D|$ in ICAL improves marginally with θ_{13}
- Sensitivity to $|D|$ in SK50 improves remarkably with θ_{13}

BACK

Resolving the θ_{23} Octant Ambiguity

Resolving the θ_{23} Octant Ambiguity with ICAL



1 MTonyr

S.C and P. Roy hep-ph/0509197

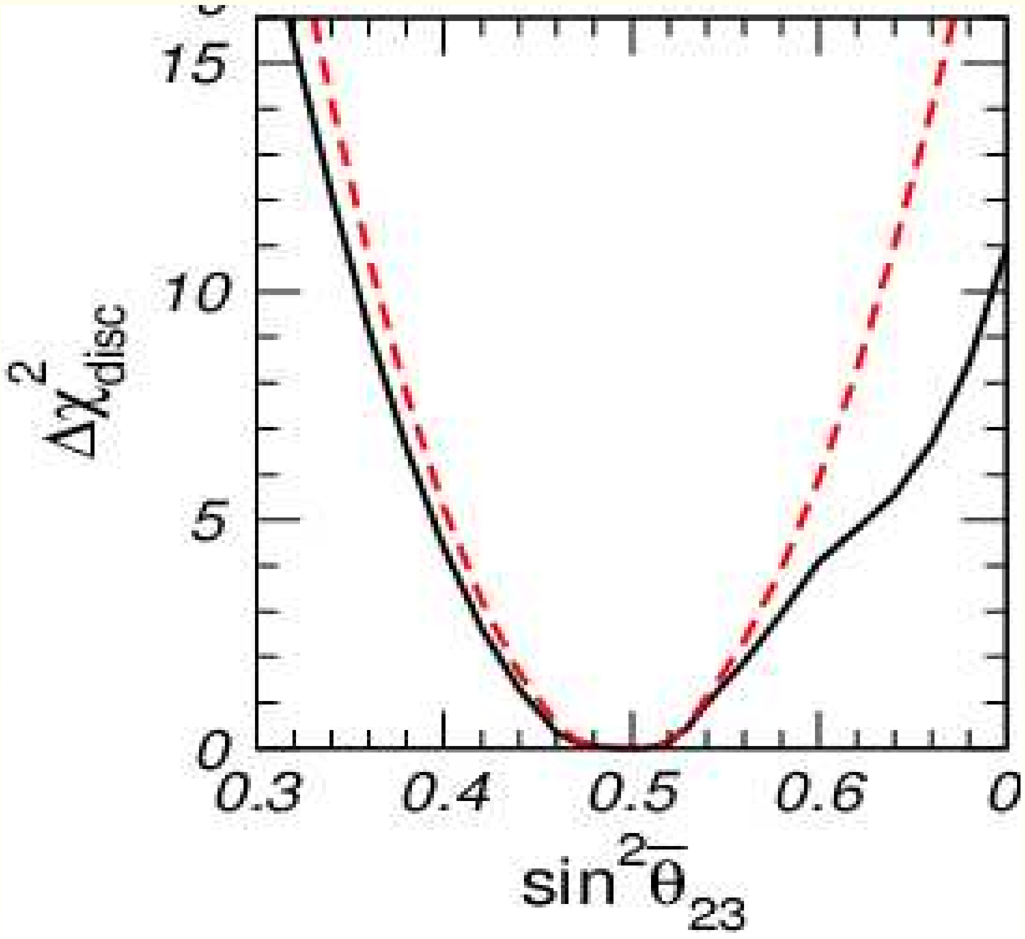
- $\sin^2\theta_{23}$(false) can be excluded at 3σ if:

$$\sin^2\theta_{23}(\text{true}) < 0.402 \text{ or } > 0.592 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.02,$$

$$\sin^2\theta_{23}(\text{true}) < 0.421 \text{ or } > 0.573 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.04.$$

BACK

Resolving the θ_{23} Octant Ambiguity with SK50



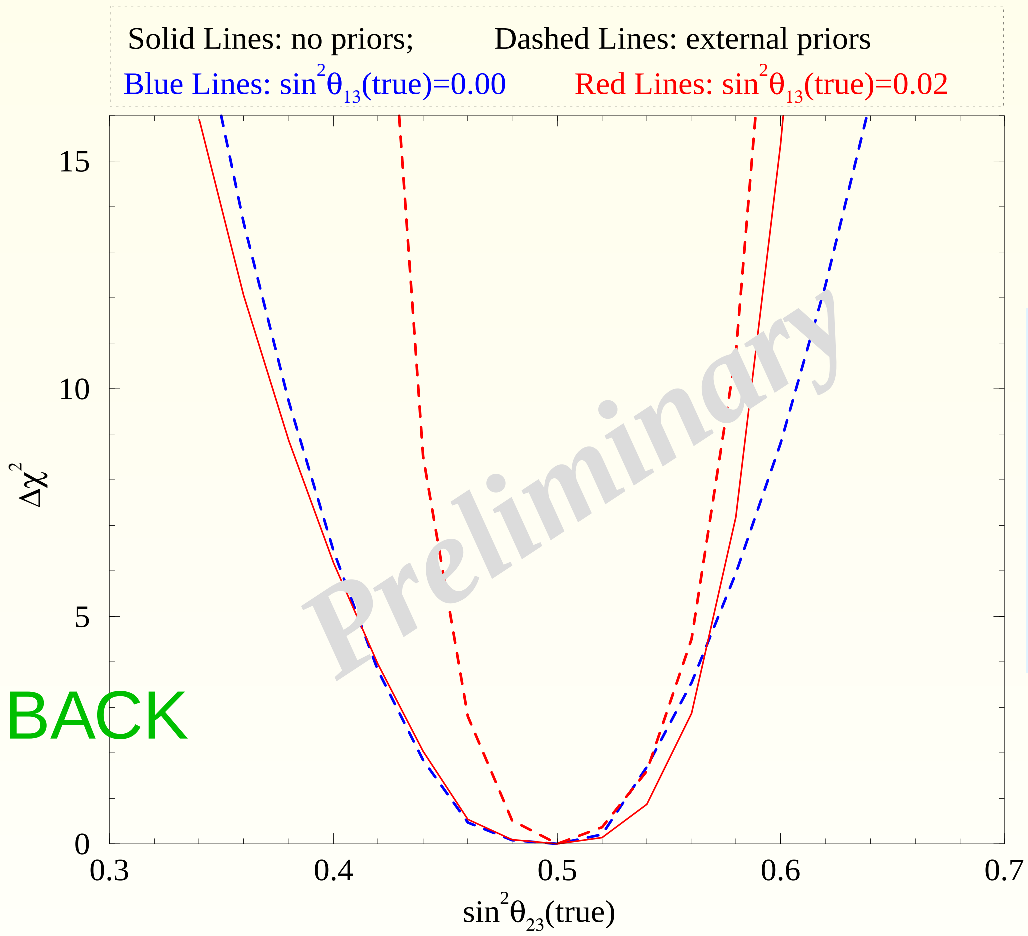
Gonzalez-Garcia et al, hep-ph/0408170

BACK

- $\sin^2\theta_{23}$(false) can be excluded at 3σ if:

$\sin^2\theta_{23}(\text{true}) < 0.36$ or > 0.62 for $\sin^2\theta_{13}(\text{true}) = 0.00$.

Resolving the θ_{23} Octant Ambiguity with SK50



Sensitivity to octant of θ_{23} improves remarkably as θ_{13} increases from zero.

BACK

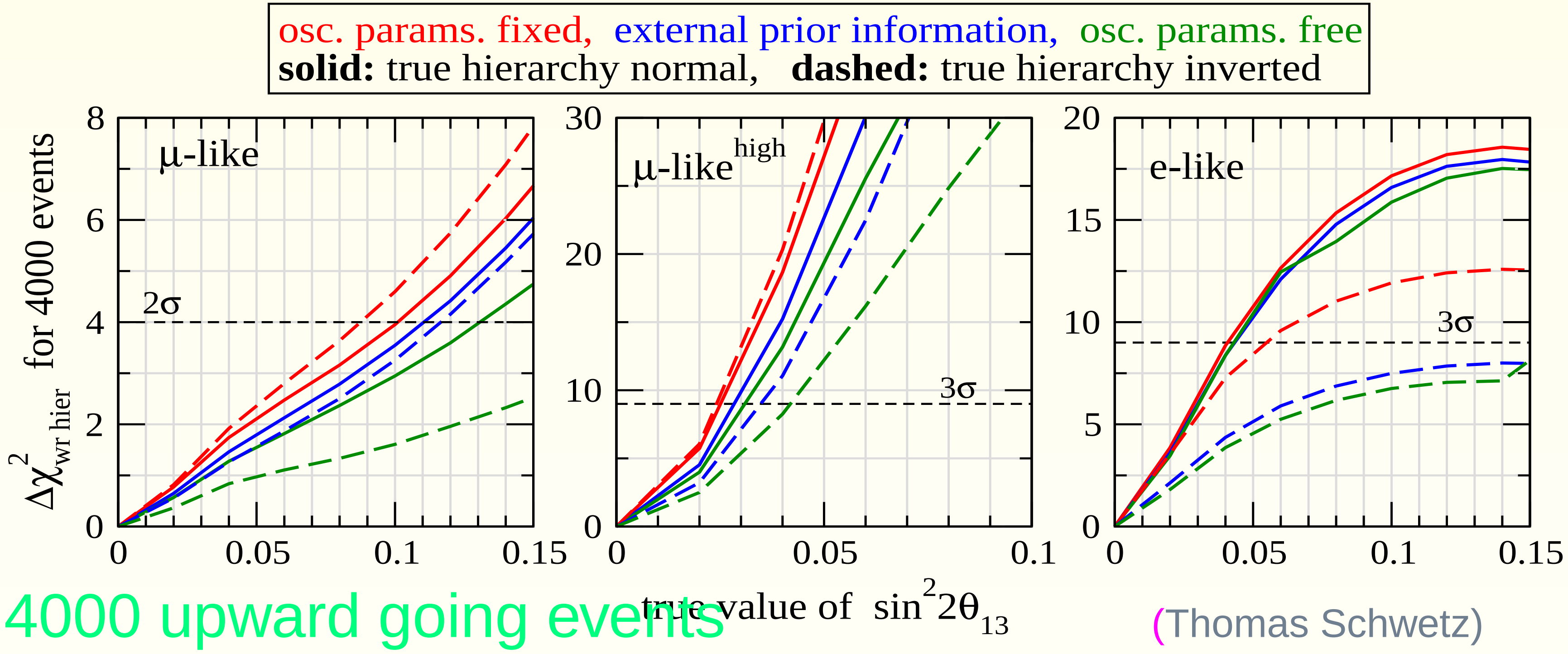
- $\sin^2\theta_{23}$(false) can be excluded at 3σ if:

$$\sin^2\theta_{23}(\text{true}) < 0.384 \text{ or } > 0.601 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.00.$$

$$\sin^2\theta_{23}(\text{true}) < 0.438 \text{ or } > 0.574 \text{ for } \sin^2\theta_{13}(\text{true}) = 0.02.$$

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with ICAL

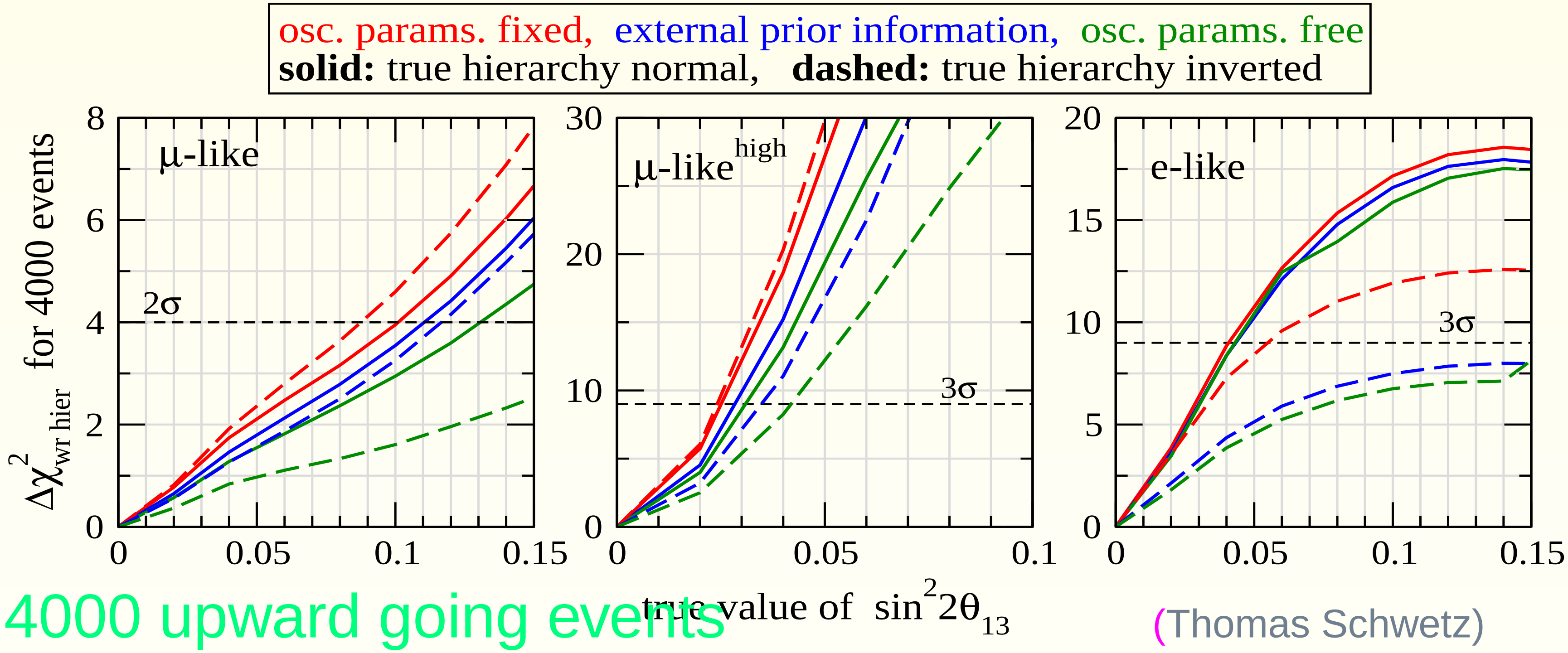


4000 upward going events

(Thomas Schwetz)

Petcov and Schwetz, hep-ph/0511277

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with ICAL



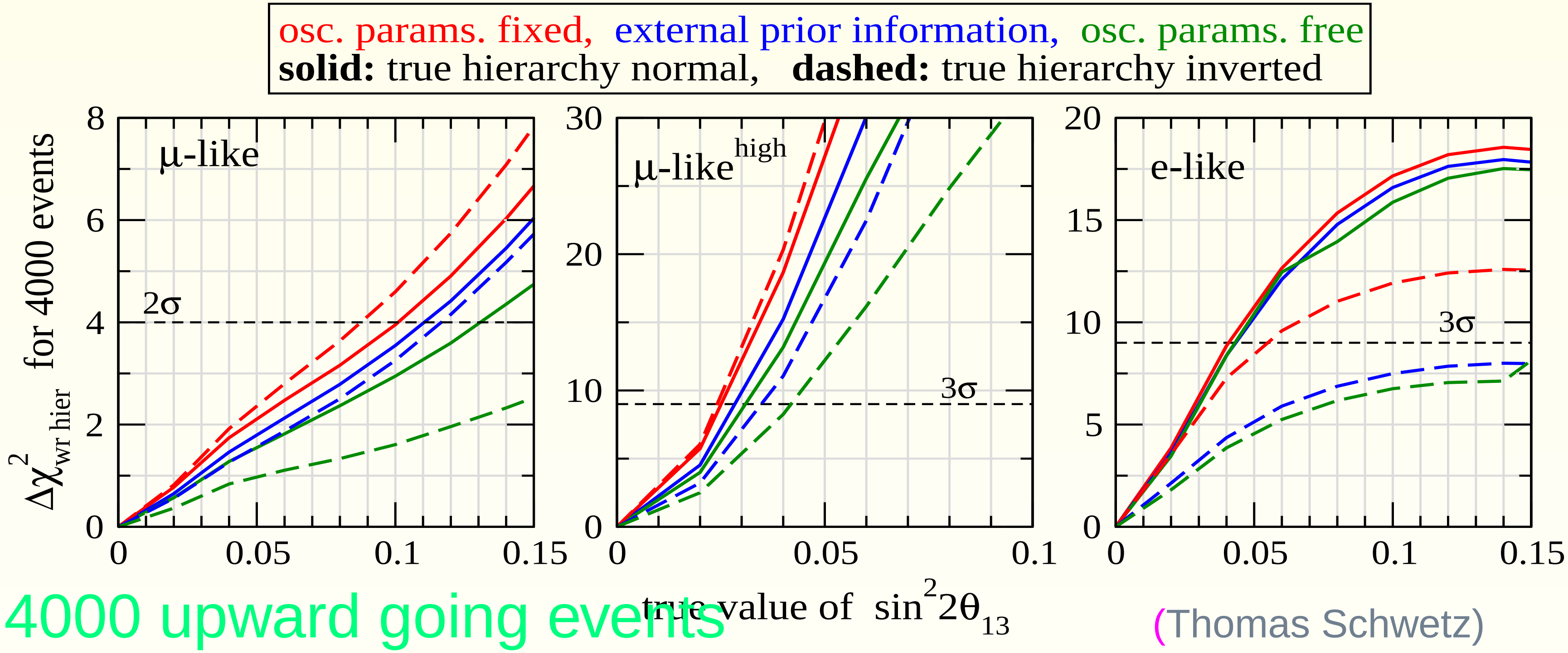
4000 upward going events

(Thomas Schwetz)

Petcov and Schwetz, hep-ph/0511277

- The wrong hierarchy can be ruled out at 2σ with 4000 upward going events for $\sin^2 2\theta_{13} = 0.1 (\sin^2 \theta_{13} = 0.026)$ and $\sin^2 \theta_{23} = 0.5$

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with ICAL



4000 upward going events

(Thomas Schwetz)

Petcov and Schwetz, hep-ph/0511277

- The wrong hierarchy can be ruled out at 2σ with 4000 upward going events for $\sin^2 2\theta_{13} = 0.1 (\sin^2\theta_{13} = 0.026)$ and $\sin^2\theta_{23} = 0.5$

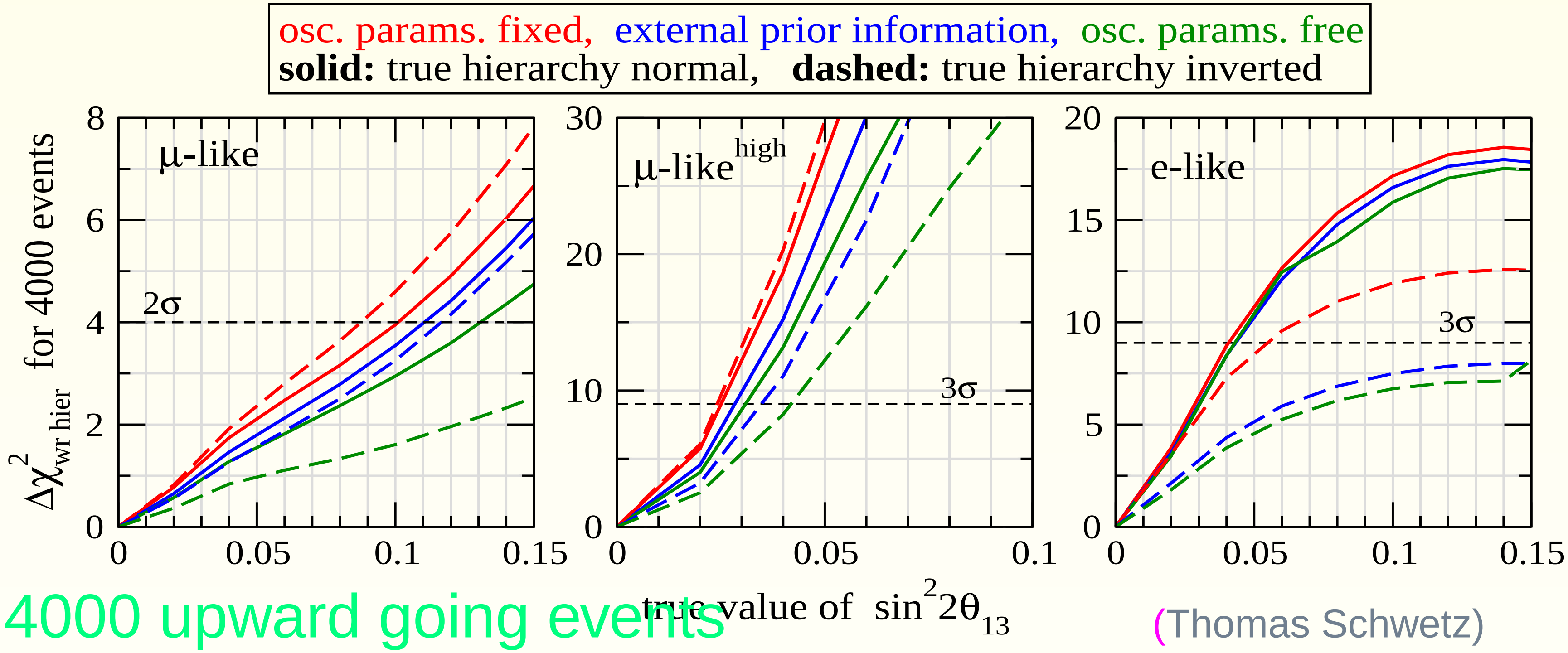
Polamares-Ruiz,Petcov (2003)

Indumathi, Murthy (2004)

Ghoshal,Gandhi,Goswami,Mehta,UmaSankar (2004)

Samanta (2006)

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with ICAL



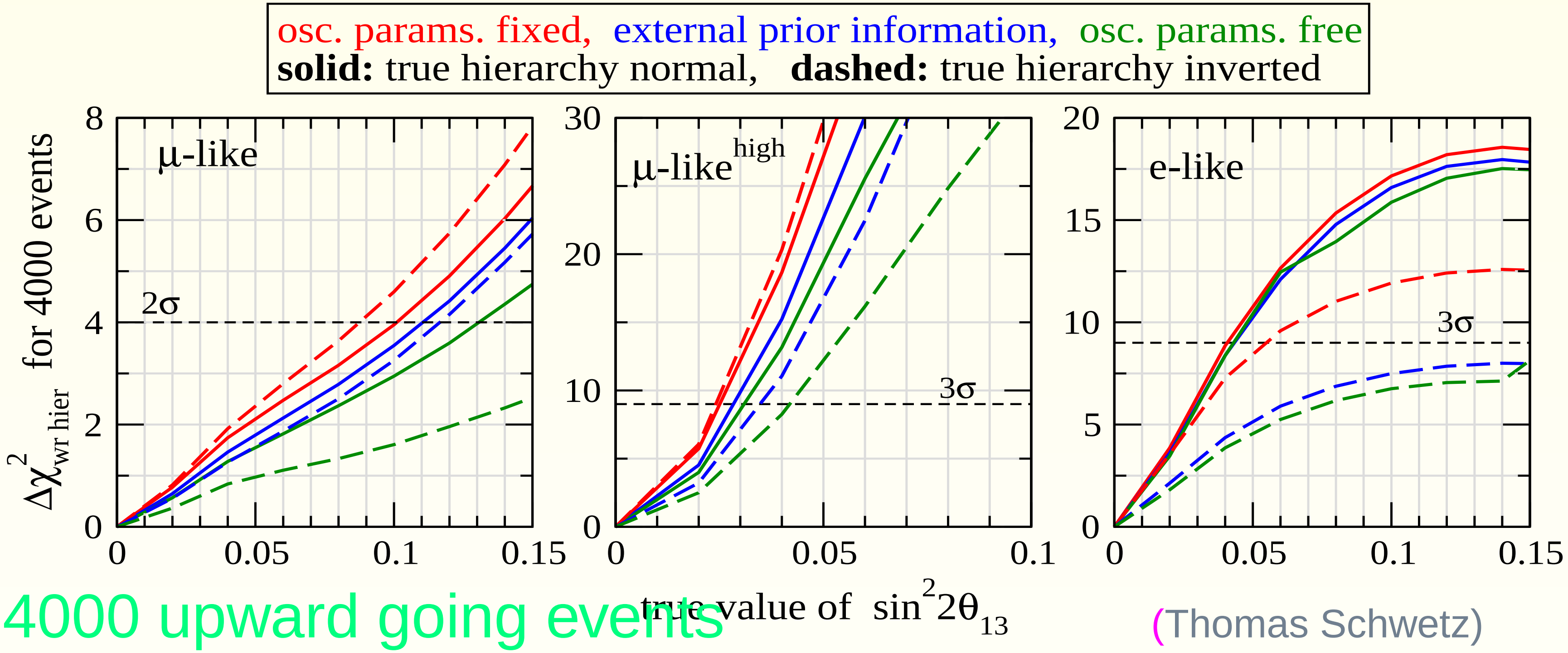
4000 upward going events

(Thomas Schwetz)

Petcov and Schwetz, hep-ph/0511277

- The wrong hierarchy can be ruled out at 2σ with 4000 upward going events for $\sin^2 2\theta_{13} = 0.1 (\sin^2\theta_{13} = 0.026)$ and $\sin^2\theta_{23} = 0.5$
- Sensitivity increases with E and L resolution

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with ICAL



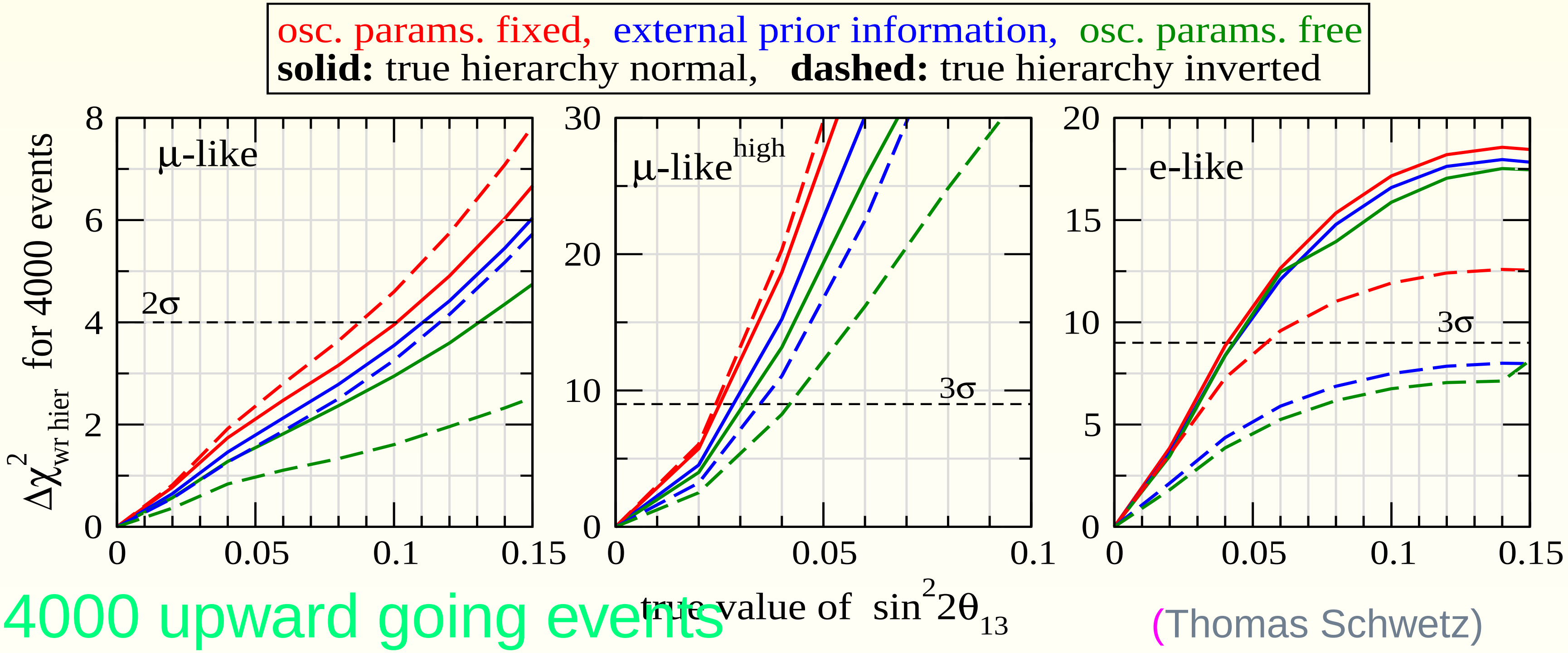
4000 upward going events

(Thomas Schwetz)

Petcov and Schwetz, hep-ph/0511277

- The wrong hierarchy can be ruled out at 2σ with 4000 upward going events for $\sin^2 2\theta_{13} = 0.1(\sin^2\theta_{13} = 0.026)$ and $\sin^2\theta_{23} = 0.5$
- Sensitivity increases with electron detection

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with ICAL



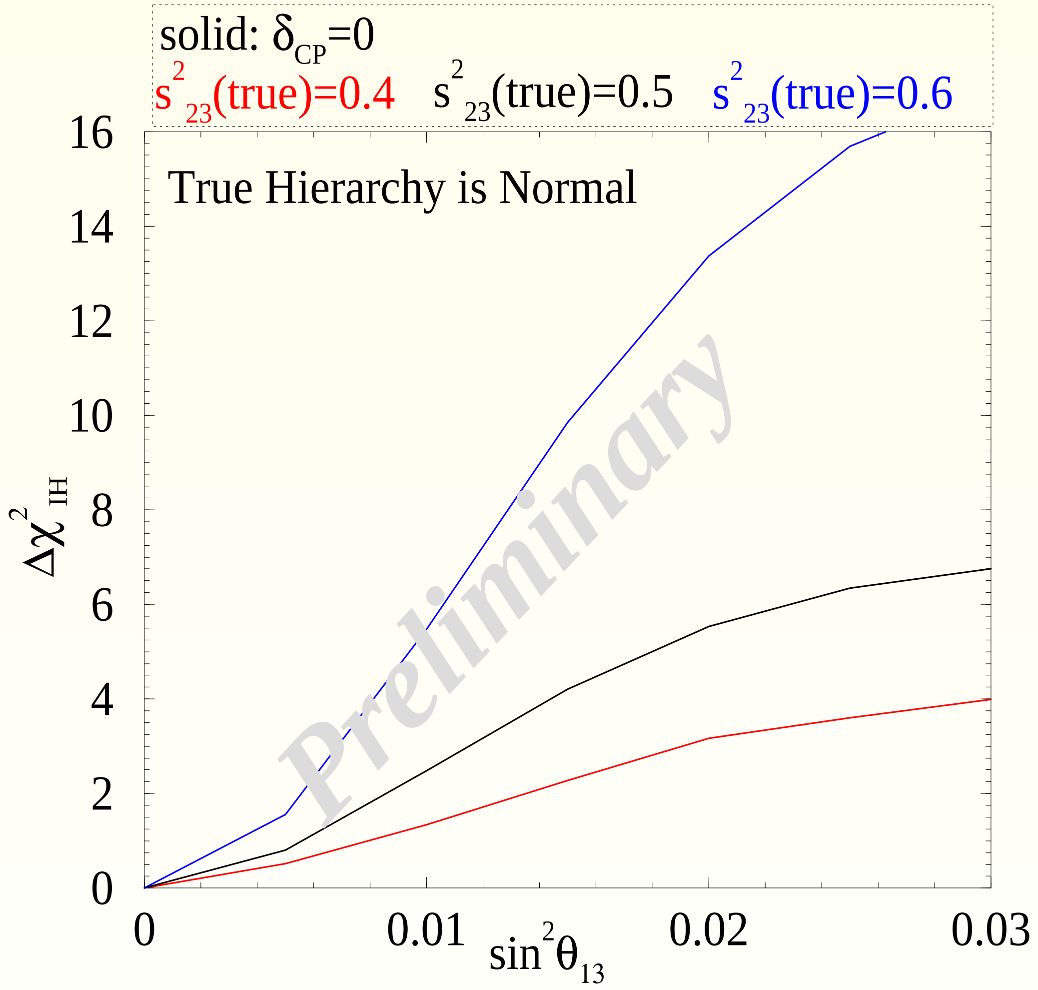
4000 upward going events

(Thomas Schwetz)

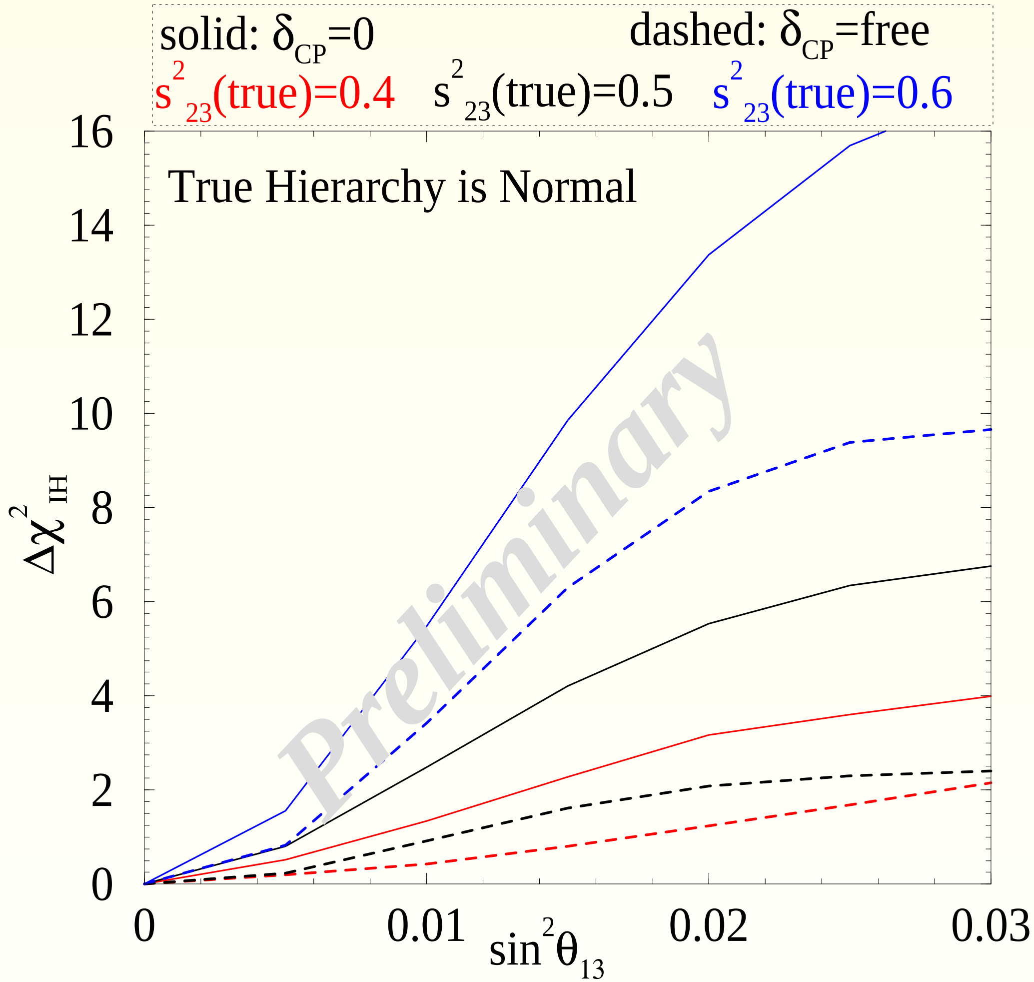
Petcov and Schwetz, hep-ph/0511277

- The wrong hierarchy can be ruled out at 2σ with 4000 upward going events for $\sin^2 2\theta_{13} = 0.1 (\sin^2 \theta_{13} = 0.026)$ and $\sin^2 \theta_{23} = 0.5$
- Sensitivity increases with $\sin^2 \theta_{23}$ BACK

Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with SK50



Resolving the $sgn(\Delta m^2_{31})$ Ambiguity with SK50



- Sensitivity drops appreciably due to δ_{CP} BACK