Neutrino Oscillation Phenomenology - III

Sandhya Choubey

Harish-Chandra Research Institute, Allahabad, India

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- Why disappearance experiments?
- Because statistics there are very large

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- $\nu_\mu \to \nu_e$ or $\nu_e \to \nu_\mu$: $P_{\mu e}$ or $P_{e\mu}$; Appearance Expts
- $sgn(\Delta m^2_{31})$ can be done using matter effects (other effects also – “interference terms”)

The $\nu_e \to \nu_\mu$ "Golden" Channel

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\begin{aligned}
P_{app}^{vac} &\simeq \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Delta \\
&\pm (\alpha\Delta)\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta_{CP}\sin\Delta\sin\Delta \\
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- Third term gives CP conserving part of the prob

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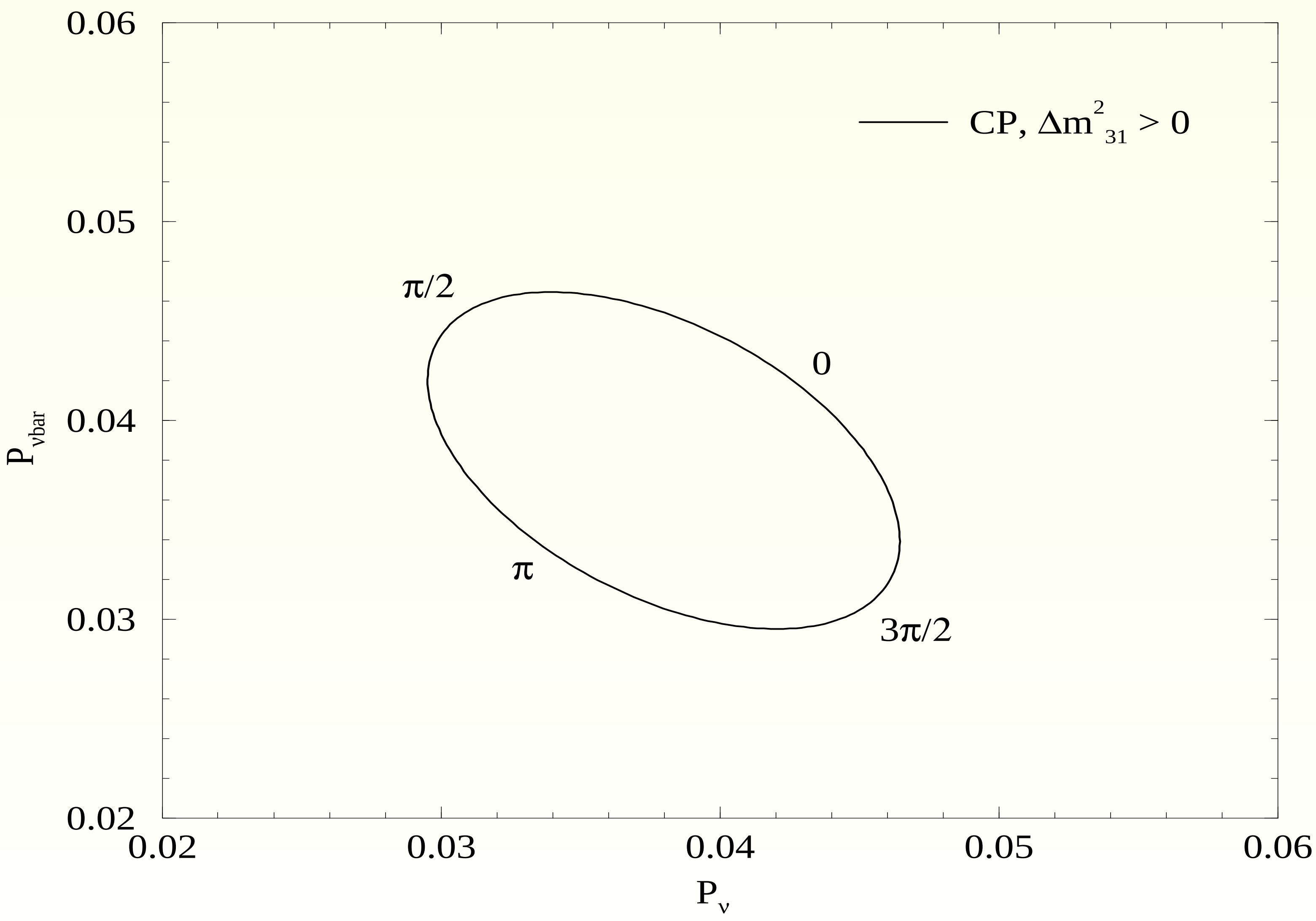
- $$\left(\frac{P_\nu + P_{\bar\nu} - 2C}{2A}\right)^2 + \left(\frac{P_\nu - P_{\bar\nu}}{2B}\right)^2 = 1$$

- This is the eqn of an ellipse in the $P_\nu - P_{\bar\nu}$ plane. These are called "bi-probability plots". Major (minor) axes measure the amplitude of $\sin\delta_{CP}$ ($\cos\delta_{CP}$) terms.

Bi-Probability Plots



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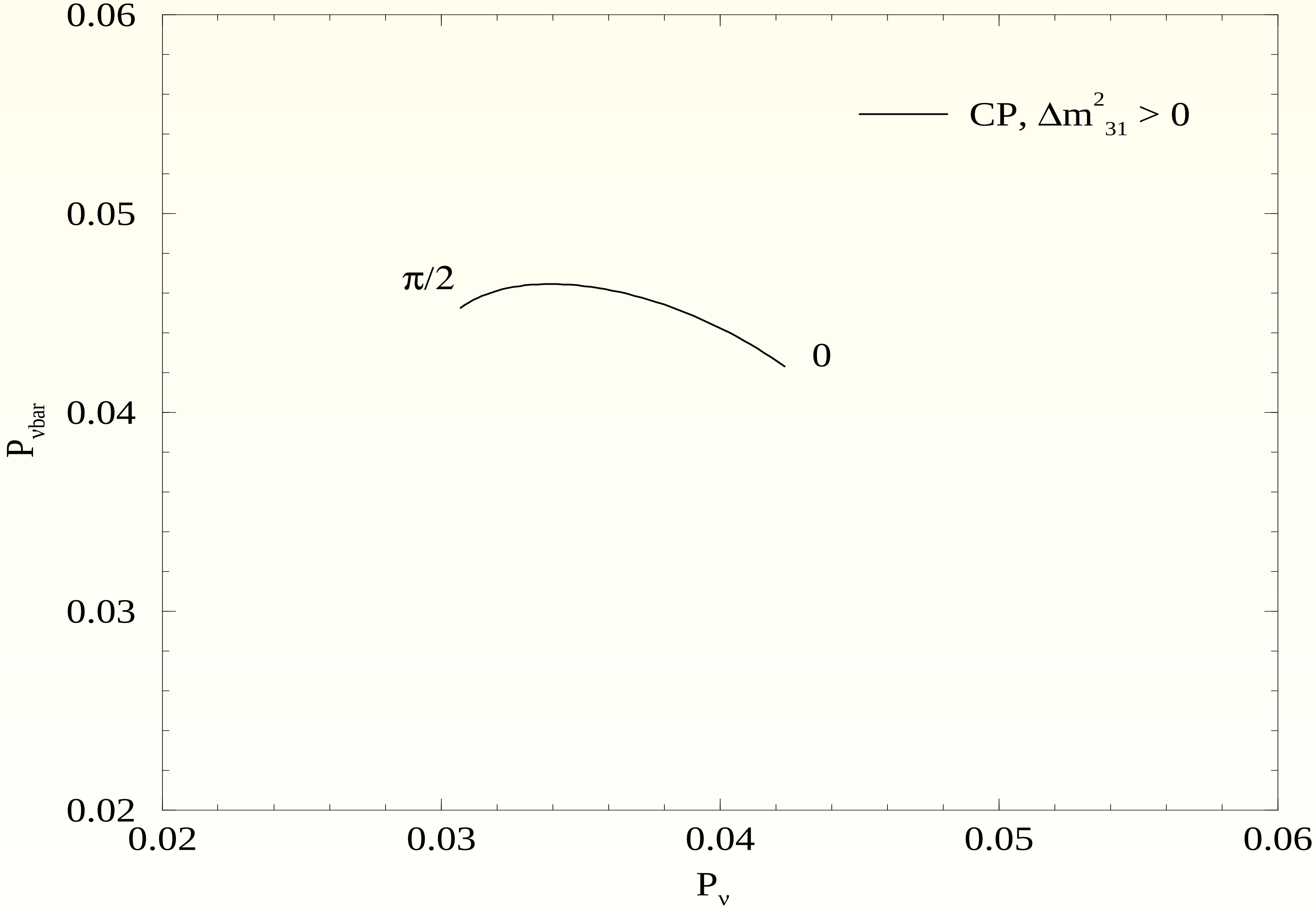

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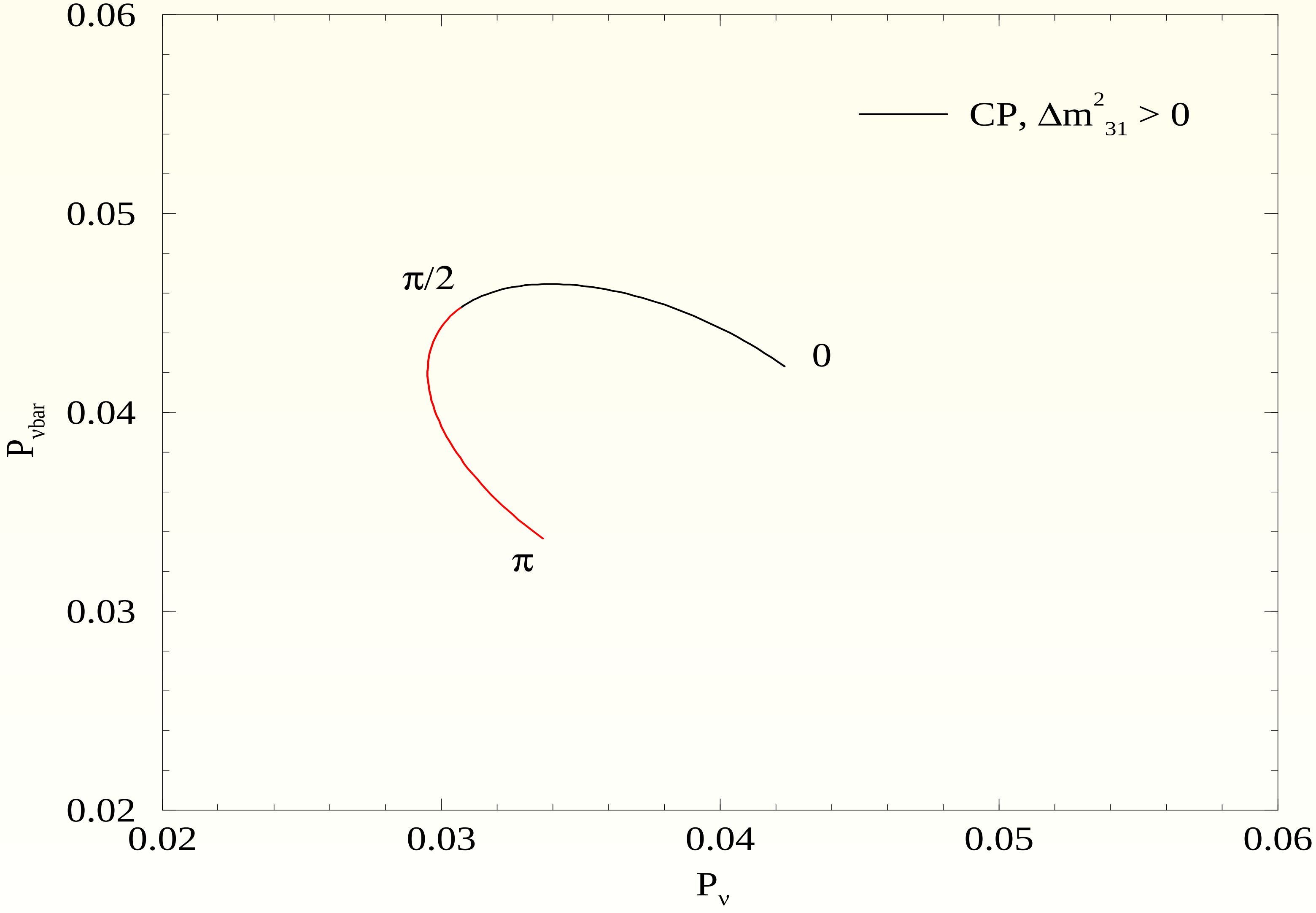

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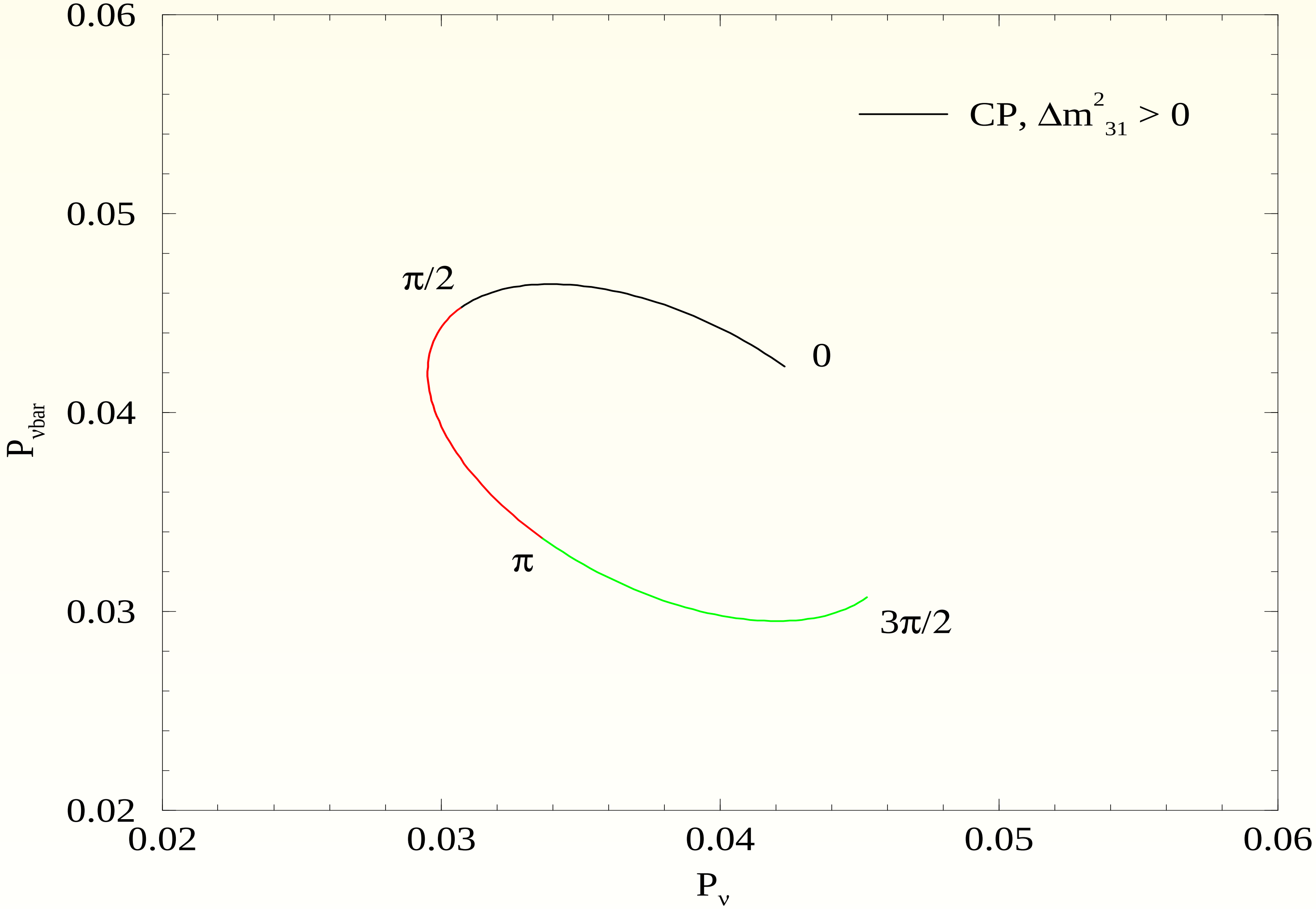

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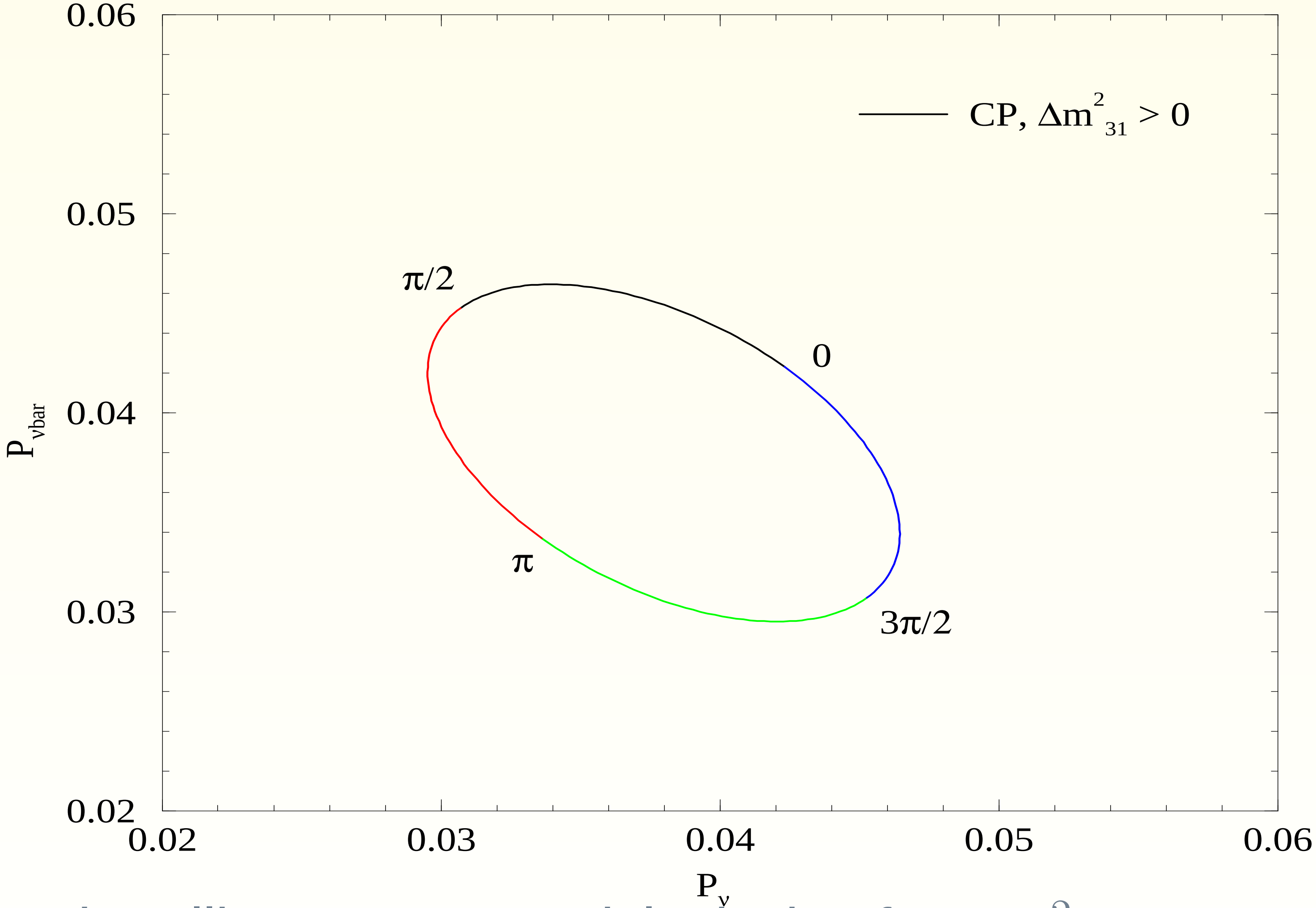

- The ellipse moves anticlockwise for $\Delta m^2_{31} > 0$.
- You can check that it goes clockwise for $\Delta m^2_{31} < 0$.

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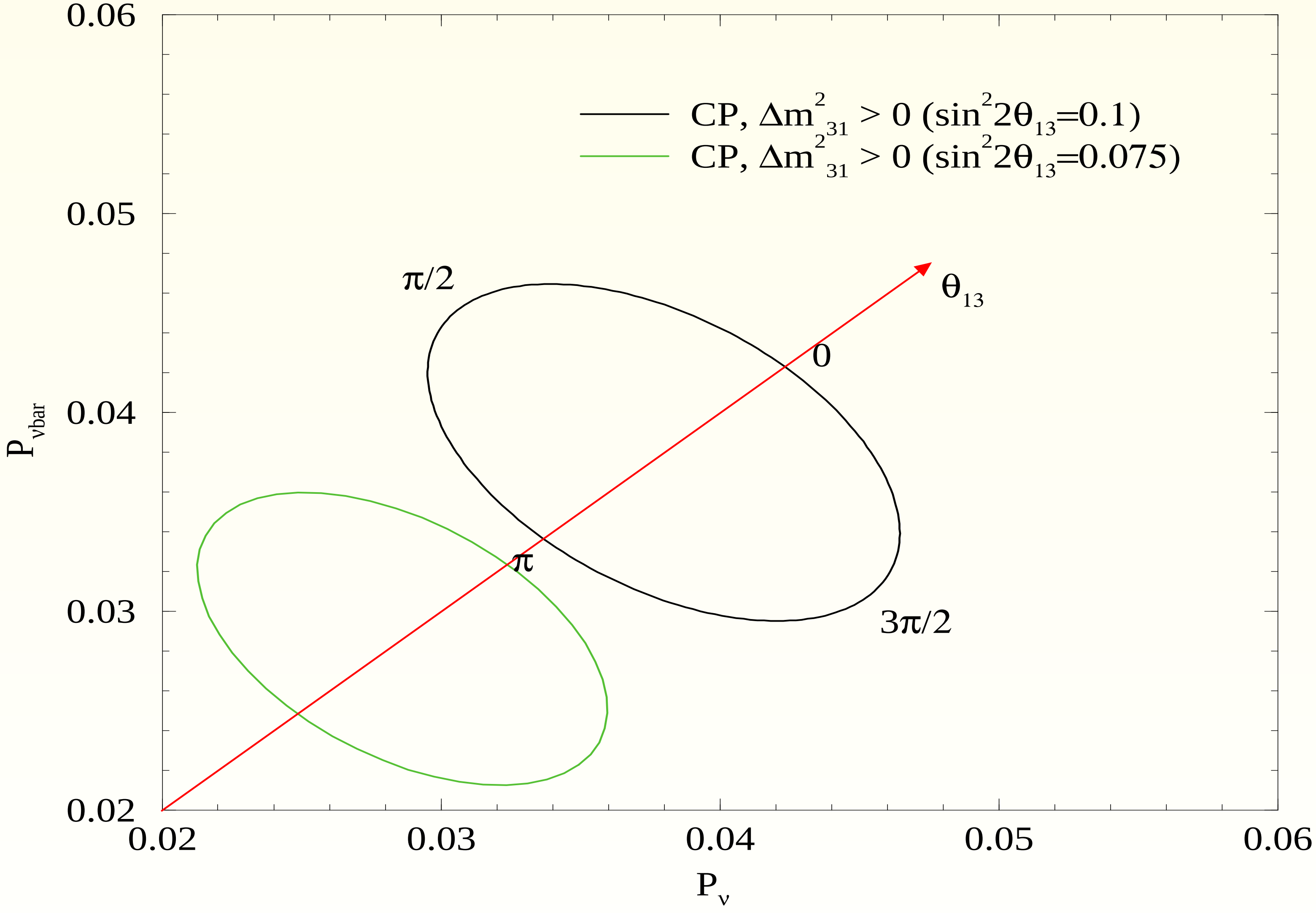

- Smaller $\theta_{13} \Rightarrow \delta_{CP}$ measurement more difficult .

Degeneracies

$Sgn(\Delta m^2_{31})$ Degeneracy

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- Do the simultaneous transformation

$$\delta_{CP} \to \pi - \delta_{CP}$$
$$\Delta m^2_{31} \to -\Delta m^2_{31}$$

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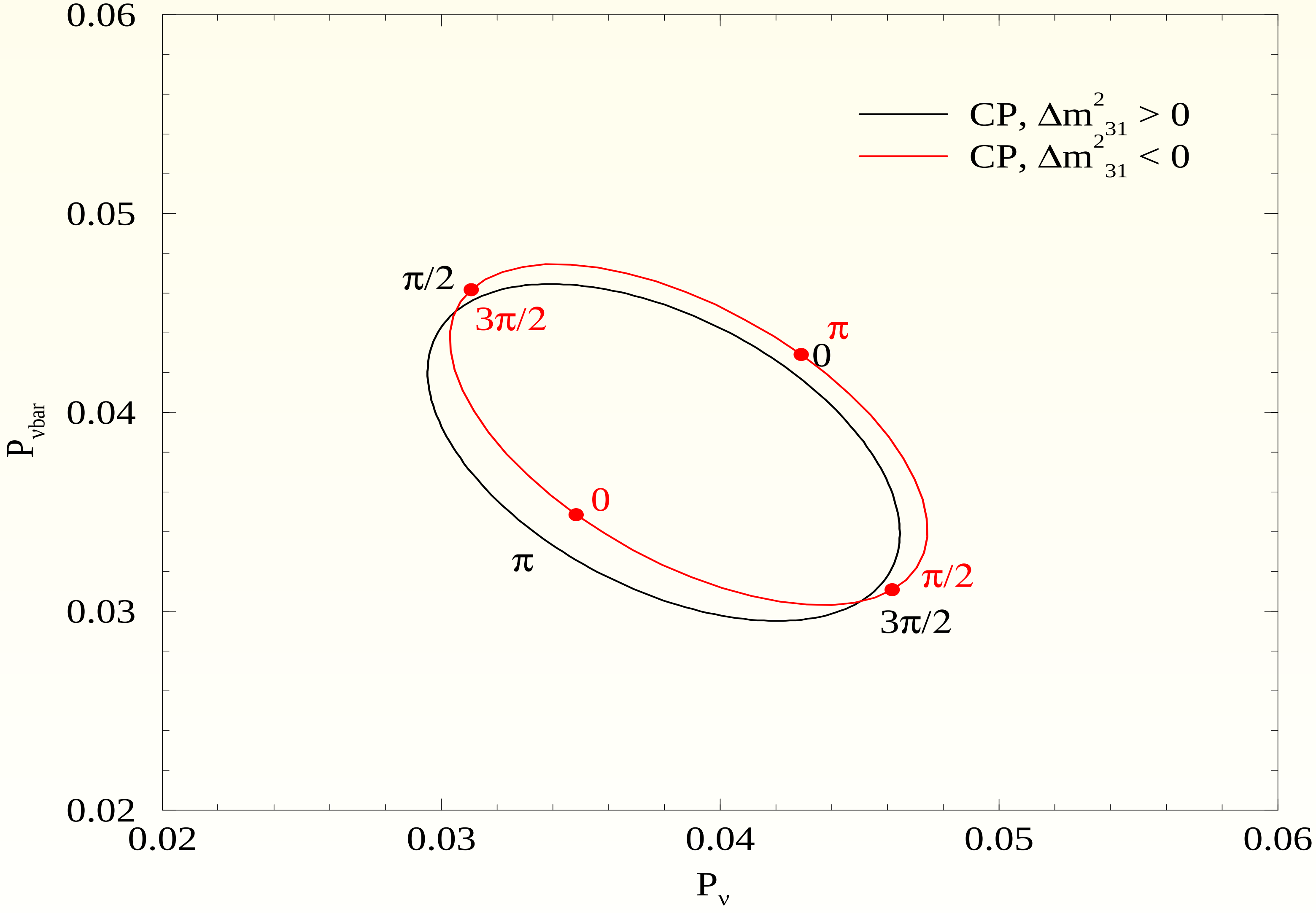
- The expression for $P_{appearance}$ remains invariant
- Since $Sgn(\Delta m^2_{31})$ is unknown, there will always be an ambiguity in the measured value of δ_{CP}

$Sgn(\Delta m^2_{31})$ **Degeneracy**



Octant of θ_{23} Degeneracy

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- We only have measurement on $\sin^2 2\theta_{23}$.

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&+ \cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2(\alpha\Delta)
\end{aligned}
$$

- We only have measurement on $\sin^2 2\theta_{23}$.
- For every non-maximal $\sin^2 2\theta_{23}$, there are 2 possible $\sin^2\theta_{23}$

Octant of θ_{23} Degeneracy

- $$\begin{aligned} P_{appearance} &= \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Delta \\ &\pm (\alpha\Delta)\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta_{CP}\sin\Delta\sin\Delta \\ &+ (\alpha\Delta)\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos\delta_{CP}\cos\Delta\sin\Delta \\ &+ \cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2(\alpha\Delta) \end{aligned}$$
- We only have measurement on $\sin^2 2\theta_{23}$.
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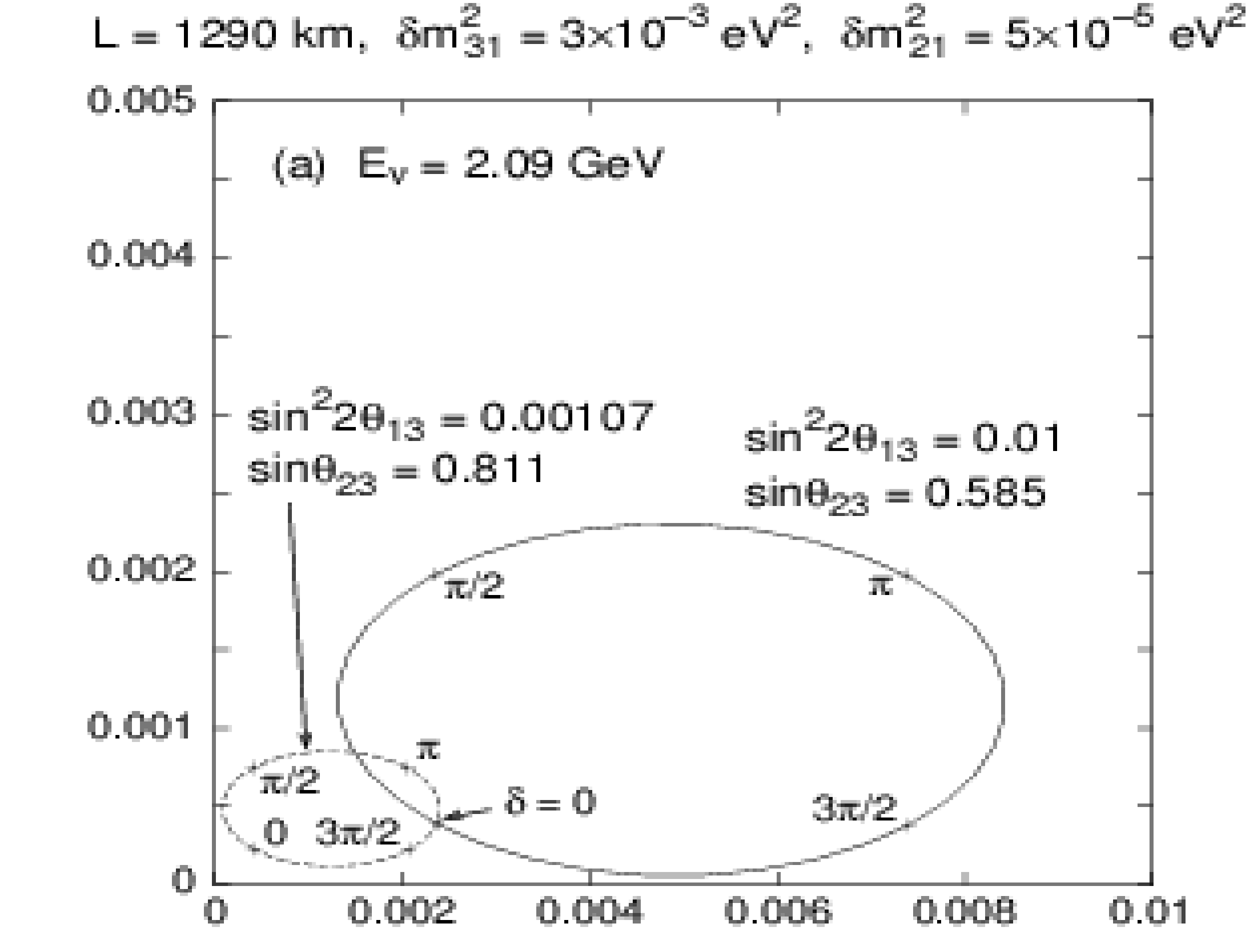
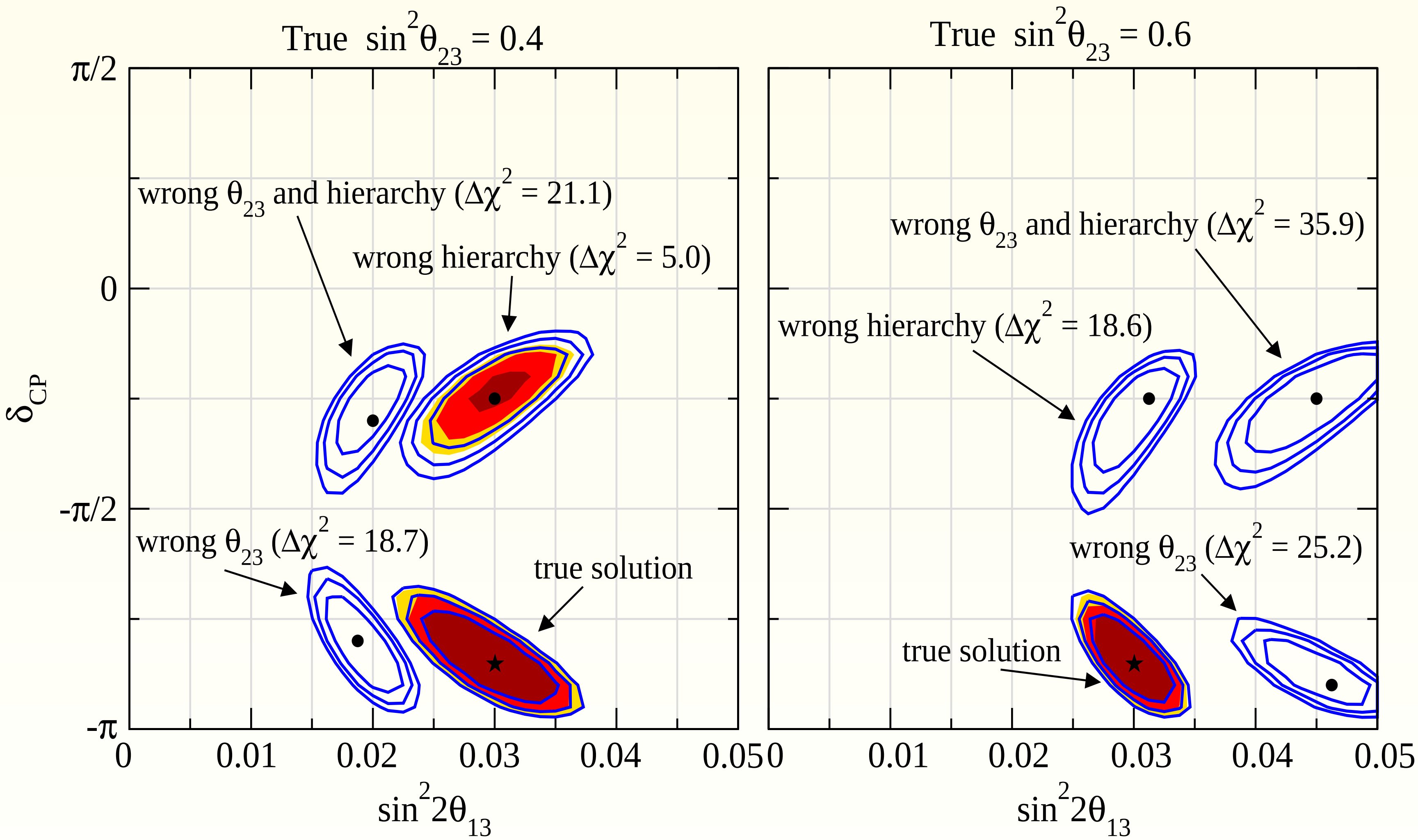
- This will give 2 disjoint fitted value for θ_{13}.

Octant of θ_{23} Degeneracy



Barger et al, hep-ph/0112119

- If $\sin^2 2\theta_{23} = 0.9$ then both $\sin^2 2\theta_{13} = 0.01$ and 0.0011 would fit the data from a given LBL expt.



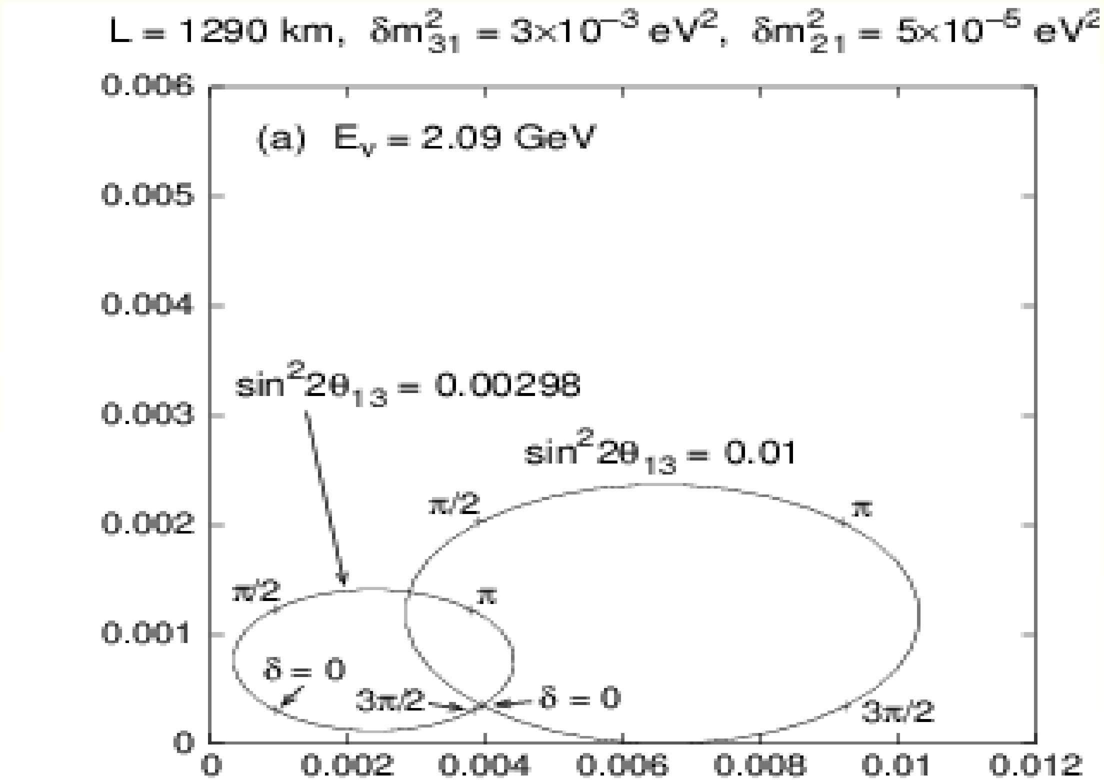
Huber et al, hep-ph/0501037

Intrinsic $(\delta_{CP}, \theta_{13})$ Degeneracy

- For a fixed value of E, it might happen that

$$P_{app}(\theta_{13}, \delta_{CP}) = P_{app}(\theta'_{13}, \delta'_{CP})$$
$$\bar{P}_{app}(\theta_{13}, \delta_{CP}) = \bar{P}_{app}(\theta'_{13}, \delta'_{CP})$$

- $(\theta_{13}, \delta_{CP})$ (true solution) & $(\theta'_{13}, \delta'_{CP})$ (fake solution)



Up to Eight-Fold Degeneracy Expected

The $\nu_e \to \nu_\mu$ Channel in Matter

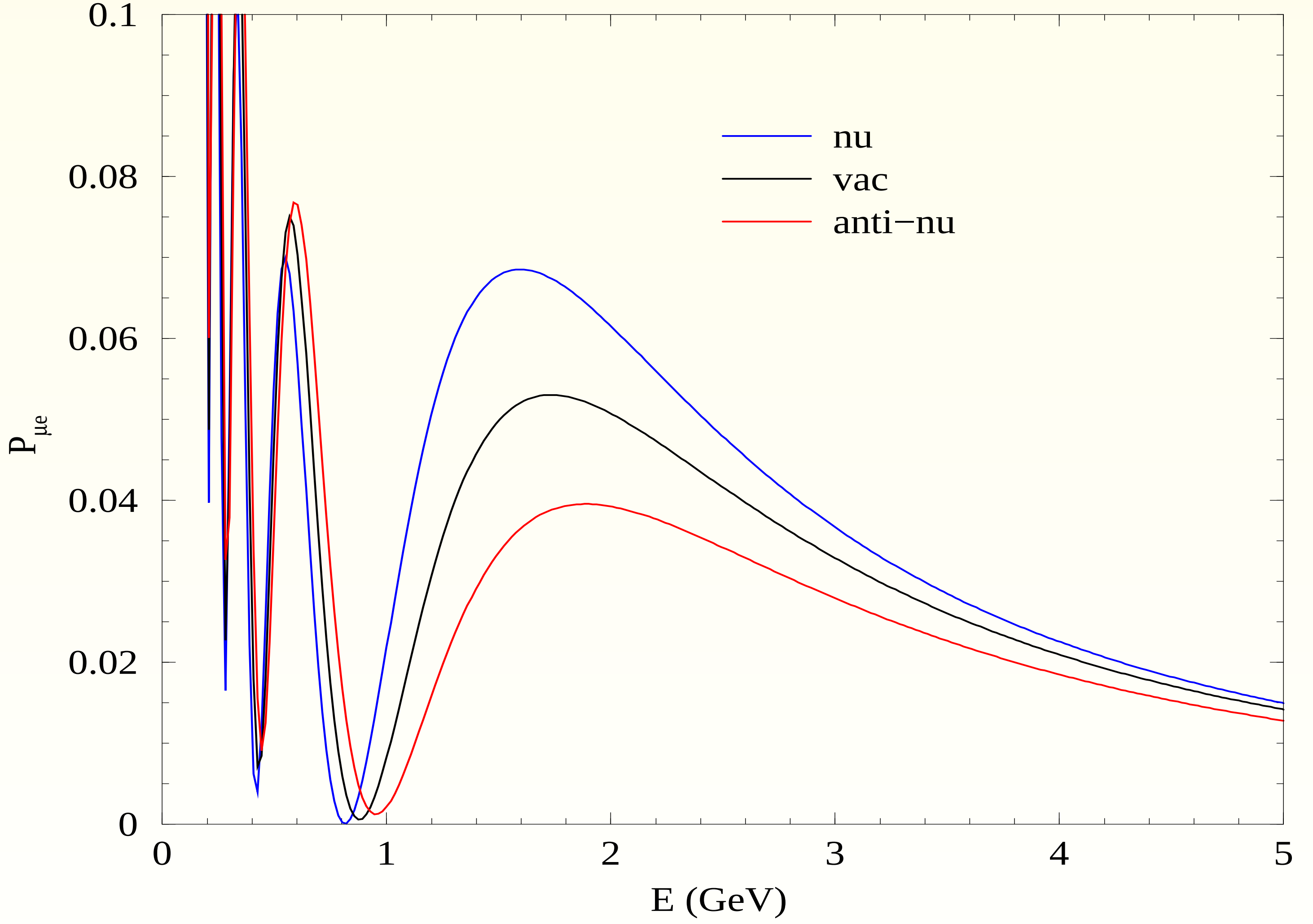
The $\nu_e \to \nu_\mu$ Channel in Matter

$$
\begin{aligned}
P_{app} \quad \simeq \quad & \sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \\
\pm \quad & \alpha\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta_{CP}\sin(\Delta)\frac{\sin(\hat{A}\Delta)}{\hat{A}}\frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
+ \quad & \alpha\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos\delta_{CP}\cos\Delta\frac{\sin(\hat{A}\Delta)}{\hat{A}}\frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
+ \quad & \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\end{aligned}
$$

- $\hat{A} = \frac{\pm A}{\Delta m^2_{31}}$
- A is positive for neutrinos
- A is negative for antineutrinos

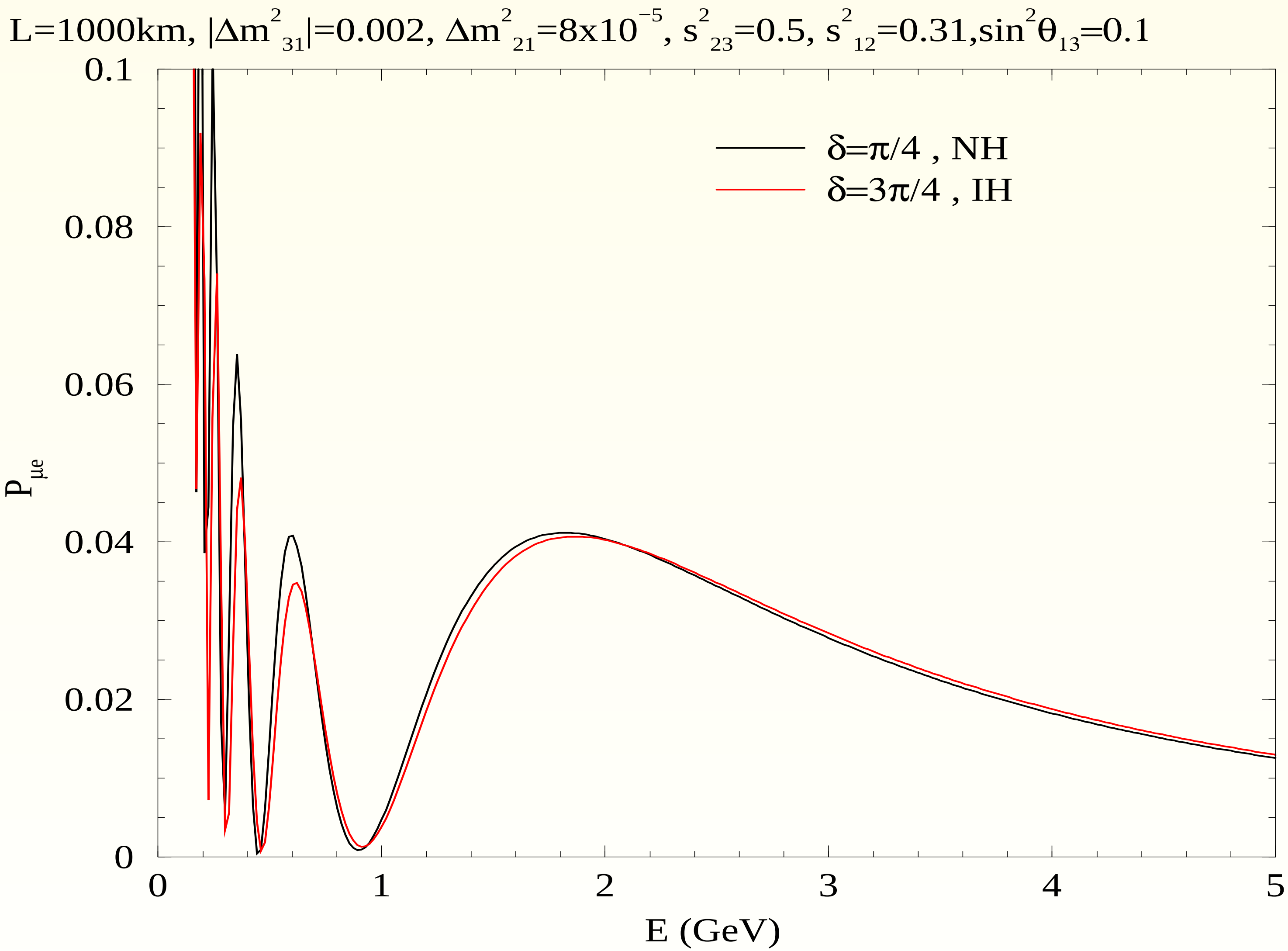
The $\nu_e \to \nu_\mu$ Channel in Matter

$L=1000km, \Delta m^2_{31}=0.002, \Delta m^2_{21}=8x10^{-5}, s^2_{23}=0.5, s^2_{12}=0.31, \sin^2\theta_{13}=0.1, \delta=0$



Resolving the Eight-Fold Degeneracy

Resolving the $Sgn(\Delta m^2_{31})$ Degeneracy



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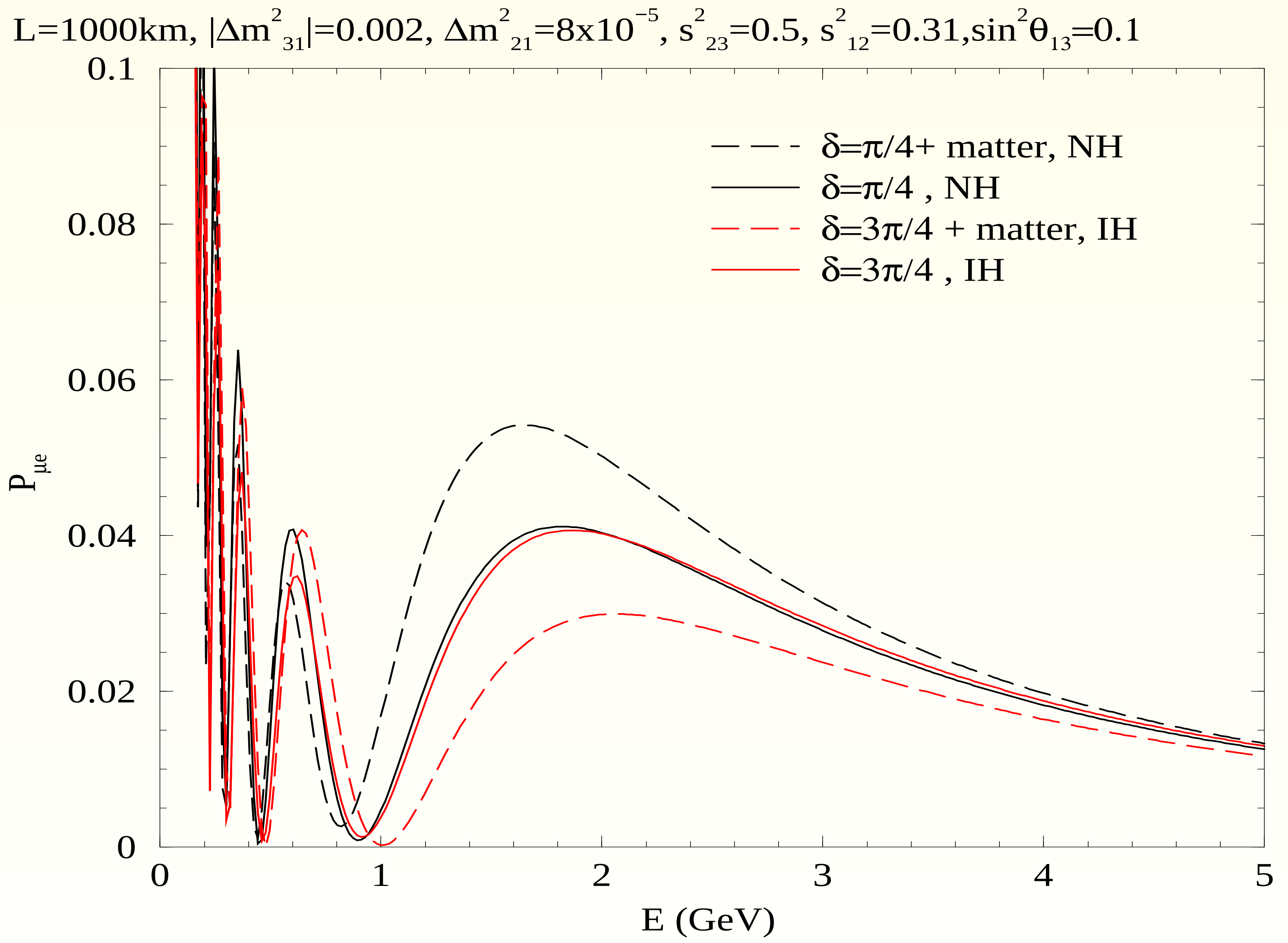

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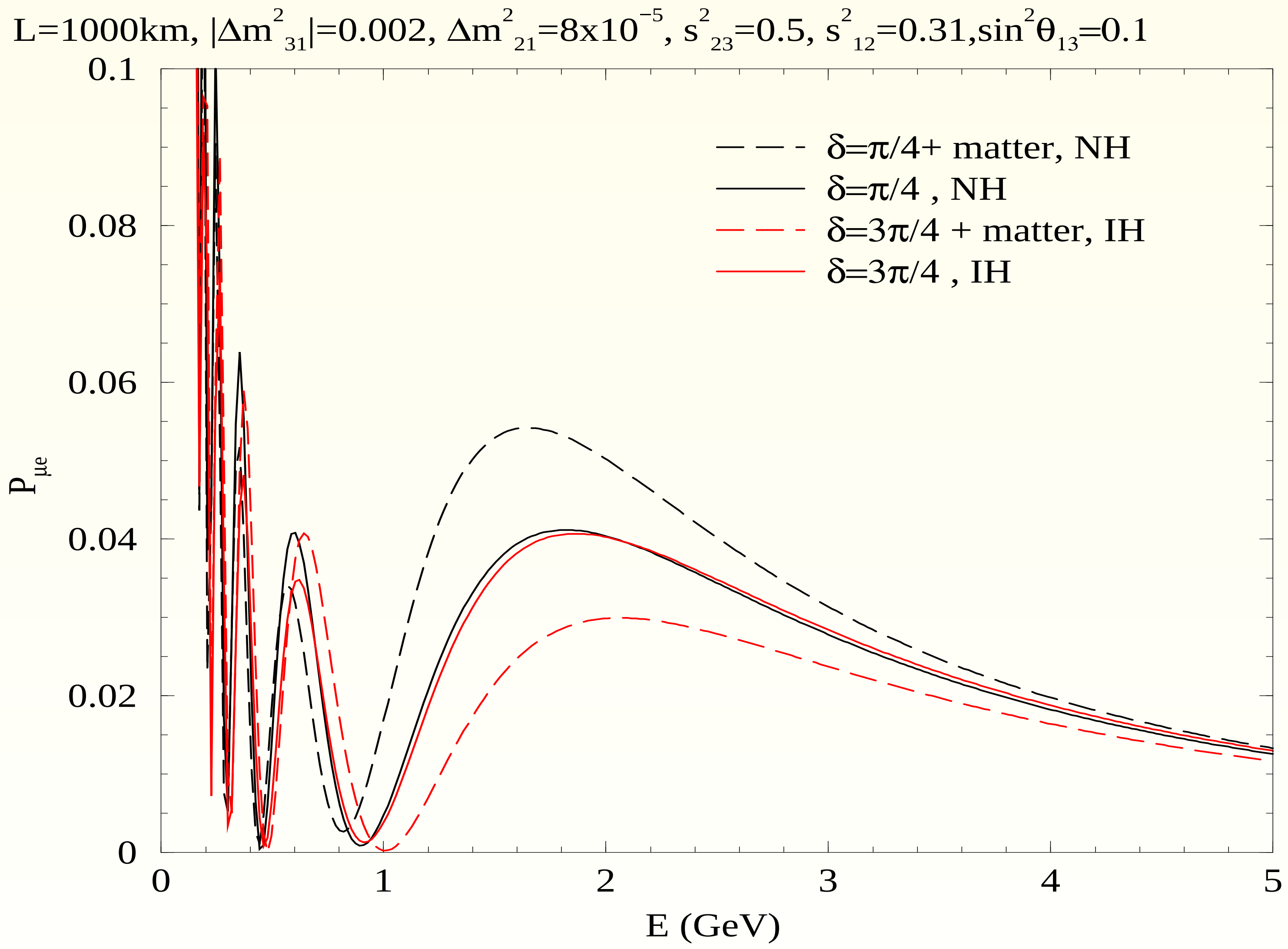

- Matter effects break the $Sgn(\Delta m^2_{31})$ Degeneracy

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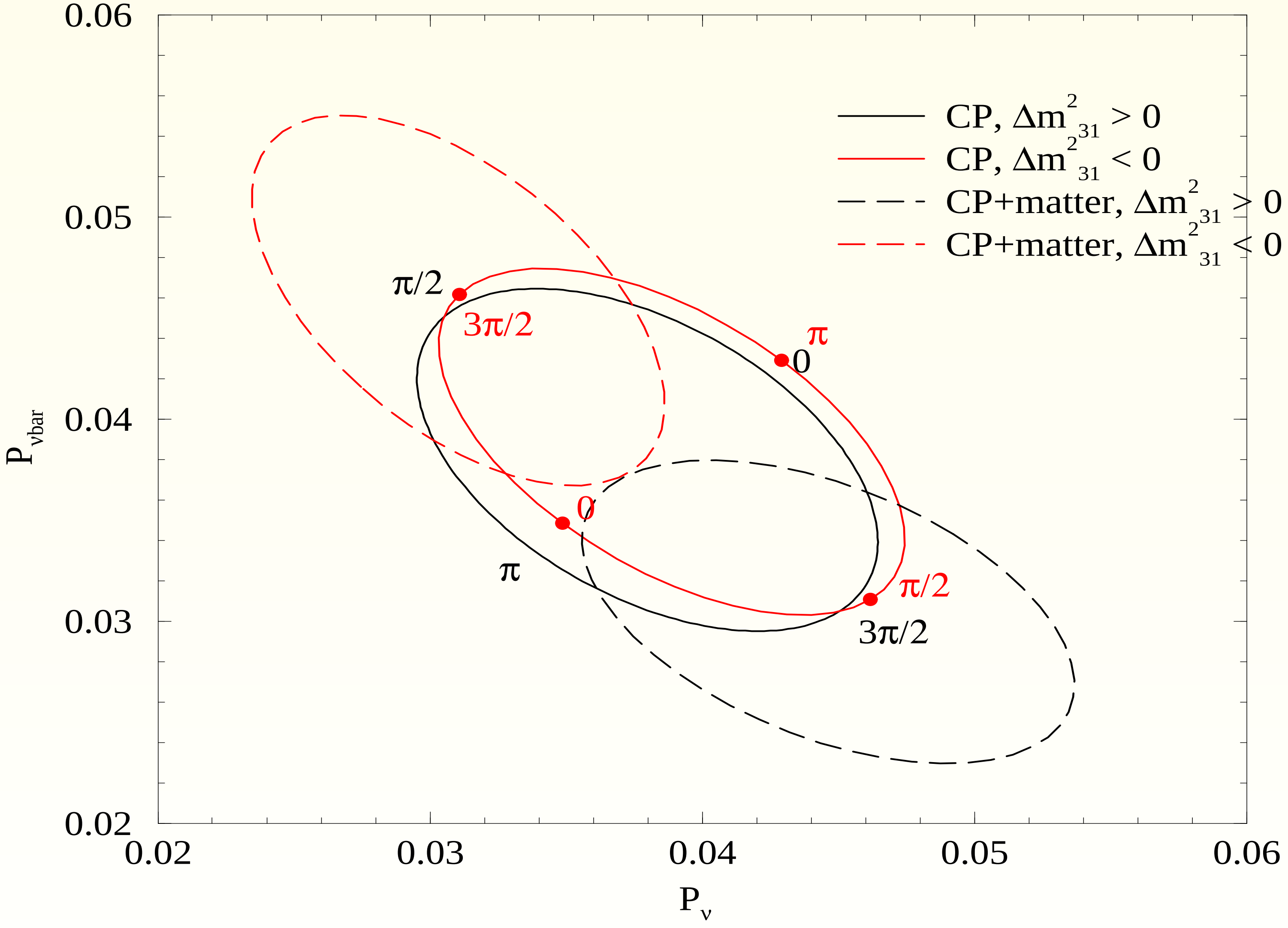
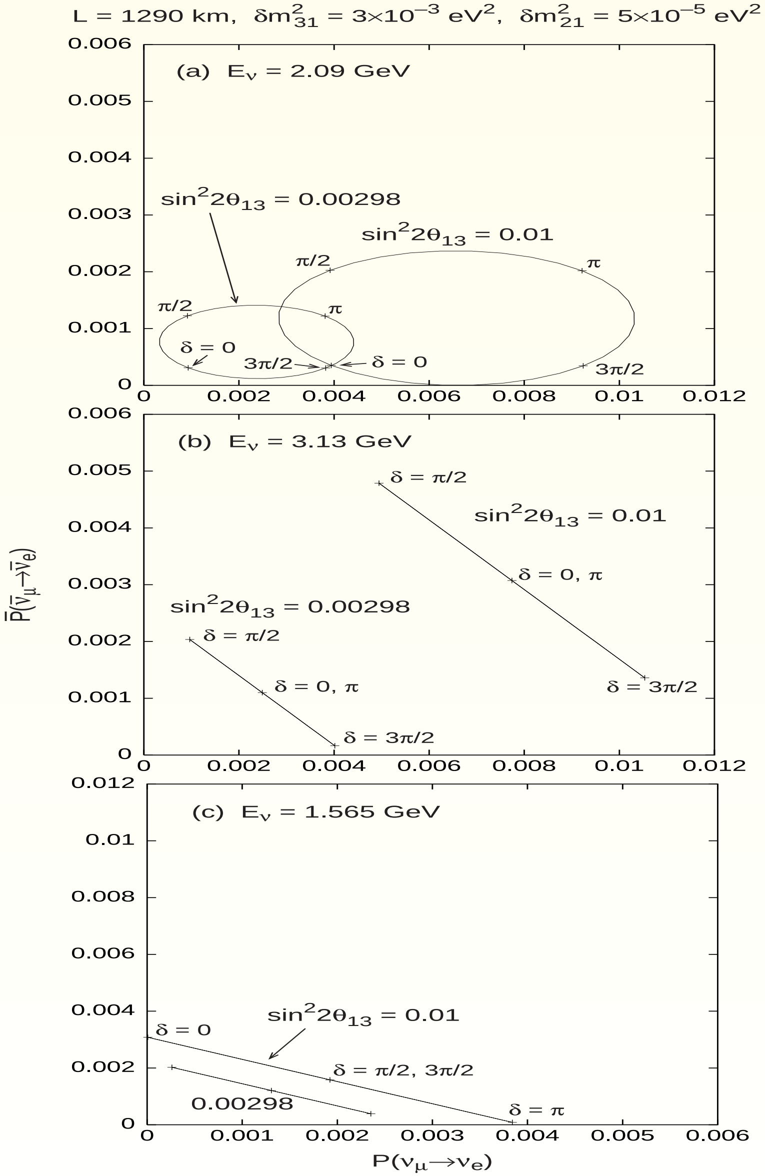


- Matter effects break the $Sgn(\Delta m^2_{31})$ Degeneracy

Resolving the Intrinsic $(\delta_{CP}, \theta_{13})$ Degeneracy



- Choose $\Delta = m\pi/2$.
- The $\cos\delta_{CP}$ term vanishes for $\Delta = (n - \frac{1}{2})\pi$.
- The $\sin\delta_{CP}$ term vanishes for $\Delta = n\pi$.
- Ellipse collapses to a line.
- Ambiguity resolved.
- Better to work with $\Delta = (n - \frac{1}{2})\pi$.
- That is where we will directly see CPV.
- Thats where we get the oscillation maxima.

Barger et al, hep-ph/0112119

Resolving the Octant of θ_{23} Degeneracy

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- Using atmospheric neutrino data in
  - $\surd$ Megaton water detectors:
    - $\star$ Δm^2_{21} driven osc effects in sub-GeV electrons
    - $\star$ θ_{13} driven matter effects in multi-GeV electrons
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- Other ways using LBL expts have been suggested.

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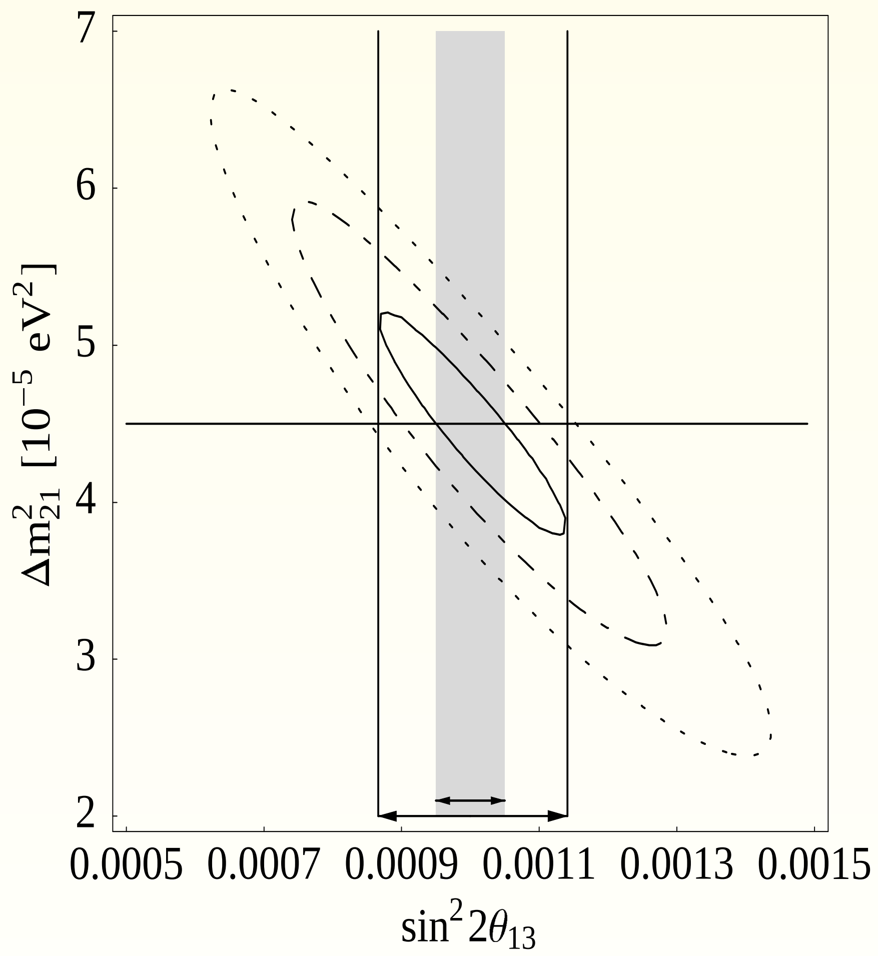
- This correlation between parameters leads to increase in the error in the measured value of the individual parameters.

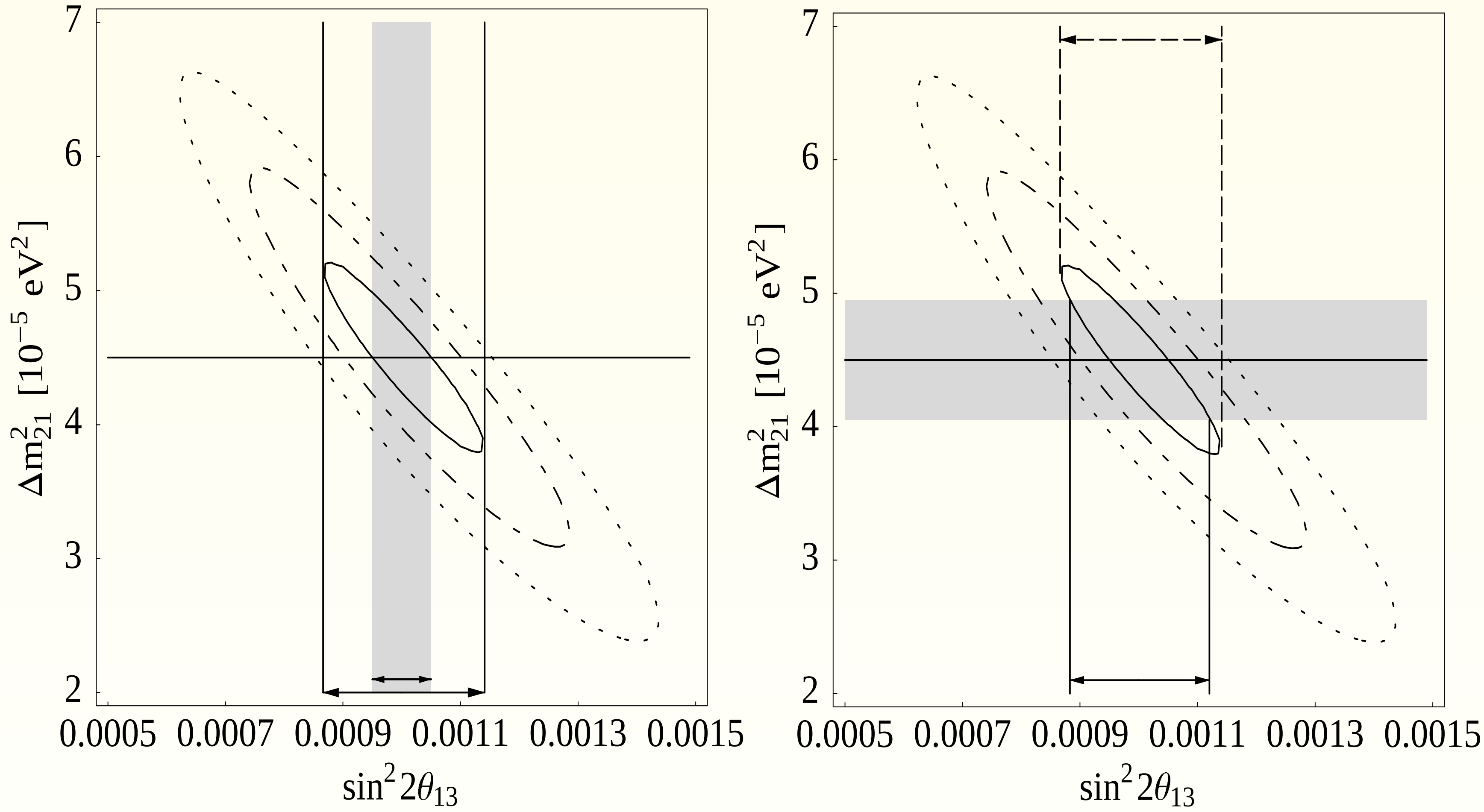
Correlations



Huber et al, hep-ph/0204352

- Note how correlations are increasing the spread.

Correlations



Huber et al, hep-ph/0204352

- Input from an external measurement helps.
- Note how correlations are increasing the spread.

Differentiating Correlations and Degeneracies

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- We discussed some ways to tackle degeneracies.

- The only way to reduce impact of correlations is to combine experiments with different characteristics.