Multiple Spectral Splits of Supernova Neutrinos

:: arXiv: 0904.3542 – with Dighe, Raffelt, Smirnov ::
:: arXiv: 0904.3542 – with Mirizzi, Tamborra, Tomas ::
:: in progress – with Choubey, Dighe, Mirizzi ::

Basudeb Dasgupta
Max Planck Institute for Physics, Munich
• Inverse SN neutrino problem
• A new layer of difficulty: Collective Oscillations
• A new player: Flux Models
• Rich (Complicated) phenomenology: Spectral Splits
• What should we look for? What could we learn?
Neutrino Oscillations in SN

• Neutrino oscillation usually involves only 2 terms
  ➤ Mass matrix / 2E
  ➤ MSW potential (due to electrons)

• In a SN, neutrinos are very dense and therefore create a similar MSW-like potential.
  ➤ Flavor non-trivial.
  ➤ Coupled neutrino oscillations a.k.a “Collective Effects”.

• Neutrino flavor spectra swap in some energy ranges.

• These are called “Collective Effects”.

The SN neutrino program

- Calculate an initial neutrino spectrum
- Calculate the changed spectrum due to oscillation effects
- Calculate flux at detector
- Construct variables that distinguish different physics/astro scenarios
- Wait for a SN…

- \( r \sim 10R_{\text{sun}}, \rho \sim 0.1\text{g/cc} \): MSW effects
- \( r \sim 100\text{ km}, \rho \sim 10^7\text{g/cc} \): Collective effects
- \( r \sim 10\text{ km}, \rho \sim 10^{10}\text{g/cc} \): Neutrinos trapped
- \( r \sim Kpc, \rho \sim 0\text{g/cc} \): Vacuum oscillations
- \( r \sim 10Kpc, \rho \sim 1-10\text{g/cc} \): Earth effects

Galactic SN

Earth
For IH:
- Exchange $\nu_e$ and $\nu_y$ above $E_c$.
- Exchange anti-$\nu_e$ and anti-$\nu_y$.

For NH:
- No collective effects.

Seminal papers by: Duan, Fuller, Carlson, and Qian (2005, 2006)
Almost 100 papers on collective effects by:
Abazajian, Balantekin, Beacom, Bell, Blennow, Carlson, Dasgupta, Dighe, Dolgov, Duan, Esteban-Pretel, Fogli, Friedland, Fuller, Gava, Goswami, Hannestad, Hansen, Kneller, Kostelecky, Lisi, Lunardini, Marrone, McLaughlin, Mirizzi, Pantaleone, Pastor, Pehlivan, Qian, Raffelt, Samuel, Serpico, Semikoz, Sigl, Smirnov, Stodolsky, Tomas, Volpe, Wong...
We get a spectral split in neutrinos, and the antineutrinos swap their energy spectra between flavors.

Actually there is a split in antineutrinos too…

Fogli, Lisi, Marrone, Mirizzi (2007)
SN Simulations: Garching 2003

- Almost thermal
- Pinching

- Burst of $\nu_e$
- Crossover
- Cooling

- $E_e < E_{e\text{bar}} < E_x$

Garching group, astro-ph/0303226

Slightly different fluxes

• What happens if initial fluxes are changed a bit?

• Let’s check out the case of the spectra predicted by Garching group for the cooling phase.

• The essential change: neutrino number fluxes are taken to be \( \nu_e : \nu_{\text{ebar}} : \nu_x = 0.85 : 0.75 : 1.00 \) and not equi-partitioned, as was commonly assumed.
Many spectral splits

- 4 splits in IH.
- 2 splits in NH.
- Why?

Clearly there is something missing in our understanding.

This could be observationally important…
Part I: How to predict the final spectra, given the initial spectra.
• We have the flux spectrum $f(E)$ for each flavor.

• However, let’s use $\omega = \Delta m^2 / 2E$ as the x-axis variable.

• Moreover, let’s label antineutrinos with $-\omega$.

• Define

$$g(\omega) = \begin{cases} f_x(E) - f_e(E) & \text{for neutrinos} \\ f_x(E) - f_e(E) & \text{for antineutrinos} \end{cases}$$

• Now we have put the all the relevant spectral information in a single function $g(\omega)$.

• How does this function look? Let’s see…
In the $g(\omega)$ variable...

- $g(\omega)=0$ where fluxes equal
- "Swaps" around every "± crossing"
- Each swap flanked by two "splits"
- Splits not always washed out completely by multi-angle effects
- Let's answer some questions now...
  - Why are there swaps around a crossing?
  - Why the ± for IH/NH?
  - What is the width of the swap?
Fixed initial neutrino density $\mu$

- "Box" spectrum at finite $\mu$.
- Spectrum oscillates to the dotted lines and back.
- Swap function looks like a Lorentzian centered at the crossing at any instant!
  - Collective motion.
  - May be we can solve this analytically?
  - Let’s try…

“Deriving” the Lorentzian

- The system has EOM

\[ \dot{P}_\omega = (\omega B + \lambda L + \mu \int d\omega_1 P_{\omega_1}) \times P_\omega \]

- Ansatz:

\[ P_\omega(t) = \begin{pmatrix} -\sin \varphi L(\omega) \\ -\frac{\omega}{\Gamma} \frac{2\sqrt{1 - \cos \varphi} L(\omega)}{1 - (1 - \cos \varphi) L(\omega)} \\ \end{pmatrix} g(\omega) \]

\[ L(\omega) = \frac{\Gamma^2}{\Gamma^2 + \omega^2} \]

- This is a merely a parametrization, and putting it back in EOMs we get

\[ \dot{\varphi} = \Gamma \sqrt{2(1 - \cos \varphi)} \]

\[ \Gamma = \frac{\delta}{\sqrt{-1 + e^{2\delta}/\mu}} \]

- EOM of a pendulum.

- Width is exponential in \( \mu \).
Changing neutrino density $\mu$

- We know that as we decrease $\mu$ (mimicking decreasing neutrino density away from the core) the pendulum damps and relaxes to lowest energy configuration.

- This system involves an adiabatic invariant that roughly relates the width of split $\omega_s$ to width of Lorentzian $\Gamma$.

$$\omega_s = \frac{\pi}{4} \Gamma \frac{2}{1 + \sqrt{1 + \frac{\pi^2}{4} \frac{\mu}{2\delta}}} = \frac{\delta}{\sqrt{-1 + e^{2\delta/\mu}}} \frac{\pi/2}{1 + \sqrt{1 + \frac{\pi^2}{4} \frac{\mu}{2\delta}}}$$

Some comments

• We showed that there is a pendulum like oscillation about a crossing.

• As $\mu$ decreases, this pendulum eventually tips over if it is inverted, i.e. +ive crossings for IH, -ive crossings for NH.

• Thus there are swaps around a crossing (B.P conserved).

• Width of the swap is related to $\Gamma$ and depends on initial $\mu$: wider swaps for larger initial $\mu$. Exponentially thin swaps for small. Also depends on $\delta$, i.e. box width.
Analogy to Spin Magnetic Resonance

• We break the collective magnetic field into a parallel and perpendicular component, drop the former.

\[ \dot{P}_\omega = (\omega B + \mu \int d\omega_1 P^\parallel_{\omega_1} + \mu \int d\omega_1 P^\perp_{\omega_1}) \times P_\omega \]

• For \( \omega=0 \), \( P \) is on-resonance (the mode has the same frequency as the transverse magnetic field)!

• Others are slightly off-resonance by \( \omega \), and their amplitude falls off as a Lorentzian, as in SMR.
• What happens if two copies of the box are put far apart in $\omega$-space?

• Each box acts like an independent pendulum; the transverse field due to the other is averaged to zero.

$$\dot{P}_\omega = (\omega B + \mu \int d\omega_1 P_{\omega_1} + \mu \int d\omega_2 P_{\omega_2}) \times P_\omega$$
What happens when they are close

• What happens if two “boxes” are put close together in the $\omega$-space

• The inner block acts like a superimposed oscillator on the bigger one. The inner swap-width is exponentially small.
What’s special about the box?

- Short answer: Nothing!
- Long answer: Although any function around the crossing works fine, doing the integrals is harder/impossible. Also, the uniqueness and stability of the solution is not guaranteed.
Great expectations

• The basic picture …

  - One swap for every ± crossing for IH/NH.
  - Width of each swap depends exponentially on $\mu$ and also on the $\delta$ for the block around that crossing.
  - Each swap approx. preserves lepton number B.P locally.

• When blocks are close more complicated things can happen, and it would be interesting to study…
Cooling phase fluxes: Recap

- Swaps around every “± crossing”
- Each swap flanked by two splits
- Splits not always washed out completely by multi-angle effects

- We have answered the questions...
  - Why are there swaps around a crossing?
  - Why the ± for IH/NH?
  - What is the width of the swap?
Accretion phase example: Recap

- We should have seen 4 splits, but we see 2 only, because the inner swap is exponentially narrower.
- In fact even in NH we should get two splits (but again they are narrow and the flux is low at low-$|\omega|$ to see anything).
• Three-flavor effects?
• Do a survey of various SN flux models and check what kind of split patterns one gets.
• Is there a simple picture to this?
• Can one show that this will/won’t have experimental relevance?
Part II:
Three Flavor Effects
Three-flavor effects

Inverted Hierarchy, 2 flavors, $\nu_e$

- $\nu_e$ at 1000 km
- initial $\nu_e$
- initial $\nu_x$

neutrino energy [MeV]

Inverted Hierarchy, 2 flavors, $\bar{\nu}_e$

- $\bar{\nu}_e$ at 1000 km
- initial $\bar{\nu}_e$
- initial $\bar{\nu}_x$

neutrino energy [MeV]

Inverted Hierarchy, full 3 flavors, $\nu_e$

- $\nu_e$ at 1000 km
- initial $\nu_e$
- initial $\nu_x$

neutrino energy [MeV]

Inverted Hierarchy, full 3 flavors, $\bar{\nu}_e$

- $\bar{\nu}_e$ at 1000 km
- initial $\bar{\nu}_e$
- initial $\bar{\nu}_x$

neutrino energy [MeV]

Alexander Friedland, arXiv:1001:0996

Solar $\Delta m^2$ driven effects

- Usually not adiabatic, i.e.
  - $\omega=\Delta m^2/E \approx \Gamma$ (pendulum frequency) less than rate at which $\mu$ is decreasing.

- Some initial disturbance helps to kick-start swaps.

Try anything twice

- $g(\omega)$ is processed twice
- Step 1: by atmospheric $\Delta m^2$ (NH/IH)
- Step 2: by solar $\Delta m^2$
- Interplay of these two steps
  - NH: cooperate
  - IH: compete with each other
- Step 1 gives required disturbance.
Inverted Hierarchy

- Atmospheric swaps (e, y)
- Solar swaps (e, x)
- Higher energy split is transferred from e to x
- Non-adiabatic effects
- In short: It’s a mess! But a mess that we understand!
Normal Hierarchy

- Almost same as 2-flavors.
- Solar driven conversions are too slow to compete.
- Simple prediction
  - High energy spectrum of \( e \) and \( \nu \) flavors are swapped.
- Let’s get a bit more ambitious...
Part III:
Survey of Flux Models and Pattern Hunting
Ternary Diagrams

• Luminosity of 3 species
• Typically $L_e = L_{ebar}$

• A pattern of splits

Fogli, Lisi, Marrone, Tamborra, arXiv:0907.5115
Flux Models: Garching 2003

- Luminosity:

- Average Energy:
Flux Models: Basel 2009

• Luminosity:

• Average Energy:

Basel group, arXiv:0908.1871
A given model at time, is a point on this plane

- Include MSW effects, vacuum mixing, Earth matter effects…
- Look at $\nu_e$ and anti-$\nu_e$ spectra at various detectors…
- See if there are some simple ways to extract NH/IH or SN physics…
NH/IH determination

- Look at the early signal (< 1 sec) in antineutrinos using ratio of events at two WC detectors.

Dasgupta, Dighe, Mirizzi, arXiv:0802.1481
More complicated time-dependent signatures
Conclusions

• Rich phenomenology…important to remember that we will have a time-dependent signal.
• Collective effects can be very different over these times.
• Theory still not complete…but in good shape.
• Lots to do…