Melting of three-sublattice order

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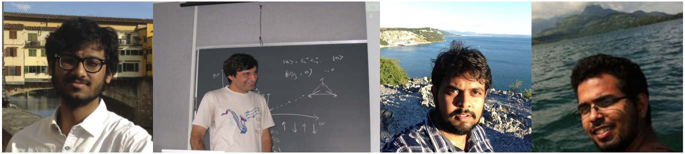
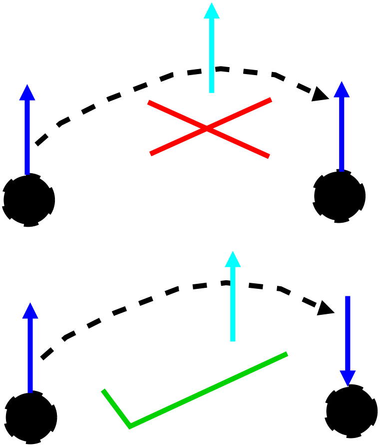


S. Biswas, D. Heidarian, G. Rakala (TIFR), S. Shivam

Recap: Local moments in Motterials

- Band-theory of solids $\rightarrow$ electron-waves occupying modes determined by electrostatic potential of nuclear array
- Strong e-e interactions $\rightarrow$ Failure of band picture
- Electron particles localized on lattice sites
  charge frozen, spin remains dynamical

Recap: Antiferromagnetic exchange



Recap: A Goodenough description

- Without spin-orbit: Isotropic exchange interactions.
  $E = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \quad J > 0$
  When is $J > 0$, large?
  Are nearest neighbour interactions dominant?
  Difficult (quantum chemistry/ab-initio studies) questions
  Thumb-rule answers: Goodenough-Kanamori-Anderson rules
  J.B. Goodenough, *Magnetism and the Chemical Bond (1963)*

Complications

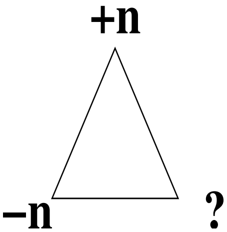
- Spin-orbit coupling λ
  spin anisotropy terms
- Orbital degeneracy
  Interplay between orbital structure and spin physics
  e.g. Vanadium spinels (Tsunetsugu & Motome 2003)

Recap: Néel order

- Bipartite lattice and nearest neighbour $J > 0$
  Spins spontaneously pick axis $\mathbf{n}$ and $\langle \vec{S}_{\vec{r}} \rangle = (-1)^{\vec{r}} \mathbf{n}$
  Néel (antiferromagnetic) Order

Geometric Frustration

Triangles on my mind...



- Triangles in nearest-neighbour connectivity *frustrate* Néel order
- Geometry induces *competition* between leading exchange interactions

Frustration spawns novel states

- Quenching of the leading exchange J
  J cannot pick ground state at classical level
- Sub-dominant interactions & quantum fluctuations play major role
  Opens the door for variety of novel low temperature states

Frustrated magnets: Plethora of lattices and materials

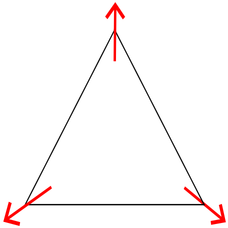
- Triangular lattice: $S=1$ $AgNiO_2$ (Ni^{2+}), $S=1/2$ Cs_2CuCl_4 (Cu^{2+})...
- Kagome: $S=5/2$ Fe jarosite (Fe^{3+}), $S=1/2$ *Herbertsmithite* $ZnCu_3(OH)_6Cl_2$ (Cu^{2+}), $S=1$ $Ni_3V_2O_8$ (Ni^{2+})...
- Pyrochlore, pyrochlore-slapb $S=3/2$ $SrCr_{9p}Ga_{12-9p}O_{19}$ Cr^{3+} (SCGO)...

Single ion anisotropy can be large

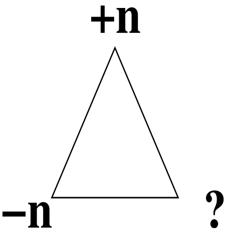
- Single ion anisotropy $-D(\mathbf{S}\cdot\mathbf{n})^2$can dominate over J
- Pyrochlore *spin ice* $Ho_2Ti_2O_7$ (Ho^{3+}, $(L+S)=8$)
  Easy axes $\mathbf{n}$ point outward from center of each tetrahedron
  $D\sim 50K$, $J\sim 1K$
  Harris *et. al.*, Phys. Rev. Lett. 79, 2554 (1997)
- Kagome Nd-langasite $Nd_3Ga_5SiO_{14}$ (Nd^{3+}, $(L+S)=9/2$)
  Easy axis perpendicular to lattice plane, $J\sim 2K$, $D\sim 10K$
  Robert *et. al.*, Physica B 2006
- So it makes sense to study leading quantum effects in a J/D expansion
  Not our focus today

Anisotropy amplifies frustration

- Isotropic spins on a triangle



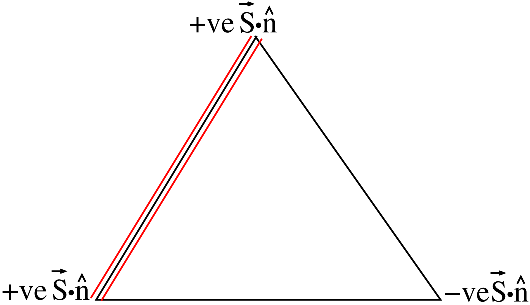
- Easy-axis $\mathbf{n}$ and triangular motifs...



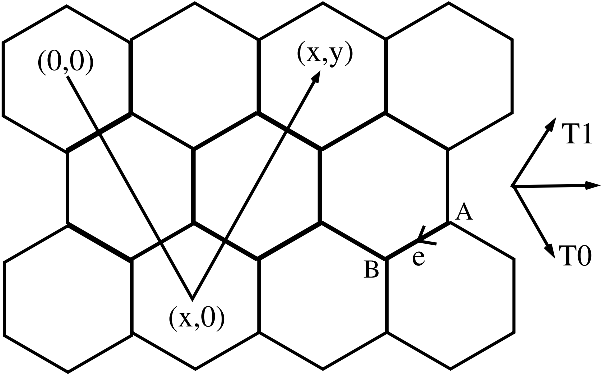
Wannier's triangular lattice model

- $H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - D \sum_i (S_i^z)^2$, with $D >> J$ on the triangular lattice.
- To leading order $S_i^z = \pm S \to \sigma = \pm 1$
  $H \approx JS^2 \sum_{\langle ij \rangle} \sigma_i \sigma_j$
- Minimum energy configurations?

Minimally frustrated configurations



- One frustrated bond per triangle



Honeycomb lattice dimer model: One dimer touching each honeycomb vertex

Classic problem in graph-theory/combinatorics/statistical mechanics

Ising 'liquid' in $T \to 0$ limit

- Calculation of Stephenson (64) gives

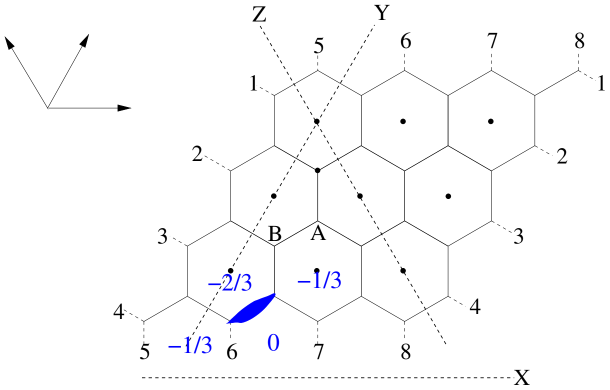
$$\langle \sigma(\mathbf{r})\sigma(0)\rangle \quad \sim \quad \frac{A}{r^{9/2}} + \frac{B\cos\left(2\pi(x+y)/3\right)}{\sqrt{r}}$$

- Spins neither freeze, nor fluctuate independently.
- Instead, form highly correlated "spin liquid".

Understanding this result:

- Dimers, heights, and Ising models of frustration
- (Obvious) connection to odd Ising gauge theories
- Connection to Kosterlitz-Thouless theory

From dimers to microscopic heights $H(R)$



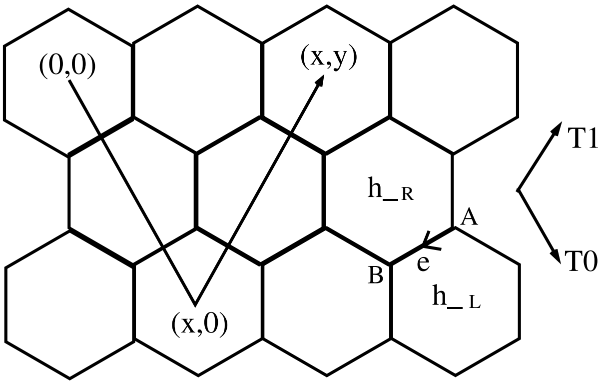
From microscopic $H(R)$ to coarse-grained $h(r)$

- Locality: What happens "outside" cannot affect what happens "inside".
  $h(r) \to h(r) + 1$
  More of a redundancy than a symmetry. (Field theorists: "compactification radius")
- Translation symmetry: $h(r) \to h(r) + 1/3$
- Rotation by $2\pi/6$ about triangular site: $h(r) \to -h(r)$

Effective action for coarse-grained $h(r)$

- Fewer flippable plaquettes $\rightarrow$ larger "tilt"
  $\mathcal{S}_{\text{eff}} = \frac{\pi}{g}(\nabla h)^2 + \lambda_6 \cos(6\pi h) + \ldots$

Ising spins in terms of $h(r)$

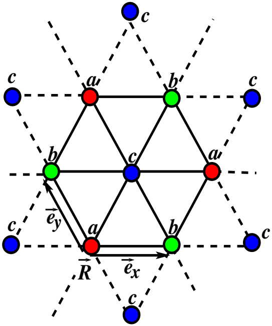


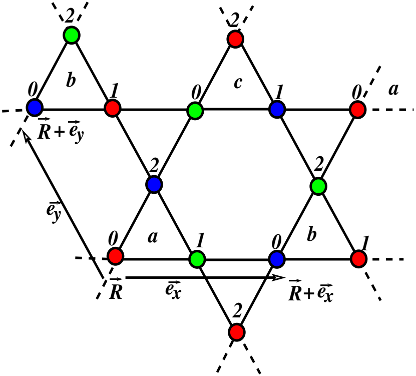
$$\sigma(R) = \exp(-3\pi i H(R)) = \exp(\tfrac{2\pi}{3} i(X+Y) - i\pi H(R))$$
$$\sigma(r) \sim A e^{i\mathbf{Q}\cdot r} e^{-i\pi h(r)} + B e^{-3i\pi h(r)} + h.c.$$

KT vortices and "odd Ising gauge theory"

- Nonzero temperature: Heights no longer single valued
  Vortex: $h \to h \pm 2$ ambiguity when three dimers touch honeycomb site (fully frustrated Ising triangle)
- Configuration space not dimer model, but model with odd number of dimers touching each honeycomb site
- "Electric field $E_{\mathrm{A}\to\mathrm{B}} = n_{AB} - 1/3$ no longer divergence-free
  But violations are 0 mod 2
  Field-theory language: Configuration space of odd-Ising gauge theory

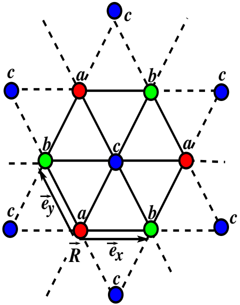
Our focus: Easy-axis antiferromagnets on Kagome and triangular lattices





Natural tripartite structure

Three-sublattice order on the triangular lattice



θ	red	green	blue
0	−S	+S	+S
$\frac{\pi}{6}$	−S	0	+S

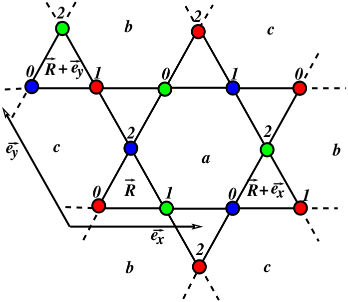
$\psi = |\psi| e^{i\theta} = -\sum_{\vec{R}} e^{i\mathbf{Q}\cdot\vec{R}} S^z_{\vec{R}}$

Ferri vs antiferro order distinguished by the choice of phase θ

Ferri: $\theta = 2\pi m/6$, Antiferro: $\theta = (2m+1)\pi/6$ ($m = 0, 1, 2 \ldots 5$)

Three-sublattice order on the Kagome lattice

θ	🔴	🟢	🔵
0	–S	+S	+S
$\frac{\pi}{6}$	–S	0	+S



$$\psi = |\psi| e^{i\theta} = -\sum_{\vec{R}} \sum_{\alpha=0,1,2} e^{i\mathbf{Q}\cdot\vec{R} - 2\pi i \frac{\alpha}{3}} S^z_{\vec{R},\alpha}$$

Again: Ferri vs antiferro distinguished by the choice of phase θ

Ferri: $\theta = 2\pi m/6$, Antiferro: $\theta = (2m+1)\pi/6$ $(m = 0, 1, 2 \ldots 5)$

On general symmetry grounds:

- Three ways in which this order can melt on heating
  Two-step melting (intermediate KT phase with power-law order for $T \in (T_{c1}, T_{c2})$)
  OR
  Three-state Potts transition followed by Ising transition
  OR
  Single first-order transition (always possible!)

Our results—Landau-Ginzburg analysis:

- Thermodynamic signature of KT phase:
  $\chi_{\hat{n}}(B) \sim 1/|B|^{p(T)}$ with $p(T) \in (\frac{2}{3}, 0)$ for $T \in (T_{c1}, T_{c2})$.
- KT phase can
  Pinch-off at multicritical point $\mathcal{M}_?$, giving way to three-state Potts criticality. $c_{\mathcal{M}_?} = ?$
  OR
  Pinch-off at multicritical point $\mathcal{M}_{\text{Clock}}$, giving way to first-order transition line.
- $\mathcal{M}_{\text{Clock}}$ previously known, not $\mathcal{M}_?$
  Note: Conjecture (Dorey-Tateo-Thompson '96) relates $\mathcal{M}_{\text{Clock}}$ to self-dual Z_6 $c = 1.25$ CFT (Zamolodchikov-Fateev '85)
  $\rightarrow c_{\mathcal{M}_{\text{Clock}}} = 1.25$

Our results—Computations for microscopic models:

- Existence of KT phase in $S = 1$ triangular lattice antiferromagnets with moderate easy-axis (single-ion) anisotropy
- Quantitative verification of predicted singular susceptibility in KT phase in several cases
- How does the KT phase pinch-off for specific cases?
  - Preliminary evidence for $\mathcal{M}_{\text{Clock}}$ on the triangular lattice
    Similar, more preliminary results on Kagome lattice systems
  - Conjecture for $\mathcal{M}_{?}$ in triangular bilayers

Incipient three-sublattice order in triangular Ising AFM

- Recall: Power-law correlator in $T \to 0$ limit:
  $\langle \sigma_r^z \sigma_0^z \rangle \sim \frac{\cos(\mathbf{Q}\cdot\mathbf{r})}{r^{1/2}}$
  Incipient order at three-sublattice wavevector $\mathbf{Q} = (2\pi/3, 2\pi/3)$
  Stephenson (1964)
- Perturbations can stabilize this order...

Triangular lattice-gas models for monolayer films on graphite

- Three-sublattice long-range order of noble-gas monolayers on graphite
  Birgeneau, Bretz, Chan, Vilches, Wiechert...(1970—1990)
  $H_{\mathrm{J_1J_2}} = J\sum_{\langle ij\rangle}\sigma_i^z\sigma_j^z - J_1\sum_{\langle\langle ij\rangle\rangle}\sigma_i^z\sigma_j^z - J_2\cdots - B\sum_i\sigma_i^z$
  Long-range three-sublattice ordering (wavevector $\mathbf{Q}$) at low temperature
  D. P. Landau (1983)

Ising models for “Artificial Kagome-ice”

- $H_{\text{Kagome}} = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - J_1 \sum_{\langle\langle ij \rangle\rangle} \sigma_i^z \sigma_j^z - J_2 \dots$
- Only nearest-neighbour couplings $\rightarrow$ classical short-range spin liquid (Kano & Naya 1950)
- Second-neighbour ferromag. couplings destabilize spin liquid (Wolf & Schotte 88)
  Ferrimagnetic three-sublattice order at low T.
- “Artificial Kagome-ice: Moments $\mathbf{M}_i = \sigma_i^z \mathbf{n}_i$
  ($\mathbf{n}_i$ at different sites non-collinear)
  Expt: Tanaka *et. al.* (2006), Qi *et. al.* (2008), Ladak *et. al.* (2010,11)
  Theory: Moller, Moessner (2009), Chern *et. al.* (2011)

Prototypical example of order-by-(quantum) disorder

- $H_{\text{TFIM}} = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - \Gamma \sum_i \sigma_i^x$ on the triangular lattice
  Long-range order at three-sublattice wavevector $\mathbf{Q}$
- Ordering of “antiferro” type $\rightarrow (+, -, 0)$
  antiferro order provides maximum “room” for quantum fluctuations
  Moessner, Sondhi, Chandra (2001), Isakov & Moessner (2003)

Prospects for experimental realization slim? (large moments, dipolar couplings...)

$S = 1$ triangular lattice antiferromagnets with single-ion anisotropy (more promising)

- $H_{\mathrm{AF}} = J\sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j - D\sum_i (S_i^z)^2$ on triangular lattice
- Low-energy physics for $D \gg J$:
  $H_b = -\frac{J^2}{D}\sum_{\langle ij \rangle}(b_i^\dagger b_j + h.c.) + J\sum_{\langle ij \rangle}(n_i - \frac{1}{2})(n_j - \frac{1}{2}) + \ldots$
  KD & Senthil (06)
- Low-temperature state for $D \gg J$: "supersolid" state of hard-core bosons at half-filling.
  Auerbach & Murthy (97), Heidarian & KD, Melko, Wessel...(05)
- Implies: Three-sublattice order in S^z + "ferro-nematic" order in $\vec{S}_\perp^2$
  (Simple easy-axis version of Chandra-Coleman (1991) "spin-nematic" ideas)
  (also related to Tsunetsugu-Arikawa (2006) proposal for $NiGa_2S_4$)

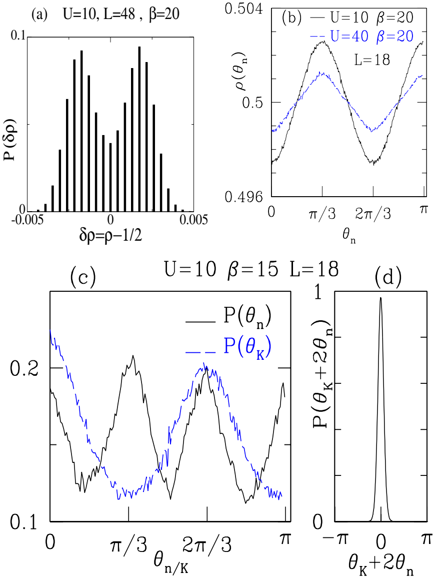
Is three-sublattice ordering of S^z in H_{AF} ferri or antiferro?

- Natural expectation: Quantum fluctuations induce antiferro order (like in the transverse field Ising model)

  $\rightarrow$

  Initial confusion: Ordering will be antiferro three-sublattice order *e. g.* Melko *et. al.* (2005)

Actual state has ferrimagnetic three-sublattice order



Heidarian & KD (2005)

Symmetry breaking transitions: Generalities

- Symmetry-breaking state characterized by long-range correlations of "order-parameter" $\hat{O}$
- phenomenological Landau free energy density $\mathcal{F}[\hat{O}]$
  Expanding $\mathcal{F}$ in powers of $\hat{O}$ (symmetry allowed terms)
- Neglecting spatial variation & fluctuations:
  phase transition $\rightarrow$ change in minimum of $\mathcal{F}$

Fluctuation effects at continuous transitions:

- More complete description of long-wavelength physics:
  Include (symmetry allowed) gradient terms in $\mathcal{F}$
  Integrate over all possible order parameter configurations
- In most cases: Corrections to mean-field exponents

Symmetries are (usually) decisive:

- Transformation properties of $\hat{O}$ determine nature of continuous transition

Landau-theory for melting of three-sublattice order

- $\mathcal{F} = K|\nabla\psi|^2 + r|\psi|^2 + u|\psi|^4 + \lambda_6(\psi^6 + \psi^{*6}) + \ldots$
  Connection with six-state clock models
  $Z = \sum_{\{p_i\}} \exp[\sum_{\langle ij\rangle} V(\frac{2\pi}{6}(p_i - p_j))]$
  Each $p_i = 0, 1, 2, \ldots 5$
  $V(x) = K_1 \cos(x) + K_2 \cos(2x) + K_3 \cos(3x)$
  Cardy (1980)

Simplest lattice model

$$H_{\rm xy} = -J_{\rm xy} \sum_{\langle \vec{r}\vec{r}' \rangle} \cos(\theta_{\vec{r}} - \theta_{\vec{r}'}) - h_6 \sum_{\vec{r}} \cos(6\theta_{\vec{r}}) \ .$$

(higher harmonics $J^{(p)}$ ($p = 2, 3$) left out of $H_{\rm xy}$ for simplicity)

Melting scenarios for three-sublattice order

- Analysis (Cardy 1980) of generalized six-state clock models
  $\rightarrow$ Three generic possibilities of relevance here:
  Two-step melting, with power-law ordered intermediate phase
  OR
  3-state Potts transition to ferromagnetic phase followed by loss of ferromagnetism via Ising transition at higher temperature..
  or vice-versa...
  OR
  First-order transition (always possible!)

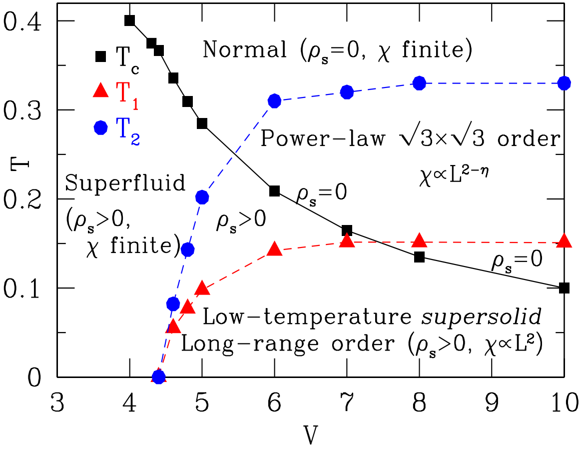
Melting of three-sublattice order in various examples

- Antiferro three-sublattice order in triangular lattice transverse field Ising model
  Two-step melting
  Isakov & Moessner (2001)
- Ferrimagn. three-sublattice order in triangular lattice-gas models of monolayer films
  Two-step melting
  D.P. Landau (83)
- Ferri. three-sublattice order in Kagome Ising antiferromagnets
  With second-neighbour ferro couplings: Two step melting
  Wolf & Schotte (88)
  With long-range dipolar couplings: Three-state Potts transition
  Moller & Moessner (09), Chern, Mellado, Tchernyshyov (11)

Nature of melting transition in $S = 1$ H_{AF}?

- Prediction of Boninsegni & Prokofiev (2005)
  Three-state Potts transition
  Prediction based on argument about relative energies of different kinds of domain walls
  hard to get right at quantitative level

Our answer from large-scale QMC simulations



Heidarian & KD (*submitted to PRB*)

Detecting power-law order?

Need scattering experiment to detect power-law version of Bragg peaks

Or

Real-space data by scanning some local probe $+$ Lots of image-processing

Alternate thermodynamic signature(!)

- Singular thermodynamic susceptibility to *uniform* easy-axis field B:
  $\chi_u(B) \sim \frac{1}{|B|^{p(T)}}$
- $p(T) = \frac{4-18\eta(T)}{4-9\eta(T)}$ for $\eta(T) \in (\frac{1}{9}, \frac{2}{9})$
  So $p(T)$ varies from $2/3$ to 0 as T increases from T_{c1} to just below T_{c2}

KD (PRL 2015)

Recall: picture for power-law ordered phase

- In state with long-range three-sublattice order, θ feels $\lambda_6 \cos(6\theta)$ potential.
  Locks into values $2\pi m/6$ (resp. $(2m+1)\pi/6$) in ferri (resp. antiferro) three-sublattice ordered state for $T < T_{c1}$
- In power-law three-sublattice ordered state for $T \in (T_{c1}, T_{c2})$, λ_6 does not pin phase θ
  θ spread uniformly $(0, 2\pi)$
- But vortices continue to be irrelevant
  Distinction between ferri and antiferro three-sublattice order lost for $T \in (T_{c1}, T_{c2})$

More formal RG description

- Fixed point free-energy density: $\frac{\mathcal{F}_{\mathrm{KT}}}{k_B T} = \frac{1}{4\pi g}(\nabla\theta)^2$
  with $g(T) \in (\frac{1}{9}, \frac{1}{4})$ corresponding to $T \in (T_1, T_2)$
- $\lambda_6 \cos(6\theta)$ *irrelevant* along fixed line
- $\langle \psi^*(r)\psi(0) \rangle \sim \frac{1}{r^{\eta(T)}}$
  with $\eta(T) = g(T)$

Jose, Kadanoff, Kirkpatrick, Nelson (1977)

General argument—I

Starting point: Ferrimagnetic three-sublattice order also involves uniform magnetization m

More complete theory should treat m and ψ on equal footing

- Symmetries allow coupling term $\tilde{\lambda}_3 m(\psi^3 + \psi^{*3})$
  augment $\frac{\mathcal{F}_{\mathrm{KT}}}{k_B T}$ with gapped free-energy density $\mathcal{F}_{\mathrm{ferro}}(m)$:
  $\mathcal{F}_{\mathrm{ferro}}(m) + \lambda_3 m \cos(3\theta)$
- λ_3 formally irrelevant along fixed line $\mathcal{F}_{\mathrm{KT}}$
  $\rightarrow$
  Physics of two-step melting unaffected—m "goes for a ride..."

But ...

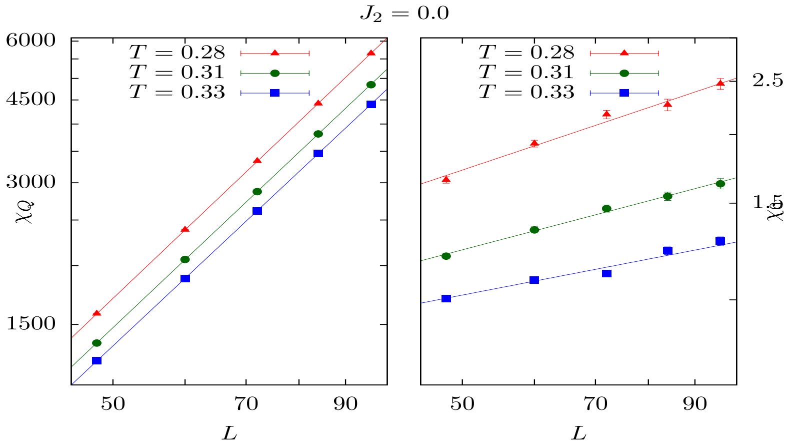
General argument—II

- m “inherits” power-law correlations of $\cos(3\theta)$:
  $C_m(r) = \langle m(r)m(0)\rangle \sim \frac{1}{r^{9\eta(T)}}$
- $\chi_L \sim \int^L d^2r C_m(r)$ in a finite-size system at $B = 0$
- $\chi_L = \chi_{\text{reg}} + bL^{2-9\eta(T)}$ for $\eta(T) \in (\frac{1}{9}, \frac{2}{9})$
  Diverges with system size at $B = 0$

General argument—III

- Uniform field $B > 0 \rightarrow$ additional term $h_3 \cos(3\theta)$ in $\mathcal{F}_{\mathrm{KT}}$
- Strongly relevant along fixed line, with RG eigenvalue $2 - 9g/2$
- Implies finite correlation length $\xi(B) \sim |B|^{-\frac{2}{4-9\eta}}$
- $\chi_u(B) \sim |B|^{-\frac{4-18\eta}{4-9\eta}}$ for $\eta(T) \in (\frac{1}{9}, \frac{2}{9})$

Test in prototypical example



In power-law ordered phase of H_{TFIM} on triangular lattice
Biswas, KD (*submitted to PRB*)

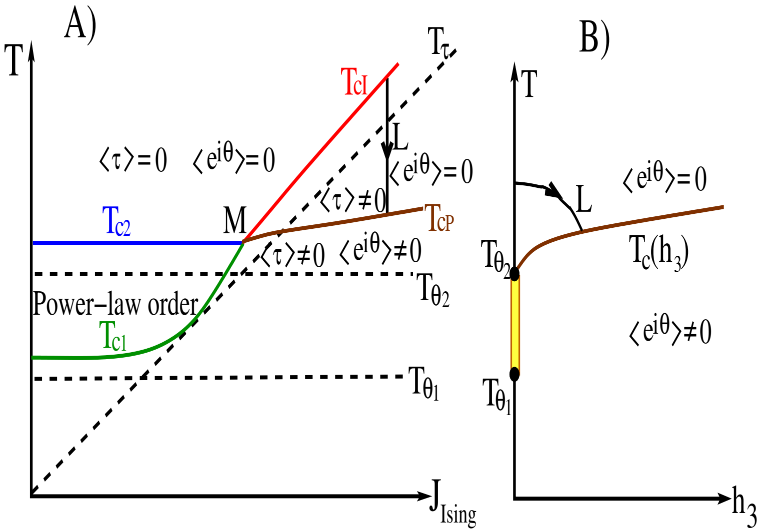
More complete coarse-grained description

$$
\begin{aligned}
H_{\text{eff}} &= H_{\text{xy}} + H_{\text{Ising}} - J_{\theta\tau} \sum_{\vec{r}} \tau_{\vec{r}} \cos(3\theta_{\vec{r}}) \ , \\
\text{where} \quad H_{\text{Ising}} &= -J_{\text{Ising}} \sum_{\langle \vec{r}\vec{r}' \rangle} \tau_{\vec{r}} \tau_{\vec{r}'} - h \sum_{\vec{r}} \tau_{\vec{r}} \ , \\
H_{\text{xy}} &= -J_{\text{xy}} \sum_{\langle \vec{r}\vec{r}' \rangle} \cos(\theta_{\vec{r}} - \theta_{\vec{r}'}) - h_6 \sum_{\vec{r}} \cos(6\theta_{\vec{r}}) \ ,
\end{aligned}
$$

with $h \propto B$.

KD (PRL 2015)

Phase diagram of H_{eff}



KD (PRL 2015)

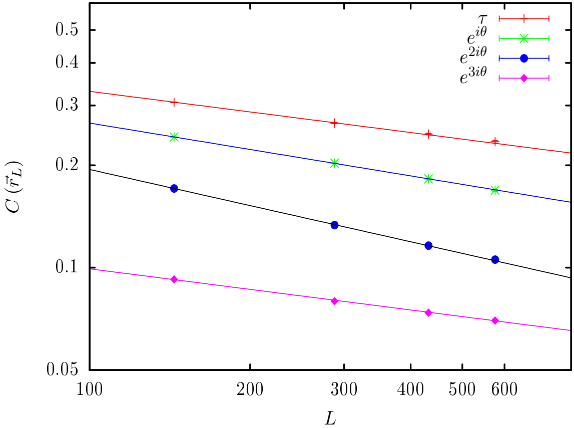
The argument...

- Start with known phase diagrams of $H_{\rm xy}$ and $H_{\rm Ising}$ and build in effects of $J_{\theta\tau}$
- When τ orders, $H_{\rm xy}$ sees effective three-fold symmetric perturbation $h_{\rm 3eff}\cos(3\theta_{\vec{r}})$ with $h_{\rm 3eff} \sim \langle\tau\rangle$
- When $e^{i\theta}$ orders, $H_{\rm Ising}$ sees effective field $h_{\rm eff}\tau_{\vec{r}}$ with $h_{\rm eff} \sim \langle\cos(3\theta)\rangle$

The “new” multicritical point $\mathcal{M}_?$

- c-theorem argument: $1 \leq c \leq \frac{3}{2}$
- To search:
  $J_{\mathrm{xy}} = h_6 = 1.0$, $J_{\theta\tau} = 0.25$
  Parametrize: $J_{\mathrm{Ising}} = f_{xy} T_{\theta_1} / T_\tau$ and $T = f_I f_{xy} T_{\theta 1}$ [with $T_{\theta 1} = 1.04$ and $T_\tau = 3.6409$]

Multicritical melting at $\mathcal{M}_?$



$[f_{\rm xy}^{\mathcal{M}_?}, f_I^{\mathcal{M}_?}] \approx [1.5570(8), 1.0061(5)]$

$C_{2\theta}$ [$C_{3\theta}$] rescaled by a factor of 7 [factor of 10]

$\eta_{3\theta} = \eta_\tau = 0.201(20)$, $\eta_\theta = 0.258(5)$, and $\eta_{2\theta} = 0.353(6)$.

KD (PRL 2015)

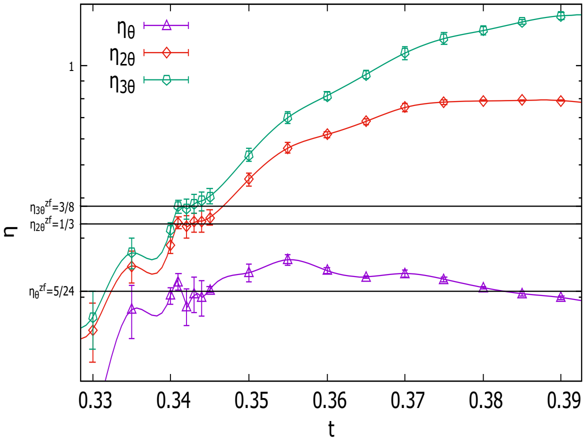
Speculation (aka wishful thinking?)

- If relative strength of first/second neighbour exchange tunable relative to long-range dipolar part in artificial kagome-ice: Could tune melting to multicritical point $\mathcal{M}_?$...
- Computations challenging due to long-range interactions

$\mathcal{M}_?$ vs $\mathcal{M}_{\mathrm{clock}}$

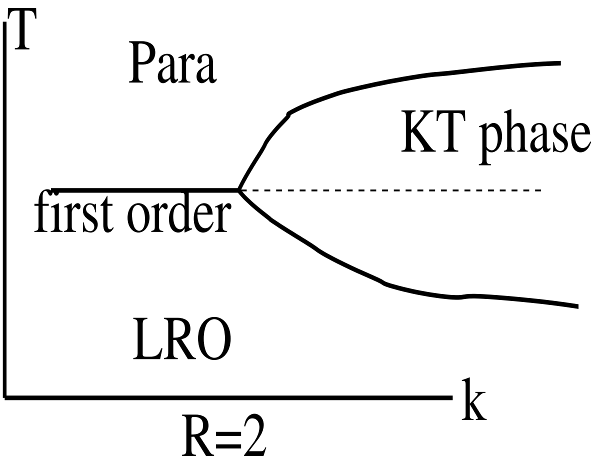
- Conjecture (Dorey '96): $\mathcal{M}_{\mathrm{clock}}$ corresponds to $c = 1.25$ self-dual Z_6 CFT constructed by Zamolodchikov-Fateev ('85).
- Conjecture yields exponents at $\mathcal{M}_{\mathrm{clock}}$: $\eta_{3\theta} = 3/8$, $\eta_{2\theta} = 1/3$, and $\eta_\theta = 5/24$.
  $\eta_{2\theta}$ and $\eta_{3\theta}$ very different from values at $\mathcal{M}_?$
  Recall: at $\mathcal{M}_?$, $\eta_{3\theta} = \eta_\tau = 0.201(20)$, $\eta_\theta = 0.258(5)$, and $\eta_{2\theta} = 0.353(6)$.

Test of conjectured exponents for $\mathcal{M}_{\mathrm{clock}}$



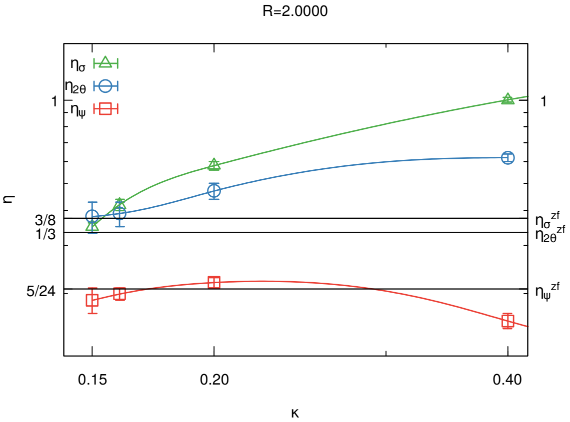
Preliminary results on Cardy's six-state clock model (S. Shivam)

Schematic of pinch-off in triangular lattice Ising AFM



$J_1 = 1$, $R = J_2 + J_3$, $\kappa = J_2 - J_3$

Evidence for $\mathcal{M}_{\mathrm{clock}}$ in triangular Ising AFM



Preliminary results (Geet Rakala)

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  Computations for H_{TFIM}: Geet Ghanshyam and Sounak Biswas (TIFR)
  Ongoing computations for $H_{\mathrm{J_1J_2}}$: Geet Ghanshyam (TIFR $\rightarrow$ Okinawa)
  Ongoing computations for six-state clock models: Sounak Shivam (IITB $\rightarrow$ EPFL Lausanne)
  + Closely related multicritical point: Master's thesis of Nisheeta Desai (BITS-Goa $\rightarrow$ Kentucky)
- Computational resources at TIFR