

Statistical Physics Spring 2010: Tutorial Sheet 1 (Instructor: K. Damle)

Due April 13 2010 in class

1. Consider a simple harmonic oscillator with oscillation frequency ω and mass m in 1-dimension.
 - Characterize the surface of constant energy E in phase space: What does it look like in geometric terms?
 - Find the ‘volume’ in phase-space occupied by states with energy $\leq E$ in classical mechanics. What are the dimensions of this ‘volume’?
 - Find the number of states with energy $\leq E$ in the corresponding quantum problem
 - When E is large, show that the two answers are proportional to each other. What is the proportionality constant?
2. As we have discussed, the entropy of an equilibrium system considered as a function of energy, *i.e.* $S(E)$, can be calculated by taking the logarithm of the number of states that ‘have total energy E ’ for a macroscopic system [question: why is ‘have total energy E ’ in quotes above?]. Apply this to a system of N independent oscillators having total energy

$$E = \frac{1}{2}\hbar\omega N + M\hbar\omega$$

where M is a large integer, and do the following

- Calculate the entropy $S(E)$
 - Determine the temperature T , or rather, find E as a function of T .
 - Now, find the probability that a given oscillator out of these N (say the first oscillator) is in a particular quantum state n . To do this, assume that all states of total energy E are equally likely.
3. Consider an insulating ionic solid in which each ion carries a net magnetic moment $g\mu_B m_J$ in the \hat{z} direction, where the quantum number

m_J is of course allowed to take values $-J, -J + 1 \dots + J$ appropriate for a ground state ionic multiplet of total angular momentum J . An external field $B\hat{z}$ couples to these moments to give the magnetic contribution E_M to the energy of the solid

$$E_M = - \sum_{i=1}^N g\mu_B m_J(i) B$$

where i indexes the lattice sites on which the ions live.

- Assuming that weak dipolar interactions between the ions are enough to equilibrate the system but not strong enough to affect the thermodynamics, calculate the linear magnetic susceptibility (*i.e.* for weak fields B).
 - Take the high temperature limit of the formula you derive. Is the resulting expression familiar?
4. The measured heat capacity per mole $c = \frac{1}{N} \frac{dE}{dT}$ of a system has been fit to an empirical formula

$$c(T) = \frac{T}{\Delta} \exp\left(-\frac{(T - T_f)^2}{\Delta^2}\right)$$

Assuming that the entropy per mole $s(T)$ obeys the third law of thermodynamics, calculate the $T \rightarrow \infty$ limit of $s(T)$ under the assumption that T_f/Δ is very very large.