Statistical Physics Spring 2010: Tutorial Sheet 2 (Instructor: K. Damle)

Due April 15 2010 in class

1. Let P_m be the probability that a system is in eigenstate with energy E_m. If we define the entropy by $S = -\sum_m P_m \log(P_m)$, and demand that S be as large as it can be subject to the constraint that the mean energy $\langle E \rangle$ (with mean $\langle ... \rangle$ taken with respect to the probabilities P_m) be some fixed value $\bar{E}$, then what choice of P_m satisfies this demand?

2. The density matrix of a free particle is given as

$$\hat{w} = \exp(-\hat{H}/T)$$

with $\hat{H} = \hat{p}^2/2m$. Assume that the particle is in a box of linear dimension L with periodic boundary conditions. In the $L \to \infty$ limit, what is the coordinate space representation of $\hat{w}$?

3. Assuming a classical canonical distribution

   - calculate the classical partition function of an ideal gas of N indistinguishable particles of mass m enclosed in an infinitely tall cylinder of cross-sectional area A placed in a uniform gravitational field.
   - Also calculate the mean energy and heat capacity
   - Now calculate the Helmholtz free energy F.
   - Compare your result for the heat capacity with that of an ideal gas without an external gravitational field. Explain the difference.

4. For a system with the Gibbs distribution function at temperature T, show that:

   - $\langle (E - \bar{E})^2 \rangle = T^2 C_v$ where $C_v = \frac{dE}{dT}$ at fixed V, N.
   - $\langle (E - \bar{E})^3 \rangle = T^4 \frac{dC_v}{dT} + 2T^3 C_v$
   - What this say about the energy fluctuations considered as a fraction of the mean energy?

   Here all averages $\langle \ldots \rangle$ are taken in the Gibbs distribution and $\bar{E} = \langle E \rangle$.

5. A substrate has N_0 trapping sites at which a gas molecule can be trapped on its surface. Only one gas molecule can be trapped in a given trapping site at a time. However, the trapped molecule can be in one of three different trapped states with three different energies ϵ_1, ϵ_2, and ϵ_3.

   Assuming that there are negligible interactions between trapped molecules, assuming that the surrounding volume of gas is big enough that the number of trapped molecules is a negligible fraction of the total, and assuming that the process of trapping and release happens fast enough that the entire system can reach equilibrium, calculate the chemical potential μ of the molecules in terms of the mean number $\bar{N}$ of trapped molecules, the temperature T, and the energy levels $\epsilon_{1/2/3}$