

Statistical Physics Spring 2010: Tutorial Sheet 3 (Instructor: K. Damle)

Due April 20 2010 in class

1. A substrate has N_0 trapping sites at which a gas molecule can be trapped on its surface. Only one gas molecule can be trapped in a given trapping site at a time. However, the trapped molecule can be in one of three different trapped states with three different energies ϵ_1 , ϵ_2 , and ϵ_3 .

Assuming that there are negligible interactions between trapped molecules, assuming that the surrounding volume of gas is big enough that the number of trapped molecules is a negligible fraction of the total, and assuming that the process of trapping and release happens fast enough that the entire system can reach equilibrium, calculate the chemical potential μ of the molecules in terms of the mean number \bar{N} of trapped molecules, the temperature T , and the energy levels $\epsilon_{1/2/3}$

2. A polymer chain is made of N monomer units. Each monomer unit can be in one of two conformational states γ and δ . Corresponding energies are ϵ_γ and ϵ_δ , and the linear extent that each monomer unit occupies when in this conformational state is correspondingly l_γ , l_δ . So each configuration of the polymer chain can be described in terms of the conformational states of the individual monomer units that make up the polymer chain (*e.g.* $\gamma\delta\gamma\gamma\delta\dots$). Assume that the polymer experiences a constant tension T_n , that the polymer is in contact with the surrounding medium at temperature T long enough to equilibrate, and that individual monomers can change their conformational state fast enough to be in equilibrium with respect to their conformational states as well.

Under these assumptions, calculate the mean length $\langle L \rangle = \langle \sum_{i=1}^N l_i \rangle$ as a function of T and T_n in equilibrium.

3. Consider a gas of N anharmonic oscillators, where the potential $V(q)$ of each oscillator is

$$V(q) = cq^2 + aq^3 + bq^4$$

Assuming a and b are both small in the appropriate sense, calculate the mean displacement $\langle q \rangle$ and the heat capacity C at temperature

T assuming that the system is described by the Gibbs ensemble, and working classically. [Hint: Expand $\exp(-V) \approx \exp(-cq^2/T) \times (1 - \dots)$]

4. Consider the ‘1-dimensional Ising model’ as an example of an interacting system of spins in one dimension. The energy of this model system is written as

$$E(\sigma_1, \sigma_2 \dots \sigma_N) = -J \sum_{i=1}^N \sigma_i \sigma_{i+1}$$

where each $\sigma_i = \pm 1$ and $\sigma_{N+1} \equiv \sigma_1$ (periodic boundary conditions, *i.e.* the system ‘lives on a circle’).

- Using the Gibbs distribution at temperature T , calculate the free energy F .
- Take the large N limit

[Hint: Find out how to set up the transfer matrix for the problem]