

Statistical Physics Spring 2010: Tutorial Sheet 4 (Instructor: K. Damle)

Due April 22 2010 in class

1. If two systems I and II are in equilibrium with each other at temperature T , show that

$$\langle (E_I - \langle E_I \rangle)^2 \rangle = \langle (E_{II} - \langle E_{II} \rangle)^2 \rangle = T^2 / (C_I^{-1} + C_{II}^{-1})$$

where C_I and C_{II} are the heat capacities of the two systems.

What does this formula reduce to when one of the systems is very large compared to the other?

2. For a gas of N non-interacting particles occupying total volume V (*i.e.* with mean density $\rho \equiv N/V$), show that the number n of particles in a macroscopic sub-volume Ω is given by the Poisson distribution

$$P(n) = \frac{(\rho\Omega)^n}{n!} \exp(-\rho\Omega)$$

3. Gruneisen's model for the dependence of the Helmholtz free energy F of a solid on its volume V is as follows:

If the potential energy of the solid with all the atoms at rest in their equilibrium position is denoted by $U(V)$, and the normal modes of vibration about this equilibrium configuration are given by $\omega_j(V)$ ($j = 1, 2, \dots, 3N - 6$ where N is the number of atoms in the solid), then Gruneisen's model postulates

$$\frac{\partial \omega_j}{\partial V} = -\gamma \frac{\omega_j}{V}$$

with γ a positive constant.

- With this model, calculate the pressure $P = -(\partial F / \partial V)_{T,N}$ as a function of $\partial U / \partial V$, γ and $\bar{\epsilon}$, the mean energy density of atomic vibrations at temperature T .
- Now, assume $U(V) = u_0(V - V_0)^2 / 2V_0$, where u_0 and V_0 are phenomenological constants with dimensions of energy density and

volume respectively. With this assumption, calculate α , the volume expansion coefficient defined as

$$\alpha = \left(\frac{\partial \bar{V}}{\partial T} \right)_{P,N} / \bar{V}$$

[Assume $\gamma \sim O(1)$, while $u_0 V_0 \ll C_V T$, and P is small]

4. If the specific heat C_V of a gas is a constant [temperature independent], show that the entropy can be written as

$$S(T, V) = N \log(V/N) + C_V \log(T) + C_V + \text{constant}$$

and also as

$$S(T, P) = -N \log(P) + C_P \log(T) + C_P + \text{different constant}$$