Quantum Mechanics 2 – February-May 2011

Course instructor: Sunil Mukhi.

Home Assignment 1

Assigned: March 8, 2011, Due: March 17, 2011

Instructions: (Sorry to be bureaucratic, but it is very hard for both me and the tutor to correct several assignments. Your cooperation will greatly help us.)

(i) Every new problem should start on a new page.

(ii) The three problems should be stapled together in order.

(iii) Please keep the exposition brief and to the point. If the problem becomes long and complicated, try to break it up into parts and solve each part separately (this advice is due to Enrico Fermi.)

(iv) We try to read all types of handwriting, but if challenged sufficiently we will give up.

(v) These problems are assigned to help you learn. By copying the solutions from someone else, you would be effectively refusing to be helped.

1. Consider a free particle moving in one dimension. In class, the prefactor of the propagator $K(x'', t''; x', t')$ was computed to be:

$$\sqrt{\frac{m}{2\pi i\hbar(t''-t')}}$$

To show this, we imposed the requirement

$$\lim_{t''\to t'} K(x'', t''; x', t') = \delta(x''-x')$$

on the classical phase.

Instead of using the above property, suppose we implement the transitive property:

$$\int_{-\infty}^{\infty} dx\ K(x'', t''; x, t)K(x, t; x', t') = K(x'', t''; x', t')$$

Show that this determines some part of the prefactor, and also find the general form of the part which *cannot* be determined by this method.

(25 marks)

2. (a) Show that the propagation kernel for a particle in a constant external field:

$$L = \tfrac{1}{2}m\dot{x}^2 + \alpha x$$

where α is constant, is:

$$K(x'',t'';x',t') = \sqrt{\frac{m}{2\pi i\hbar(t''-t')}}\exp\left(\frac{i}{\hbar}\left[\frac{m(x''-x')^2}{2(t''-t')} + \tfrac{1}{2}\alpha(t''-t')(x''+x') - \frac{\alpha^2(t''-t')^3}{24\,m}\right]\right)$$

(b) Try to find the ground-state energy by taking a suitable limit of the above kernel. Explain the physical reason for any difficulty you may encounter.

(20+20=40 marks)

3. Consider a charged particle in a constant external magnetic field. Choosing the vector potential to be $A_i = \frac{1}{2}B(-y,x,0)$, the Lagrangian is:

$$L = \tfrac{m}{2}\dot{\vec{x}}^2 + \tfrac{qB}{2}(x\dot{y} - y\dot{x})$$

where m, q are the mass and charge respectively. Show that the propagation kernel is:

$$K(\vec{x}'',t'';\vec{x}',t') = \left(\frac{m}{2\pi i\hbar(t''-t')}\right)^{3/2}\frac{\omega(t''-t')/2}{\sin\omega(t''-t')/2}\times$$

$$\exp\left(\frac{im}{2\hbar}\left\{\frac{(z''-z')^2}{t''-t'} + \frac{\omega}{2}\cot\frac{\omega(t''-t')}{2}[(x''-x')^2+(y''-y')^2] + \omega(x'y''-y'x'')\right\}\right)$$

where $\omega = qB/m$.

(35 marks)