Quantum Mechanics 2 – February-May 2011

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Home Assignment 2

Assigned: March 31, 2011, Due: April 12, 2011

1. We saw in class that the WKB energies for the harmonic oscillator come out exactly correct, but the wave functions are not exact. Plot the ground state, first excited state and 5th excited state wave-functions of the 1d harmonic oscillator and superpose the WKB wave functions for the same energies to bring out how good or bad they are.

   (25 marks)

2. In 1974 two new particles called the ψ and ψ' were discovered, with rest energies 3.1 and 3.7 GeV, respectively (1 GeV=10^9 eV). These are believed to be nonrelativistic bound states of a "charmed" quark with mass 1.5 GeV/c^2 (i.e. mc^2=1.5 GeV) and an antiquark of the same mass, in a linear potential $V = V_0 + kr$. By assuming that these are the $n = 0$ and $n = 1$ bound states of zero orbital angular momentum, calculate V_0 using the WKB formula. What do you predict for the mass of ψ'', the $n = 2$ state? (The measured value is $\simeq 4.2$ GeV/c^2).
   Hints: (i) Work with GeV instead of eV, (ii) There is no need to determine k explicitly, (iii) you may benefit from reading Page 448-449 of Shankar's book which explains the small differences in the Bohr-Sommerfeld quantisation rule when one or both turning points are at an infinite barrier rather than a linear potential.

   (20+20 marks)

3. Consider the 1d quartic oscillator with $V(x) = \frac{\lambda}{4!}x^4$:
   (i) Calculate the approximate energy levels E_0, E_1, E_2 using the WKB approximation,
   (ii) estimate the ground state energy using the variational principle starting with a trial wave function of your choice,
   (iii) write the Schrödinger equation and redefine the coordinate x so that it becomes independent of all constants. In this form its ground state energy must be of order 1. Use this to estimate the ground state energy of the original problem.
   (iv) compare the three answers for the ground state energy.

   (35 marks)