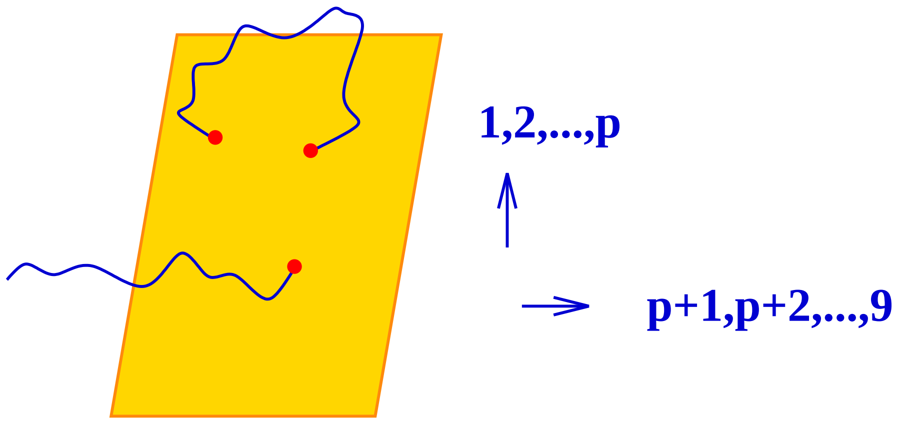
6. D-Branes and Nonabelian Gauge Symmetry

We have seen that open strings are allowed to have Neumann or Dirichlet boundary conditions independently in each direction.

Since there are altogether 9 spatial directions, we can allow p Neumann and $9 - p$ Dirichlet boundary conditions:

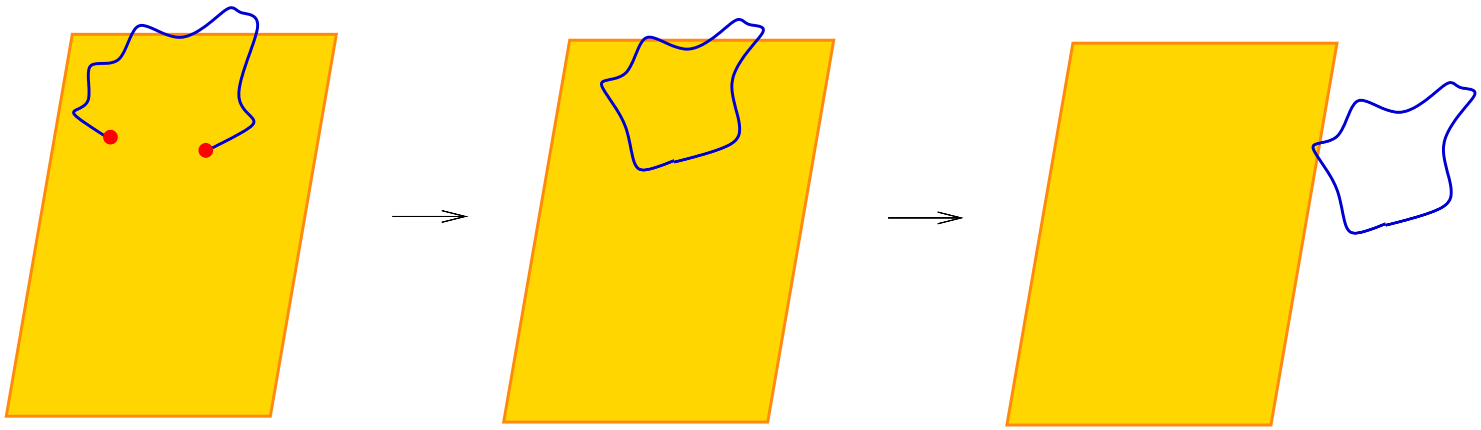


This defines a p-dimensional hypersurface in spacetime. It is called a D-brane, generalising the notion of a membrane.

What is the physical interpretation of this?

Open strings interact locally at their end points.

Consider an open string with both ends on the same brane. If the two ends meet, it can become a closed string and leave the brane.



Thus we expect open and closed strings to coexist in the same theory.

But closed strings live in 10 dimensions and have Lorentz invariance. So a D-brane, on which open strings live, must be an excited state of the closed string theory.

It is not one of the closed-string excited states that we've already seen. Indeed, it turns out to be a nonperturbative state of the theory.

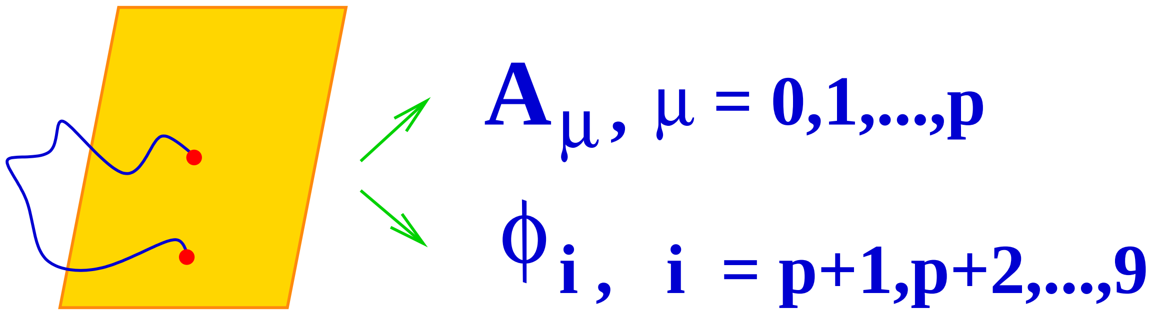
From the point of view of supergravity, it is known that there are extended solitonic excitations similar to the kinks, vortices and domain walls of ordinary quantum field theory.

A D-brane is identified as such a solitonic excitation. The new thing is that it can be explicitly studied using open strings.

Earlier we quantised open strings with NN boundary conditions. The massless states were a photon A_μ in 10 dimensions, as well as a fermion λ_α.

Now let us quantise strings with DD boundary conditions in $9 - p$ spatial directions. Since the ends of the string only propagate in $p + 1$ spacetime dimensions, the string states will be confined to the D-brane.

We find that the massless states are now a photon A_μ in $p+1$ dimensions, as well as $9-p$ scalar fields ϕ_i. (We ignore the fermionic partners).



In particular, there is one scalar field for each direction transverse to the brane.

It is tempting to assume that the vacuum expectation value of these scalars represent the transverse locations of the branes.

When these scalars become time-dependent, that physically represents transverse motion of the brane.

The low-energy effective field theory on a Dp-brane is a $p+1$-dimensional field theory.

It can be shown, either using supergravity or using open-string scattering amplitudes, that some Dp-branes carry a charge under Ramond-Ramond gauge fields.

As an explicit example, consider type IIA string theory, which has an RR gauge field $C^{(1)}_\mu$ with a field strength:

$$G^{(2)}_{\mu\nu} = \partial_\mu C^{(1)}_\nu - \partial_\nu C^{(1)}_\mu$$

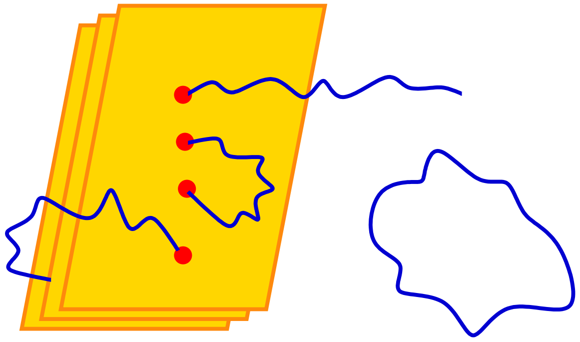
In the presence of a D0-brane, this field strength satisfies the equation

$$\partial^I G^{(2)}_{0I} \sim q\,\delta^9(x)$$

with $q = \pm 1$. Thus we have both D0 and anti-D0 branes. Because of their charge, such branes are individually stable.

There are other, uncharged, branes that are unstable. They have tachyons on their world-volume, even in superstring theory.

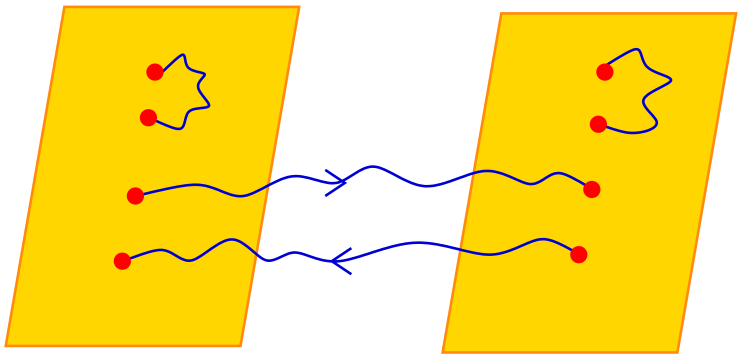
But now we can finally see our way to finding nonabelian gauge symmetry in string theory. Simply assemble a collection of N parallel D-branes:



Now an open string can start on any one of the branes and end on any other. So there are N^2 species of open strings. That means the massless gauge field is an $N \times N$ matrix A_μ^{ab}.

Is string theory clever enough to construct non-abelian gauge symmetry in this situation? Well, of course it is.

Let's consider the simplest example, $N = 2$:



We see that there are four species of strings. Of these, two are localised on individual branes, so they clearly represent the abelian gauge field of that brane. Together, they provide $U(1) \times U(1)$ gauge fields.

Now, the endpoint of a string ending on a D-brane can be shown to behave as a point charge on the brane world-volume.

So the two strings that are stretching across the branes are charged under $U(1) \times U(1)$. They provide the extra gauge fields to enhance:

$$U(1) \times U(1) \to U(2)$$

If the two D-branes are precisely coincident, then the strings stretching from one to the other can have zero length. At this point, all the four gauge fields are massless.

If we now separate the branes, two of the four strings acquire a minimum length and therefore a classical energy. So the corresponding gauge fields must be massive.

But we claimed that transverse motion of the branes is represented by giving a VEV to the transverse scalar fields.

Therefore this is string theory's realisation of the Higgs mechanism!

As usual, things now fall into place.

We can quantise the N^2 strings on a stack of D-branes and perform amplitude calculations using vertex operators.

The result is as expected. The low-energy effective theory is the Yang-Mills theory of a $U(N)$ gauge field $A_\mu^{ab},\ a, b = 1, 2, \ldots N$, coupled to scalars and fermions in the adjoint representation of $U(N)$.

Amplitude calculations reveal the famous "three-gluon" and "four-gluon" interactions, the signature of Yang-Mills theory:

$$\text{tr}\, \partial_\mu A_\nu [A^\mu, A^\nu], \qquad [A_\mu, A_\nu][A^\mu, A^\nu]$$

D-branes are extremely useful objects. Besides introducing non-Abelian gauge symmetries into string theory, they also help us to reformulate familiar notions from field theory and mathematics in a new way. This has led to several new insights about gauge theory.