7. Compactification and Physics

We have discovered that type IIA/B superstring theory is a consistent ten-dimensional theory with local $\mathcal{N} = 2$ supersymmetry.

What does this have to do with reality?

Our quantisation of the theory in 10 flat extended spacetime dimensions has perhaps been slightly misleading. We could have chosen to have the string propagate in any 10-dimensional spacetime.

All such choices need not be consistent. But there is one very simple choice that is always consistent.

Let us use new labels for the spacetime directions:

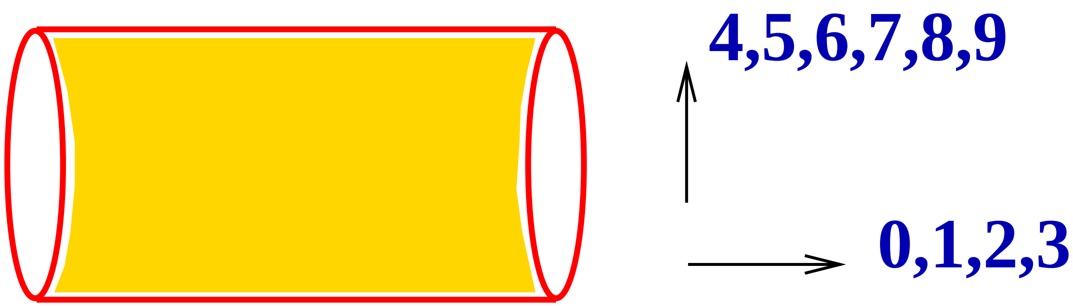
$$0, 1, 2, 3 \longrightarrow \mu, \nu \cdots$$

$$4, 5, 6, 7, 8, 9 \longrightarrow i, j, \cdots$$

Now suppose that the 6 coordinates X^i are periodic:

$$X^i \sim X^i + 2\pi R^i$$

This has nothing to do with worldsheet boundary conditions. It says that some directions of physical space are curled up:



If we probe such a world through experiments whose available energy E satisfies:

$$E << \frac{1}{R_i} \quad \text{for all } i$$

this world will not appear 10-dimensional, but rather 4-dimensional.

This is because, for its Fourier modes to fit into the compact dimension, an elementary particle needs an energy of order the inverse radius.

What would change if we formulated superstring theory in this kind of "toroidally compactified" spacetime?

(i) The periodicity of the six X^i's breaks the Lorentz group

$$SO(9,1) \rightarrow SO(3,1)$$

This is, of course, a good thing.

(ii) The mode expansion of the closed string changes and we get additional modes. Instead of

$$X^i = x^i + 2\alpha' p^i t + \text{oscillators}$$

we now have

$$X^i = x^i + 2\alpha' p^i t + 2L^i \sigma + \text{oscillators}$$

where L^i is quantised.

Originally the L^i mode was prohibited by the requirement:

$$X^i(t, \sigma + \pi) = X^i(t, \sigma)$$

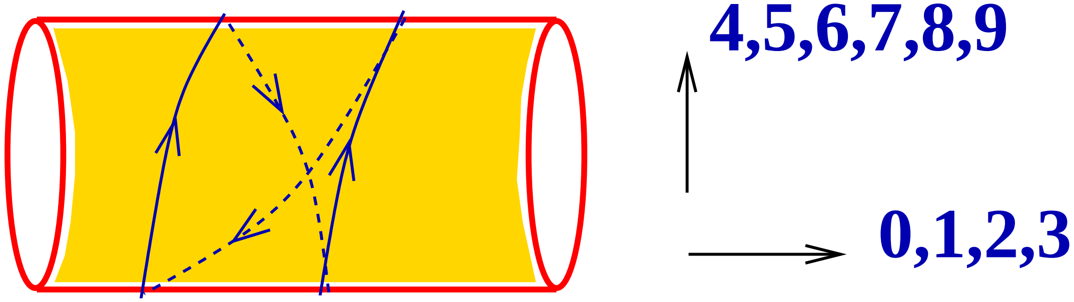
But because X^i itself is periodic, this requirement is now relaxed to:

$$X^i(t, \sigma + \pi) = X^i(t, \sigma) + 2\pi\, n_i R^i$$

from which we get

$$L^i = n_i R^i, \quad n_i \text{ integers}$$

A mode of nonzero L^i is a winding mode of the string.



In this example the string is winding twice around a compact direction.

Note that the centre-of-mass momentum is also quantised:

$$p^i = \frac{m^i}{R^i}, \quad m^i \text{ integers}$$

just as for an ordinary particle in a compact space.

The contributions to the energy from momentum and winding modes are:

$$E \;\sim\; \sum_i (p^i)^2 + \frac{(L_i)^2}{\alpha'^2} \;\sim\; \left(\frac{m^i}{R^i}\right)^2 + \frac{(n_i R^i)^2}{\alpha'^2}, \quad n_i, m^i \text{ arbitrary integers}$$

This formula has a symmetry under

$$R^i \leftrightarrow \frac{\alpha'}{R^i}, \quad n_i \leftrightarrow m^i$$

This is an exact symmetry of string theory, called T-duality or target-space duality. Physically it tells us there is a minimum length in string theory, the string length $\sqrt{\alpha'}$. Any length smaller than that can be "dualised" into a larger length.

(iii) The massless string states that we found in type IIA/B supergravity continue to exist after toroidal compactification.

However, their 10-momentum will be restricted to a 4-momentum p^μ, with the other 6 components being zero unless we supply enormous energies.

It no longer makes sense to think of them as 10-dimensional tensors. Their indices can be divided into $\mu = 0, 1, 2, 3$ and $i = 4, 5, \ldots, 9$. The latter indices are scalars under the 4-dimensional Lorentz group.

This is called Kaluza-Klein reduction.

As an example, the 10-dimensional metric or graviton field becomes:

$$G_{\mu\nu}, \quad G_{\mu i}, \quad G_{ij}$$

which means a metric, 6 gauge fields and 21 scalars in 4 dimensions.

Performing this process for all the fields of type IIA/B supergravity, we find that both theories reduce to the same supergravity theory in 4 dimensions.

The 2 gravitinos in 10 dimensions become 8 gravitinos in 4 dimensions. Thus we have 8 local supersymmetry charges in 4 dimensions.

Indeed, this is the fabled $\mathcal{N} = 8$ supergravity.

Unfortunately, $\mathcal{N} = 8$ supergravity has far too many things wrong with it to be the right low energy theory describing nature.

But the compactified string is certainly a theory in 4 spacetime dimensions, with gravity, gauge fields, fermions...

Theoretically this is just as natural a background of string theory as open 10-dimensional spacetime. And it enjoys the good properties of string theory in 10 open dimensions, such as UV finiteness of amplitudes.

Moreover, this also indicates how we should fix the scale α' of string theory. It should be chosen so that the 4-dimensional theory has the right value of the Planck mass. Then the infinite tower of string excitations would have masses of order 10^{19} GeV, and be unobservable.

As we know, to get a more realistic theory of low-energy physics, it is better to have $\mathcal{N} = 1$ supersymmetry in 4 dimensions. Hence we must search for string backgrounds with lower supersymmetry.

Besides type IIA/B superstrings, there are 3 more superstring theories in 10 dimensions, with $\mathcal{N} = 1$ supersymmetry.

One of these, called the type I superstring, arises by quotienting the closed string by orientation reversal.

This creates a kind of mirror called an orientifold plane in 10 dimensions. The latter object turns out to have -32 units of charge with respect to a D-brane. So we must put 32 D-branes in as well.

These would give the gauge group $U(32)$, but the orientifold plane breaks this to a subgroup, $SO(32)$.

This theory can also be toroidally compactified to four dimensions, but it gives us $\mathcal{N} = 4$ supergravity coupled to $\mathcal{N} = 4$ super-Yang-Mills theory. This is a slight improvement on $\mathcal{N} = 8$, but not good enough.

Another two superstring theories in 10 dimensions are called heterotic strings. They arise by fusing the left movers of the bosonic string and the right movers of the fermionic string.

For this to be possible, the bosonic string has to be compactified on a torus from 26 to 10 dimensions. Only two 16-tori are consistent (with modular invariance) and they give rise to Yang-Mills multiplets of $SO(32)$ or $E_8 \times E_8$ respectively.

The resulting theories are $\mathcal{N} = 1$ superstrings in 10 dimensions: the $SO(32)$ and $E_8 \times E_8$ heterotic strings.

Again, toroidal compactification of these theories is no use since we end up with $\mathcal{N} = 4$ supersymmetry in 4 dimensions.

Essentially this means we have to consider more complicated compactifications than just toroidal.

One can for example take an arbitrary differentiable manifold that is 6-dimensional, compact, and small.

Then one can ask whether the theory compactified on this is

(i) consistent,

(ii) $\mathcal{N} = 1$ supersymmetric,

(iii) has a zero or small cosmological constant,

(iv) parity-violating,

and so on.

Roughly, requirements (i),(ii) and (iii) rule out most 6-manifolds!

The ones that survive are a class called Calabi-Yau manifolds. But even these do not work with type IIA/B strings, because we end up with $\mathcal{N} = 2$ supersymmetry after compactifying to 4 dimensions.

With type I and heterotic $SO(32)$ superstrings, Calabi-Yau compactification gives the right supersymmetry, but no parity violation.

But with the $E_8 \times E_8$ heterotic string we can satisfy all the above basic requirements.

This is the basis of early string phenomenology. But we will not be able to touch on any of the details here.

We have been talking about the Kaluza-Klein reduction within perturbative string theory.

Many nonperturbative properties of string theory are known, though they could not be discussed here. Nonperturbative compactifications satisfying the four physical requirements listed above do exist for all the 5 superstring theories.

Typically, these involve D-branes and orientifold planes, as well as geometric singularities at which light states are pinned.

Even more remarkably, nonperturbative investigations reveal that all the 5 superstring theories are really the same. In some sense they are all deformations of a common theory. So, which one we choose as the starting point for compactification is largely a matter of convenience.