Mini-Course on String Theory (Sunil Mukhi)

Exercise Sheet, No. 4

1. Check that the superconformal gauge action for the fermionic string:

$$S = -\frac{T}{2}\int d\sigma\, dt\left(\partial_a X^\mu \partial_a X_\mu - i\bar{\psi}^\mu \rho^a \partial_a \psi_\mu\right)$$

is invariant under the global supersymmetry transformations:

$$\delta X^\mu = \bar{\epsilon}\psi^\mu, \qquad \delta\psi^\mu = -i\rho^a \partial_a X^\mu$$

Use Noether's theorem to find the corresponding conserved supercurrent:

$$J_{\alpha a} = \tfrac{1}{2}\,\rho^b \rho_a\, \psi^\mu_\alpha\, \partial_b X_\mu$$

and show that it is conserved when the equations of motion are satisfied.

2. Use the same procedure as we used for the bosonic string (involving the notorious formula $\sum_1^\infty n = -1/12$) to show that the normal-ordering constant a in the NS sector of the open fermionic string is:

$$a = \frac{D-2}{16}$$

Also check explicitly that the normal-ordering constant is zero in the R sector.

3. (Difficult, unless you know group theory!) In $SO(8)$, show that

$$\mathbf{8}_s \times \mathbf{8}_c = \mathbf{8}_v + \mathbf{56}$$
$$\mathbf{8}_s \times \mathbf{8}_s = \mathbf{1} + \mathbf{28} + \mathbf{35}$$

Note that the first relation is an expression of the famous *triality* property of $SO(8)$, according to which tensoring any two of s, c, v (spinor, conjugate spinor, vector) produces the third. Also the second relation tells us that the spinorial representation $\mathbf{8}_s$ is *real.*

An informal introduction to these and related issues may be found in the book of Green, Schwarz and Witten.