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New Ways to Make Old Strings

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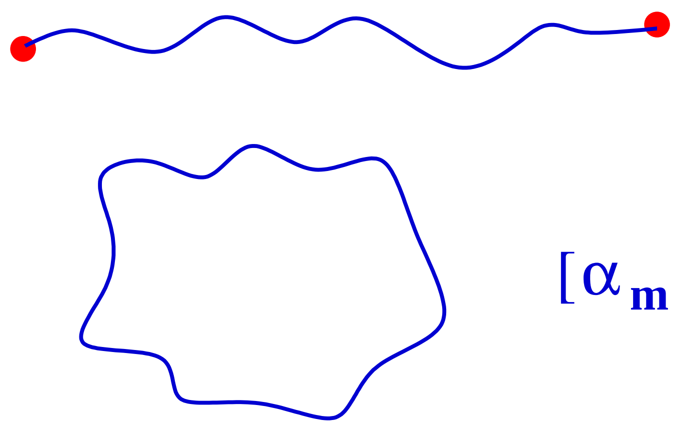
University of Louisville, September 6 2002

The important thing in science is not so much to obtain new facts, as to discover new ways of thinking about them.

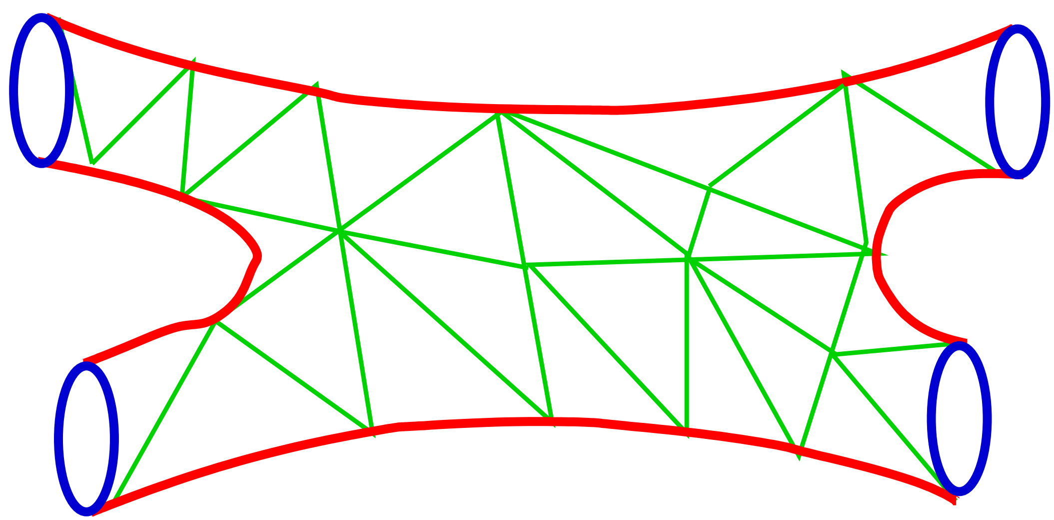
Sir William Bragg

1. Strings from First Principles:



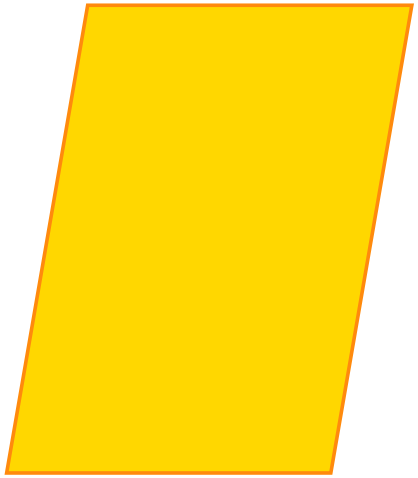
$$[\alpha_m , \alpha_n] = \delta_{m+n,0}$$

2. Strings from Matrices:

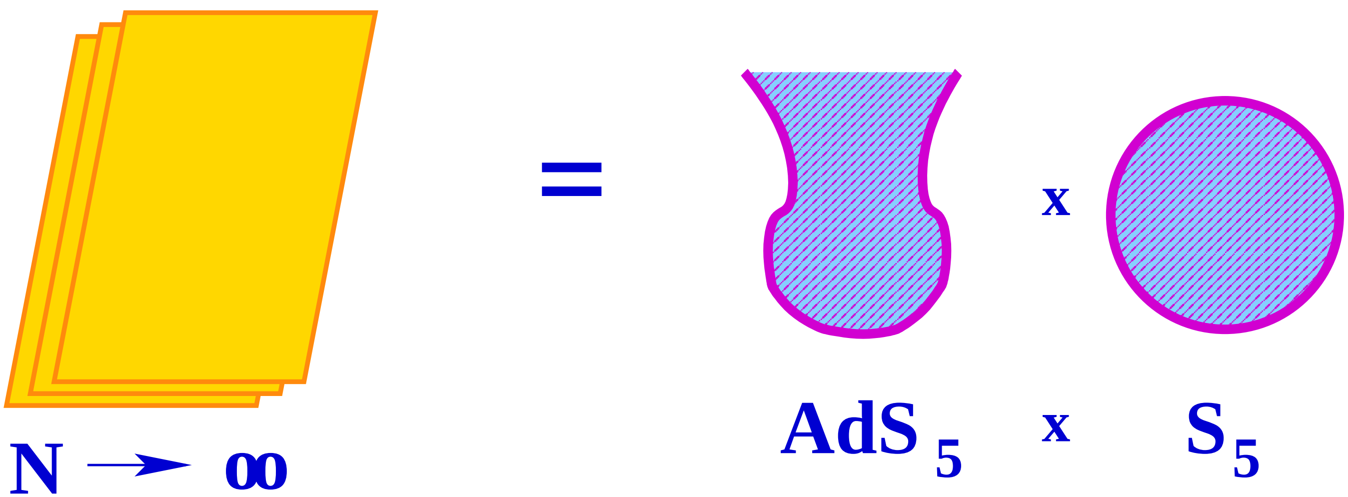


3. Strings from Branes (I):

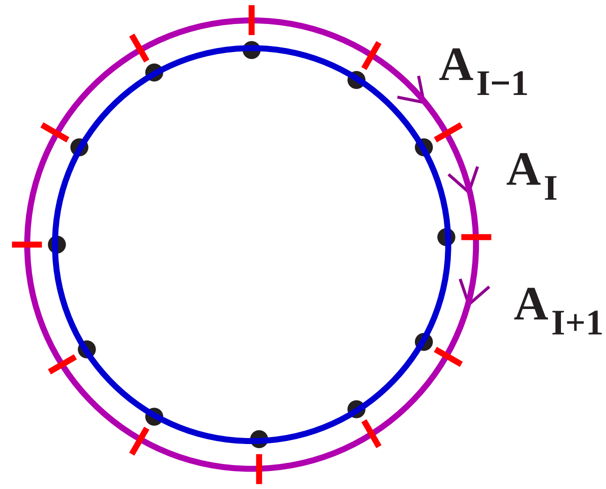
$$[X^1, X^2] = i$$



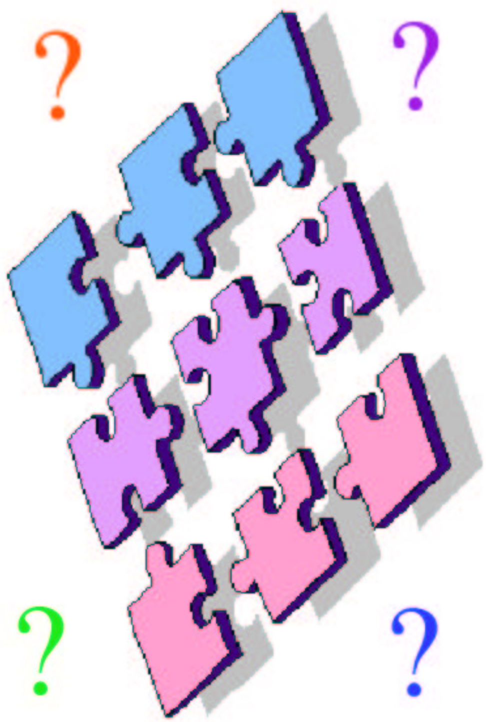
4. Strings from Branes (II):



5. Strings from Bits:



6. Conclusions and Outlook:



But first:

Why do we want to make strings???

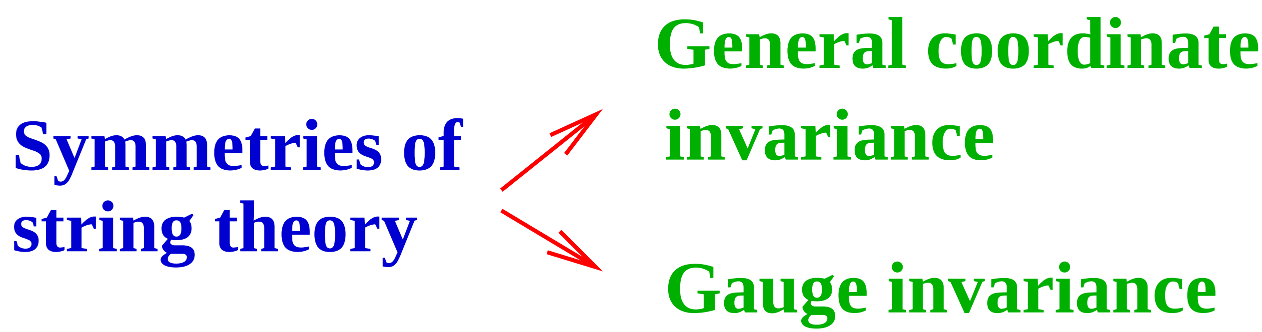
- Strings play a central role in our understanding of two apparently unrelated physical phenomena:

  - **Gravitation**
  - **Quark confinement**

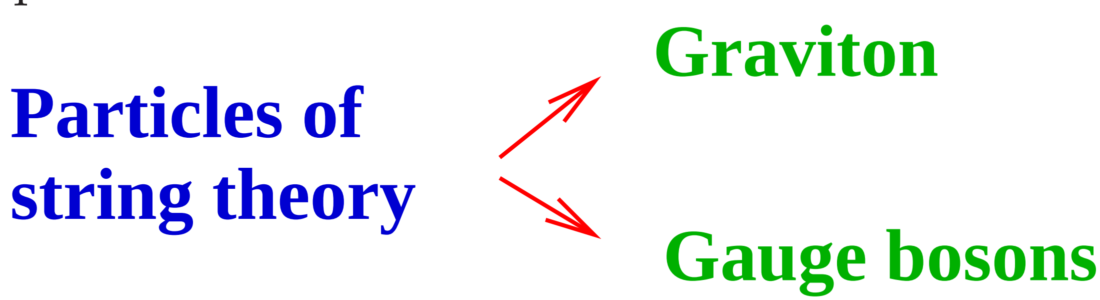
- Let us briefly review the reasons for this.

Gravitation

- Fundamental strings, when quantized, possess a spectrum of light particle-like excitations at low energy. These excitations come with symmetries that are determined by consistency.



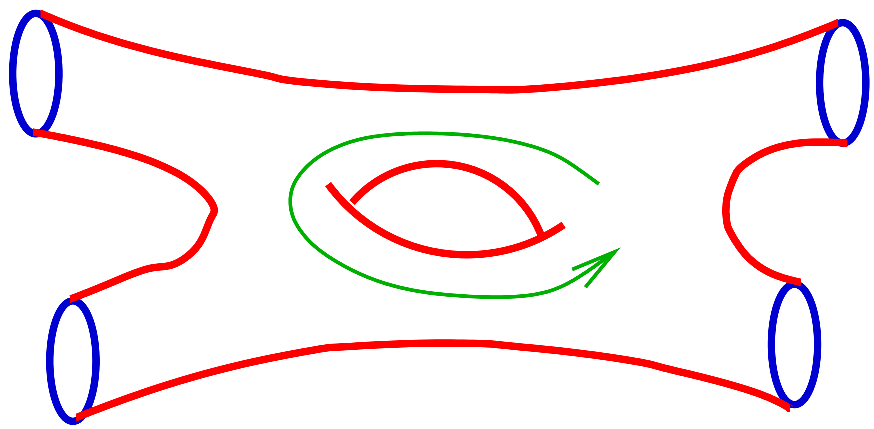
which implies that:



String theories describe gravity.

In addition, string excitations include infinitely many massive particles, whose mass scale is governed by a parameter α' of dimension L^2.

- It would seem that string theory is a complicated way of describing gravity. But the payoff comes when we scatter strings against each other:



By the laws of quantum mechanics, all possible virtual particles circulate in the loop.

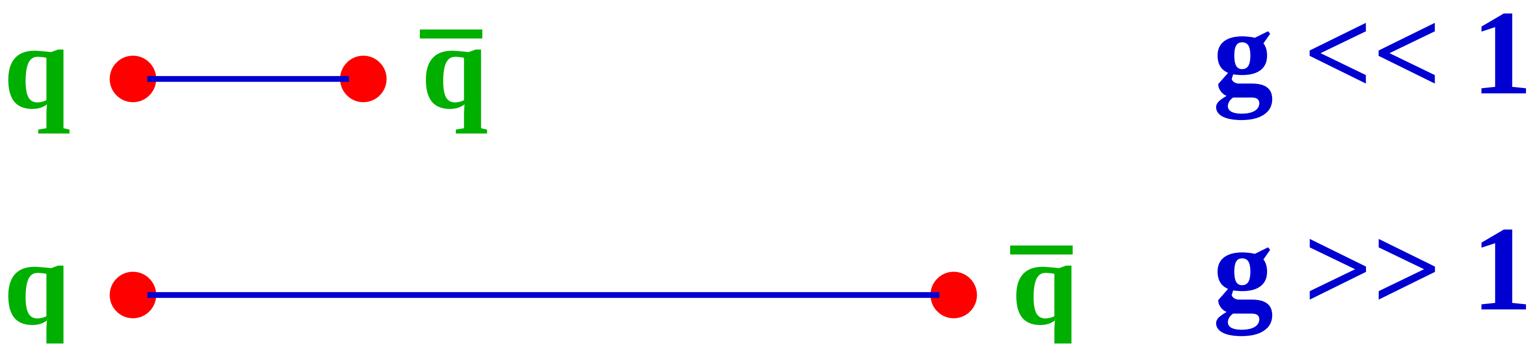
Such amplitudes are generically divergent in ordinary field theory, but in string theory they are finite, thanks to the massive particles.

So string theory is a consistent quantum theory of gravity.

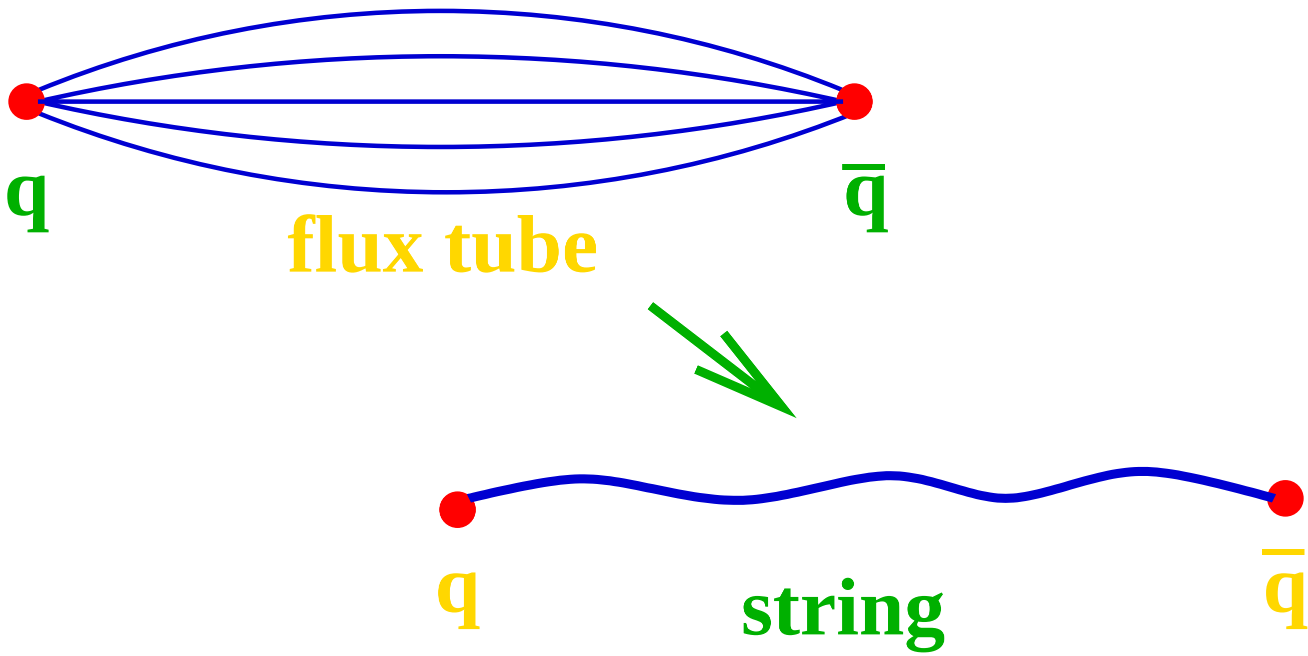
Quark Confinement

- The fundamental theory of strong interactions, Quantum Chromodynamics (QCD), is useful at short distances where the coupling constant is weak.

  However, at long distances one cannot use perturbation theory, because the coupling is strong.



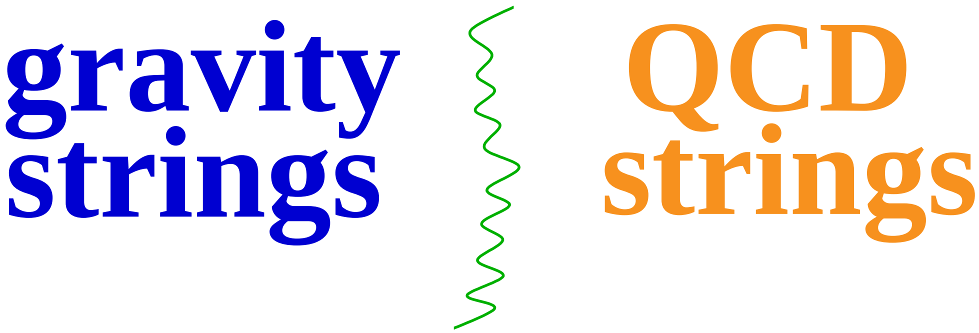
It is thought that there is a dual description of QCD:



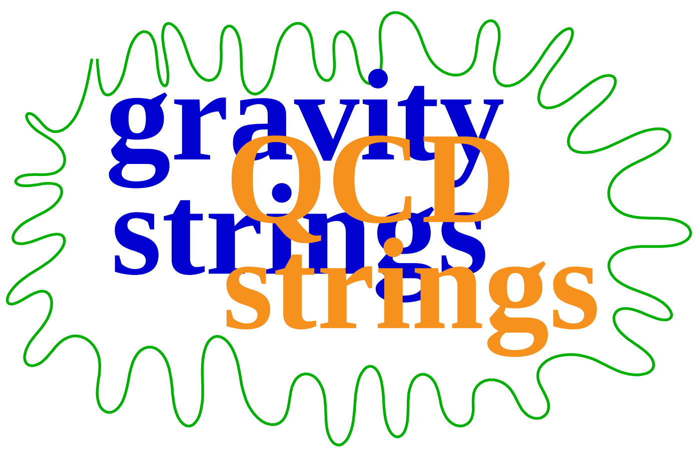
- In the dual description, the strings are fundamental.

  String perturbation theory would capture nonperturbative effects in QCD, including quark confinement.

- The two possible roles of strings, to describe gravity and to describe QCD flux lines, were assumed to be mutually exclusive:



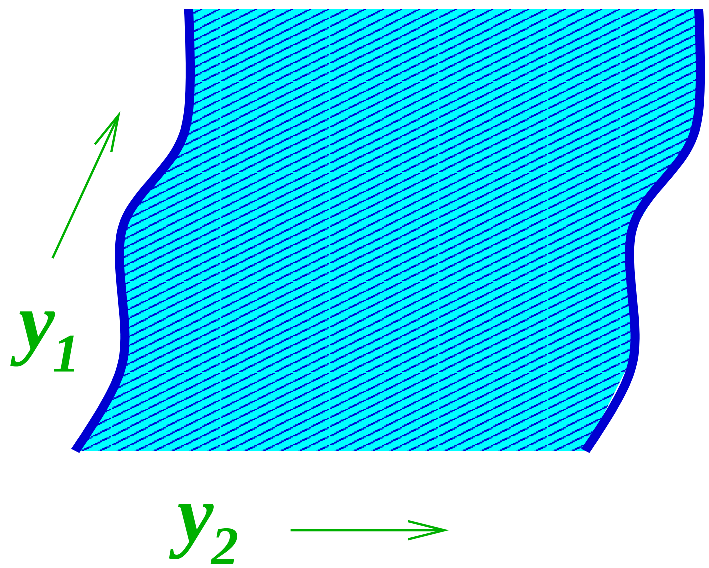
But recent developments indicate that they are in fact interlinked:



1. Strings from First Principles

- The quantization of strings from first principles is fairly straightforward. It proceeds by the following steps:

  (i) The string sweeps out a worldsheet as it propagates:



  (ii) This worldsheet is embedded in spacetime, whose coordinates are:

$$X^\mu(y_1, y_2), \quad \mu = 0, 1, \ldots, D-1$$

(iii) The string action is the area of this embedded surface:

$$S = \int dy_1 dy_2 \sqrt{\det g} \;\; g^{\alpha\beta} \frac{\partial X^\mu}{\partial y^\alpha}\frac{\partial X^\mu}{\partial y^\beta}$$

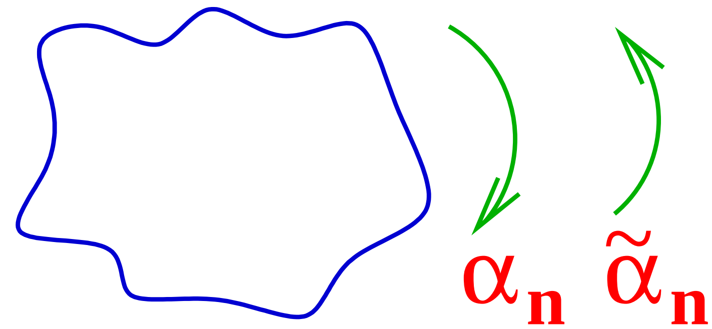
(iv) After gauge-fixing the symmetries, one reduces to the light-cone degrees of freedom

$$X^i(y_1, y_2), \quad i = 1, 2, \ldots, D-2$$

which can be expanded in terms of normal modes:

$$X^i(y_1, y_2) = X_0^i + P^i y_1 + \sum_n \frac{1}{n}\,\alpha_n^i\, e^{2\pi i n(y_1+y_2)} + \sum_n \frac{1}{n}\,\widetilde{\alpha}_n^i\, e^{2\pi i n(y_1-y_2)}$$

(v) Finally we quantize the modes:



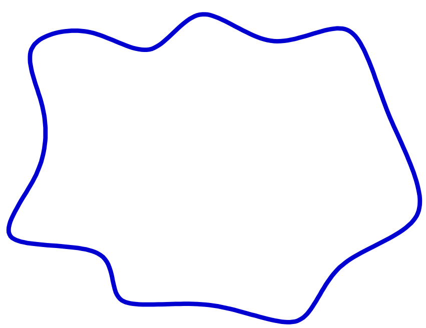
X_0^i : centre-of-mass position

P^i : total momentum

α_n^i : left-moving excitation, n-th mode

$\tilde{\alpha}_n^i$: right-moving excitation, n-th mode

With a given set of modes excited, the string behaves at low energies as an elementary particle with corresponding quantum numbers:

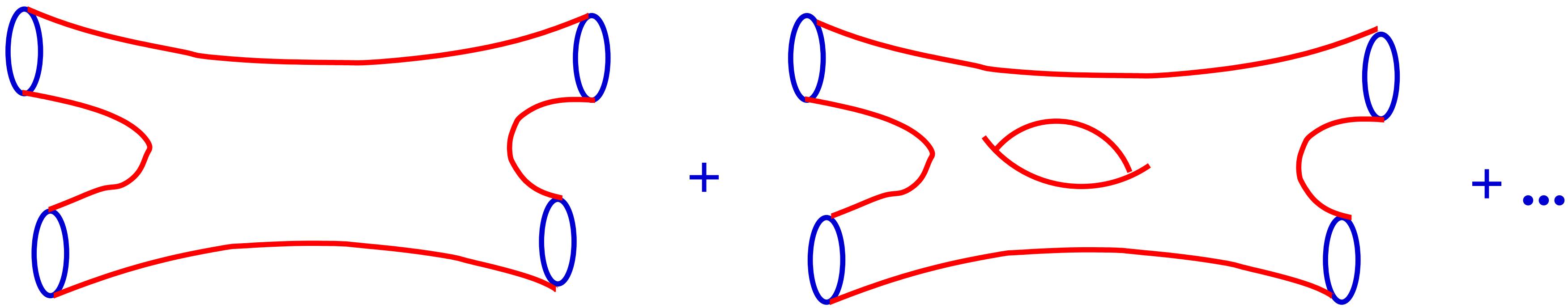


$$= \alpha^i_{-1}\,\tilde{\alpha}^j_{-1}\,|0\rangle$$

$$= g^{ij} \quad \text{(graviton)}$$

Having constructed string modes, there is then a prescription to compute scattering amplitudes, using worldsheets like:

“tree level” “one loop”



order 1 order $(g_{\text{string}})^2$

- For gravitons, these amplitudes are finite, unlike the corresponding amplitudes in Einstein’s gravity.

There are some important subtleties. Quantization of strings leads to a consistency condition that

$$D = 26$$

and also that the lowest state of the string is tachyonic.

Both these problems are mitigated by switching to the supersymmetric string. In this case there is no tachyon, and we have

$$D = 10$$

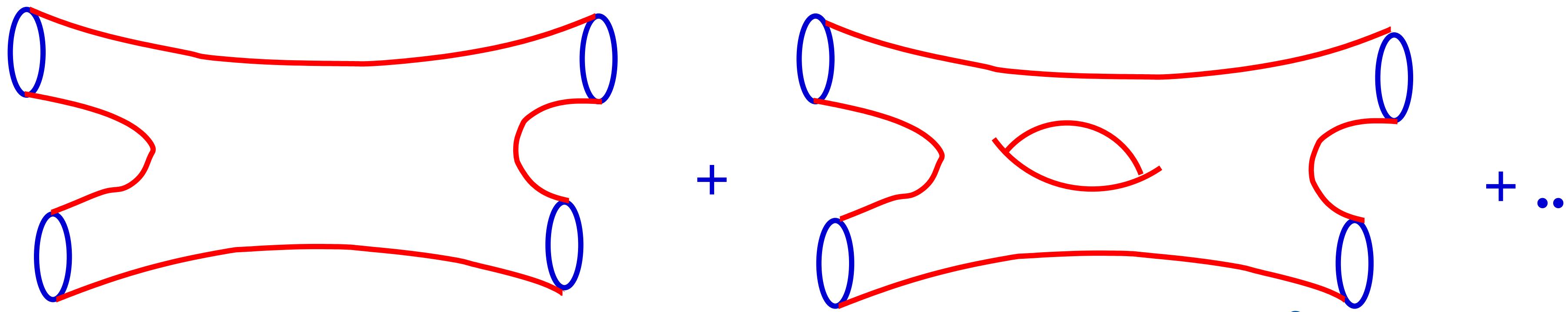
The real world is supposed to arise when the string theory is compactified on a 6-dimensional space, leaving the four open spacetime directions that we observe.

The phenomenology of such a compactification is largely determined by the shape and size of the internal 6-manifold.

To reduce the technicalities of our story, we will not always mention supersymmetry, but it will usually be present.

2. Strings from Matrices

- We saw that the perturbation series for strings was described by a set of surfaces. For each topology, we have to sum over all possible shapes of the surface.

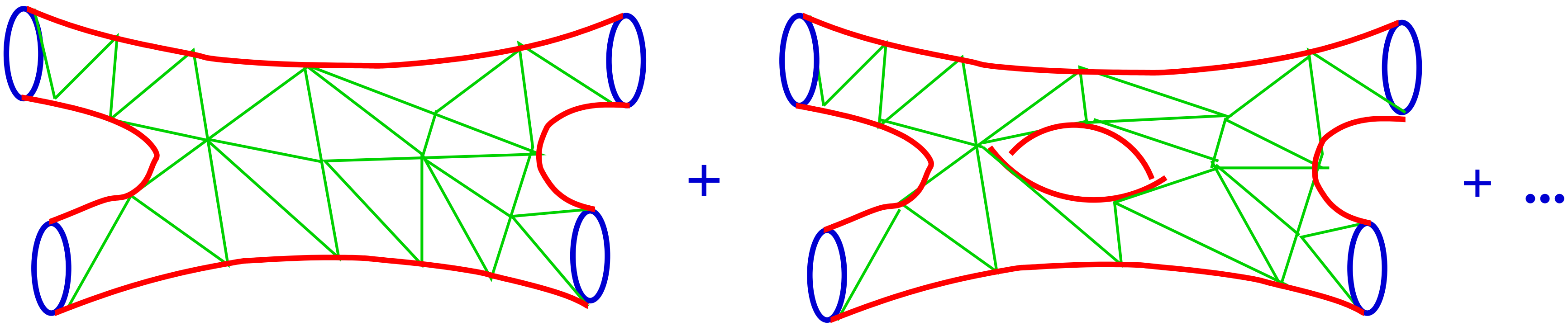


order 1 order $(g_{\text{string}})^2$

This description is intrinsically perturbative in the string coupling.

- 12 years ago, it was found that that we can describe strings in a way inspired by statistical mechanics, which is intrinsically non-perturbative.

This can be done by discretizing the surface:



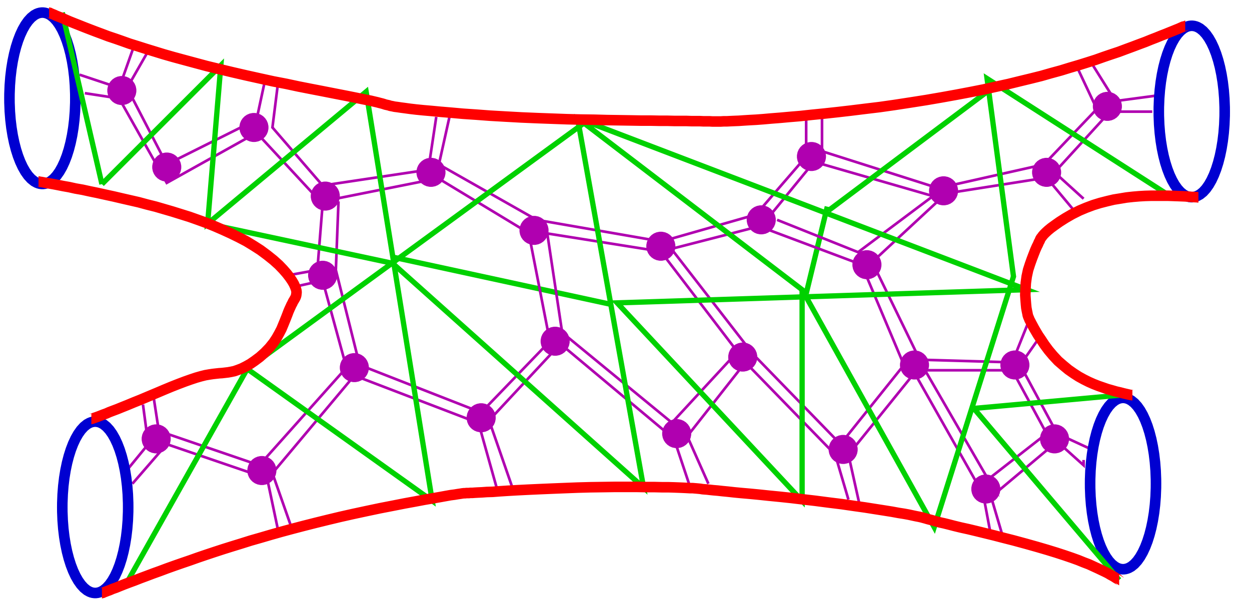
The surface is divided into triangles. We sum over all ways of doing so, which implements the sum over all surfaces.

The triangulation knows about the genus of the surface (number of handles, h) by virtue of the relation:

$$\mathrm{v}\,(\mathrm{ertices}) - \mathrm{e}\,(\mathrm{dges}) + \mathrm{f}\,(\mathrm{aces}) = 2 - 2\,\mathrm{h}\,(\mathrm{andles})$$

- A triangulation defines a lattice, to which we can associate a dual lattice. This is done by placing dots in the faces and connecting them.

  If the original lattice is made of triangles then the dual lattice is made of 3-point vertices.



The dual graph also satisfies:

$$\mathrm{v}\,(\mathrm{ertices}) - \mathrm{e}\,(\mathrm{dges}) + \mathrm{f}\,(\mathrm{aces}) = 2 - 2\,\mathrm{h}\,(\mathrm{andles})$$

- The key to studying triangulated surfaces is to write a random matrix integral:

$$\int dM\, e^{-N\,\mathrm{tr}\left(\frac{1}{2}M^2 + g\,M^3\right)}$$

where

$$M_{ij} : \; N \times N \text{ Hermitian matrix}, \qquad dM \;\equiv\; \prod_{i,j} dM_{ij}$$

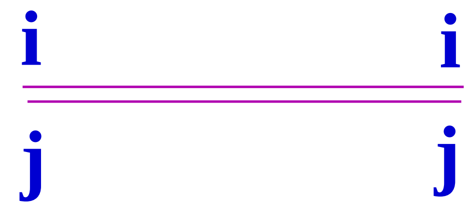
This can be expanded in a Feynman diagram expansion:

$$\int dM\, e^{-\frac{N}{2}\,\mathrm{tr}\,M^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left(-gN\,\mathrm{tr}\,M^3\right)^n$$

which, as usual, is drawn in terms of propagators and vertices.

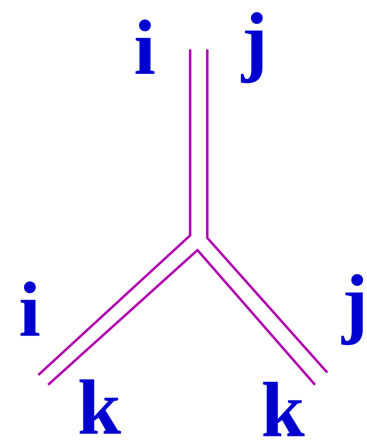
The propagator in a diagram can be drawn:

$$\langle M_{ij} M_{ji} \rangle =$$



while the vertex is:

$$\mathrm{tr}\, M^3 = M_{ij}\, M_{jk}\, M_{ki} =$$



So, expansion of this matrix integral in Feynman diagrams generates triangulated random surfaces!

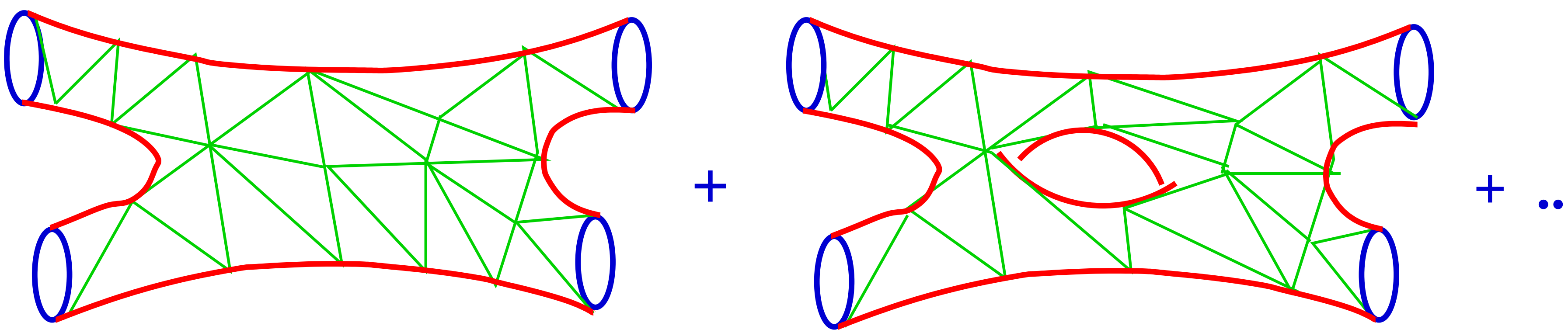
- The size N of the matrix helps us count the genus.

$$v\,ertex \to gN, \qquad e\,dge \to N^{-1}, \qquad f\,ace \to N$$

Hence, the total N–dependence of a given diagram is:

$$N^{v-e+f} = N^{2-2h} = N^2 \left(\frac{1}{N^2}\right)^h$$

It follows that $\frac{1}{N^2}$ is the genus expansion parameter!



order 1 order $\frac{1}{N^2}$

In other words:

$$g_{\text{string}} = \frac{1}{N}$$

The moral of the story is that random-matrix theory at large N describes string theory with coupling $g_s \sim \frac{1}{N}$.

- An exact solution of the matrix model (as opposed to a Feynman-diagram expansion) would determine the entire dependence on N, including nonperturbative string theory effects.

Unfortunately this simple matrix model describes strings propagating in strange low-dimensional spacetimes.

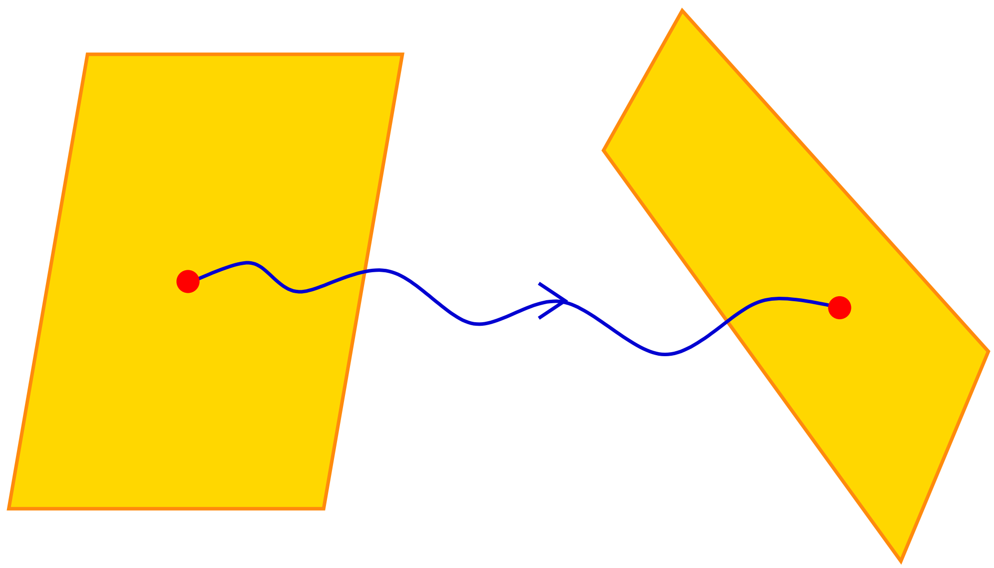
However, it provides an insight that we will frequently use.

3. Strings from Branes (I)

- Let us now study open strings. These have end-points.

  In superstring theory, the bulk of a string must propagate in 9 spatial dimensions. What about the ends?

  A very general solution is to confine each end, independently, to a hypersurface in 9 dimensions:



- The hypersurface to which the ends of the string are glued is called a:

Dirichlet brane

There are compelling arguments that such D-branes are dynamical. They are analogous to interacting membranes.

In our 3-dimensional world we can have:

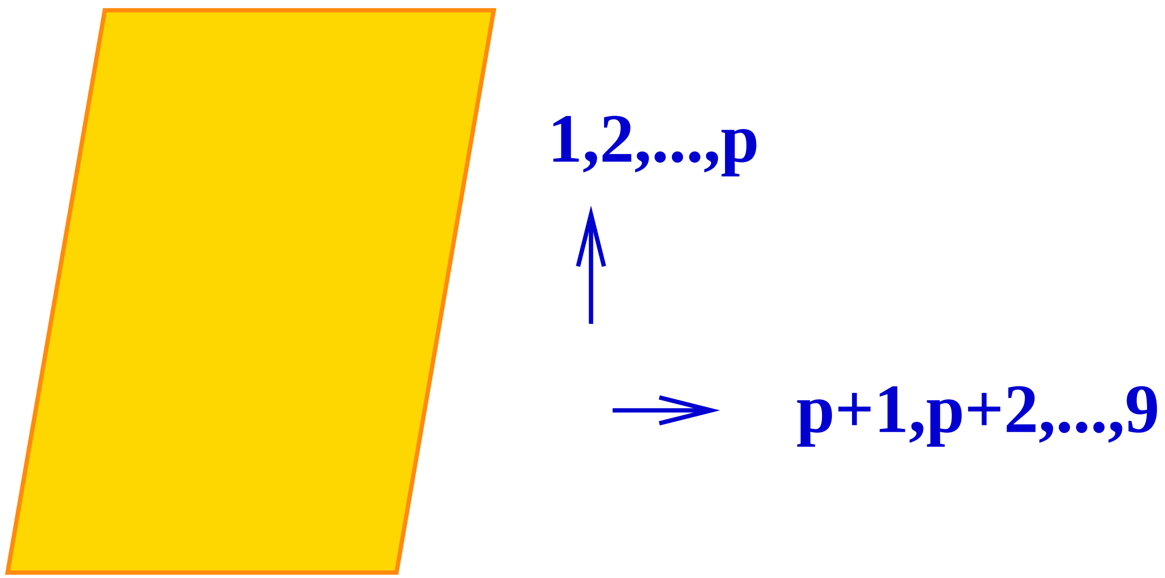
0-branes: particles
1-branes: strings
2-branes: membranes

But in open string theory in 9 spatial dimensions, we can have:

$$\mathrm{D}p - \text{branes for } p = 0, 1, 2, 3, 4, 5, 6, 7, 8$$

Some of these are stable, while others decay rapidly.

- A Dp-brane has p spatial dimensions along itself and $9 - p$ dimensions transverse to itself:

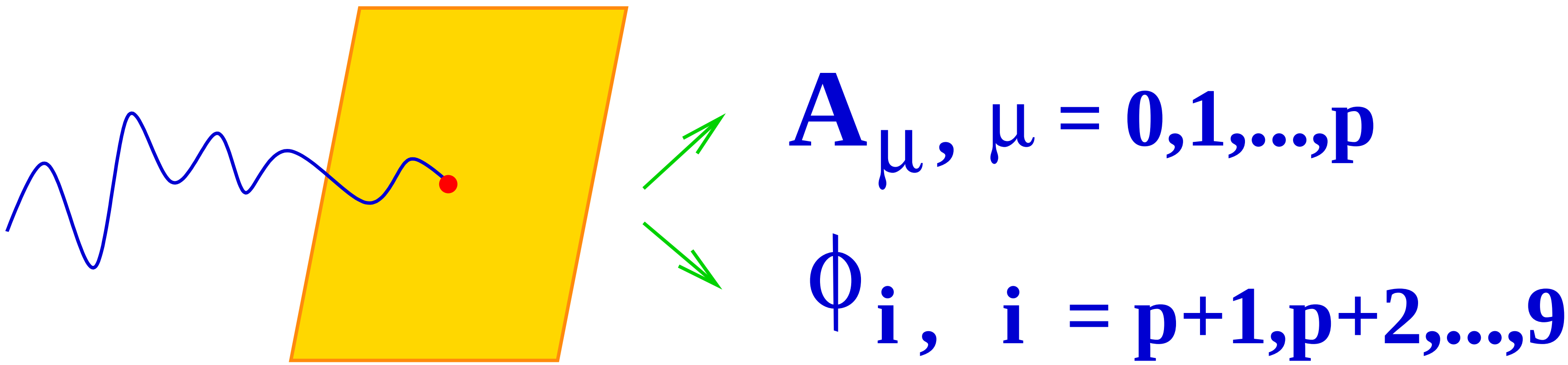


In what follows, we will be interested in the special cases:

p=0: 0-branes (9 transverse dimensions)

p=3: 3-branes (6 transverse dimensions)

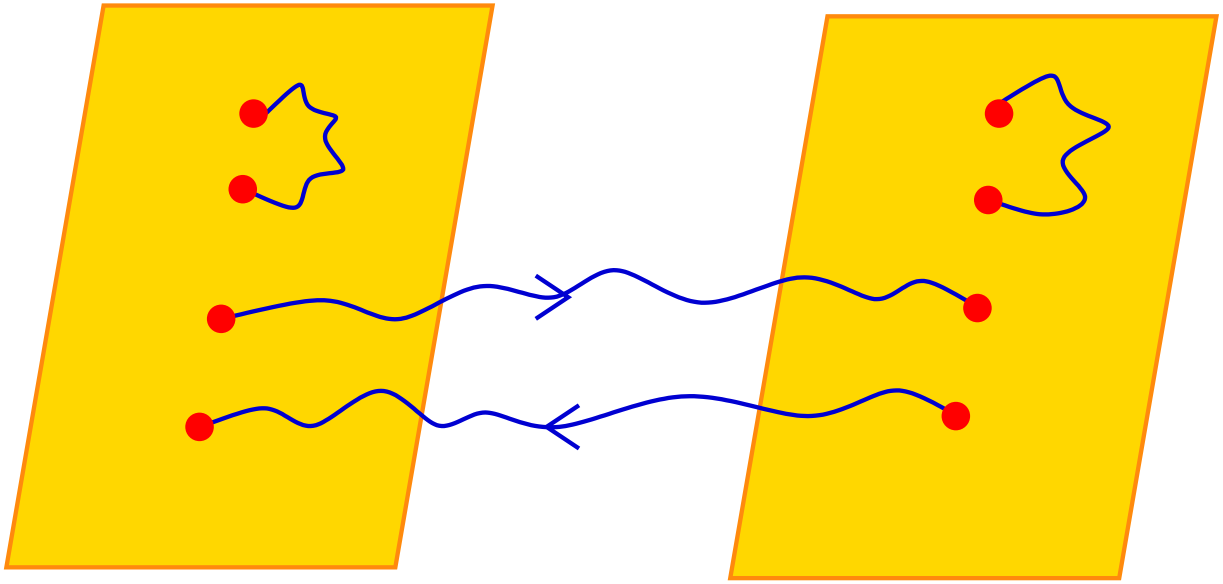
- Just as closed strings have gravitons as their light bosonic excitations, open string excitations produce vector fields like photons, as well as scalar fields.



The vectors and scalars propagate along the D-brane. (There are also fermions, which we will not discuss.)

Hence, we can realize gauge theories as the low-energy limit of open string theories.

A key property of gauge theory, nonabelian gauge symmetry, arises in a very elegant way:



Between a pair of D-branes, there are four kinds of open strings. These give rise to the gauge fields for $U(2)$ gauge theory.

In general,

$$N \text{ D-branes} \longrightarrow U(N) \text{ gauge theory}$$

- If D-branes are dynamical, what parameter labels their position?

  A clue comes from strings stretching from one brane to another. These have a minimum length proportional to the separation of the branes.

  Thus for separated D-branes, two of the four U(2) gauge fields are massive.

  Gauge fields can become massive only through a Higgs mechanism, where a scalar field acquires a vacuum expectation value. It follows that

The position of a D-brane is the
VEV of a scalar field localized on it.

This fits perfectly with our observation that there are $9-p$ scalar fields ϕ_i propagating on a Dp-brane.

That is exactly the number of transverse directions along which the brane can move.

- Now consider N coincident D-branes. The field theory on their worldvolume is a $U(N)$ gauge theory. In such a theory the gauge fields and scalars are matrix-valued:

$$A_\mu(x),\ \phi_i(x) \ \longrightarrow\ A_\mu^{ab}(x),\ \phi_i^{ab}(x), \quad a,b=1,2,\ldots,N$$

It follows that the motion of a system of N D-branes is encoded in matrix-valued coordinates:

$$\phi_i^{ab}=(X^i)^{ab}, \quad i=p+1,p+2,\ldots,9$$

- Now we can state the proposal.

  String theory and all its D-branes can be constructed starting with:

  infinitely many D0-branes

  This is the same as the quantum mechanics of matrices:

$$(X^i)^{ab}, \quad i = 1, 2, \ldots, 9, \qquad N \to \infty$$

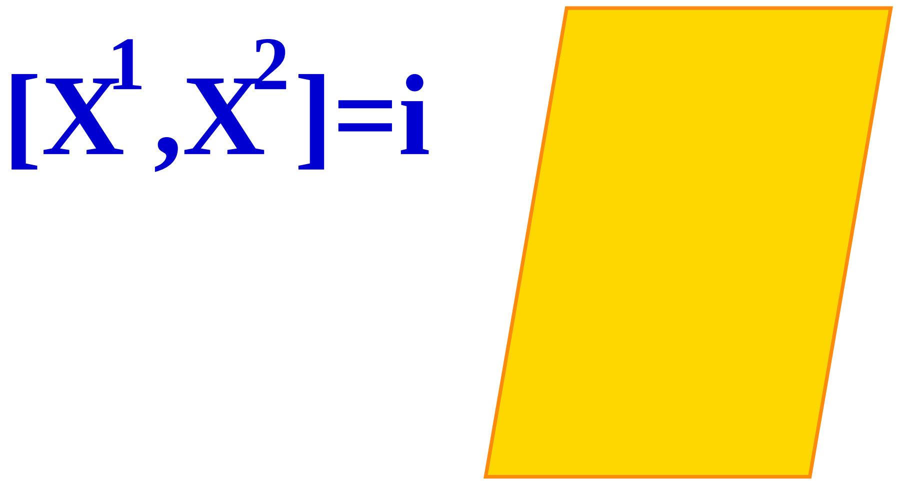
  The quantum mechanics of this system is very elegant. Among the quantum states of this system, we find all the light excitations of closed strings: for example, gravitons.

- Even more remarkably, we can make open-strings. Choose the following classical configuration of infinitely many D0-branes:

$$[X^1, X^2] = i$$

(which we can only do because the matrices are infinite!).

This configuration has energy localized over the entire X^1, X^2 plane. In fact, it is a D2-brane!



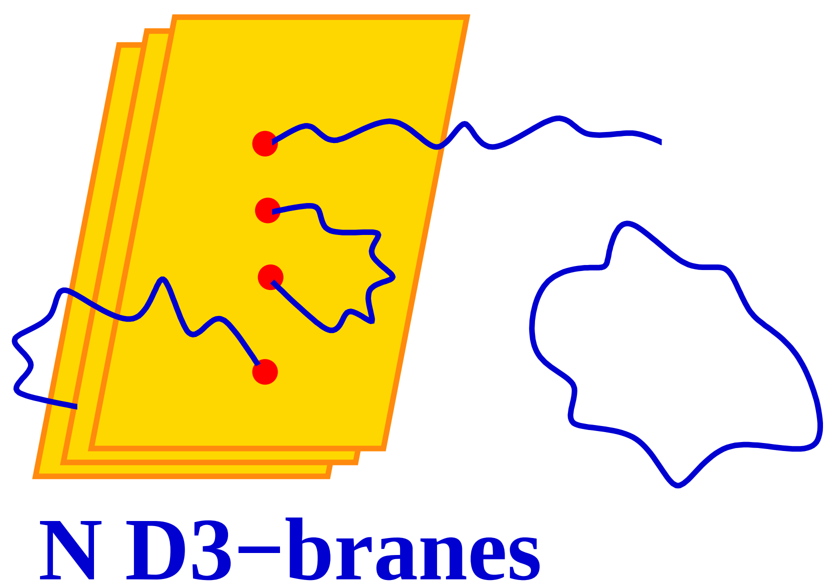
Infinitely many D0-branes seem to encode all the dynamics of strings in a non-perturbative way.

4. Strings from Branes (II)

A different, and more powerful, way to obtain strings from branes arose 5 years ago.

It is called the AdS/CFT correspondence, and arises as follows.

We start with a system of N D3-branes. It has open-strings ending on it, and closed strings propagate in the bulk around it.



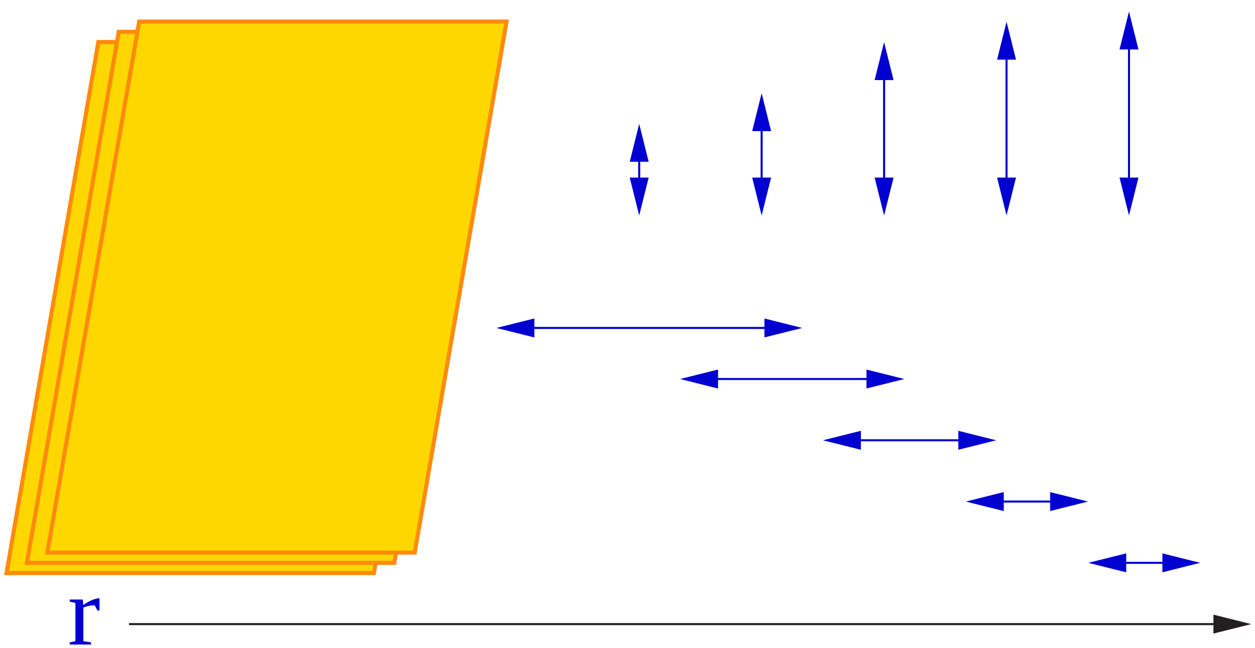
Let us study the gravitational field of this system.

The gravitational field of a system of N D-branes can be obtained as a classical solution of low-energy gravity:

The spacetime metric of the system is given by:

$$ds^2 = \frac{1}{\sqrt{1+\frac{R^4}{r^4}}}\left(-dt^2 + \sum_{i=1}^{3}(dx^i)^2\right) + \sqrt{1+\frac{R^4}{r^4}}\left(dr^2 + r^2 d\Omega_5^2\right)$$

where r is the transverse distance from the brane. Pictorially:



Here, $R^2 = \sqrt{4\pi g_s N}\alpha'$, where g_s is the string coupling and α' is the square of the string length.

Thus we see that a set of N coincident D3-branes has two descriptions:

(i) a hypersurface where open string excitations propagate

(ii) a gravitating object which curves spacetime

We now take a low-energy limit of the system.

By comparing both descriptions, we will obtain a "duality".

At low energies, the massless gauge theory on the brane decouples from all the other modes of string theory.

Thus we are left with a pure $3+1$ dimensional supersymmetric gauge field theory, with gauge group $U(N)$.

In other words,

(i) a hypersurface where open string excitations propagate

$$\xrightarrow{\text{low energy limit}}$$

(i)' a supersymmetric gauge field theory in (3+1)d

In the gravitating description, the redshift in energies tells us that the low energy limit is the same as the near-brane limit: $r \to 0$. In this limit, the metric becomes:

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + \sum_{i=1}^{3}(dx^i)^2\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad}_{AdS_5} \qquad \underbrace{\qquad}_{S^5}$$

This is the metric of Anti DeSitter spacetime times a 5-sphere: $AdS_5 \times S^5$.

In other words,

(ii) strings near a gravitating object which curves spacetime

$$\xrightarrow{\text{low energy limit}}$$

(ii)' string theory in curved $AdS_5 \times S^5$ spacetime

- The fact that there are two ways to describe the same low-energy limit motivates the conjecture:

(i)' a supersymmetric gauge field theory in (3+1)d

=

(ii)' string theory in curved $AdS_5 \times S^5$ spacetime

We have found a field theory that describes strings!

We are not just saying that the field theory is a limit of the string theory.

The field theory **is** the string theory, in different variables!

Reminder: Some important details have been skipped.

(i) The theories being discussed are supersymmetric.

(ii) There are some electric/magnetic fluxes passing through AdS_5 and S^5 which contribute the energy-momentum required to solve the Einstein equations.

The Good News:

- This correspondence has been extensively tested and works amazingly well.
- If we consider a string in $AdS_5 \times S^5$ spacetime, with its ends at infinity, this can be identified with the Wilson loop variable in gauge theory:

$$e^{i \oint A \cdot dx}$$

This is the operator that would describe confined flux tubes in QCD.

Thus, in a model situation (our gauge theory does not actually exhibit confinement), we see that:

QCD strings and gravitating strings are the same thing.

The Bad News:

- However, in practice the AdS/CFT correspondence applies only to a limited class of states in the string theory.

  String theories have two types of states:

light, particle-like states	$\leftrightarrow$	described by gravity field theory
genuinely "stringy" states	$\leftrightarrow$	need the full string theory

  It turns out that we can only apply the above correspondence to the light particle-like states.

  The principal reason is that it is very difficult to quantize strings propagating in curved spacetimes with flux.

  In practice, the correspondence has led to new information about gauge theory, but little detailed insight about how strings arise from gauge fields.

5. Strings from Bits

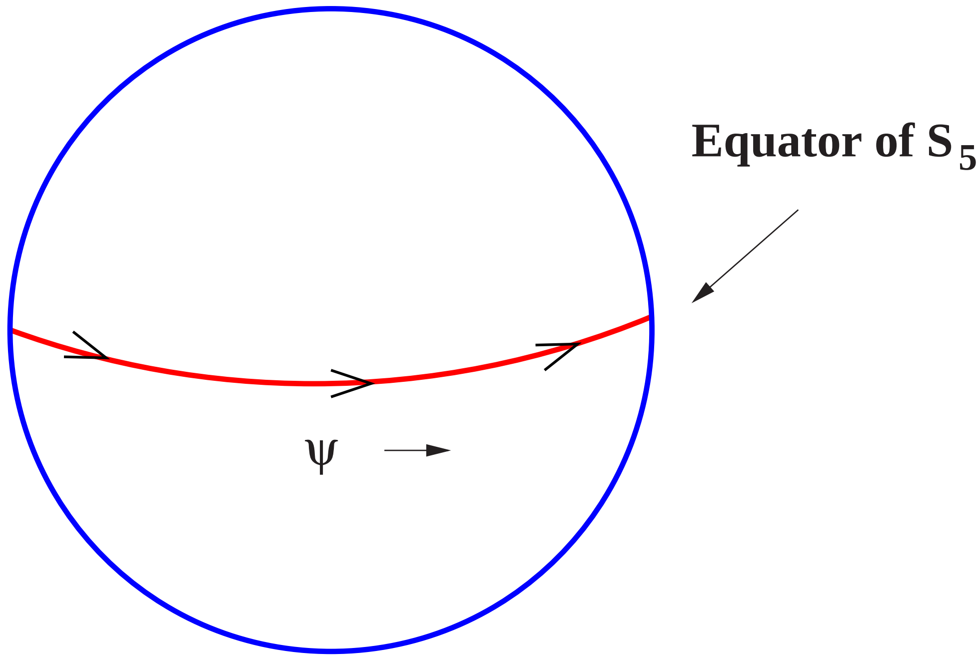
One way to progress beyond this limitation might be to take a limit of the Anti-deSitter spacetime which "flattens" it in some way.

There would be a corresponding limit in the gauge theory.

In that case, this limit of gauge theory would hopefully provide an explicit construction of a string propagating in some spacetime.

Such a limit exists, and is called the Penrose limit.

It consists of "zooming in" on the trajectory of a lightlike particle moving along an equator of S^5, while sitting at the origin of AdS_5.



In this limit, the $AdS_5 \times S^5$ spacetime reduces to a so-called pp-wave:

$$ds^2 = -4dx^+dx^- + \sum_{i=1}^{8} dx_i^2 - \mu^2 \sum_{i=1}^{8} x_i^2\,(dx^+)^2,$$

If we think of x^+ as the "time" direction, this has the appearance of a harmonic oscillator potential along 8 of the 9 spatial directions.

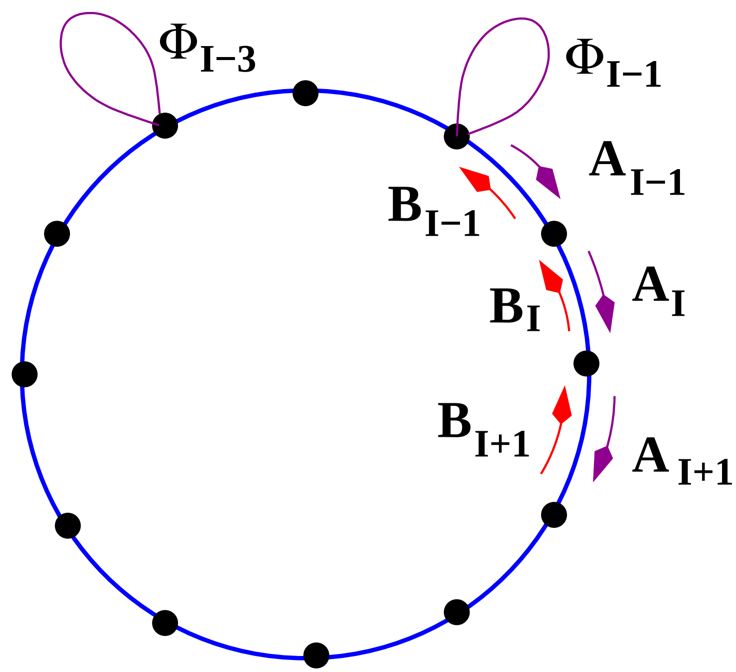
Unlike for AdS_5, string theory in this background is exactly solvable.

- I will briefly describe an approach to this problem which constructs non-relativistic strings.

  In this approach, we compactify one spatial direction on a circle and place an electric field along the circle. Interactions of the string with the field render it very light, so long as it wraps around the circle in one direction:

We have shown that these winding strings can be explicitly constructed out of the operators in a special supersymmetric gauge field theory.

The theory can be summarized by drawing a "quiver diagram".



The diagram describes a field theory with a product gauge group,

$$SU(N) \times SU(N) \times \cdots \times SU(N)$$

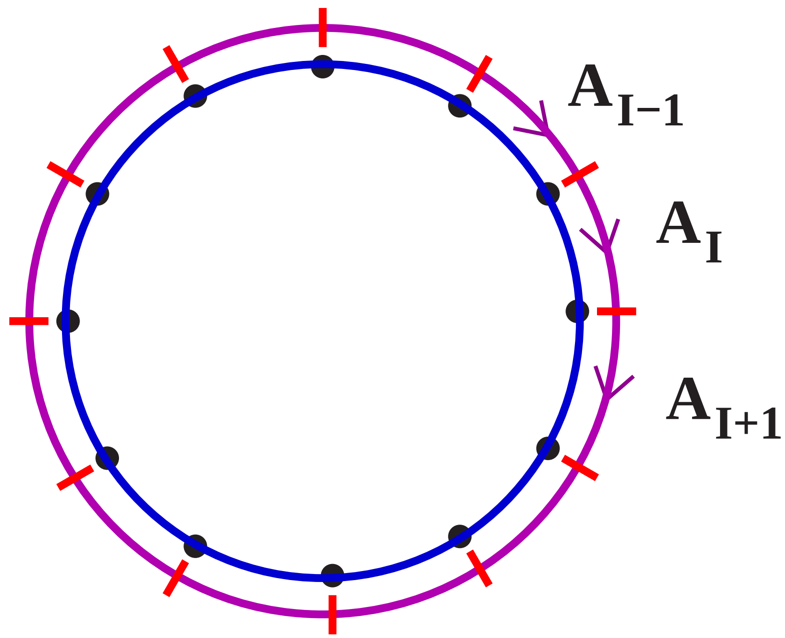
There are three kinds of (matrix valued) scalar fields: adjoints ϕ_i, bi-fundamentals A_i, and conjugate bi-fundamentals B_i.

We need to take $N \to \infty$ in the gauge theory.

Consider the gauge-invariant operator:

$$\mathrm{tr}\,(A_1 A_2 \cdots A_N)$$

Pictorially, this operator is a "string" of fields that are "winding" around the quiver diagram:



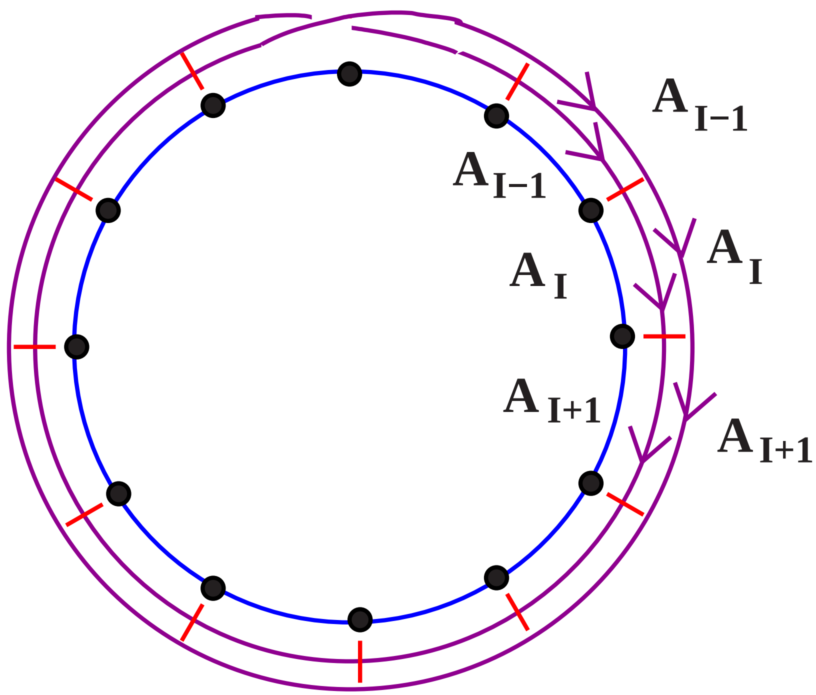
By comparing quantum numbers, one can identify this with the ground state of the singly-wound string.

- It is easy to construct the string ground states with arbitrary winding. We have:

$$|k\rangle = \frac{1}{\sqrt{\mathcal{N}^k}}\mathrm{tr}\left(A_1 A_2 \cdots A_{N_2}\right)^k$$

for any positive integer k.

For example, the state $|k=2\rangle$ looks like:



and represents the ground state of a doubly-wound string.

- Using the other scalars and fermions, one can construct a rich spectrum of operators that are in perfect correspondence with oscillator modes for a non-relativistic string moving in a pp-wave background with a compact direction.

  The correspondence between gauge theories and strings looks rather physical in this example. We see that:

  $$\text{quantum fields in gauge theory} = \text{pieces of string}$$

  In this formalism, one can try to compute scattering amplitudes, both in field theory and in string theory language.

  If they agree, as expected, this would provide a very physical and explicit construction of strings from fields.

6. Conclusions and Outlook

We have seen that there are many ways to construct strings.

Apart from the first-principle approach, the others all rely on matrices in some form.

The most recent development is the construction of strings propagating in a pp-wave spacetime from field operators.

The time seems to be ripe to address the two main unsolved puzzles in string theory:

(i) what is the QCD string, and does it give a useful description of hadron physics?

(ii) what is the correct unified theory of all interactions, and can it be described nonperturbatively by matrices?