# Stable Non-BPS States and Their Holographic Duals

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*Based on:*

S.M. and N.V. Suryanarayana, in progress.
S.M. and N.V. Suryanarayana, hep-th/0003219
S.M., N.V. Suryanarayana, D. Tong, hep-th/0001066

## 1. Introduction

- Type II string theory has various stable, BPS D$p$-branes:

$$IIA: \qquad p = 0, 2, 4, 6, 8$$
$$IIB: \qquad p = -1, 1, 3, 5, 7, 9$$

and unstable non-BPS D$p$-branes:

$$IIA: \qquad p = -1, 1, 3, 5, 7, 9$$
$$IIB: \qquad p = 0, 2, 4, 6, 8$$

- The spectrum on the latter branes is the spectrum of a single open string, but without GSO projection. Hence there is a real tachyon.

- The BPS branes are of course stable, while the non-BPS branes can decay, via tachyon condensation, into the vacuum, or into lower (BPS or non-BPS) branes.

- A pair of a BPS brane and its antibrane is also unstable and can decay similarly.

- This is quite a general paradigm. In flat backgrounds, type IIA branes are either BPS and stable, or non-BPS and unstable.

- It is interesting to look for backgrounds which admit non-BPS but stable branes. In this situation, masses are not protected by BPS formulae. We can hope to disentangle effects of duality from effects of supersymmetry.

- If the backgrounds are themselves non-supersymmetric then things rapidly become difficult. The most accessible situations are those where the backgrounds are supersymmetric, but the states that we study are not.

- Some examples are: orbifolds, orientifolds, Calabi-Yau compactifications. Another class of examples is provided by *suspended brane constructions*. These all have lower supersymmetry than flat space, which helps to find stable non-BPS states.

- In the following I will make extensive use of the *conifold singularity* and its brane-construction dual. ALE spaces will also play an auxiliary role.

## 2. Singularities, Brane Duals and Fractional Branes

- Let us start with type IIB on a $Z_2$ ALE singularity along the (6789) directions.

- Via T-duality along $x^6$, the *ALE* singularity turns into a pair of NS5-branes in type IIA string theory, extending along the (12345) directions and located at different points along $x^6$:

- The ALE singularity hides a 2-cycle $\Sigma$ of zero size, which can be resolved to get an Eguchi-Hanson space. But at the orbifold point, the NS-NS $B$-field has a flux of $\frac{1}{2}$ through this 2-cycle. In the brane dual, the NS5-branes are symmetrically located along the $x^6$ circle.

- This duality extends beyond the orbifold point. Varying the $B$-flux in the ALE corresponds to varying the relative $x^6$ separations of the NS5-branes.

- If we bring a D3-brane into the plane of an ALE singularity, it can split into a pair of fractional D3-branes $f3, f3'$ of charge and tension $\alpha$ and $1-\alpha$ where $\alpha = \int_\Sigma B$ is the $B$-flux.

- The fractional branes are interpreted as:

$$\begin{aligned} f3:& \quad \text{D5 wrapped on } \Sigma \\ f3':& \quad \overline{\text{D}}\text{5 wrapped on } \Sigma, \quad \int_\Sigma F = 1 \end{aligned}$$

- In the dual brane construction, a D4-brane wrapped on $x^6$ can be brought in to touch the NS5-branes, where it can break into two pieces:

- The gauge group $U(1) \times U(1)$ and the presence of bi-fundamental matter is also evident from the brane construction.

- An analogous relation holds for the *conifold* singularity along the (456789) directions. It is dual to a similar brane construction but with rotated NS5-branes:

- This model too has bi-fundamental matter, but also a quartic superpotential.

## 3. Fractional Branes and a Stable Non-BPS Configuration

- An interesting class of non-BPS brane configurations is obtained from the system of an *adjacent brane-antibrane pair*. In some cases, this can be analysed using perturbative string theory, via duality to ALE or conifold singularities.

- The configuration of interest contains a pair of parallel NS5-branes oriented as was just discussed. In the two intervals between the NS5-branes, we place a D4-brane and a $\overline{\text{D}}$4-brane:

- The NS5-brane configuration is T-dual to an ALE singularity. The D4 and $\overline{\text{D}}$4-brane in the intervals T-dualise into a fractional brane and a fractional antibrane. Let us try to understand this correspondence in more detail.

- A $\mathrm{D3} - \overline{\mathrm{D}}3$ pair at a $Z_2$ ALE singularity splits into 4 distinct types of fractional branes, which we call $f3, f3', \overline{f3}, \overline{f3}'$.

- These are interpreted as follows:

$$\begin{aligned}
f3: &\quad \text{D5 wrapped on } \Sigma, \quad \int_\Sigma F = 0 \\
f3': &\quad \overline{\text{D}}\text{5 wrapped on } \Sigma, \quad \int_\Sigma F = 1 \\
\overline{f3}: &\quad \overline{\text{D}}\text{5 wrapped on } \Sigma, \quad \int_\Sigma F = 0 \\
\overline{f3}': &\quad \text{D5 wrapped on } \Sigma, \quad \int_\Sigma F = 1
\end{aligned}$$

- Introducing a $\mathrm{D4} - \overline{\mathrm{D}}4$ pair in the brane construction, we see that it too can break into four distinct pieces:

- This is the Coulomb branch, and we can identify the four fractional branes as in the figure.

- Since we are interested in studying an adjacent D4$-\overline{\text{D}}$4 pair, we see that the dual fractional branes are $f3$ and $\overline{f3}'$.

- This system has a net D5-brane charge of $+2$, and a net D3-brane charge of $2\alpha - 1$.

- The open strings connecting adjacent branes correspond in the ALE dual to the following Chan-Paton factors:

$$\begin{aligned}
f3 - \overline{f3}' : &\quad \frac{1}{2}(\sigma_3 + i\sigma_2) \otimes (\sigma_1 + i\sigma_2) \\
\overline{f3}' - f3 : &\quad \frac{1}{2}(\sigma_3 - i\sigma_2) \otimes (\sigma_1 - i\sigma_2) \\
f3' - \overline{f3} : &\quad \frac{1}{2}(\sigma_3 - i\sigma_2) \otimes (\sigma_1 + i\sigma_2) \\
\overline{f3} - f3' : &\quad \frac{1}{2}(\sigma_3 + i\sigma_2) \otimes (\sigma_1 - i\sigma_2)
\end{aligned}$$

- These are all odd under the ALE projection. Therefore the strings connecting $f3$ to $\overline{f3}'$ have no tachyonic or massless bosonic states. In fact, these strings only give massless fermions.

- Next we construct the boundary states corresponding to the fractional D3-branes, and use them to compute the force between the adjacent pair of interest.

- There are four independent consistent boundary states for D3, $\overline{\mathrm{D}}3$, which can be identified with the four fractional branes $f3, f3', \overline{f3}', \overline{f3}$.

$$
\begin{aligned}
|\mathrm{D}3,+\rangle &= \frac{1}{2}(|U\rangle_{NSNS} + |U\rangle_{RR} + |T\rangle_{NSNS} + |T\rangle_{RR}) \\
|\mathrm{D}3,-\rangle &= \frac{1}{2}(|U\rangle_{NSNS} + |U\rangle_{RR} - |T\rangle_{NSNS} - |T\rangle_{RR}) \\
|\overline{\mathrm{D}}3,+\rangle &= \frac{1}{2}(|U\rangle_{NSNS} - |U\rangle_{RR} - |T\rangle_{NSNS} + |T\rangle_{RR}) \\
|\overline{\mathrm{D}}3,-\rangle &= \frac{1}{2}(|U\rangle_{NSNS} - |U\rangle_{RR} + |T\rangle_{NSNS} - |T\rangle_{RR})
\end{aligned}
$$

- The amplitude of interest is:

$$
\begin{aligned}
&\int_0^\infty dl\, \langle \overline{\mathrm{D}}3,+|e^{-lH_c}|\mathrm{D}3,+\rangle \\
=&\int_0^\infty \frac{dt}{2t}\, \mathrm{tr}\,_{NS-R}\left(\frac{1-(-1)^F}{2}\,\frac{1-R}{2}\,e^{-2tH_0}\right) \\
=&\frac{v^{(4)}}{32(2\pi)^4}\int_0^\infty \frac{dt}{t^3}\Big\{\frac{f_3(\tilde q)^8 + f_4(\tilde q)^8 - f_2(\tilde q)^8}{f_1(\tilde q)^8} \\
&- 4\frac{f_4(\tilde q)^4 f_3(\tilde q)^4 + f_4(\tilde q)^4 f_3(\tilde q)^4}{f_1(\tilde q)^4 f_2(\tilde q)^4}\Big\}
\end{aligned}
$$

- This simplifies to:

$$\frac{v^{(4)}}{16(2\pi)^4}\int_0^\infty \frac{dt}{t^3}\frac{f_4(\tilde{q})^8}{f_1(\tilde{q})^8}\left[1-4\frac{f_1(\tilde{q})^4 f_3(\tilde{q})^4}{f_2(\tilde{q})^4 f_4(\tilde{q})^4}\right]$$

  The integrand is strictly negative, implying that the force between the $f3$ and $\overline{f3}'$ is repulsive.

- Thus we find that the force between an adjacent suspended brane-antibrane pair is *repulsive.*

- Now consider a "twist" on the configuration of adjacent brane-antibrane pairs that we discussed earlier. We rotate one NS5-brane:

- Thus we now have an NS5 and an NS5'-brane, making up the brane dual of the conifold. The adjacent brane-antibrane pair is dual to fractional branes at a conifold.

- Physically, we expect a repulsive force between the adjacent brane and antibrane, as was shown earlier in the unrotated model. But there is also a classical attraction since the branes cannot separate without being stretched.

- This leads to a possibility of stable equilibrium at finite displacement.

- In fact we get a more complicated result exhibiting a phase transition as a function of the radius.

- The tension of the stretched D4-brane is

$$\mathcal{V}\,T_4\sqrt{L^2+2r^2}$$

  where $\mathcal{V}$ is an (infinite) volume factor, $T_4$ is the tension of a BPS D4-brane, and $L$ is the separation between the NS5 and NS5'-branes.

- We assume that the repulsion is as for the ALE (unrotated) case, since it comes from strings connecting the $\mathrm{D4}-\overline{\mathrm{D}}4$ pair across each NS5-brane.

- After a calculation, we find that the shape of the potential depends on the separation parameter $L$.

- Hence the brane and antibrane are aligned for small $L$ but they separate to a finite distance for large $L$:

An estimate gives $L_c \sim 0.28\, g_s^{-1}$.

## 4. Branes at a Conifold and Non-BPS States in $AdS_5$

- If we bring $N$ D3-branes to a conifold singularity and take the large-$N$ limit, we end up with a $\frac{1}{4}$-supersymmetric background of type IIB: $AdS_5 \times T_{1,1}$ where $T_{1,1}$ is a particular Einstein 5-manifold.

- If we T-dualise the conifold we get a model of rotated NS5-branes. $N$ D3-branes at the conifold become $N$ D4-branes wrapped round the $x^6$ circle:

- The adjacent brane-antibrane model that we have described does not have an $AdS$ dual. If we add $N$ D4-branes then the $\overline{\text{D}}4$ will annihilate against a fractional D4-brane, leaving $N-1$ whole D4-branes plus two fractional D4-branes:

- Let us now describe a stable non-BPS brane construction that, instead, does have an $AdS$ dual.

- Take $N$ D4-branes as before and introduce a D2-brane in the first interval:

- In the conifold geometry, this corresponds to the introduction of a fractional D-string in the plane of the singularity.

- This configuration is clearly non-supersymmetric. For example, the strings joining a D2-brane and $N$ D4-branes in the interval will be tachyonic. The stable result should be a bound state of the D4-branes and the D2-brane. While this is BPS by itself, the neighbouring interval still has only D4-branes:

- The $(2,4)$ bound state and the D4-branes preserve incompatible supersymmetries. Hence the whole system is non-BPS, much as for an adjacent brane-antibrane pair.

- In the conifold geometry, we have a fractional D-string bound to $N$ $f3$-branes and coincident with $N$ $f3'$ branes.

- Now we can take the large $N$ limit. What does this state become?

- The conifold geometry is replaced by its 5-manifold base, the Einstein space $T_{1,1}$. Topologically,

$$T_{1,1} \sim S^2 \times S^3$$

- The $S^2$ is the same 2-cycle that was of vanishing size before taking the large-$N$ limit. The fractional D-string was actually a D3-brane wrapped on this $S^2$.

- Hence, in the large $N$ limit, the fractional D-string can be identified with a "fat string" obtained by wrapping a D3-brane on $S^2$.

- Before going further, let us list all the unwrapped and wrapped branes of this model:

| Dim. | $--$ | $S^2$ | $S^3$ | $S^2 \times S^3$ |
|---|---|---|---|---|
| $-1$ | $D(-1)$ | $D1$ | $UD2$ | $UD4$ |
| 0 | $UD0$ | $UD2$ | $D3$ | $D5$ |
| 1 | $D1$ | $D3$ | $UD4$ | $UD6$ |
| 2 | $UD2$ | $UD4$ | $D5$ | $D7$ |
| 3 | $D3$ | $D5$ | $UD6$ | $UD8$ |
| 4 | $UD4$ | $UD6$ | $D7$ | $D9$ |

- The D5 wrapped on $S^2$ is known to be a domain wall that augments the gauge group:

$$SU(N) \times SU(N) \to SU(N+1) \times SU(N)$$

- The D3 wrapped on $S^2$ is our fat string. We would like to understand its holographic dual description.

- The Euclidean D-string wrapped on $S^2$ gives rise to a new instanton, while the (unstable) $UD2$ on $S^2$ is a new unstable D0-brane. We will examine their holographic duals too.

## 5. Some Properties of the Fat String

- The nature of the fat string depends on the B-flux through $S^2$. In general we have

$$\int_{S^2} B_{NS,NS} = \alpha, \qquad \int_{S^2} B_{RR} = \beta$$

- The $SU(N) \times SU(N)$ gauge theory on the 3-branes has couplings and $\theta$-angles given by

$$\tau_1 = \beta + \alpha \tau_s$$
$$\tau_2 = -\beta + (1-\alpha)\tau_s$$

  where $\tau_s = \frac{\chi_{RR}}{2\pi} + \frac{i}{g_s}$.

- The fat string carries D-string charge $\alpha$ and F-string charge $\beta$, by virtue of the Chern-Simons coupling

$$\int B_{NS,NS} \wedge B_{RR} \to \alpha \int B_{RR} + \beta \int B_{NS,NS}$$

  on a D3-brane.

- It is convenient to choose $\beta = 0$.

- The tension of the fat string can be estimated from integrating the DBI action of a D3-brane over $S^2$:

$$T_{\text{fat}} \sim T_3 \int_{S^2} \sqrt{detg + (B_{NS,NS})^2}$$

In the flat space limit, the $S^2$ is of zero size and this becomes

$$T_{\text{fat}} \sim T_3 \, \alpha$$

which shows that it is BPS. On the other hand at large $N$ the dominant contribution comes from

$$T_{\text{fat}} \sim T_3 \int_{S^2} \sqrt{g} \sim \frac{N}{(g_s N)^{\frac{1}{2}} \alpha'}$$

- As with fractional branes, there are really two complementary fat strings, the second one being an anti D3-brane wrapped over $S^2$ and having a magnetic flux $\int F = 1$ over the cycle. We call this a fat′ string. It has a D-string charge $(1 - \alpha)$.

- The non-BPS nature of fat strings, and their charges, imply the reaction

$$\text{fat string} + \text{fat}' \text{ string} \rightarrow \text{D-string}$$

with loss of energy.

- Recall how a D-string is understood in holography. In $AdS_5 \times S^5$, a D-string parallel to the boundary corresponds to a magnetic flux tube. As the string falls towards the horizon, the flux tube fattens and in the limit becomes a constant flux:

- The same result holds for a D-string in $AdS_5 \times T^{1,1}$, but the flux is in the diagonal of the $SU(N) \times SU(N)$ gauge group.

- The fat string is similarly a flux tube in the boundary theory, but this time the flux is *only in one* $SU(N)$ *factor*.

- This is consistent with its non-BPS nature. On a 3-brane we have nonlinearly realised supersymmetry that acts on the gauginos as:

$$\delta^* \lambda^{(1)}_\alpha = \frac{1}{4\pi\alpha'} \eta^*_\alpha, \quad \delta^* \lambda^{(2)}_\alpha = \frac{1}{4\pi\alpha'} \eta^*_\alpha$$

  and linearly realised supersymmetry:

$$\delta\lambda^{(1)}_\alpha = F^{(1)}_{23} \sigma^{23\,\beta}_\alpha \, \eta_\beta, \quad \delta\lambda^{(2)}_\alpha = F^{(2)}_{23} \sigma^{23\,\beta}_\alpha \, \eta_\beta$$

We see that, if and only if the fluxes are diagonal: $F^{(1)} = F^{(2)} = F$, there is a surviving set of linearly realised supersymmetries, described by choosing

$$\eta^*_\alpha = -4\pi\alpha' F_{23}\sigma^{23\,\beta}_{\alpha}\,\eta_\beta$$

- With this non-BPS fat string, one can now study Wilson/'t Hooft loops in the $AdS$ context and compare predictions at weak and strong 't Hooft coupling (in progress).

- A brief comment on some other wrapped branes:

  D1 wrapped on $S^2$ is a new "D-instanton". It is expected to be dual to a Yang-Mills instanton in the first factor of $SU(N) \times SU(N)$.

  It has its own associated sphaleron, the D2-brane of type IIB wrapped on $S^2$.

- The relation between the two is parallel to the one between unwrapped D-instantons and D0-branes, studied recently.

## 6. Conclusions

- The stable brane-antibrane construction could decribe an interesting non-SUSY model field theory. Microscopically it has a pair of branes separated by a finite calculable distance (brane-world model?).

- BPS brane constructions are most useful when we can use S-duality or M-theory. What do we learn from these about brane-antibrane constructions.

- Is there a physical reason why "fat" objects are associated to one $SU(N)$ factor while "thin" objects are diagonal in $SU(N) \times SU(N)$?

- A lot of interesting physical results should emerge from a closer inspection of the $AdS/CFT$ correspondence for non-BPS states.