Taylor expansion in chemical potential:

Mass dependence of the Wroblewski parameter

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Taylor expansion

Condensates

Wroblewski Parameter

Ward Identities

Results

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- Possible way out includes the reweighting techniques, analytic continuation from computations at imaginary μ_B, and Taylor expansions.
- We present Taylor series expansion of the chiral condensate and related quantities at finite chemical potential about $\mu_B = 0$

Taylor Expansion

- First results for Taylor expansion for free energy was done by Gottlieb *et.al.* $Phys.Rev.Lett.\mathbf{59}(1987)2247$ and for the meson correlators by Miyamura *et.al.* $Phys.Rev.\mathbf{D66}(2002)077502$.

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- In Taylor expansion, one can do continuum extrapolation of each term. This removes prescription dependence of putting chemical potential on the lattice $[$Gavai $\&$ Gupta,$Phys.Rev.\mathbf{D68}(2003)034506]$.
- Taylor expansion is well behaved when the coefficients of the higher order terms tend to decrease. Otherwise the expansion breaks down $\Rightarrow$ approaching a phase transition.

General Formalism

- For the light *u,d* quarks the QCD partition function is

$$Z = e^{-F/T} = \int \mathcal{D}U e^{-S(T)} \prod_{f=u,d} Det M(m_f, T, \mu_f)$$

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$$P(T, \mu_u, \mu_d) = P(T, 0, 0) + \sum_f n_f \mu_f + \frac{1}{2!} \sum_{fg} \chi_{fg} \mu_f \mu_g + ...$$

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$$n_f = \frac{T}{V}\left.\frac{\partial ln Z}{\partial \mu_f}\right|_{\mu_f=0} ; \chi_{fg} = \frac{T}{V}\left.\frac{\partial^2 ln Z}{\partial \mu_f \partial \mu_g}\right|_{\mu_f=\mu_g=0}$$

Derivatives

- Derivatives of lnZ are obtained from those of Z

$$\frac{\partial Z}{\partial \mu_f} = \int \mathcal{DU} \left[tr M_f^{-1} M_f' \right]$$

$$\frac{\partial^2 Z}{\partial \mu_f \partial \mu_g} = \int \mathcal{DU} \Big[tr M_g^{-1} M_g' tr M_f^{-1} M_f' + \left\{ -tr M_g^{-1} M_g' M_g^{-1} M_g' + tr M_g^{-1} M_g'' \right\} \delta_{fg} \Big]$$

where $M_f' = \gamma_0$, $M_f^{-1} = \psi \bar{\psi}$

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$$\mathcal{O}_1 = trM_f^{-1}M'_f$$
$$\mathcal{O}_{11} = \mathcal{O}_1\mathcal{O}_1$$
$$\mathcal{O}_2 = \frac{\partial \mathcal{O}_1}{\partial \mu_f}$$

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$$\frac{\partial^2 J}{\partial \mu_g \partial \mu_f} = \int \mathcal{DU}\left[\mathcal{O}_{11}^{(f)}\mathcal{J} + \delta_{fg}\mathcal{O}_2^{(f)}\mathcal{J} + \mathcal{O}_1^{(f)}\mathcal{J}^{(g)} + \mathcal{J}^{(fg)}\right]$$

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$$\langle \mathcal{J} \rangle = \langle J_0 \rangle + \langle J_1 \rangle \mu + \{\langle J_2 \rangle - \langle J_0 \rangle \langle Z_2 \rangle\}\frac{\mu^2}{2!} + \cdots$$

TIFR

Condensate

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$$\mathcal{J}_u^{(uu)} = tr(2M_u^{-1} M_u' M_u^{-1} M_u^{-1} M_u' - M_u^{-1} M_u'' M_u^{-1})$$

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- We construct isoscalar and isovector combinations

$$C_S = \frac{1}{2}\langle\bar{\psi}\psi\rangle \quad and \quad C_V = \frac{1}{2}\langle\bar{\psi}\tau_3\psi\rangle$$

where, $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ and $\bar{\psi} = \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}$

Condensate

- For isovector chemical potential $\mu_u = -\mu_d = \mu$

$$C_S(\mu) = C_S(0)$$

$$+[2\{\langle \mathcal{O}_2 \mathcal{J} \rangle - \langle \mathcal{O}_2 \rangle \langle \mathcal{J} \rangle\} + \langle \mathcal{O}_1 \mathcal{J}' + \mathcal{J}'' \rangle] \frac{\mu^2}{2!} + \cdots$$

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$$\langle C_V\rangle_\mu = \langle \mathcal{J}'\rangle\mu + \cdots$$

- For iso-scalar chemical potential $\mu_u = \mu_d = \mu$, C_V vanishes to all orders and,

$$C_S(\mu) = C_S(0) + \langle \mathcal{O}_1 \mathcal{J} + \mathcal{J}'\rangle\mu + [4\{\langle \mathcal{O}_{11} \mathcal{J}\rangle - \langle \mathcal{O}_{11}\rangle\langle \mathcal{J}\rangle\}$$
$$+2\{\langle \mathcal{O}_2 \mathcal{J}\rangle - \langle \mathcal{O}_2\rangle\langle \mathcal{J}\rangle\} + \langle \mathcal{O}_1 \mathcal{J}' + \mathcal{J}''\rangle]\frac{\mu^2}{2!} + \cdots$$

Condensate and Susceptibility

- We relate Taylor coefficients of condensate and susceptibility

$$C(T,\mu) = \frac{1}{V}\frac{\partial lnZ}{\partial m}$$

$$\Rightarrow C_2(T,0) = \frac{\partial^2 C(T,\mu)}{\partial \mu^2}\bigg|_{\mu=0} = \frac{1}{T}\frac{\partial \chi_{uu}}{\partial m}$$

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Chiral Ward Identity

- The pseudo-scalar correlator χ_{PS} satisfies the Chiral Ward Identity

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- For $T > T_c$ and $m < \Omega(= \pi T)$, meson screening mass is independent of m $\Rightarrow C_2 = 0$.

Wroblewski Parameter

- Strangeness enhancement in heavy-ion collisions can be expressed in terms of the Wroblewski parameter.

$$\lambda_s(T) = \frac{\langle n_s \rangle}{\langle n_u + n_d \rangle} = \frac{\chi_{ss}(T)}{\chi_{uu}(T)}$$

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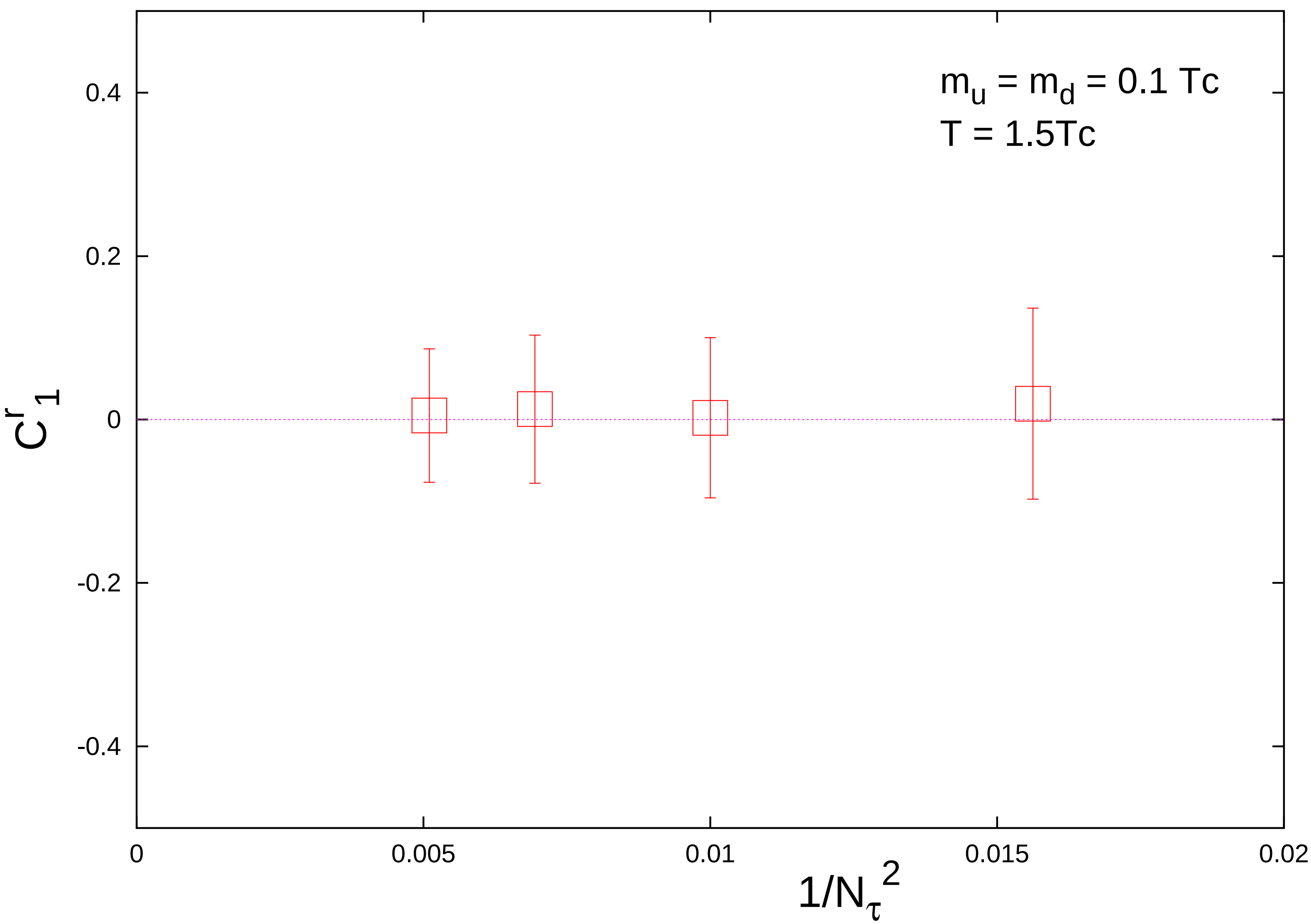
Computations

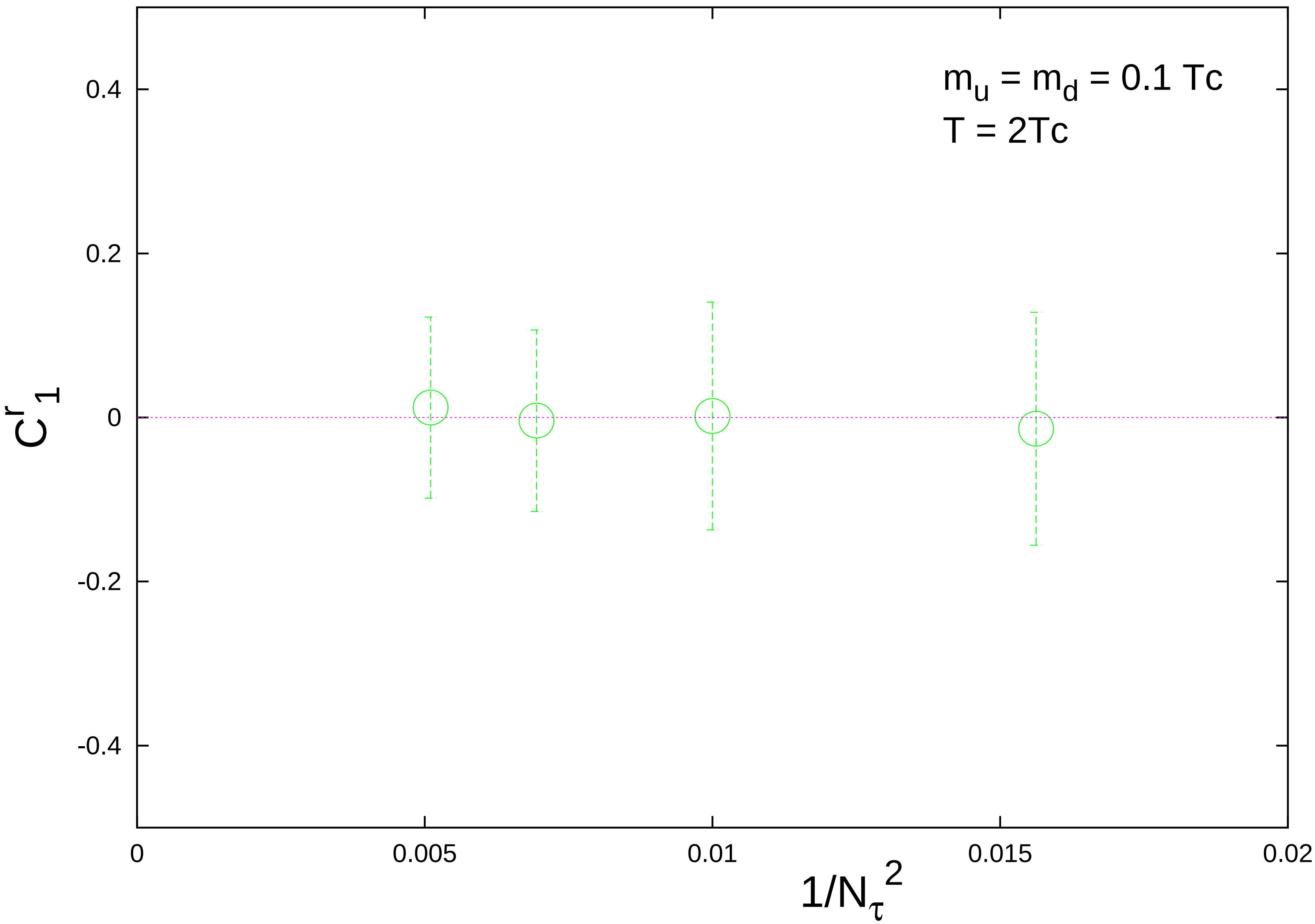
- We used 2 degenerate flavors of staggered quarks in quenched QCD.
- Measurements were done at $1.5T_c$, $2.0T_c$ and $3.0T_c$.
- Continuum extrapolations were obtained from simulations done at $N_\tau = 4, 8, 10, 12$ and 14 lattices.

Numerical Results

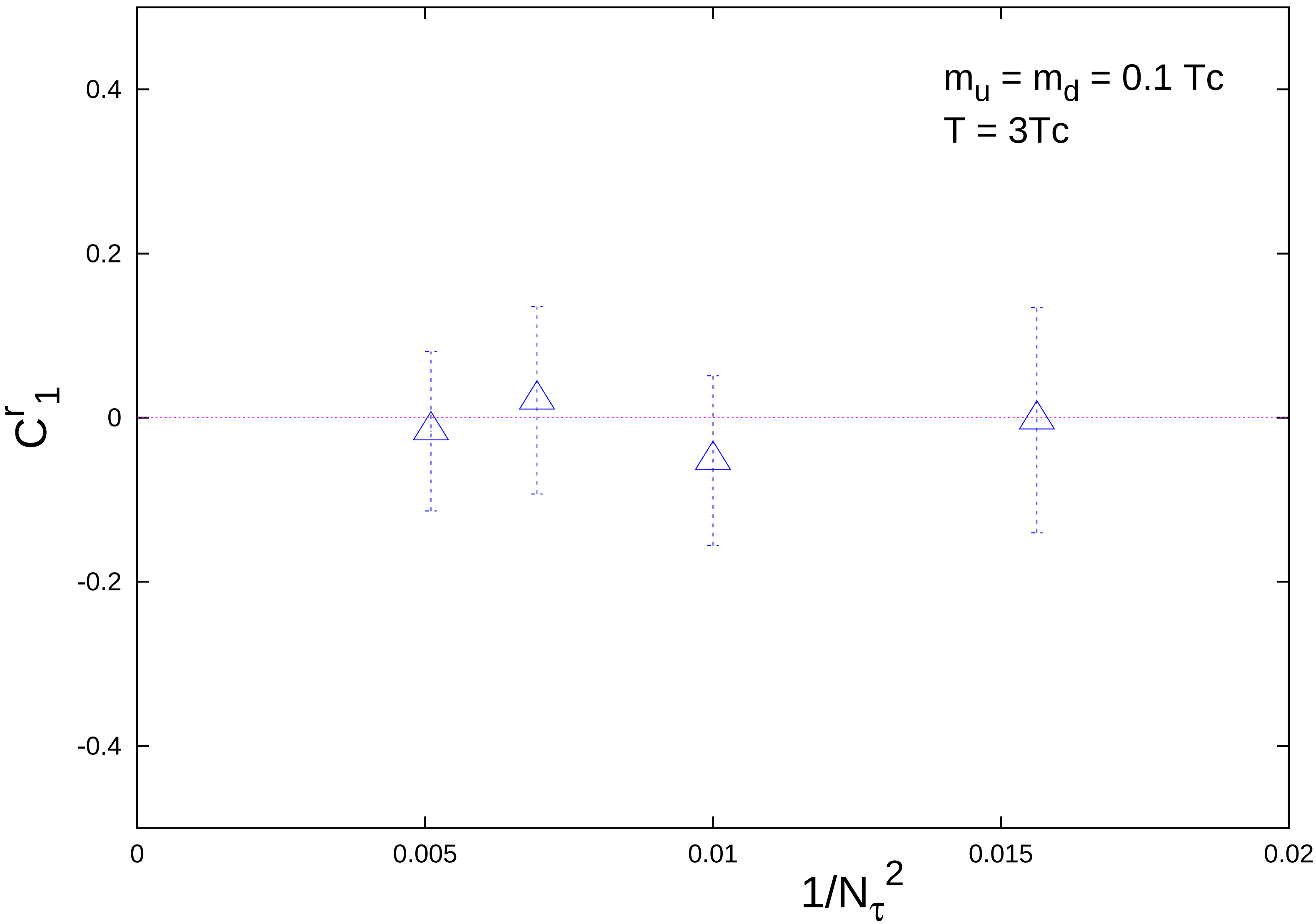
- First order derivatives are related to the mass derivative of the number density, which is zero at $\mu = 0 \rightarrow$ consistent with our computations.
- In the continuum, both the condensate and the second derivative have divergences which are to be cured by renormalization.
- Since the divergence is in the ultraviolet limit, ratios of finite temperature quantities with those at zero temperature should be well behaved.

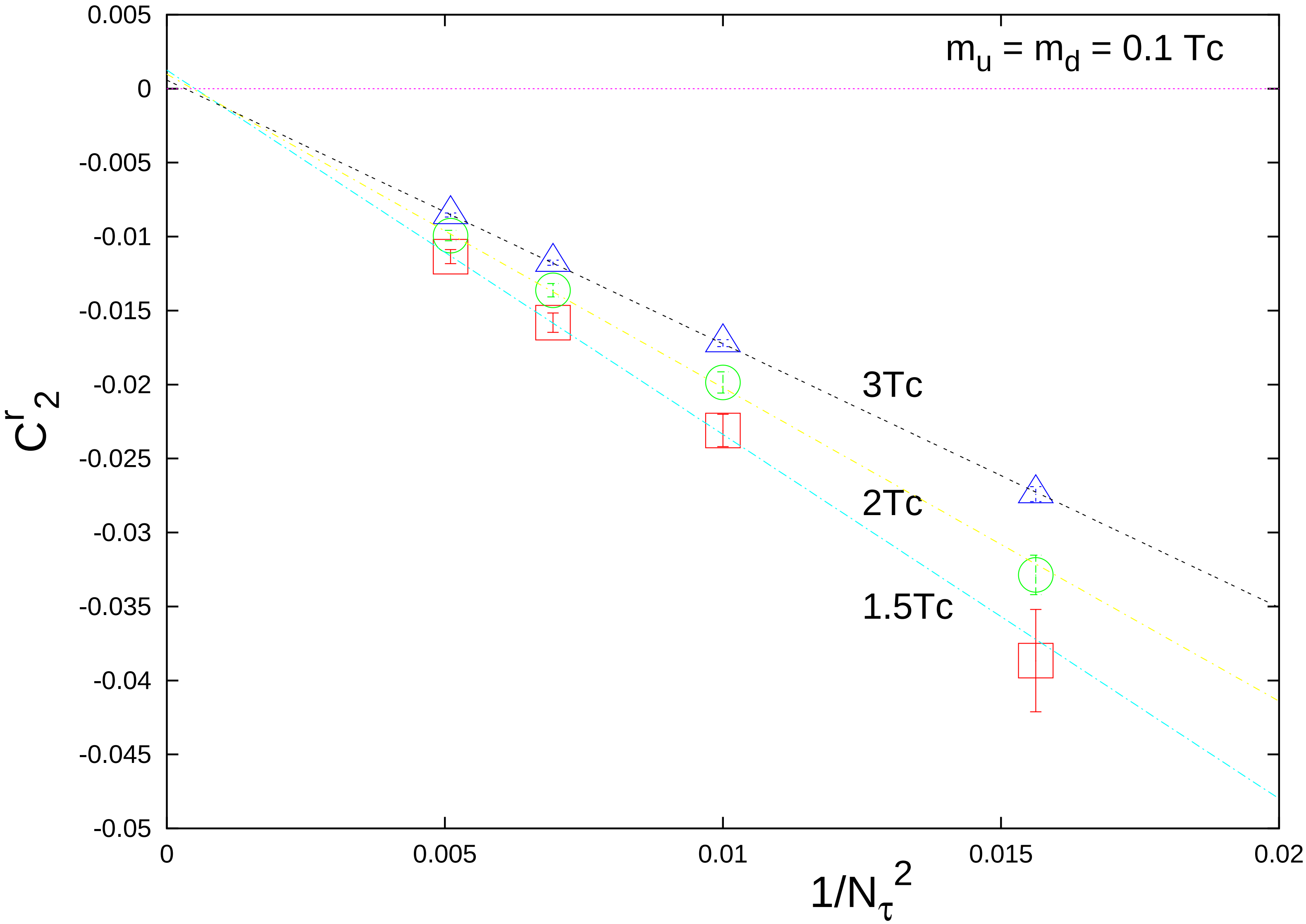


$m_u = m_d = 0.1$ Tc
T = 1.5Tc
0.4
0.2
0
-0.2
-0.4
C^r_1
0
0.005
0.01
0.015
0.02
$1/N_\tau^2$



$m_u = m_d = 0.1$ Tc
T = 2Tc
0.4
0.2
0
-0.2
-0.4
C^r_1
0
0.005
0.01
0.015
0.02
$1/N_\tau^2$


$m_u = m_d = 0.1$ Tc
T = 3Tc
0.4
0.2
0
-0.2
-0.4
C^r_1
0
0.005
0.01
0.015
0.02
$1/N_\tau^2$



$m_u = m_d = 0.1$ Tc
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
-0.04
-0.045
-0.05
c^r_2
3Tc
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- We computed the Taylor coefficients of the chiral condensate upto second order, above the critical temperature. They satisfy various Ward identities and Maxwell relations.
- Ward identities and Maxwell relations relate Taylor coefficients of various quantities.
- Through a Maxwell relation we were able to connect the Taylor coefficients of the chiral condensate with the mass dependence of the Wroblewski parameter.