SYMMETRIES OF NONHIERARCHICAL NEUTRINOS

from high to low scales

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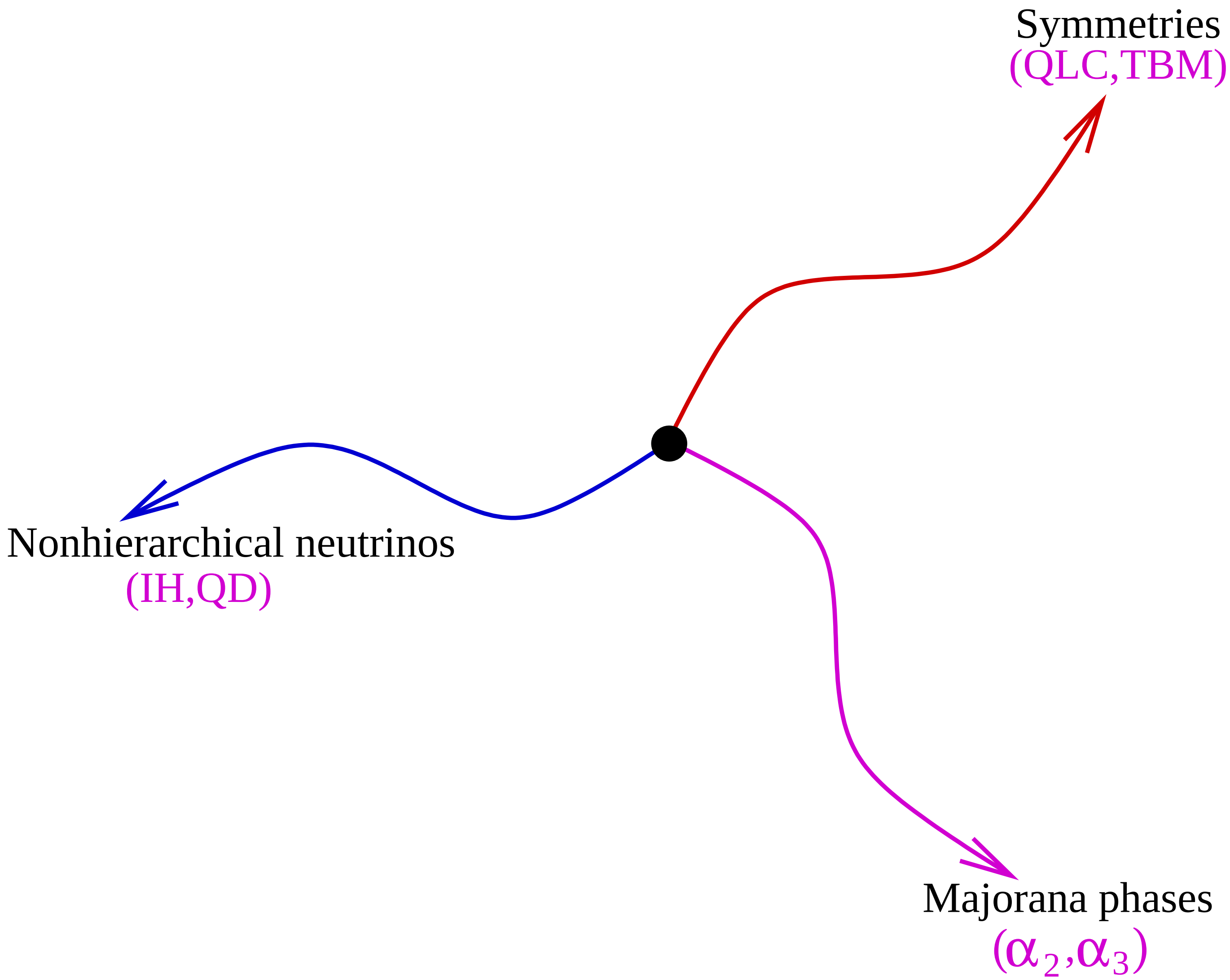
Symmetries
(QLC,TBM)
Nonhierarchical neutrinos
(IH,QD)
Majorana phases
(α_2,α_3)

- PRELIMINARIES
- NEUTRINO FACTFILE
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PRELIMINARIES

Flavor mixing

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\nu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U_\nu \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

No mixing limit : $U \to I$

$$|\nu_1\rangle \to |\nu_e\rangle$$
$$|\nu_2\rangle \to |\nu_\mu\rangle$$
$$|\nu_3\rangle \to |\nu_\tau\rangle$$

SM fermion mass term

$$\mathcal{L}_m = -\bar{f}_{La} M^f_{ab} f_{Rb} + \text{h.c.}$$ chiral $\leftrightarrow$ flavor basis

$$f_{La} = U^f_{ai} f_{Li}$$
$$f_{Rb} = W^f_{bj} f_{Rj}$$ mass basis

$$\mathcal{L}_m = -\bar{f}_{Li} \mathcal{M}^{f(D)}_{ij} f_{Rj} + \text{h.c.}$$ chiral $\leftrightarrow$ flavor basis

$$\mathcal{M}^{f(D)} = U^{f^\dagger} \mathcal{M}^f W^f$$

Flavor $\leftrightarrow$ mass biunitary transformation

cc weak interactions

Quarks

$$\mathcal{L}^q_{cc} = \bar{u}_{La}\gamma^\mu d_{La} W^+_\mu + \text{h.c.}$$

$$= \bar{u}_{Li}\gamma^\mu V^{CKM}_{ij} d_{\ell j} W^+_\mu + \text{h.c.}$$

$$V^{CKM} = U^{u\dagger} U^d$$

Leptons

$$\mathcal{L}^\ell_{cc} = \bar{\ell}_{La}\gamma^\mu \nu_{La} W^-_\mu + \text{h.c.}$$

$$= \bar{\ell}_{Li}\gamma^\mu U^{PMNS}_{ij} \nu_{Lj} W^-_\mu + \text{h.c.}$$

$$U^{PMNS} = U^{\ell\dagger} U^\nu$$

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_\ell} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_\ell} & c_{12}c_{23} - s_{12}s_{23}s_{13}c^{i\delta_\ell} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_\ell} & -c_{12}s_{13} - s_{12}c_{23}s_{13}c^{i\delta_\ell} & c_{23}c_{13} \end{pmatrix}$$

with

$$c_{ij} \equiv \cos\theta_{ij}$$

$$s_{ij} \equiv \sin\theta_{ij} \quad ?$$

No, $\exists$ an additional diagonal Majorana phase matrix factor!

Neutrino mass term

Assumed Majorana form

$$\mathcal{L}_m = -\frac{1}{2}\overline{\nu^c}\mathcal{M}^\nu \nu + \text{h.c.}$$

$\mathcal{M}^\nu$ complex symmetric matrix in family space

$$\mathcal{M}^{\nu(D)} = U^{\nu\dagger}\mathcal{M}^\nu U^{\nu\star} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \; m_i \text{ complex}$$

3 Majorana phases, one overall phase absorvable in ν

Two relative Majorana phases

convention: choose $m_1 =$ real, $m_2 = |m_2|e^{i\alpha_2}$, $m_3 = |m_3|e^{i\alpha_3}$

$\longrightarrow$ additional phase factor matrix to the right in U^{PMNS}:

$$U^{PMNS} = U^{\text{CKM-form}} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_3}{2}} \end{pmatrix}$$

Running of neutrino masses & mixing angles

Loop renormalization effects $\longrightarrow$

coupling strengths $g_i \equiv g_i(t)$,

$t \propto ln\frac{Q}{\Lambda}$ with Q as running & Λ as fixed scale, chosen high

True, in particular, of Yukawa couplings $\longrightarrow$ masses.

$\Rightarrow$

Fermion masses and mixing parameters become functions of t. Specifically, in the neutrino sector

$$\begin{aligned} m_i &\equiv m_i(t), \\ \theta_{ij} &\equiv \theta_{ij}(t), \\ \delta_\ell &\equiv \delta_\ell(t). \end{aligned}$$

Our idea :

Choose some symmetry at a high scale ($Q = \Lambda$, i.e. $t = 0$) and see how they look at the laboratory scale ($Q = \lambda \sim$ TeV).

NEUTRINO FACTFILE

Masses :

At least two of the three known light neutrinos are massive!

$$[|m_2|^2 - |m_1|^2]^{1/2} \equiv \sqrt{\delta m_s^2} \sim 0.009 \text{ eV} \quad :$$

$$: \quad \text{neutrino oscillation}$$

$$||m_3|^2 - |m_{2,1}|^2|^{1/2} \equiv \sqrt{|\delta m_A^2|} \sim 0.05 \text{ eV} \quad :$$

$$\sum_{i=1}^{3} |m_i| < \mathcal{O}(1) \text{ eV} \qquad \text{cosmology}$$

Mixing

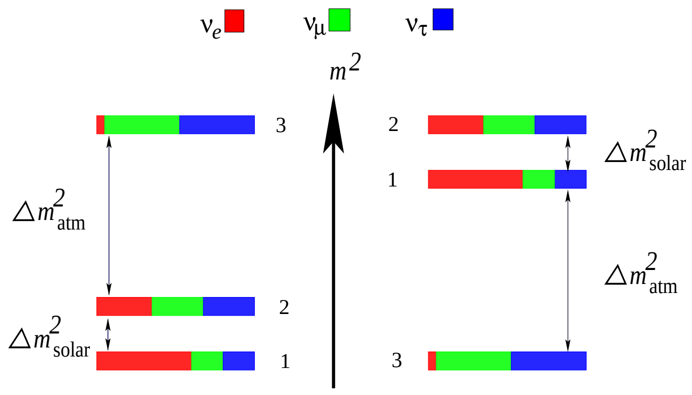
$$U \equiv U^{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta_\ell)$$

$$\theta_{12} \sim 34^\circ,\ \theta_{23} \sim 45^\circ,\ \theta_{13} < 12^\circ,\ \delta_\ell = ?$$

$$1\sigma: \quad \theta_{12} = 33.8^\circ - 1.8^\circ + 2.4^\circ, \theta_{23} = 45^\circ \pm 4^\circ \ .$$

$$U^{PMNS} \sim \begin{pmatrix} 0.8 & 0.5 & < 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}, \text{cf. } V^{CKM} \sim \begin{pmatrix} 0.97 & 0.22 & 0.003 \\ 0.22 & 0.97 & 0.04 \\ 0.01 & 0.04 & 0.99 \end{pmatrix}$$

Mass Pattern



Normal ordering

$$|m_3|^2 > |m_2|^2 > |m_1|^2$$

Inverted ordering

$$|m_3|^2 < |m_1|^2 < |m_2|^2$$

3 patterns

Normal hierarchical (NH) $|m_1| \sim 0$, $|m_2| \sim 0.009$ **eV**, $|m_3| \sim 0.05$ **eV**

Inverted hierarchical (IH) $|m_3| \sim 0$, $|m_1| \sim |m_2| \sim 0.05$ **eV**

Quasi Degenerate (QD) 0.05 **eV** $< |m_1| \sim |m_2| \sim |m_3| \lesssim 0.33$ **eV**

Nonhierarchical $= \{IH\ QD\}$

RGE from Λ to λ controlled by $\dfrac{|m_i + m_j|^2}{|m_i|^2 - |m_j|^2}\Delta_\tau$ **where** $\Delta_\tau \leq 10^{-2}$.

Only nonhierarchical neutrinos have significant RGE effects!

MASS PARAMETRIZATION WITH MAJORANA PHASES

Three real parameters m_0, ρ_A, ϵ_S

$m_1 = |m_1|$

$m_{2,3} = |m_{2,3}| e^{i\alpha_{2,3}}$

$|m_1| = m_0(1-\rho_A)(1-\epsilon_S)$ $\qquad$ m_0, ρ_A, ϵ_S real

$|m_2| = m_0(1-\rho_A)(1+\epsilon_S)$ $\qquad$ $m_0, \epsilon_S > 0, m_0 \lesssim 0.33 eV$

$|m_3| = m_0(1+\rho_A)$ $\qquad$ $-1 \lesssim \rho_A \lesssim 1, \rho_A \gtrless 0$ for $\substack{\text{normal}\\ \text{inverted}}$ ordering

$\delta m_S^2 = 4m_0^2(1-\rho_A)^2\epsilon_S$

$|\delta m_A^2| = 4m_0^2|\rho_A|$ $\qquad$ $\Gamma \equiv \rho_A^{-1} - \rho_A \gtrless 0$ for $\substack{\text{normal}\\ \text{inverted}}$ ordering

$\Sigma_i |m_i| = 3m_0(1 - \frac{\rho_A}{3})$ $\qquad$ Γ dimensionless no. between 0 and ± 182.

NH : $m_0 \simeq 0.025$ eV, $\rho_A \simeq 1, \epsilon_S \simeq 1, \Gamma \simeq 0+$

IH : $m_0 \simeq 0.025$ eV, $\rho_A \simeq -1, \epsilon_S \simeq 1.6 \times 10^{-2}, \Gamma \simeq 0-$

QD : $0.025 \ll m_0 \lesssim 0.33$ eV, $|\rho_A| \lesssim .0056, \epsilon_S \gtrsim 2 \times 10^{-6}, |\Gamma| \lesssim 182$

Nonhierarchical = $\{IH, QD\}$.

HIGH SCALE NEUTRINO SYMMETRIES

QLC → 1 Minakata, Smirnov (2004)
QLC → 2 Raidal (2004)

TBM → 1 Harrison-Perkins-Scott (2002)
TBM → 2 King (2005)

$$V_{\mathrm{CKM}} \supset \theta_c \sim 12.6^\circ, U^{\mathrm{BM}} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}, U^{\mathrm{TBM}} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ -1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

$\downarrow$ $\downarrow$

$$\theta_{23}^{\Lambda} = 45^\circ = \theta_{12}^{\Lambda}, \ \theta_{13}^{\Lambda} = 0 \qquad \theta_{23}^{\Lambda} = 45^\circ, \theta_{12}^{\Lambda} = 35.26^\circ, \ \theta_{13}^{\Lambda} = 0$$

QLC1	QLC2	TBM1	TBM2
$U^{\mathsf{PMNS}} = V^{\mathsf{CKM}^\dagger} U_\nu^{\mathsf{BM}}$	$U^{\mathsf{PMNS}} = U_\ell^{\mathsf{BM}^\dagger} V^{\mathsf{CKM}}$	$U^{\mathsf{PMNS}} = U^{\mathsf{TBM}}$	$U^{\mathsf{PMNS}} = \tilde{V}^{\mathsf{CKM}^\dagger} U^{\mathsf{TBM}}$, where
$U_u = 1$ basis $\rightarrow U_d = U_\ell$	$U_d = 1$ basis $\rightarrow U_u = U_\nu$	Family A_4 or S_3	$\tilde{V}^{\mathsf{CKM}} \simeq \begin{pmatrix} 1 & \theta_{C/3} & 0 \\ \theta_{C/3} & 1 & \vert V_{cb} \vert \\ 0 & \vert V_{cb} \vert & 1 \end{pmatrix}$
$\theta_{12}^{\wedge} \simeq \frac{\pi}{4} - \frac{\theta_c}{\sqrt{2}} \simeq 35.4^\circ$	$\theta_{12}^{\wedge} \simeq \frac{\pi}{4} - \theta_c \simeq 32.4^\circ$	$\theta_{12}^{\wedge} = \sin^{-1}\frac{1}{\sqrt{3}} \simeq 35.3^\circ$	$\theta_{12}^{\wedge} \simeq \sin^{-1}\frac{1}{\sqrt{3}} - \frac{\theta_c}{3\sqrt{2}} \simeq 32.3^\circ$
$\theta_{23}^{\wedge} \simeq \frac{\pi}{4} - \vert V_{cb} \vert - \frac{\theta_c^2}{4} \simeq 42.1^\circ$	$\theta_{23}^{\wedge} \simeq \frac{\pi}{4} - \frac{\vert V_{cb} \vert}{\sqrt{2}} \simeq 43.4^\circ$	$\theta_{23}^{\wedge} = 45^\circ$	$\theta_{23}^{\wedge} \simeq \frac{\pi}{4} - \vert V_{cb} \vert \simeq 42.7^\circ$
$\theta_{13}^{\wedge} \simeq \frac{\theta_c}{\sqrt{2}} \simeq 8.9^\circ$	$\theta_{13}^{\wedge} \simeq \frac{\vert V_{cb} \vert}{\sqrt{2}} \simeq 1.6^\circ$	$\theta_{13}^{\wedge} = 0^\circ$	$\theta_{13}^{\wedge} \simeq \frac{\theta_c}{3\sqrt{2}} \simeq 3.1^\circ$

RGE FROM Λ to λ IN MSSM

Generation of Majorana ν-mass

Dim 5 operator in SM

$$\mathcal{O} = c^{ab}\frac{(\ell^a \cdot H)(\ell^b \cdot H)}{\Lambda}, \qquad \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$\Longrightarrow (\mathcal{M}^\nu)^{ab} = c^{ab}\frac{v^2}{2\Lambda} \qquad v \simeq 246 \text{ GeV}$$

Evolve c^{ab} from high scale Λ to laboratory scale λ in MSSM.

1-loop contributions from: gauge bosons, gauginos, fermions, sfermions.

Log-sensitive to Λ/λ.

Chankowski & Pokorski (2000, 2002)

$$\mathcal{M}^{\nu,\lambda} = I_K I_{\mathcal{K}}^T \mathcal{M}^{\nu,\Lambda} I_{\mathcal{K}}$$

with symmetry properties imposed on $\mathcal{M}^{\nu,\Lambda}$.

$$\begin{aligned} t &= \frac{1}{16\pi^2} ln \frac{Q}{\Lambda}, \\ I_K &= \exp\left[-\int_0^{t(\lambda)} d\tau \{6g_2^2(\tau) + 2g_Y^2(\tau) - 6\mathrm{Tr}(Y_u^\dagger Y_u)(\tau)\}\right], \\ I_{\mathcal{K}} &= \exp\left[-\int_0^{t(\lambda)} d\tau (Y_\ell^\dagger Y_\ell)(\tau)\right]. \end{aligned}$$

In the basis with $U^\ell = I$, i.e.

$$\mathcal{M}^\ell = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix},$$

with $m_{e,\mu}^2$ **neglected,**

$$I_{\mathcal{K}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-\Delta_\tau} \end{pmatrix}, \quad \begin{array}{c} \Delta_\tau = m_\tau^2(\tan^2\beta + 1)(8\pi^2 v^2)^{-1} ln\frac{\Lambda}{\lambda} \\ \lesssim 10^{-2} \text{ for } v_u/v_d \equiv \tan\beta \lesssim 30 \\ N.B.\ v^2 = v_u^2 + v_d^2 \end{array} .$$

Neglect $\mathcal{O}(\Delta_\tau^2)$**. Then**

$$\mathcal{M}^{\nu,\lambda} \simeq I_K \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_\tau \end{pmatrix} \mathcal{M}^{\nu,\Lambda} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\Delta_\tau \end{pmatrix} + \mathcal{O}(\Delta_\tau^2).$$

$\mathcal{M}^\nu \longrightarrow U^\nu \longrightarrow U^{\mathrm{PMNS}}$. **Work in the basis** $U^\ell = I$**.**

cf. $\mathcal{M}^{\nu\Lambda} \longrightarrow U^{\Lambda\mathrm{PMNS}}$.

Get

$$\theta_{ij} = \theta_{ij}^{\Lambda} + k_{ij}\Delta_\tau, \alpha_{2,3} = \alpha_{2,3}^{\Lambda} + a_{2,3}\Delta_\tau, \delta_\ell = d_\ell \Delta_\tau \quad \text{upto } \mathcal{O}(\Delta_\tau^2).$$

$$k_{12} = \frac{1}{2}\sin 2\theta_{12}^{\Lambda}\sin^2\theta_{23}^{\Lambda}\frac{|m_1^{\Lambda}+m_2^{\Lambda}|^2}{|m_2^{\Lambda}|^2-|m_1^{\Lambda}|^2} \simeq \frac{1}{4\epsilon_S^{\Lambda}}\sin 2\theta_{12}^{\Lambda}\sin^2\theta_{23}^{\Lambda}\left[1+\cos\alpha_2^{\Lambda}+(\epsilon_S^{\Lambda})^2(1-\cos\alpha_2^{\Lambda})\right] + \mathcal{O}(\theta_{13}^{\Lambda}),$$

$$k_{23} = \frac{1}{2}\sin 2\theta_{23}^{\Lambda}\left(\cos^2\theta_{12}^{\Lambda}\frac{|m_2^{\Lambda}+m_3^{\Lambda}|^2}{|m_3^{\Lambda}|^2-|m_2^{\Lambda}|^2}+\sin^2\theta_{12}^{\Lambda}\frac{|m_1^{\Lambda}+m_3^{\Lambda}|^2}{|m_3^{\Lambda}|^2-|m_1^{\Lambda}|^2}\right)$$
$$\simeq \frac{\Gamma^{\Lambda}}{4}\sin 2\theta_{23}^{\Lambda}\left[1+\cos^2\theta_{12}^{\Lambda}\cos(\alpha_2^{\Lambda}-\alpha_3^{\Lambda})+\sin^2\theta_{12}^{\Lambda}\cos\alpha_3^{\Lambda}\right]+\frac{\rho_A^{\Lambda}}{2}\sin 2\theta_{12}^{\Lambda}\sin 2\theta_{23}^{\Lambda}, + \mathcal{O}(\theta_{13}^{\Lambda},\epsilon_S^{\Lambda}),$$

$$k_{13} = \frac{1}{4}\sin 2\theta_{12}^{\Lambda}\sin 2\theta_{23}^{\Lambda}\left(\frac{|m_2^{\Lambda}+m_3^{\Lambda}|^2}{|m_3^{\Lambda}|^2-|m_2^{\Lambda}|^2}-\frac{|m_1^{\Lambda}+m_3^{\Lambda}|^2}{|m_3^{\Lambda}|^2-|m_1^{\Lambda}|^2}\right)$$
$$\simeq \frac{\Gamma^{\Lambda}}{8}\sin 2\theta_{12}^{\Lambda}\sin 2\theta_{23}^{\Lambda}\left[\cos(\alpha_2^{\Lambda}-\alpha_3^{\Lambda})-\cos\alpha_3^{\Lambda}\right]+\mathcal{O}(\theta_{13}^{\Lambda},\epsilon_S^{\Lambda}),$$

$$|m_i^{\lambda}| = I_K|m_i^{\Lambda}|(1+\mu_i\Delta_\tau+\mathcal{O}(\Delta_\tau^2)),\ \ \mu_i=\mathcal{O}(1),$$

$$\alpha_i^{\lambda} = \alpha_i^{\Lambda}+a_i\Delta_\tau+\mathcal{O}(\Delta_\tau^2),\ \ a_2\simeq -4\frac{|m_1^{\Lambda}m_2^{\Lambda}|}{|m_2^{\Lambda}|^2-|m_1^{\Lambda}|^2}\cos 2\theta_{12}^{\Lambda}\sin^2\theta_{23}^{\Lambda}\sin\alpha_2^{\Lambda}.$$

CORRELATED CONSTRAINTS

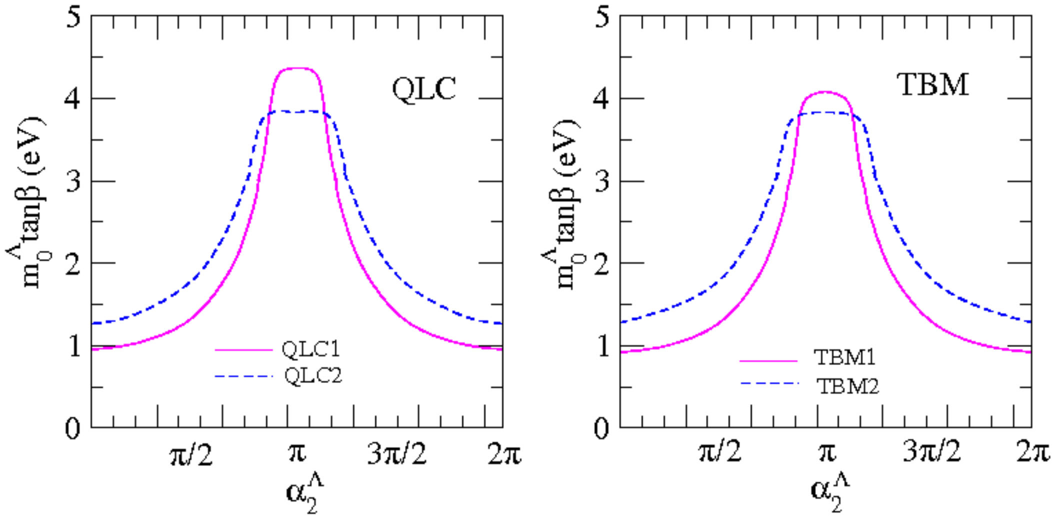
$$\begin{aligned}
\text{QLC1} &: \quad \theta_{ij} = \theta_{ij}^{\wedge} + k_{ij}^{\text{QLC1}}\Delta_\tau + \mathcal{O}(\theta_c^3, \theta_{13}^{\wedge}\Delta_\tau, \Delta_\tau^2), \\
\text{QLC2} &: \quad \theta_{ij} = \theta_{ij}^{\wedge} + k_{ij}^{\text{QLC2}}\Delta_\tau + \mathcal{O}(\cdots), \\
\text{TBM1} &: \quad \theta_{ij} = \theta_{ij}^{\wedge} + k_{ij}^{\text{TBM1}}\Delta_\tau + \mathcal{O}(\cdots), \\
\text{TBM2} &: \quad \theta_{ij} = \theta_{ij}^{\wedge} + k_{ij}^{\text{TBM2}}\Delta_\tau + \mathcal{O}(\cdots).
\end{aligned}$$

Use 3σ allowed ranges for neutrino mass and mixing parameters

$$\begin{aligned}
&7 \times 10^{-5}\ \text{eV}^2 < \delta m_S^2 < 9.1 \times 10^{-5}\ \text{eV}^2, \\
&1.7 \times 10^{-3}\ \text{eV}^2 < |\delta m_A^2| < 3.3 \times 10^{-3}\ \text{eV}^2, \\
&30^\circ < \theta_{12} < 39.2^\circ, \\
&35.5^\circ < \theta_{23} < 55.5^\circ, \\
&\theta_{12} < 12^\circ.
\end{aligned}$$

Tightest constraints from θ_{12}

$m_0 \tan\beta - \alpha_2$ exclusion regions for QLC(1,2), TBM(1,2)



$\alpha_2 \simeq \pi$, i.e. $m_1 \simeq -m_2$ **at all scales**

also supported by leptogenesis (Buchmiller, de Bari, Plumacher).

$$\theta_{12} > \theta_{12}^{\Lambda} = \begin{cases} 35.4^\circ & QLC1, \\ 32.4^\circ & QLC2, \\ \\ 35.26^\circ & TBM1, \\ 35.3^\circ & TBM2, \end{cases}$$

$$\Delta\theta_{23} \simeq \frac{\Delta\tau}{\rho_A^{\Lambda}} \sin 2\theta_{23}^{\Lambda}(1 - \cos 2\theta_{12}^{\Lambda} \cos \alpha_3^{\Lambda}).$$

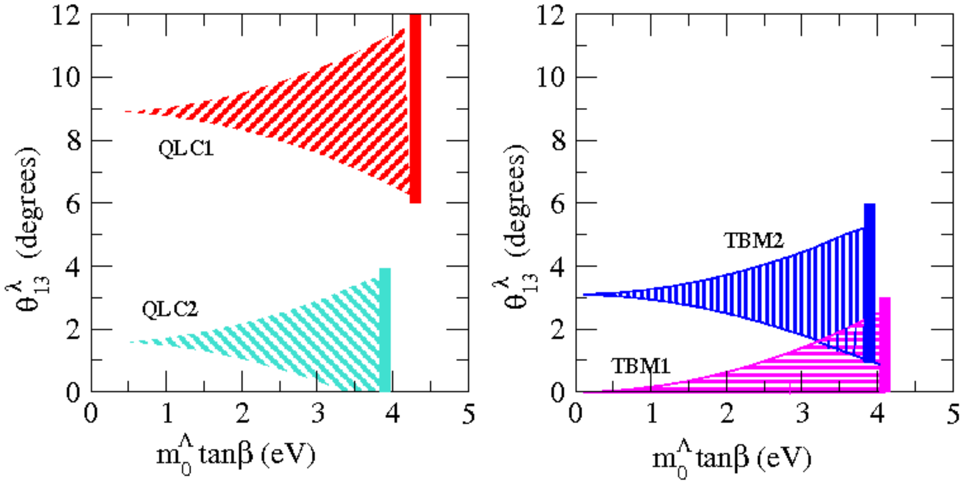
For $\substack{\text{normal} \\ \text{inverted}}$ **ordering,**

$$\theta_{23} \gtrless \theta_{23}^{\Lambda} = \begin{cases} 42.5^\circ & QLC1, \\ 42.7^\circ & QLC2, \\ \\ 45^\circ & TBM1, \\ 42.5^\circ & TBM2. \end{cases}$$

θ_{13} can discriminate among scenarios!

$$\theta_{13}^{\Lambda} = \begin{array}{ll} 8.9^\circ & QLC1, \\ 1.6^\circ & QLC2, \\ & \\ 0^\circ & TBM1, \\ 3.1^\circ & TBM2. \end{array}$$

$\Delta\theta_{13} \equiv \theta_{13}^{\Lambda} - \theta_{13}$ **depends on** $m_0 \tan\beta$**.**



$\theta_{13} < 6^\circ$ **will exclude QLC1.**

If $m_0^{\Lambda} \tan\beta < 2$ **eV** ($m_0^{\lambda} \tan\beta < 1.4$ **eV), TBM2 will be distinguishable from QLC2 and TBM1. Await results on** θ_{13} **from DCHOOZ & DBAY.**

CONCLUSIONS

- **High scale symmetries, QLC (1,2) and TBM (1,2) compatible with data for nontrivial $\alpha_{2,3}$ in the case of nonhierarchical neutrinos.**

- **Correlated constraints in the $m_0 \tan\beta - \alpha_2$ plane. Specifically need $\alpha_2 \simeq \pi$, i.e. $m_1 \simeq -m_2$, also supported by leptogenesis.**

- **Killing predictions**

$$\theta_{12} \geq 35.4^\circ \qquad QLC1,$$

$$\theta_{12} \geq 32.4^\circ \qquad QLC2,$$

$$\theta_{12} \geq 35.26^\circ \qquad TBM1,$$

$$\theta_{12} \geq 35.3^\circ \qquad TBM2.$$

$SNO3$?

- $\theta_{23} - \theta_{23}^{\Lambda}$ **correlated to normal or inverted ordering for each scenario.**

- θ_{13} **can in principle distinguish among the scenarios, DCHOOZ and DBAY ?**