Fluctuation and Other Probes of Quark Matter and Critical End Point

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in collaboration with C. Nonaka, B. Müller, S.A. Bass (PRL in press)

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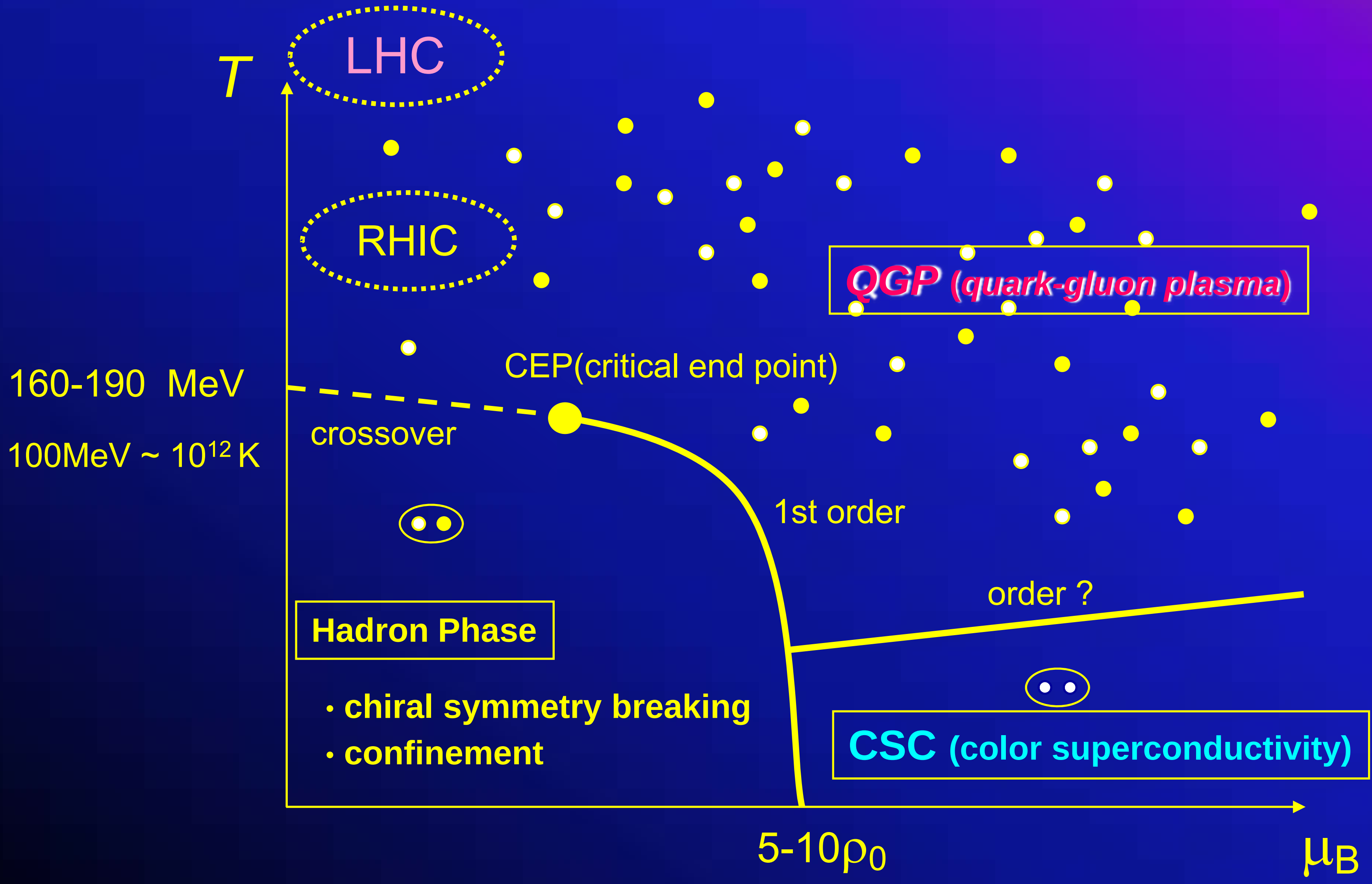
@ Goa Workshop

QCD Phase Diagram



20th Anniversary of CEP in QCD

Nuclear Physics **A504** (1989) 668–684
North-Holland, Amsterdam

CHIRAL RESTORATION AT FINITE DENSITY AND TEMPERATURE

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(Revised 24 April 1989)

M. Asakawa, K. Yazaki / Chiral restoration

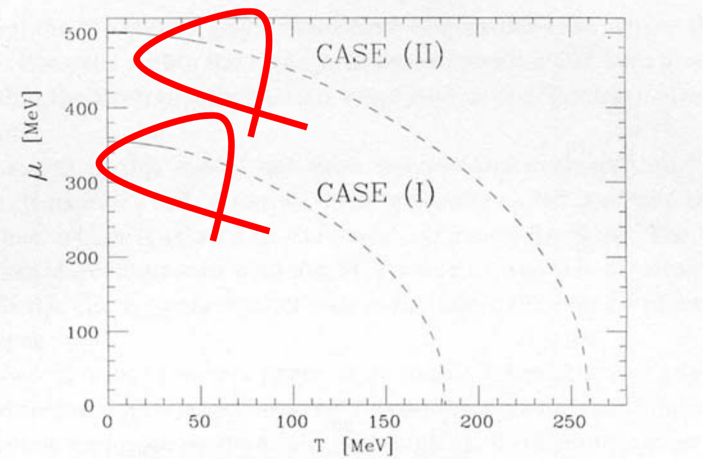
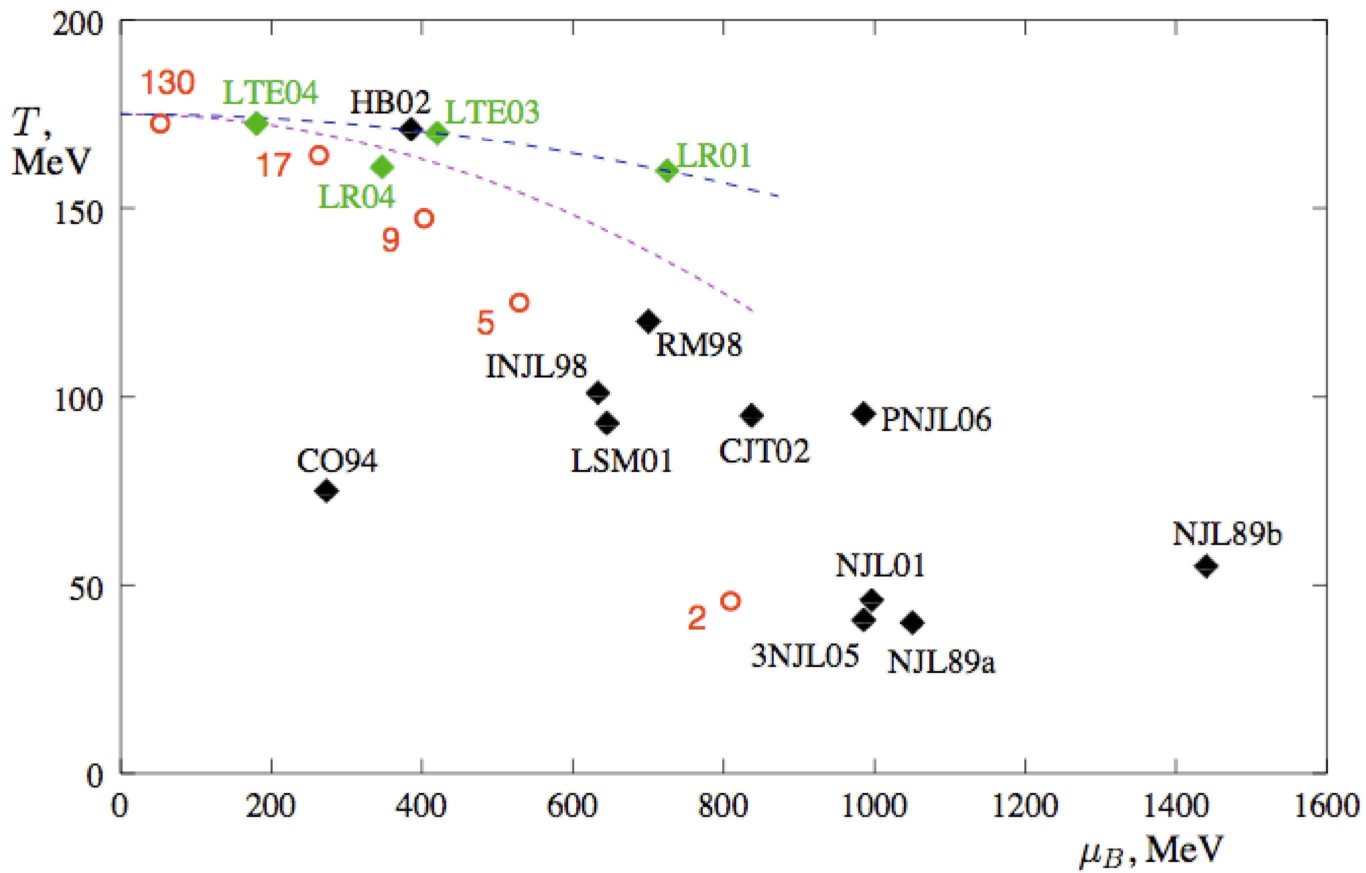


Fig. 8. The phase transition line in the cases (I) and (II).

Where is CEP, if any?



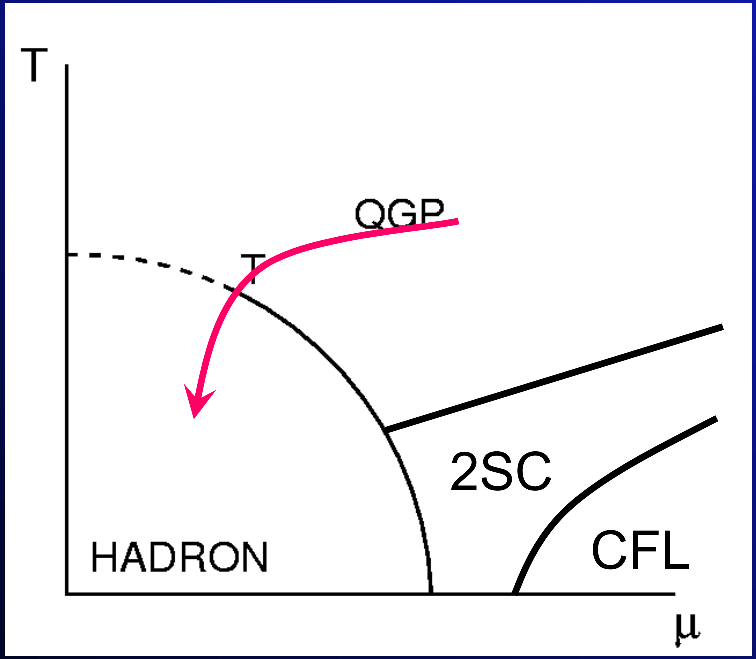
Stephanov,hep-lat/0701002

CEP = 2nd order phase transition, but...

CEP =
2nd Order Phase Transition Point

Divergence of Fluctuation
Correlation Length
Specific Heat ?

If expansion is adiabatic



even if the system goes right through the critical end point…

There is no conservation law that slows down the change of those quantities !

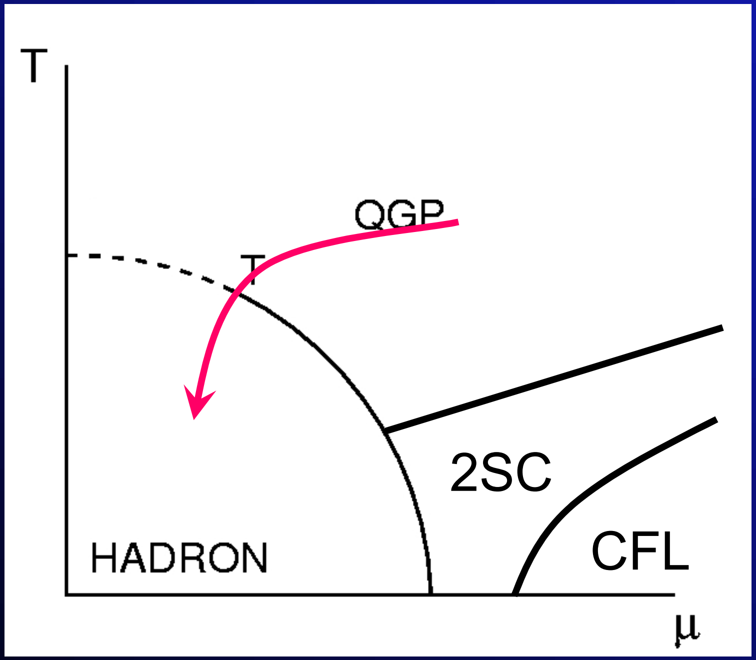
Subject to Final State Interactions

Fluctuation and Final State Interaction

Hadronic observables are, in general, easily modified in hadronic phase !

E.g., $\Delta^+ \rightarrow p + \pi^0$ (change of particle number)

$p + \pi^- \rightarrow n + \pi^0$ (change of charged particle number)



- Fluctuation of pion multiplicity
- Fluctuation of pion slope parameter T
- Fluctuation of N_+ / N_-

even if the system goes right through the critical point…

plus critical slowing down

Locally Conserved Charges

- Fluctuations of Locally Conserved Quantities

  In addition,

- Quantities sensitive to the microscopic structure of the dense matter

■ What satisfies the conditions, Conserved and Observable ?

i. Net Baryon Number
ii. Net Electric Charge
iii. Net Strangeness

i) and ii) are sensitive to the microscopic structure of matter

but

i) and iii) are difficult to measure

Hadron Gas vs. QGP

Hadron Gas Phase

QGP Phase

■ Electric Charge

- ~2/3 of hadrons carry electric charge ± 1

- only ~ 1/2 of d.o.f., i.e., quarks and anti-quarks carry electric charge $\pm\frac{1}{3}, \pm\frac{2}{3}$

⇨ **Larger Charge Fluctuation**

■ Baryon Charge

- Baryon charge is only carried by heavy and less abundant baryons

  But all of them carry baryon charge ± 1

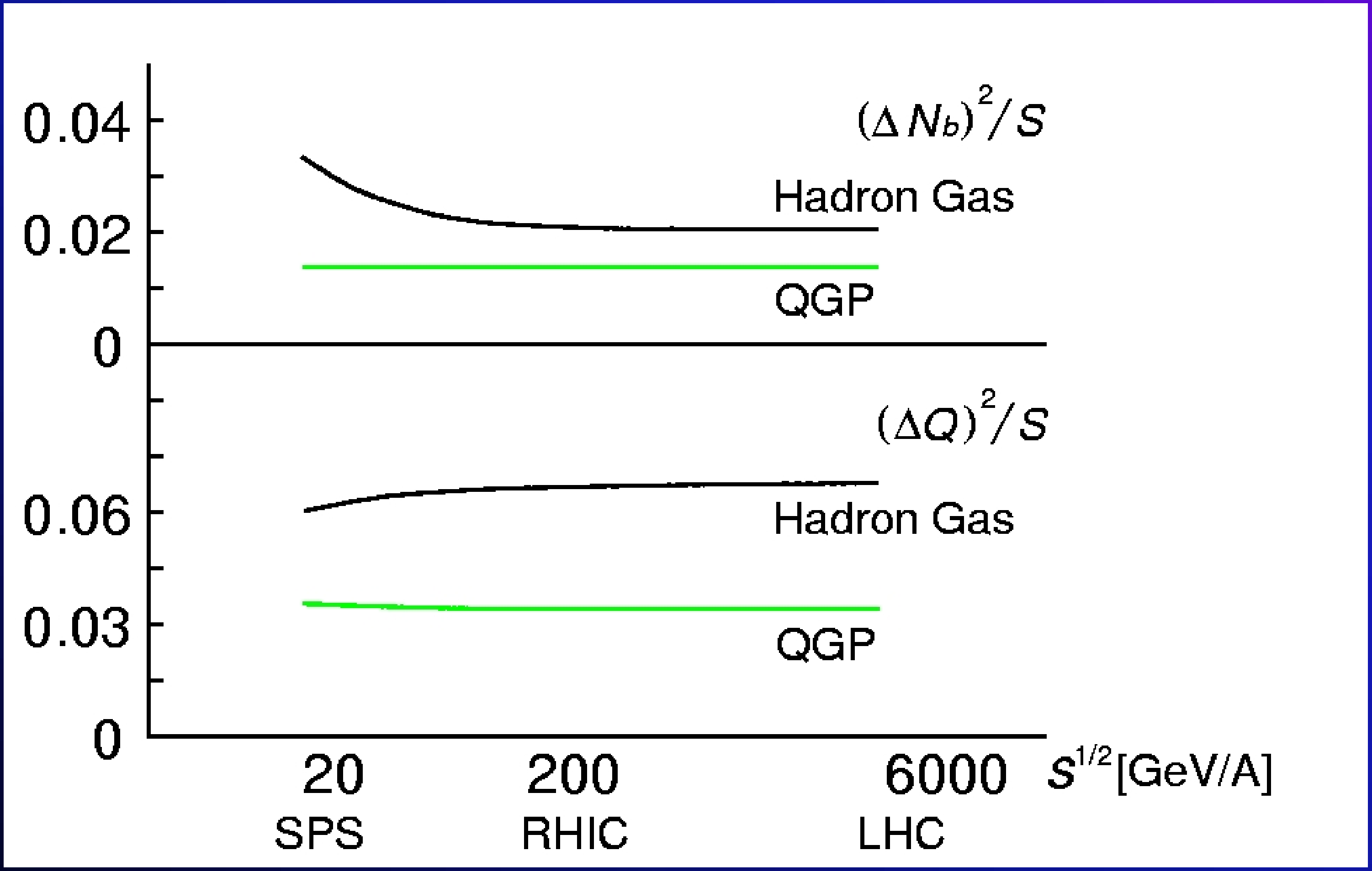
- All quarks and anti-quarks carry baryon charge

  But each of them carries baryon charge only $\pm\frac{1}{3}$

⇨ **Fluctuation is again larger in Hadron Gas**

M. Asakawa (Osaka University)

Fluctuations per Entropy



Heinz, Müller, and M.A., PRL (2000)
Also, Jeon and Koch, PRL (2000)

Charge Fluctuation at RHIC

- D-measure

$$D = 4\frac{\left\langle (\Delta Q)^2 \right\rangle}{N_{ch}}$$

Predictions

QGP phase

$$D \sim 1$$

Hadron phase

$$D \sim 4$$

($D \sim 3$: hadron resonance gas)

Experimental Value

$D = 2.8 \pm 0.05$ (STAR)

$D \sim 4$ (PHENIX)

Charge Fluctuation in ReCo

- Estimate with Quark Recombination Model

Białas, PLB(2002)
Nonaka, Müller, Bass, M.A., PRC (2005)

$$\langle \delta Q^2 \rangle \equiv \langle Q^2 \rangle - \langle Q \rangle^2$$
$$= \sum_i q_i^2 \langle n_i \rangle + \underbrace{\sum_{i,j} c_{ij}^{(2)} q_i q_j \langle n_i \rangle \langle n_j \rangle}_{}$$

→ correlation term

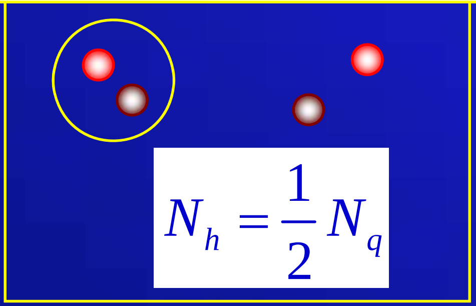
If correlation can be neglected (2 flavor case)

$$\langle \delta Q^2 \rangle = \frac{1}{9}(4N_u + 4N_{\bar{u}} + N_d + N_{\bar{d}}) = \frac{5}{18} N_q$$

$$N_{ch} = \frac{2}{3} N_h = \frac{1}{3} N_q$$

$$\boxed{D = 4\frac{\langle (\Delta Q)^2 \rangle}{N_{ch}} = 3.3}$$

Quark recombination



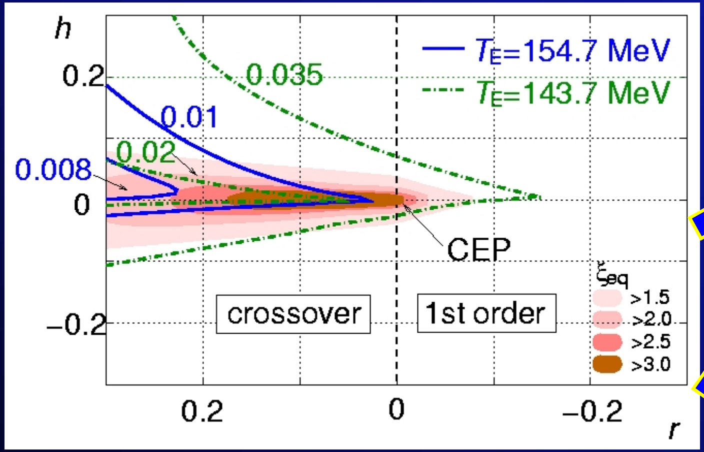
$$N_h = \frac{1}{2} N_q$$

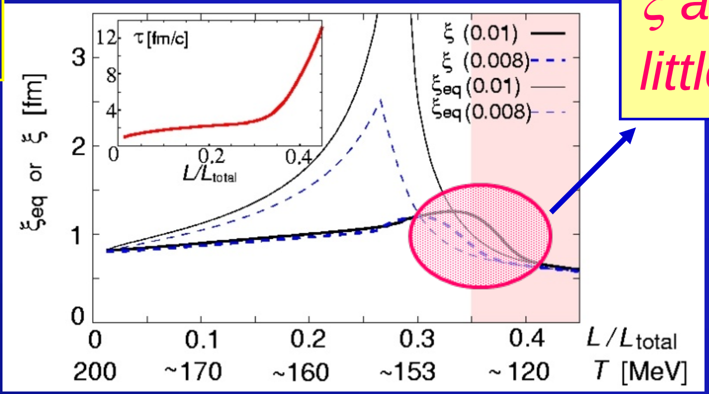
close to experimental data

Critical Slowing Down and Final State Int.

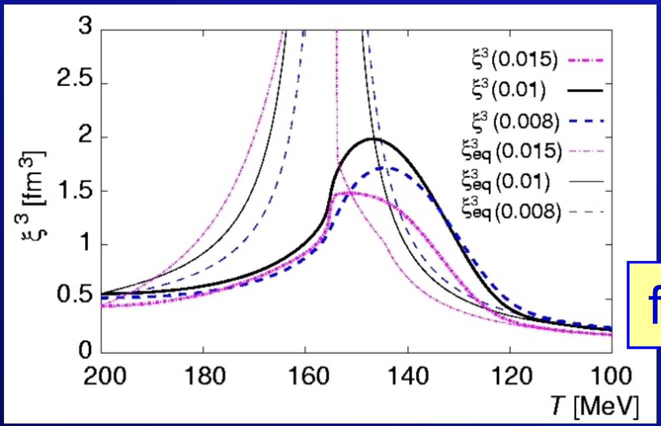
Furthermore,
critical slowing down limits the size of fluctuation, correlation length !

Time Evolution along given isentropic trajectories (n_B/s : fixed)





ξ and ξ_{eq}
little difference



fluctuation ~ ξ^3

$$\frac{d}{d\tau} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)]\left(m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)}\right)$$

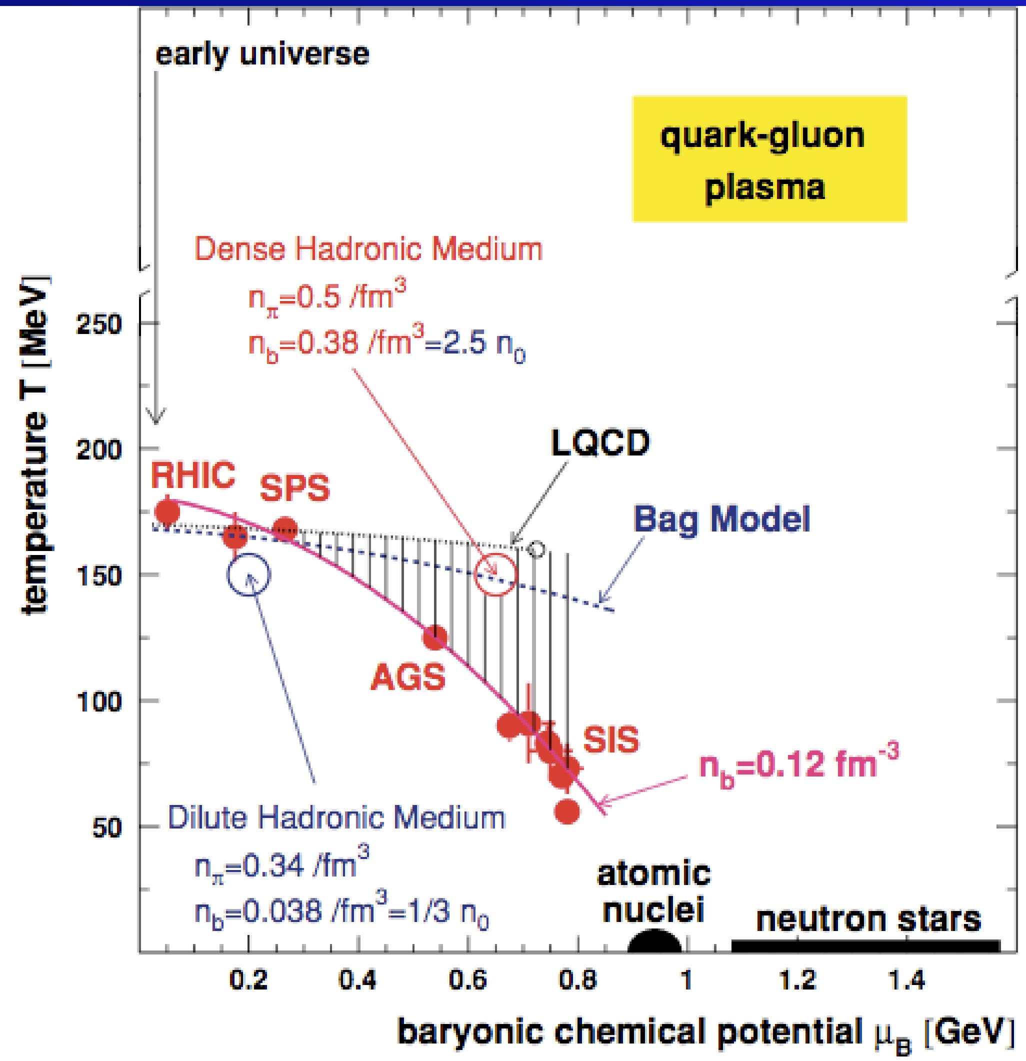
$$\Gamma[m_\sigma(\tau)] = \frac{A}{\xi_0}(m_\sigma(\tau)\xi_0)^z, \; m_\sigma(\tau) = \frac{1}{\xi(\tau)}$$

$z \approx 3$ Model H (Hohenberg and Halperin RMP49(77)435)

Principles to Look for Other Observables

■ We are in need of observables that are not subject to final state interactions

⇨ After Freezeout, no effect of final state interactions



Chemical Freezeout

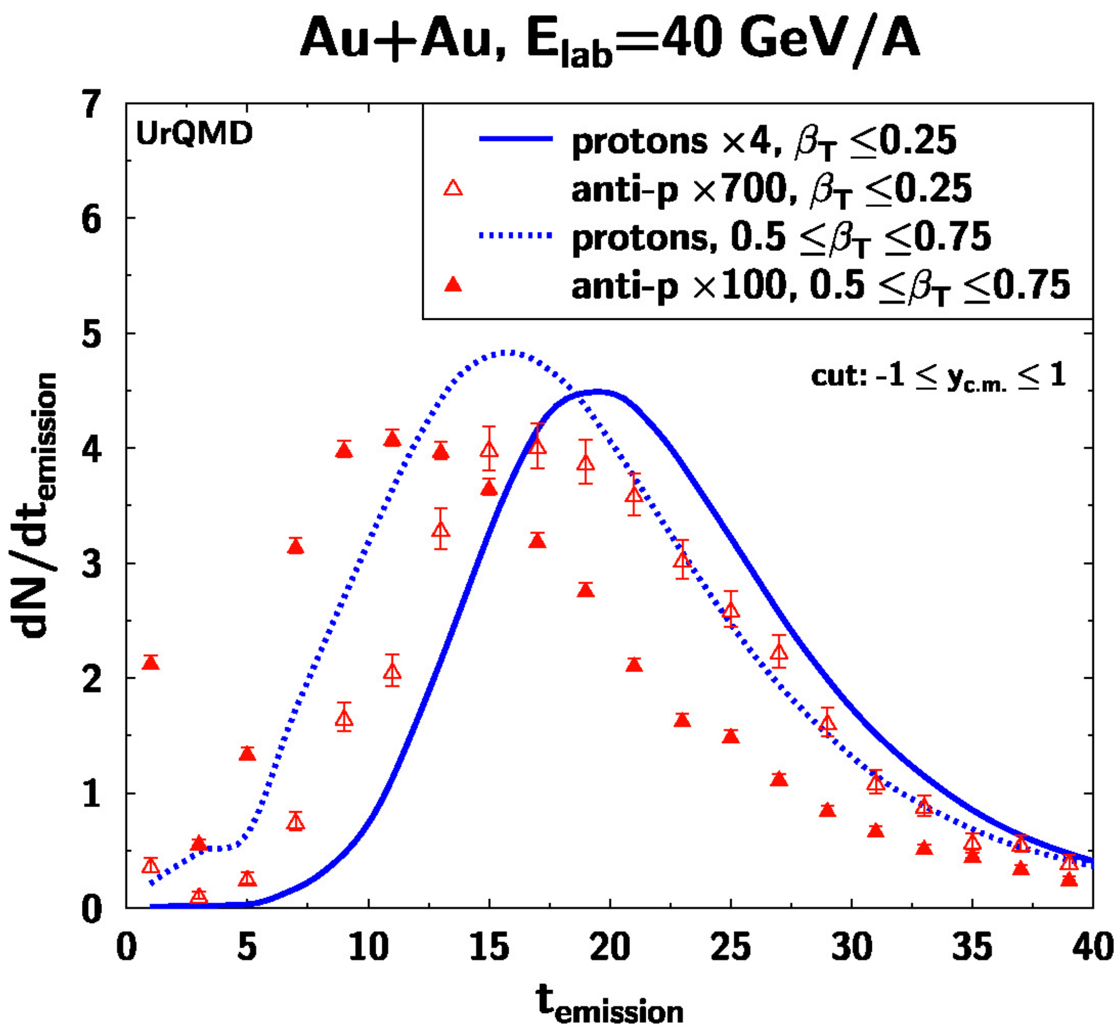
- usually assumed momentum independent
- but this is not right

chemical freezeout time: p_T (or y_T) dependent

✓ Larger p_T (or y_T), earlier ch. freezeout

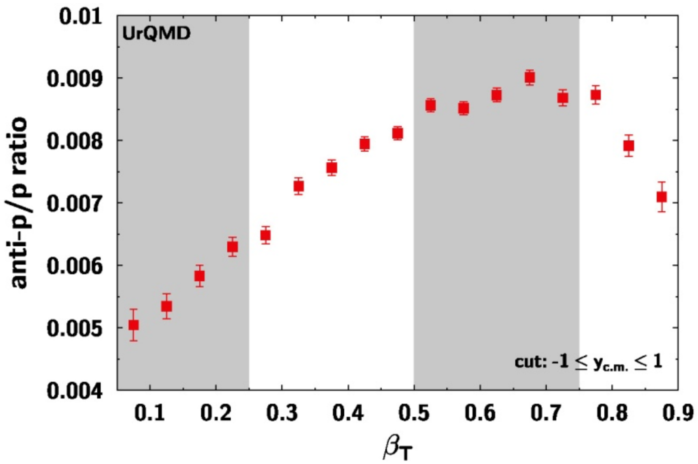
⇨ ***Principle I***

Emission Time Distribution



Emission Time

- Larger β_T, earlier emission
- No CEP effect (UrQMD)



Principle II

Universality:

QCD CEP belongs to the same universality class as 3d Ising Model

Lattice QCD at finite density: still in its infancy

For critical behavior: need to carry out $V \longrightarrow \infty$ limit

same universality class

QCD Critical End Point ⟺ *3d Ising Model End Point*

T and μ_B

$$(T,\mu_B) \longleftrightarrow (r,h)$$

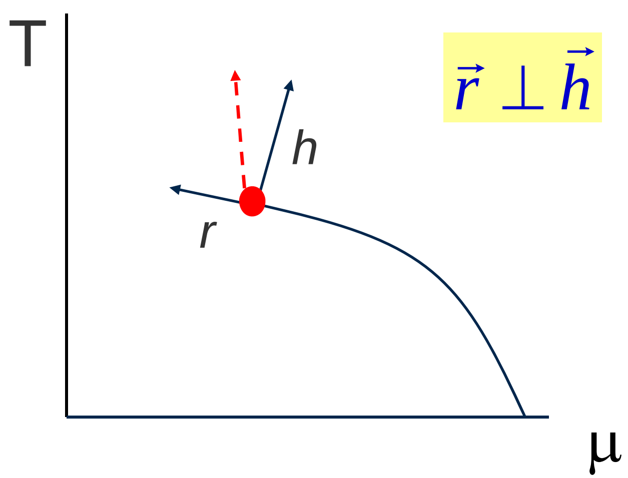
$$r = \frac{T - T_C}{T_C}$$

h : external magnetic field

What is not universal

- Further Assumptions
  - Size of Critical Region
    - No general universality
    - Lattice calculation: not yet $V \to \infty$ limit
    - Renormalization group analysis in Effective Models ?

    ⇨ need to be treated as an input, at the moment
  - Mapping



$\vec{r} \perp \vec{h}$

$\vec{r}$ || 1st order PT line
is *not* an assumption

EOS on Ising Side

■ Critical Behavior on Ising Side

- parametric representation

$$M = M_0 R^{\beta}\theta$$

$$r = R(1-\theta^2)$$

$$h = h_0 R^{\beta\delta} h(\theta) = h_0 R^{\beta\delta}(\theta - 0.76201\theta^3 + 0.00804\theta^5)$$

$$(R \geq 0, -1.154 \leq \theta \leq 1.154)$$

Condition for M_0 and h_0

$M(r,h) = 1$ at $(r,h) = (0,1)$

$M(r,h) = 1$ at $(r,h) = (-1,+0)$

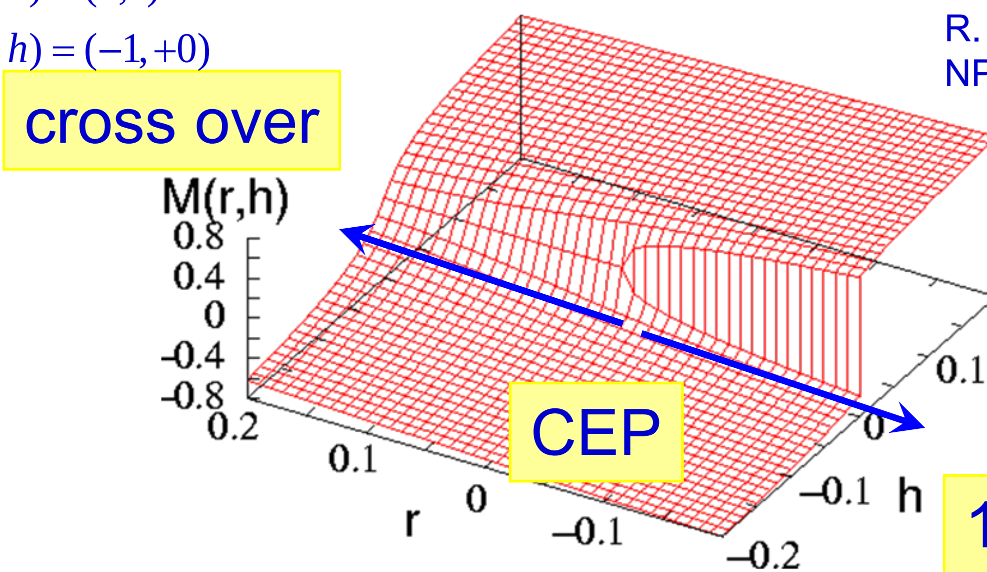
$$r = \frac{T - T_C}{T_C}$$

h : external magnetic field

$\beta = 0.326$

$\delta = 4.8$ (Critical Exponents)

R. Guida and J. Zinn-Justin, NPB486 (1997) 626



1st order

Singular Part + Non-singular Part

- Matching between Hadronic and QGP EOS
  - Entropy Density consists of *Singular* and *Non-Singular* Parts
    - **Only Singular Part** shows universal behavior
- Requirement:

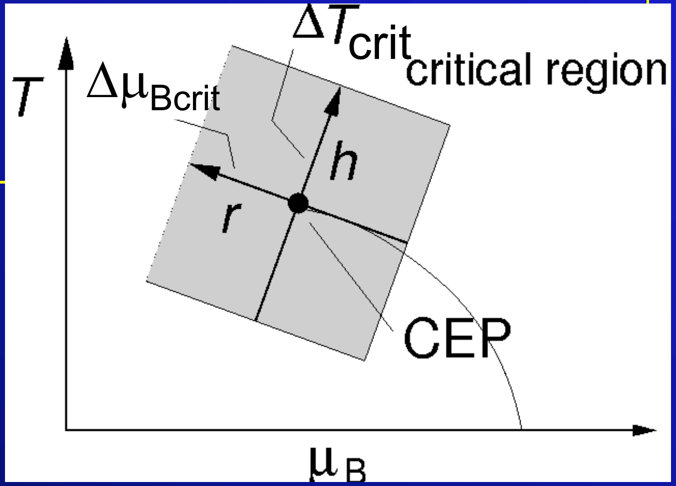
  reproduce both the singular behavior and known asymptotic limits

- Matched Entropy Density

$$s_{\text{real}} = (T, \mu_B) = \frac{1}{2}\left\{1 - \tanh\left[S_c(T, \mu_B)\right]\right\} s_H(T, \mu_B) + \frac{1}{2}\left\{1 + \tanh\left[S_c(T, \mu_B)\right]\right\} s_Q(T, \mu_B)$$

$s_H(T, \mu_B)$: Hadron Phase (excluded volume model)

$s_Q(T, \mu_B)$: QGP phase



- Dimensionless Quantity: S_c

$$S_c(T, \mu_B) = s_c(T, \mu_B)\sqrt{\left(\Delta T_{\text{crit}}\right)^2 + \left(\Delta \mu_{\text{crit}}\right)^2} \times D$$

D: related to extent of critical region

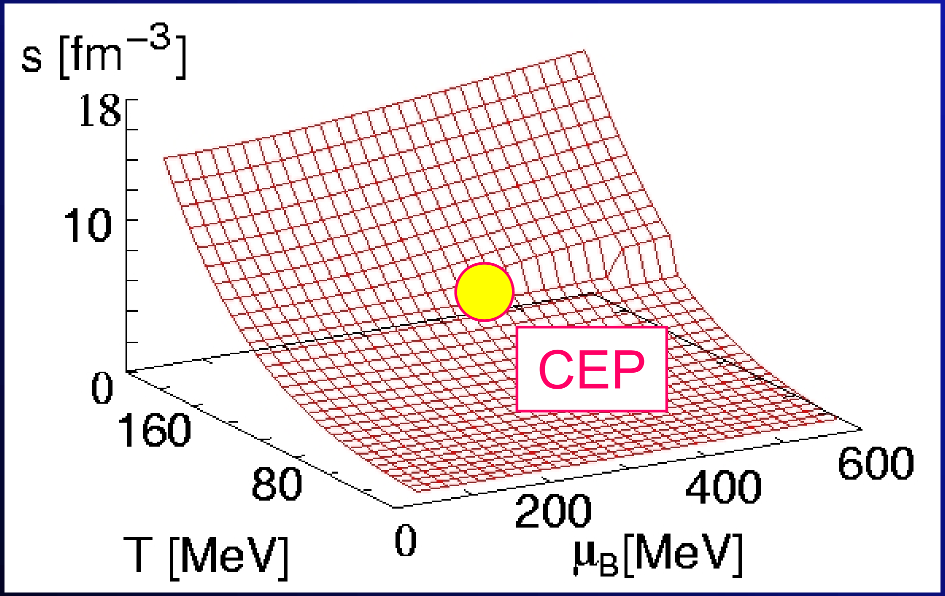
M. Asakawa (Osaka University)

Isentropic Trajectories

- In each volume element, Entropy (S) and Baryon Number (N_B) are conserved, as long as entropy production can be ignored (= when viscosities are small)

Isentropic Trajectories (n_B/s = const.)

An Example



Near CEP s and n_B change rapidly

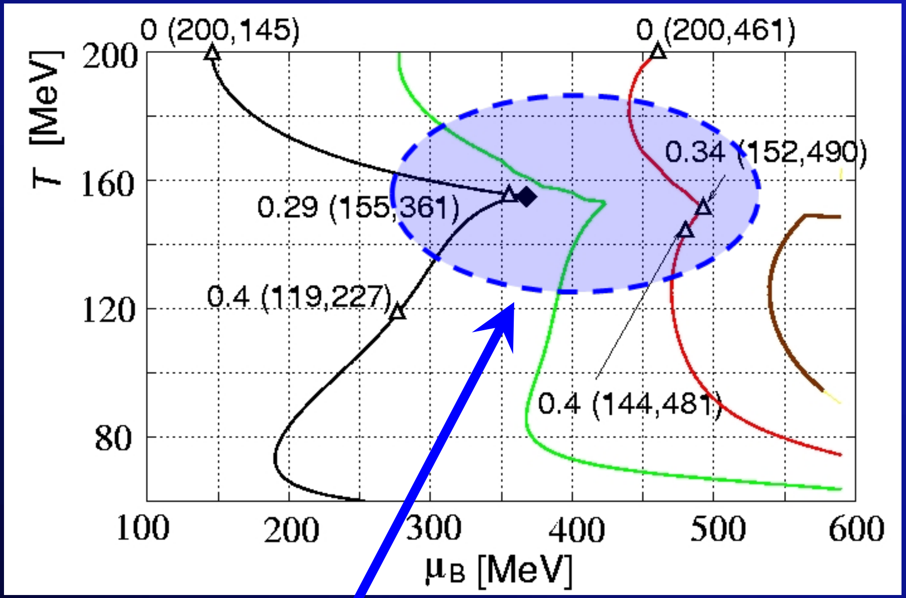
→ *isentropic trajectories show non-trivial behavior*

↕

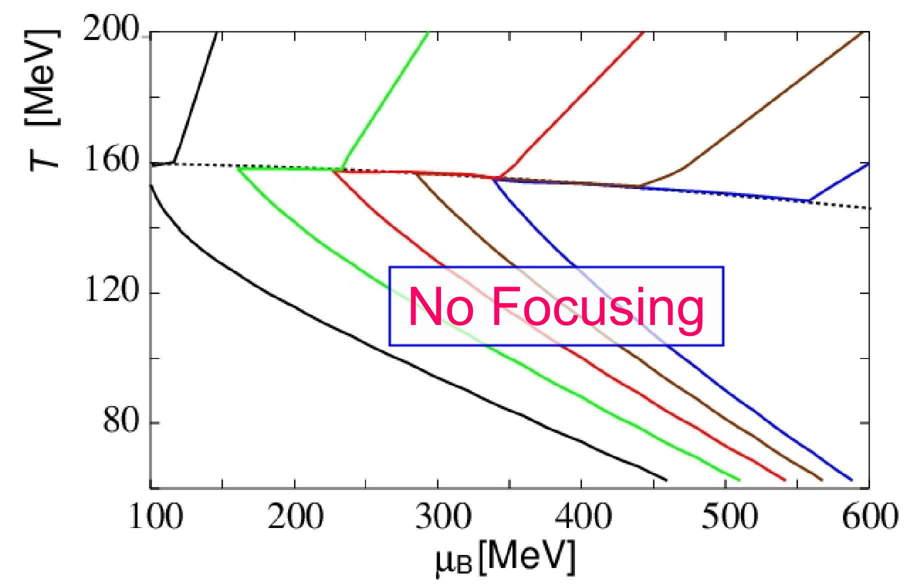
Bag Model EOS case

With Large Critical Region

with CEP



without CEP (EOS in usual hydro calculation)

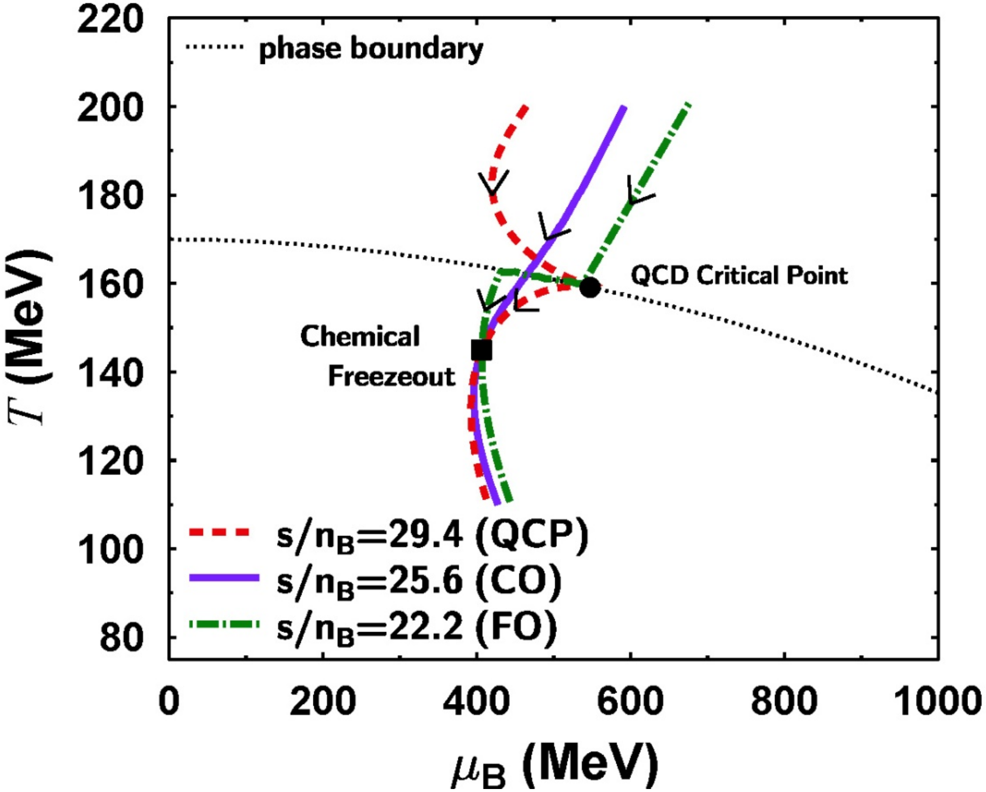


Focusing of Isentropic Trajectories

Excluded Volume Approximation
+ Bag Model EOS

used in most hydro calculations

Consequence



For a given chemical freezeout point, prepare three isentropic trajectories: w/ and w/o CEP

Along isentropic trajectory:

- FO, CO $\frac{\mu_B}{T}$ ↗
- QCP $\frac{\mu_B}{T}$ →

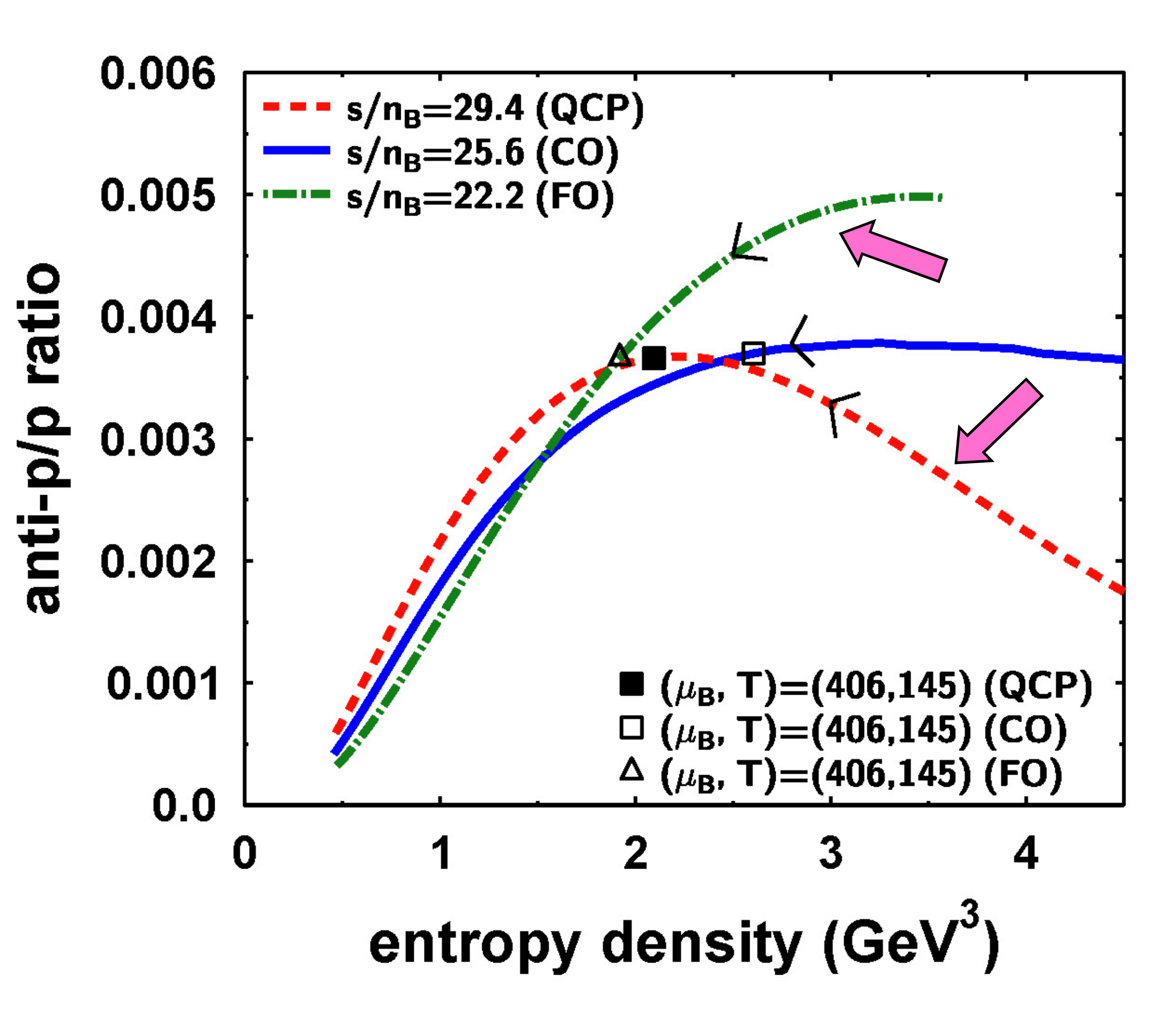
Principle I

As a function of $p_T(y_T)$:

- FO, CO $\frac{\mu_B}{T}$ ↘
- QCP $\frac{\mu_B}{T}$ ↗

➡ $\bar{p}/p$ ratio : near CEP steeper

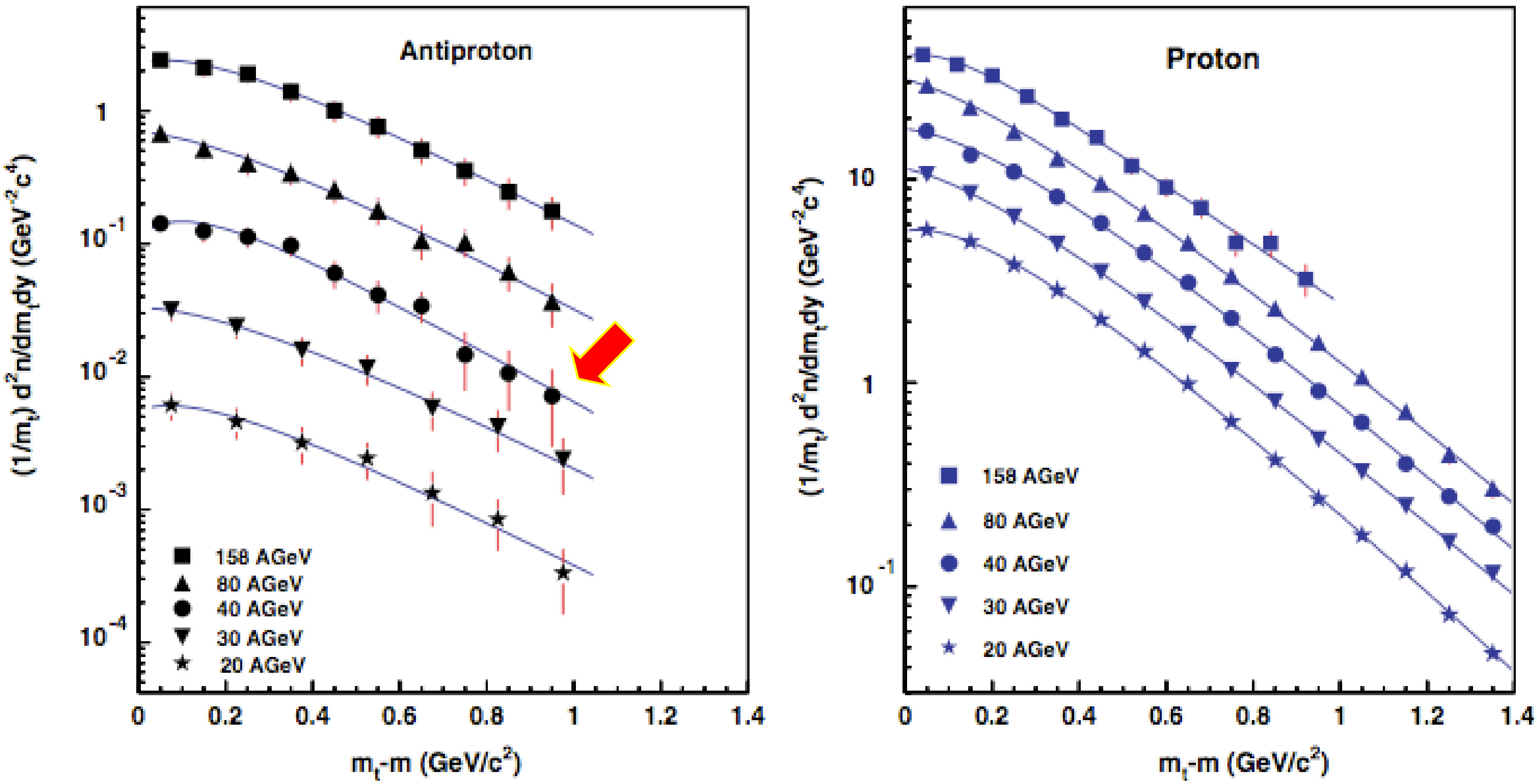
Evolution along Isentropic Trajectory



$$\bar{p}/p \sim \exp\left(-\frac{2\mu_B}{T}\right)$$

with CEP
steeper $\bar{p}$ spectra at high P_T

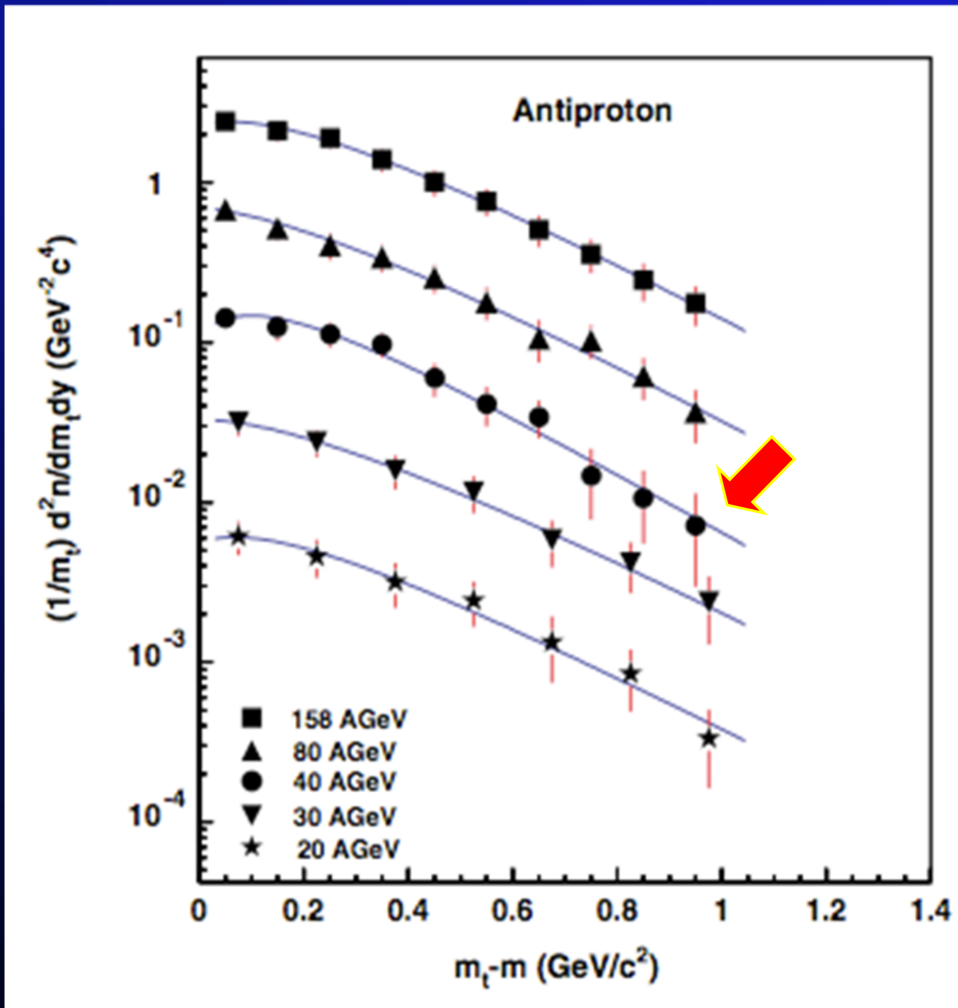
Effect on Spectra ?



steeper $\bar{p}$ spectra at high P_T

NA49, PRC73, 044910(2006)

Result of One Temperature Fit



NA49, PRC73, 044910(2006)

	$E_{\rm beam}$ (A GeV)	dn/dy	T (MeV)	$\langle m_t\rangle - m$ (MeV/c^2)
$\bar{p}$	158	1.66 ± 0.17	291 ± 15	384 ± 19
	80	0.87 ± 0.07	283 ± 30	385 ± 41
	40	0.32 ± 0.03	246 ± 35	355 ± 51
	30	0.16 ± 0.02	290 ± 45	395 ± 60
	20	0.06 ± 0.01	279 ± 64	394 ± 60
p	158	29.6 ± 0.9	308 ± 9	413 ± 13
	80	30.1 ± 1.0	260 ± 11	364 ± 16
	40	41.3 ± 1.1	257 ± 11	367 ± 16
	30	42.1 ± 2.0	265 ± 10	362 ± 14
	20	46.1 ± 2.1	249 ± 9	352 ± 13

- Only one experimental result for $\overline{p}$ slope
- Still error bar is large

Summary

- Two Principles:

  i) Chemical Freezeout is $p_T(\beta_T)$ dependent

  ii) Isentropic Trajectory behaves non-trivially near CEP (focusing)

⇒ $\bar{p}/p$ ratio behaves non-monotonously near CEP

> Information on the QCD critical point:
> such as location, size of critical region, existence…

- We then made a data search
  - turned out NA49 $\bar{p}$ data shows non-trivial behavior around 40 GeV/A
  - still error bar is large, *finer energy scans at SPS, FAIR, RHIC*: desirable
- Effect on Flow ?

  c_s changes differently from the case with EOS used in usual hydro cal.

  (3D hydro cal. with CEP + UrQMD: C. Nonaka in progress)