Finite-temperature Field Theory

Aleksi Vuorinen

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Initial Conditions in Heavy Ion Collisions
Goa, India, September 2008
Outline

Phase diagram of QCD
  Tools for finite-temperature QCD
  The phase diagram
  The phases of QCD

The deconfinement transition
  Preliminaries
  Pure Yang-Mills theory
  Dynamical quarks

Perturbative thermal QCD
  Basic thermal field theory
  IR Problems
  Effective Theory Approach

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Summary
Setting the stage

- Today, specialize to equilibrium thermodynamics of QCD, keeping $N_c$ and quark masses unfixed

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4} F_{\mu \nu}^a F_{\mu \nu}^a + \sum_f \bar{\psi}_f (\hat{D} + i m_f) \psi_f,$$

$$F_{\mu \nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c,$$

$$D_\mu \equiv \partial_\mu - ig A_\mu^a T^a, \quad \text{Tr } T^a T^b = \frac{1}{2} \delta^{ab}$$

- Want to know (among other things):
  - Structure of QCD phase diagram: Location of transition lines, critical points,...
  - Properties of deconfinement transition
  - Equation of state
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Tools available

- **Lattice QCD**
  - Only non-perturbative first principles tool: Most trustworthy whenever available
  - Limitations: $T \sim T_c$, small $\mu$ (sign problem!), realistic quark masses,...
  - Limited use with real time phenomena

- **Weak coupling methods**
  - Asymptotic freedom $\Rightarrow$ At asymptotically high $T$ or $\mu$, perturbation theory guaranteed to "converge"
  - Can probe entire parameter space of theory ($\mu$, $N_c$, $N_f$, quark masses); even real time no problem
  - Limitation: Expansions well behaved only at $T \gg T_c$ ($\mu \gg \mu_c$) — excludes most interesting regime
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Effective theories

- Ideology: Identify relevant degrees of freedom, and study simpler theory
- Chiral lagrangian, EQCD, Polyakov loop models, $Z_N$QCD,...
- Seldom obtain quantitatively new insight independent of above two methods

Gauge-gravity duality

- Conjectured duality between $SU(N_c) \mathcal{N} = 4$ Super Yang-Mills in 4$d$ and type IIB string theory in $AdS_5 \times S_5$
- Enables analytic calculations in strongly coupled large $N_c$ non-Abelian gauge theory — including real time observables
- Limitations: $\mathcal{N} = 4$ SYM $\neq$ QCD, having to take $N_c, \lambda \rightarrow \infty,...$
- Immensely active field at the moment; new progress being made every day
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Phase diagram

Most fundamental question to answer when studying equilibrium thermodynamics: What are the phases of your theory as a function of its parameters?

- Phase transitions characterized by jumps (continuous or discontinuous) in thermodynamic quantities

- Universality arguments (based on symmetries, etc.) useful, but ultimately a numerical problem
  - Lattice QCD invaluable at small $\mu$ — compare to situation at high $\mu$, small $T$

- Answer highly dependent of $N_c, N_f, \text{quark masses}$; here try to get as close to physical case as possible
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Tools for finite-temperature QCD
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The phases of QCD

1. Small $T$ and $\mu_B$: Hadron gas
   ▶ Dilute gas of color neutral bound states
   ▶ Thermodynamic description in terms of resonance gas model

2. $T \lesssim 10$ MeV, $308$ MeV $< \mu_B \lesssim 1$ GeV: Hadron liquid (including nuclear superfluid)
   ▶ Beyond nuclear ground state, hadron gas becomes dense, liquid-like
   ▶ Separated from hadron gas by first order (water-vapor-like) transition line, ending at 2nd order critical point
3. $T \gtrsim 180$ MeV or $\mu_B \gtrsim 1$ GeV, $T \gtrsim 100$ MeV: Quark gluon plasma (QGP)
   - Description in terms of deconfined quarks, gluons; much more to follow

4. Asymptotically high $\mu_B$, $T \lesssim 100$ MeV: Color-flavor locked (CFL) phase
   - Single gluon exchange provides attractive coupling between quarks on Fermi surface $\Rightarrow$ BCS pairing
   - Condensate invariant under simultaneous transformation in color and flavor space, hence CFL
   - Description via effective theories (NJL models, ...)

5. $1$ GeV $\lesssim \mu_B \lesssim ?$, $T \lesssim 100$ MeV: Non-CFL superconductor
   - Precise nature still undetermined (kaon condensation?)
   - Problem: Not many first principles method available
In the rest of the talk, look at the high $T$, small $\mu$ region in more detail: Quark gluon plasma
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Elementary considerations

▶ Old qualitative argument: When density of nuclear matter exceeds hadron density, nucleons start overlapping
  ▶ Asymptotic freedom $\Rightarrow$ Description of system in terms of quarks, gluons
▶ Hard to study experimentally — weak coupling methods only available in asymptopia
▶ Lattice QCD most important quantitative tool
  ▶ Reveals strong dependence on $N_c, N_f, m_q$
▶ Let’s fix $N_c = 3$ and start from the cleanest case...
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Pure Yang-Mills theory: The center symmetry

- Full gauge symmetry of SU(3) Yang-Mills theory
  \[ A_\mu(x) \to s(x)(A_\mu(x) + i\partial_\mu)s(x)\dagger, \quad s(x) \in SU(3) \]
  \[ s(x + \beta\hat{e}_t) = z s(x), \quad z \in Z(3) \]

- The Wilson line transforms as a Z(3) fundamental
  \[ \Omega(x) \equiv \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_0(\tau, x) \right] \]
  \[ \text{Tr} \Omega(x) \to z \text{Tr} \Omega(x) \]

- \( \Omega \) order parameter for deconfinement transition
  - \( |\langle \text{Tr} \Omega(x) \rangle| = e^{-\beta \Delta F_q(x)} \)
  - Non-zero value signals existence of free color charges
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In deconfined phase, effective potential for $\Omega$ has degenerate minima $\Omega_{\text{min}} \sim e^{i2\pi n/3}$, $n \in \{0, 1, 2\}$

- Tunnelings between different vacua important near $T_c$
- At (1st order) phase transition quadruple point with phase coexistence with the confining one
Inclusion of dynamical quarks

- Dynamical ($m_q < \infty$) quarks break $Z(3)$ symmetry explicitly
  - Wilson line no longer a strict order parameter for transition
  - Jump (rapid change) in $|\langle \text{Tr} \Omega(x) \rangle|$ nevertheless still visible in phase transition region
- With $N_f$ flavors of (nearly) massless quarks, chiral symmetry explicit at high $T$
  - At smaller temperatures, spontaneously broken via appearance of quark condensates
  - Chiral and deconfinement transitions closely related
- Huge lattice effort in determining phase diagram as function of quark masses
  - Current understanding: Physical transition cross-over at $\mu = 0$ — first order line starts from critical point at $(T, \mu) \approx (170,290)$ MeV
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Recap of thermal field theory

- For equilibrium thermodynamics, want to compute most importantly the partition function

\[ p = \lim_{V \to \infty} \frac{T}{V} \ln Z, \]

\[ Z \equiv \text{Tr} \exp \left[ - \frac{\mathcal{H} - \sum_f \mu_f N_f}{T} \right] \]

- Recipe of perturbation theory: Expand functional integral in \( g \) to obtain loop expansion (in vacuum diagrams)

- Main difference to \( T = 0 \): Space-time now \( \mathbb{R}^3 \times S^1 \)
Recap of thermal field theory

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IR Sector of QCD

Immediate problem: Strict loop expansions of thermodynamic quantities IR divergent at three-loop order

- To obtain finite result, need to resum infinite classes of diagrams \( \mathcal{O}(g^3) \) term for pressure

- Need detailed understanding of energy scales contributing to the problem
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Energy Scales in Hot QCD

At asymptotically high $T$, with $g \ll 1$, clear separation of three length scales:

- $\lambda \sim 1/(\pi T)$: Wavelength of thermal fluctuations, inverse effective mass of non-static field modes ($p_0 \neq 0$)
  - $n(E)g^2(T) \sim g^2(T) \Rightarrow$ Contributes perturbatively at high $T$

- $\lambda \sim 1/(gT)$: Screening length of static color electric fluctuations, inverse thermal mass of $A_0$
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- Strict loop expansions of thermodynamic quantities IR divergent
  - Solution: Resum contributions of scales $gT$ and $g^2T$
  - Get terms non-analytic in $g^2$ in expansions

- Two competing (and completing) approaches: Direct 4d resummations and effective 3d theories

- Resummed perturbation theory systematized with hard thermal/dense loops (HTL/HDL) (Braaten, Pisarski)
  - Reorganize perturbation expansions by treating hard and soft scales on separate footing
  - Not limited to static quantities: Classical result gluon/quark damping rates (Braaten, Pisarski)
  - In equilibrium QCD, notice improved convergence of expansions; extensive work by Andersen, Braaten & Strickland and Blaizot, Iancu & Rebhan
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Effective Theories and Hot QCD

- Scale hierarchy $\Rightarrow$ Natural to integrate out massive (non-static) modes (Appelquist, Pisarski)
  - Effective description accurate for $\lambda \gtrsim 1/(gT)$
- Integrate out heavy modes to obtain 3d effective theory EQCD for static bosonic dof's (Braaten, Nieto)

$$\mathcal{L}_{\text{EQCD}} = g^{-2}_E \left\{ \frac{1}{2} \text{Tr} F_{ij}^2 + \text{Tr} [(D_i A_0)^2] \right\} + m^2_E \text{Tr}(A_0^2) + \lambda_E \text{Tr}(A_0^4) \right\} + \delta \mathcal{L}_E,$$

$$g_E \equiv \sqrt{T} g, \ m_E \sim gT, \ \lambda_E \sim g^2$$

- Parameters available through comparison of long distance correlators in EQCD and full QCD
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Effective Theories and Hot QCD

- EQCD valuable in reorganizing perturbation theory
  - No need for resummations in full theory
- IR sensitive sector described by EQCD: Non-perturbative contributions available through simulations in a 3d theory
- Near $T_c$ theory unphysical due to loss of $Z(3)$ symmetry
  - Can be cured by integrating in some heavy dof’s (AV, Yaffe), resulting in a physical phase diagram (Kurkela)
  - Breaking of the symmetry automatic in perturbative expansions; no effect at high enough $T$
- Finite $\mu$ has only minor effects as long as $m_D \ll T$ (Hart, Laine, Philipsen; Ipp, Kajantie, Rebhan, AV)
  - One new operator generated to Lagrangian, parameter matching affected
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  - No need for resummations in full theory
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Recent Applications of Dim. Red. Approach

- Equation of state (Kajantie, Laine, Rummukainen, Schröder)
- $\mu$-dependence of $p$ & quark number susceptibilities (AV)
- Spatial ’t Hooft loop (Giovannangeli, Korthals Altes)
- Two-loop gauge coupling at high $T$ (Laine, Schröder)
- Correlation lengths (Hart, Laine, Philipsen; Laine, Vepsäläinen)
- Spatial string tension (Laine, Schröder)
- Standard model pressure (Gynther, Vepsäläinen)
- Four-loop pressure of $\phi^4$ theory (Gynther, Laine, Schröder, Torrero, AV)
Example: Spatial String Tension

Laine, Schröder (2005): Compute in EQCD

\[ \sigma_s \equiv - \lim_{R_1 \to \infty} \lim_{R_2 \to \infty} \frac{1}{R_1 R_2} \ln W_S(R_1, R_2) \]

to 2-loop order and compare to full theory lattice data.
Example: Spatial String Tension

\[ \frac{T_c}{\Lambda_{\overline{MS}}} = 1.10 \ldots 1.35 \]

1-loop

2-loop

4d lattice, \( N_t = 8 \)

Aleksi Vuorinen, CERN

Finite-temperature Field Theory, Lecture 3
Outline

Phase diagram of QCD
  Tools for finite-temperature QCD
  The phase diagram
  The phases of QCD

The deconfinement transition
  Preliminaries
  Pure Yang-Mills theory
  Dynamical quarks

Perturbative thermal QCD
  Basic thermal field theory
  IR Problems
  Effective Theory Approach

Summary
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- At present, limited tools available for studying equilibrium thermodynamics of QCD
  - Lattice QCD most fundamental, weak coupling methods most versatile
- QCD has rich phase structure, now largely determined through lattice studies
  - Open questions: Nature/location of critical endpoint, non-CFL superconducting phases, value of $T_c$ for deconfinement / chiral transitions,...
  - Deconfinement transition believed to be cross-over, critical point at $(T, \mu) \approx (170, 290)$ MeV
- At high $T$, perturbative QCD suffers from IR problems
  - Long distance properties of QCD describable through dimensionally reduced effective theory