Some Results from Causal Viscous Hydrodynamics

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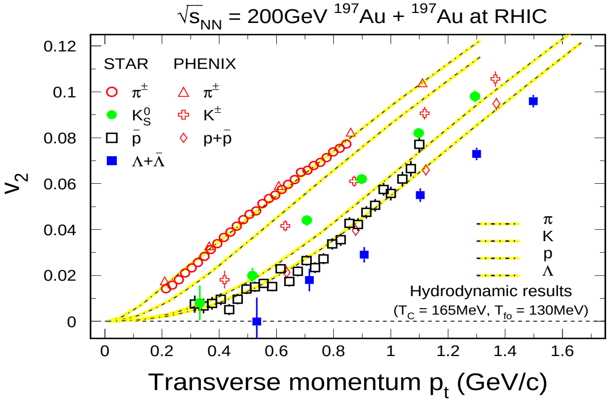
Outline

- Introduction — Discovery(?) of an almost perfect liquid & "success" of ideal hydro at RHIC
- What is the need of viscous hydro?
- Basic idea of causal dissipative hydro
- Shear & bulk viscosity vs temperature
- Some results of causal viscous hydro & other recent developments
- Our work on causal viscous hydro
- What remains to be done?

Introduction

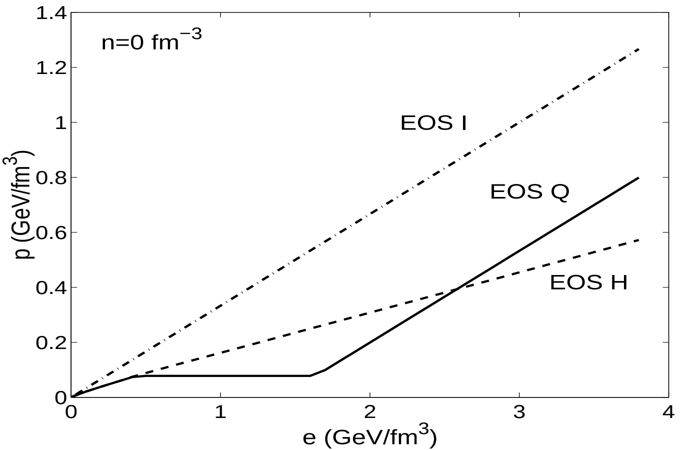
- RHIC (BNL): Claims of formation of an almost perfect liquid — strongly coupled QGP (sQGP).
- Evidence:
  - "large" elliptic flow $v_2(p_T) \Rightarrow$ (local) equilibration of matter.
  - jet quenching $\Rightarrow$ medium is dense & coloured ($\therefore$ partonic, not hadronic).
- Corroborative Evidence: constituent quark scaling v_2/n_q vs $KE_T/n_q \Rightarrow$ flow is developed at the quark level. Hadronization occurs by quark recombination.

"Large" Elliptic Flow — Success of Ideal Hydro

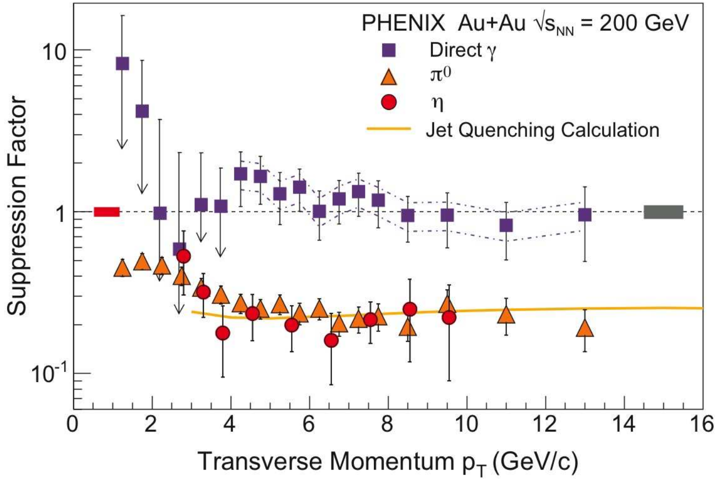


Minimum-bias data, Oldenberg (STAR), nucl-ex/0412001.
Solid lines: Huovinen et al. (2001, -04), EoS=Q. Mass ordering.

Three equations of state

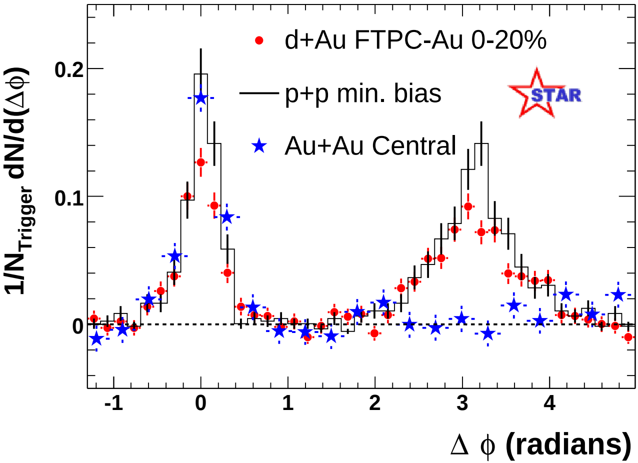


EOS H: interacting gas of all known resonances ($<$ 2 GeV).
EOS I: ideal gas of massless q, g. EOS Q: the two EOS matched via Maxwell construction: 1st-order phase transition.
... Kolb et al. hep-ph/0006129.



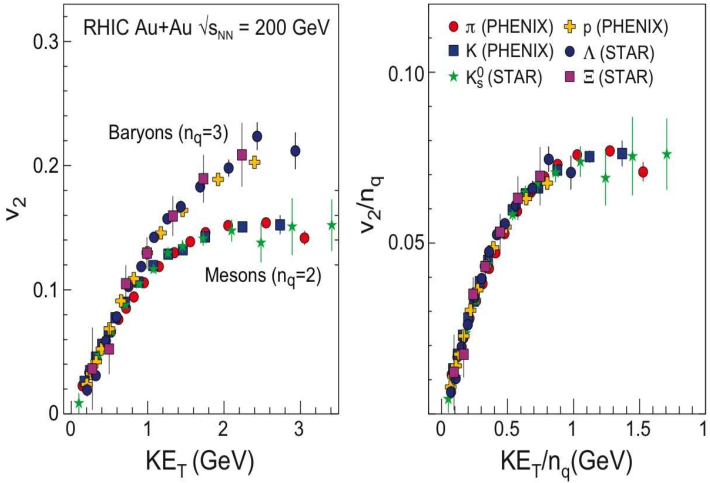
R_{AA} vs p_T. Suppression is a final-state effect. Energetic partons lose energy as they traverse the medium. Calc. by Gyulassy, Levai, Vitev. $dN^g/dy = 1150$.

Jet Quenching



Dijet fragment azimuthal correlations [nucl-ex/0306024].

Constituent Quark Scaling



Left: 2 distinct branches. Right: universal curve.

Hydrodynamics — what, where, why, how

- A set of coupled partial differential equations for n, ϵ, p, u^μ etc. In addition: transport coefficients & relaxation times also occur.
- Calc. of p_T spectra and elliptic flow v_2.
  Also calc. of jet quenching, J/ψ melting, thermal γ, ℓ^2, etc.
  Thus hydro plays a central role in modeling rhics.
- Powerful technique: Given initial conditions & EoS, hydro predicts evolution of the matter.

  Limitation: applicable at or near (local) thermodynamic equilibrium only.

Hydro calculations for rhics

- Earliest calculations: ideal/perfect/nondissipative/equilibrium/inviscid hydro.
- In broad agreement with RHIC data on p_T spectra & $v_2(p_T \lesssim 1.5$ GeV/c). Mass ordering of $v_2(p_T)$ for various hadrons was also reproduced correctly.
- What then is the need of viscous hydro?

What is the Need of Viscous Hydro?

- (Ideal) "hydro models seem to work for min-bias data but not for centrality-selected π & $\bar{p}$ data"
  ... STAR, Phys. Rev. C 72, 14904 (2005).

- Assume a quasiparticle picture
  (1) QM uncertainty principle $\Rightarrow \Delta x$ & $\therefore$ *mfp* $\not< \langle p \rangle^{-1}$
  (2) *mfp* $\not< \langle interparticle\ \ spacing \rangle$
  (3) QM unitarity $\Rightarrow$ cross sections have an upper bound
  $\Rightarrow$ mfp has a lower bound
  $\eta \propto$ mfp $\therefore$ no fluid can have exactly zero viscosity.

- AdS/CFT: KSS bound: $\frac{\eta}{s} \geq \frac{1}{4\pi}$. Violation of the bound: Kats & Petrov 0712.0743; Brigante et al. 0802.3318.
  (caveat: applying all this to QCD is speculative.)

What is the Need of Viscous Hydro? (continued)

- Initial (& final) conditions are uncertain. Ideal hydro can mimic viscous hydro if the initial (&/or final) conditions are suitably tuned. CGC & fluctuations → a larger v_2.
- Some v_2 may build-up during pre-eqlbm (i.e., pre-hydro) regime. Success of ideal hydro may be due to the neglect of this contribution to v_2.
- Dramatic rise of the bulk viscosity near T_c. sQGP is not a perfect fluid!
- Finally, to claim success for ideal hydro, one should calculate viscous corrections and show explicitly that they are indeed small.

Relativistic Viscous Hydro (a Brief History)

- Relativistic version of Navier-Stokes eq: Eckart (1940), Landau (1953). First-order or standard formalism.

- Problems: (1) Acausality (Müller 1960's, Israel & Stewart 1970's), (2) Instability (Hiscock & Lindblom 1980's), (3) Lack of relativistic covariance.

- Second-order theory or causal dissipative hydro. Israel-Stewart theory is one of the several causal theories.

- Causal hydro of gauge theory plasmas from AdS/CFT duality.

- Application to rhics: Muronga (2002, -04, -07), Heinz, Song, Chaudhury (2006, -07, -08), Baier, Romatschke^2, Wiedemann, (2006, -07, -08), Koide et al. (2006), Tsumura et al. (2006), RSB & Gupta (2007), Dusling & Teaney (2007), Huovinen & Molnar (2008), Dumitru, Molnar, Nara (2007), Pratt (2008) ...

Basic idea of causal dissipative hydro

- First a simple example of charge diffusion:

$$\begin{aligned} \partial_\mu J^\mu &= 0 && \ldots \text{ Conservation eq.} \\ J_i &= -D\partial_i \rho && \ldots \text{ Constitutive eq. (Fick's law)} \end{aligned}$$

Elimination of J_i gives

$$\partial_0 \rho - D\partial_i^2 \rho = 0 \;\; \ldots \text{ Diffusion eq. (parabolic)}$$

Solution: $\rho \sim \exp(-x^2/4Dt)/\sqrt{4\pi Dt}$ ... violates causality

- To restore causality:

$$\begin{aligned} \tau_J \partial_0 J_i + J_i &= -D\partial_i \rho && \ldots \text{ Constitutive eq.} \\ \tau_J \partial_0^2 \rho + \partial_0 \rho - D\partial_i^2 \rho &= 0 && \ldots \text{ Diff. eq. (hyperbolic). Telegrapher's eq.} \end{aligned}$$

- If $v^2 \equiv D/\tau_J < 1$, causality is restored.

Basic idea of causal dissipative hydro (contd.)

- Now consider hydro:

$$\partial_\mu T^{\mu\nu} = 0 \quad \ldots \text{ Conservation eq.}$$

$$T_{ij} = P\delta_{ij} - \eta(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k) - \zeta\delta_{ij}\partial_k u_k \quad \ldots \text{ Consti. eq.}$$

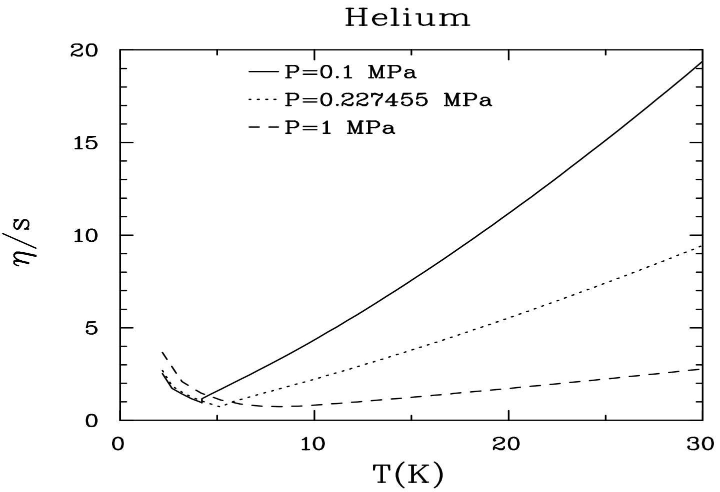
- Tensor decomposition is more complicated. But the basic idea is the same. Causality is restored by introducing higher-order terms with new set of transport coefficients $\tau_\pi, \ \tau_\Pi$, etc.

- $\tau_\pi, \ \tau_\Pi$ are important at early times or for rapid evolution.

- Effective-theory expansion.

Various causal dissipative hydro formulations

- Müller theory (1967)
- Israel-Stewart theory (1979)
- Carter's theory (1991)
- Öttinger-Grmela formulation (1997-98)
- Memory function method of Koide et al. (2007)



Fixed P. Solid: below P_c, dotted: at P_c, dashed: above P_c. Similar behaviour seen in N_2, H_2O at and near their critical points ... Csernai, Kapusta, McLerran, PRL (2006).

Shear and bulk viscosity in high-T QCD

$\eta \sim \frac{T^3}{\alpha_s^2 \ln \alpha_s^{-1}}$ ... Arnold, Moore, Yaffe (2000, '03)

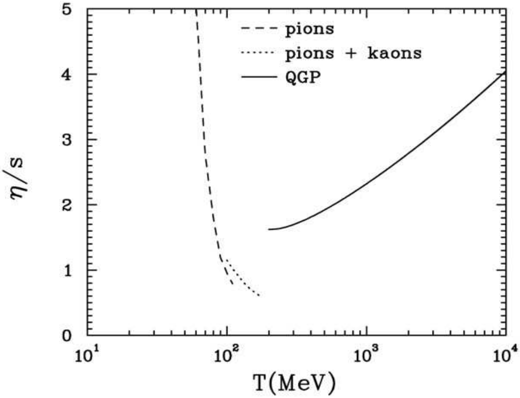
$\zeta \sim \frac{\alpha_s^2 T^3}{\ln \alpha_s^{-1}}$ ... Arnold, Dogan, Moore (2006)

As the temperature T increases:

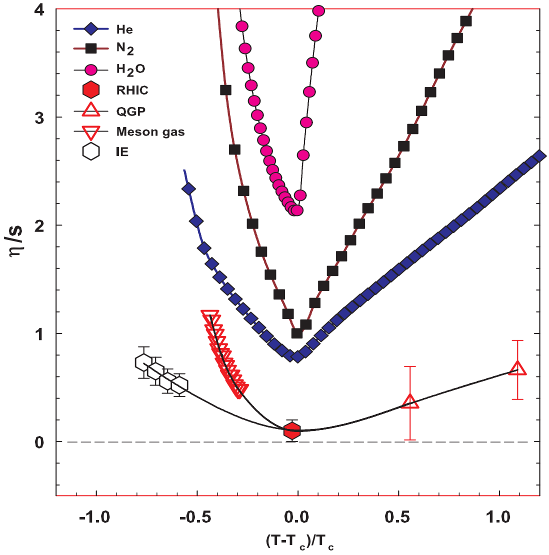
$\eta,\ \zeta$ both $\uparrow$

$\eta/T^3\ \uparrow, \qquad \zeta/T^3\ \downarrow$

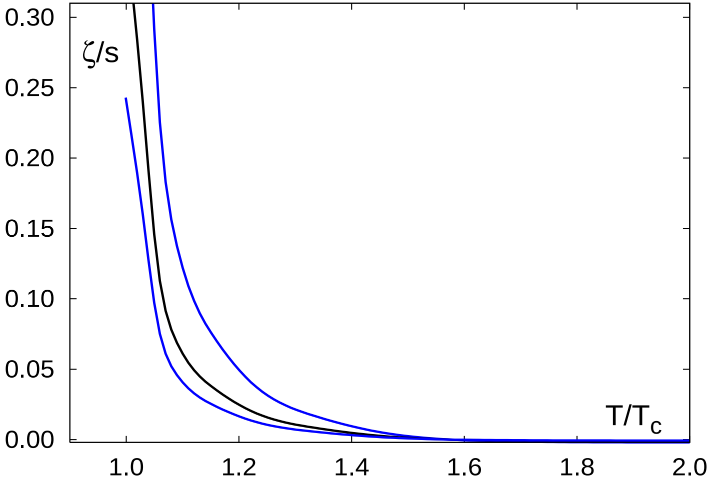
$\zeta/\eta \sim \alpha_s^4\ \downarrow$



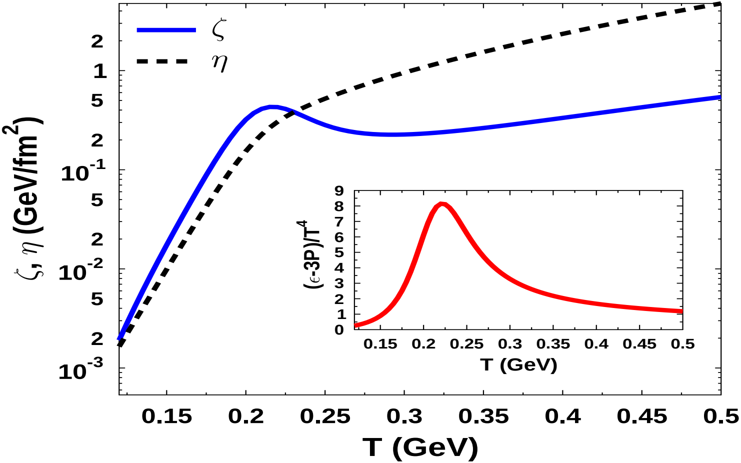
Left: $\eta/s \sim f_\pi^4/T^4$ ... Prakash et al., PRp (1993). Right: QGP based on pQCD ... Csernai, Kapusta, McLerran, PRL (2006). Both calculations are unreliable near T_c.



Lacey et al. (PRL 2007)



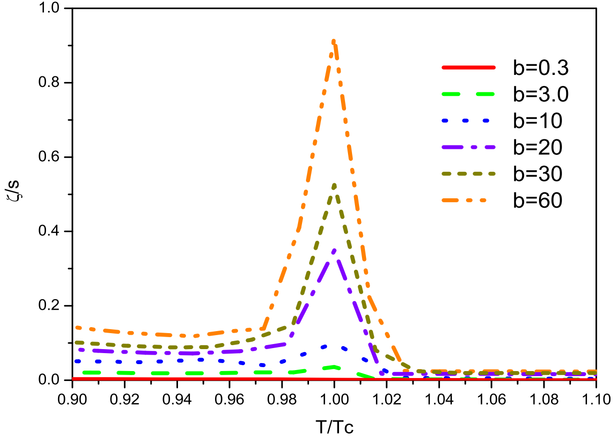
Karsch, Kharzeev, Tuchin (PLB 2008): based on lattice data. $\omega_0 = 0.5, 1, 1.5$ GeV (top to bottom). sQGP not a perfect liquid near T_c!



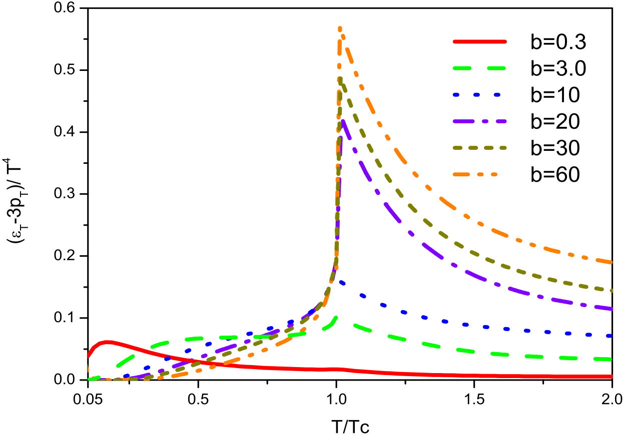
Fries et al. (0807.4333): $\eta = s/4\pi$ ... KSS bound.

ζ: fit to lattice results of H. Meyer (PRL 2008); quenched QCD.

Inset: fit to lattice results of Cheng et al. (PRD 2008); 2 light & 1 heavy flavours; unquenched QCD.



Li & Huang (0807.0292): Real scalar field theory.
ζ/s for different coupling strengths b.



Li & Huang (0807.0292): Real scalar field theory.
"Interaction measure" for different coupling strengths *b*.

Results of causal viscous hydro

Will present

- Fries, Müller, Schäfer (0807.4333): Includes bulk viscosity.

Will not present

- Romatschke & Romatschke (PRL 2007)
- Song & Heinz (PLB 2008, PRC 2008)
- Dusling & Teaney (PRC 2008)
- Molnar & Huovinen (2008)

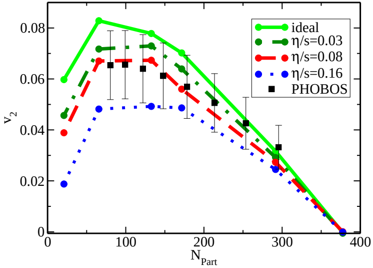
...

Assumptions:

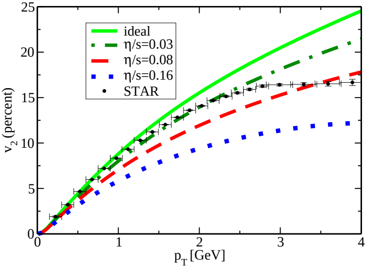
Longi. boost invariance. $2+1$ D. Israel-Stewart hydro.
$\eta/s = 0.03,\ 0.08,\ 0.16$ fixed. Bulk viscosity ignored.
EoS: semirealistic result of Laine & Schroder (2006).
Freezeout: Cooper-Frye, $T_f = 150$ MeV. Hydro used until the last scattering, instead of a more sophisticated hydro+cascade.

Initial conditions:

$\tau_0 = 1$ fm/c. Standard Glauber. $\epsilon \sim n_{\rm coll}$. Also used $s \sim n_{\rm part}$.
$\pi^{\mu\nu} = 0$. $u^x = 0 = u^y$.



p_T-integrated v_2



Minimum-bias v_2

Au-Au, 200 GeV, charged particles. PHOBOS: 90% confidence level systematic errors. STAR: only statistical errors.

Conclusion: p_T-integrated v_2 is consistent with η/s up to 0.16. Min-bias $v_2(p_T)$ favours $\eta/s < 1/4\pi =$ KSS bound.

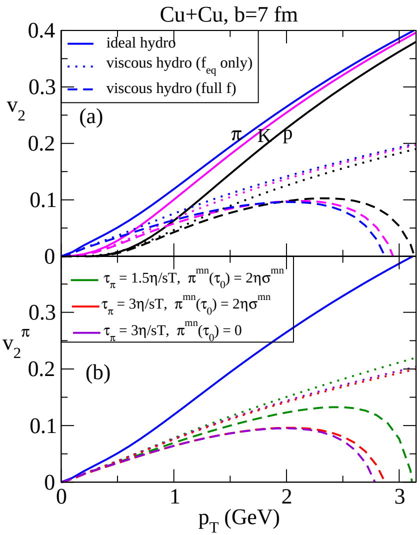
Song & Heinz (PLB 2008)

Assumptions:

Longi. boost invariance. $2+1$ D. Israel-Stewart but differs from Romatschke^2. $\eta/s = 1/4\pi$ fixed. Bulk viscosity ignored.
EoS: $\sim$ EOS-Q. Includes phase transition. $T_c = 164$ MeV.
Freezeout: Cooper-Frye. Isothermal surface at $T_{\rm dec} = 130$ MeV.

Initial conditions:

$\tau_0 = 0.6$ fm/c. Standard Glauber. $\epsilon(x, y, b) \sim$ no. of wounded N's. $\pi^{mn} = 0$ or Navier Stokes. But results are insensitive.



Even “minimal” viscosity $\eta/s = 1/4\pi$ leads to a large reduction of $v_2(p_T)$, even if freezeout is with f_{eq} only.

Tentative conclusion: RHIC data may be inconsistent with the KSS bound.

Dusling & Teaney (PRC 2008)

Assumptions:

Longi. boost invariance. $2+1$ D. Öttinger & Grmela (1997, '98).
$\eta/s = 0.05,\ 0.13,\ 0.20$ fixed. Bulk viscosity ignored.
EoS: $p = \epsilon/3$. No phase transition.
Freezeout: Cooper-Frye. Not necessarily an isotherm.

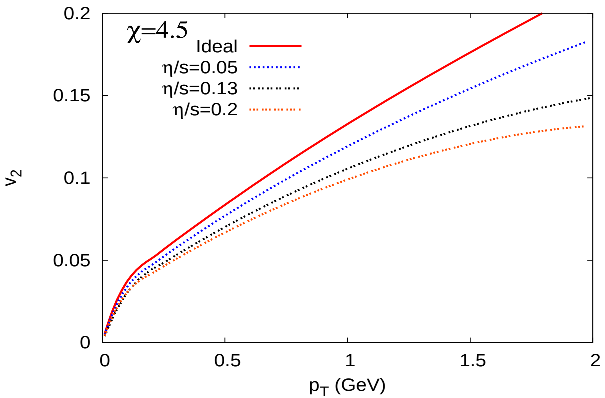
Initial conditions:

$\tau_0 = 1$ fm/c. Flow vel. à la Bjorken. Glauber.

$$s(x, y, \tau_0) = \frac{C_s}{\tau_0}\frac{dN_p}{dxdy} \quad \ldots \quad N_p : \text{ no. of participants}$$

$\pi^{ij} = \eta \left\langle \partial^i u^j \right\rangle$, i.e., Navier-Stokes.

Dusling & Teaney (contd.)



$v_2(p_T)$ for massless particles. χ defines the freeze-out surface.

Conclusion: Freezeout with f_0 only: effects of viscosity on v_2 are modest. With $f_0 + \delta f$: v_2 is strongly modified at large p_T.

Fries, Müller, & Schäfer (0807.4333)

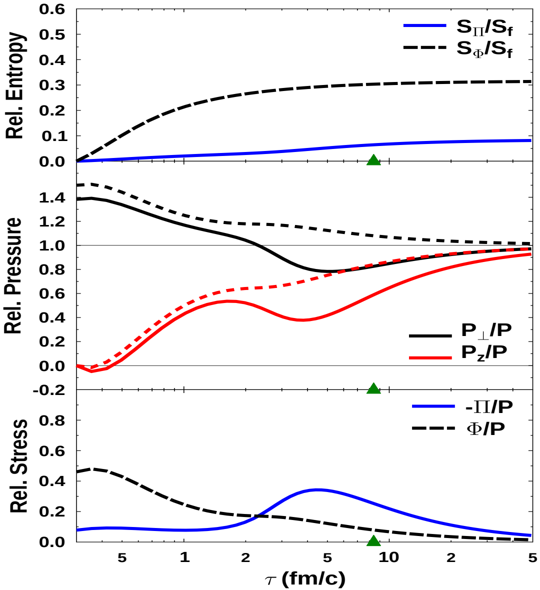
Assumptions:

Longi. boost invariance. 1D expansion. $\eta/s = 1/4\pi$ fixed.
Bulk viscosity included. $\tau_\pi = \tau_\Pi$.
EoS: 2 light & 1 heavy flavours. Unquenched lattice QCD. Cheng et al. (2008).

Initial conditions:

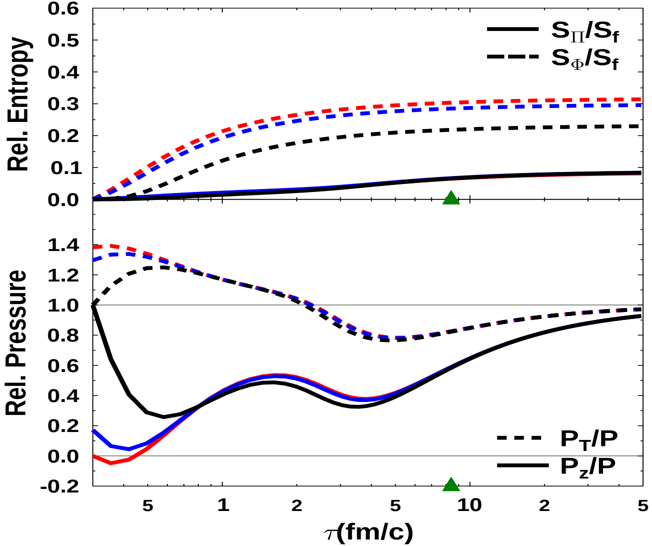
$\tau_0 = 0.3$ fm/c $\sim$ decoh. time. $\epsilon = 50$ GeV/fm^3. $T \simeq 400$ MeV.

- "Equilibration": $\Pi(\tau_0) = \Phi(\tau_0) = 0$.
- "1st order viscous hydro": $\Pi(\tau_0) = -\zeta(T_0)/\tau_0,\ \Phi(\tau_0) = 4\eta(T_0)/3\tau_0$.
- "Max. anisotropic": $\Pi(\tau_0) = -\zeta(T_0)/\tau_0,\ \Phi(\tau_0) = P(\tau_0) + \Pi(\tau_0)$ so that $P_z(\tau_0) = 0$.

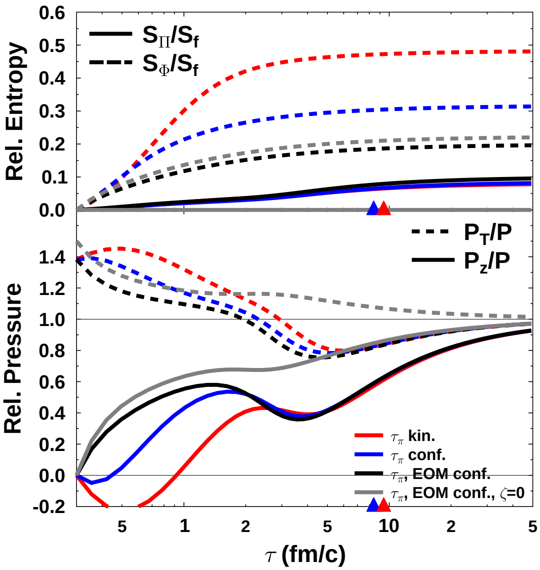


$$
\begin{aligned}
P_\perp &= P + \Pi + \Phi/2 \\
P_z &= P + \Pi - \Phi
\end{aligned}
$$

Initial cond. (iii), S, SYM.
Central panel:
dashed = no bulk viscosity.
Note: Marked anisotropy
even at T_c.
Filled $\triangle \leftrightarrow T_c$.



Impact of different initial conditions: (i) black, (ii) blue, (iii) red.



Influence of diff τ_π & conformal terms.

red, blue: long/short τ_π
blue, black: std/conformal
black, gray: bulk visco. Y/N

Effects of Bulk Viscosity — summary

- Large bulk viscosities around T_c lead to sizeable deviations from equilibrium throughout the entire lifetime of QGP.
- Bulk viscosities just slightly larger than currently favoured could easily lead to breakdown of hydro around T_c.
- The decreased pressure should slow down the expansion & increase the time spent in the vicinity of the phase transition.
- The amount of entropy produced through bulk stress around T_c is smaller than that produced by shear stress at earlier times. Hence no large increase of the final particle multiplicity is expected.

Other latest developments

- Drescher et al., PRC (2007): They fitted v_2/ϵ vs $(1/S)(dN/dy)$ data by means of a simple model based on incomplete thermalization. Extracted η/s. Found it to be twice the conjectured lower bound. Viscous corrections to the ideal fluid picture found to be about 25-30%.

- Baier et al. (0712.2451 hep-th): Müller, Israel, Stewart theory does not contain all allowed second-order terms.

- Bhattacharyya et al. (0712.2456 hep-th): Derived nonlinear fluid dynamics from gravity or Einstein's eqs.

  Recall: Policastro et al. (2001): Had derived linearized fluid dynamics from gravity.

Present Work — Various Fluid Models Studied

We studied some aspects of the Israel-Stewart (IS) theory.

Assumptions: Only longitudinal expansion. Net baryon no. = 0, Heat flow = 0, Bulk viscosity = 0.

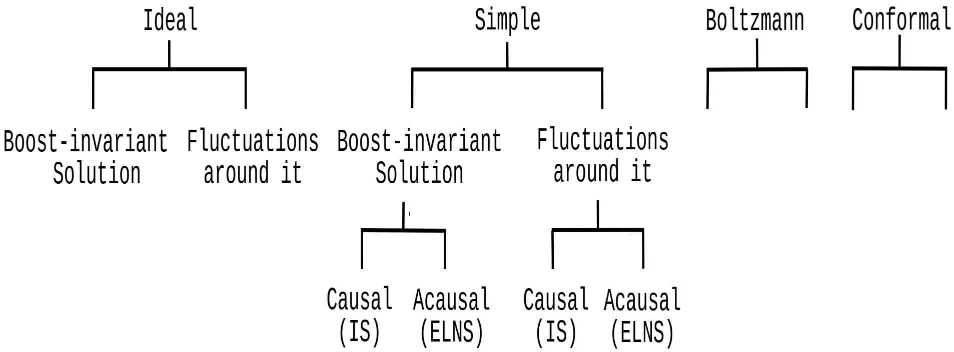
We studied ideal fluid and 3 models of viscous fluid:

• Simple fluid: material properties $\eta_V, \ \tau_\pi, \ c_s$ assumed T-independent.

• Boltzmann fluid (massless Boltzmann gas): $\eta_V, \ \tau_\pi$ depend on T, so does ϵ, but $\epsilon\tau_\pi/\eta_V = 9/2$. $c_s^2 = 1/3$.

• Conformal fluids: $\tau_\pi = a/T, \ \eta_V/s = 1/4\pi \therefore \epsilon\tau_\pi/\eta_V = 3\pi a$ where a is an arbitrary dim.less positive constant. $c_s^2 = 1/3$.

Various Cases Studied



Studied: Evolution of energy density, viscous pressure tensor, total entropy. Stability of the boost-invariant solutions: sound waves, etc. Comparison of ideal, causal & acausal hydro.

Equations of Relativistic Causal Viscous Hydro

$$
\begin{aligned}
D\epsilon &= -(\epsilon+p)\nabla_\mu u^\mu + \frac{1}{2}\pi^{\mu\nu}\langle\nabla_\mu u_\nu\rangle, && \text{Energy} \\
(\epsilon+p)Du^\mu &= \nabla^\mu p - \Delta^\mu_\sigma \nabla_\rho \pi^{\sigma\rho} + \pi^{\mu\sigma} Du_\sigma, && \text{Momentum} \\
\tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta D\pi^{\alpha\beta} + \pi^{\mu\nu} &= \eta_V \langle\nabla^\mu u^\nu\rangle - 2\tau_\pi \pi^{\alpha(\mu}\omega^{\nu)}_\alpha. && \text{Shear Pressure}
\end{aligned}
$$

Longi. expansion: 3 (coupled) tensor eqs. $\rightarrow$ 3 (coupled) scalar eqs. for (a) ϵ: energy density, (b) π_V: the only independent element of $\pi^{\mu\nu}$, (c) y: rapidity $-$ space-time rapidity.

$$
\begin{aligned}
D\epsilon + (1+c_s^2)\Theta\epsilon &= \Theta\pi_V, \\
c_s^2\widetilde{D}\epsilon + (1+c_s^2)S\epsilon &= \widetilde{D}\pi_V + S\pi_V, \\
\tau_\pi D\pi_V + \pi_V &= \frac{4}{3}\eta_V\Theta.
\end{aligned}
$$

Dimensionless Numbers in Fluid Dynamics — Nonrelativistic

Knudsen number $K \equiv \lambda/L$. $K \ll 1$: Hydro applicable. Large no. of collisions per particle. Helps equilibration. Otherwise equilibration is doubtful.

Mach number $M \equiv v/c_s$. $M \ll 1$: Incompressible flow.

Reynolds number $R \equiv \rho v L/\eta_V \sim$ Inertial stress/Viscous stress. Small R: Linear term in NS eq. dominates. Steady or Laminar flow. Large R: Complicated flow $\rightarrow$ turbulence.

$$\because \eta_V \simeq \rho c_s \lambda \quad \therefore \boxed{KR \simeq M}$$

Law of similarity (Reynolds 1883).

Relativistic analogue?

Dimensionless Numbers in Fluid Dynamics — Relativistic

Symmetries of hydro eqs $\rightarrow$ relativistic analogue of the law of similarity:

We define scaling invariants

$$\boxed{\chi \equiv \epsilon/\varpi} \quad \boxed{\varphi \equiv \pi_V/\varpi} \quad \boxed{\mathbf{S} \equiv \pi_V/\epsilon}$$

where ϵ: energy density, π_V: the only independent element of the shear pressure tensor, $\varpi \equiv \eta_V/\tau_\pi$.

If $\tau_\pi \rightarrow 0$ and in the NR limit:

$\mathbf{S} \rightarrow (\quad)M^2/R, \quad \varphi \rightarrow (\quad)MK, \quad \chi \rightarrow (\quad)KR/M.$

At RHIC, $M \sim \mathcal{O}(1)$ $\therefore$ $\boxed{\mathbf{S} \sim 1/R}$, $\boxed{\varphi \sim K}$.

Since $KR \simeq M$, $\boxed{\chi \sim \mathcal{O}(1)}$. Boltzmann fluid: $\chi = 2/3$.

Thermalization and Freezeout

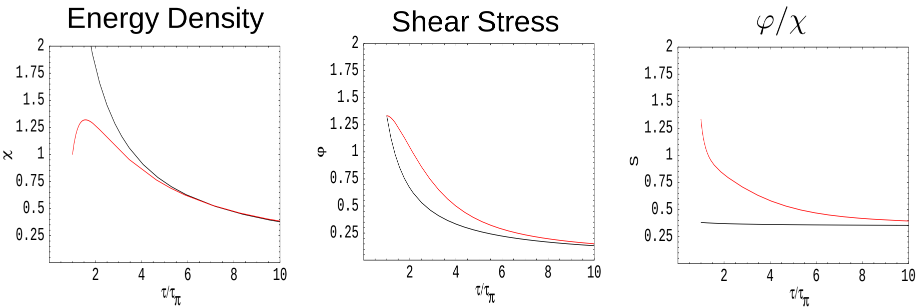
• Hydro applicable only during $\tau_{th} < \tau < \tau_{f.o.}$ if K is small enough.

• Ideal fluid: $K = 0$ $\therefore$ hydro is always applicable.
$\therefore$ Notions of thermalization and freezeout need to be imposed from the outside.

• Viscous fluid: some understanding of these phenomena may be possible.

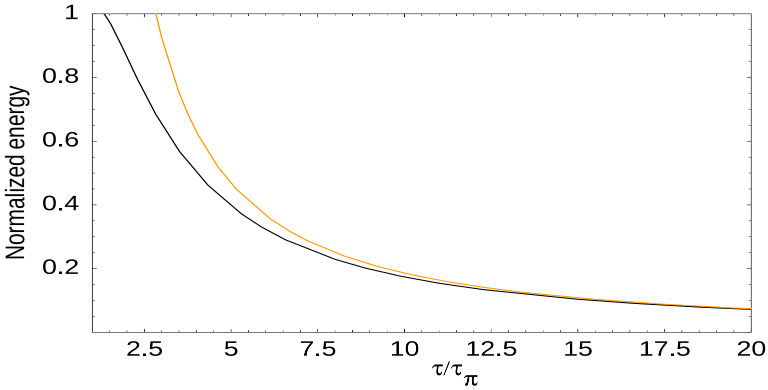
We provide a self-consistent definition of thermalization in IS hydro in terms of φ.

Scaling Solutions for “Simple” Fluid



Red curves: IS hydro, Black curves: ELNS hydro

Note: Exact analytic solutions of viscous hydro eqs.



Energy density (ϵ/ϵ_0) vs time (τ/τ_π). Black curve: Boltzmann fluid (IS hydro), Yellow curve: Ideal fluid (Bjorken solution).

Beyond Boost Invariance

Checked stability of boost-invariant solutions against small fluctuations.

• IS causal hydro: small k: fluctuations are overdamped.
large k: damped sound waves.

• ELNS acausal hydro: No sound waves. All fluctuations are diffusively damped.

Qualitative difference.

Conclusions

• Explored certain phenomena that arise in IS causal viscous hydro. Studied 3 different models of viscous fluids. Longitudinal boost invariant solutions & beyond.

• Compared with (1) ELNS acausal viscous hydro, (2) ideal hydro. Found qualitatively different features in IS hydro.

• Given ϵ(final), acausal hydro & ideal hydro grossly overestimate ϵ(initial) as compared to causal hydro.

• Presented dimensionless numbers in relativistic fluid dynamics: in analogy with K, M, R.

• Gave a self-consistent definition of thermalization in IS hydro.

What remains to be done?

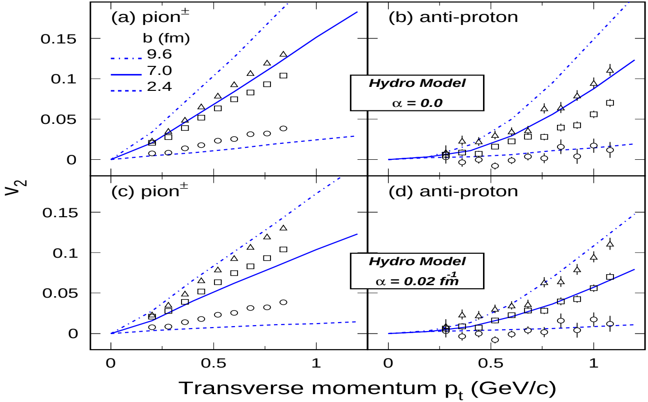
- Bulk as well as shear viscosity (together with temperature dependence of ζ/s and η/s) needs to be incorporated.
- Can causal viscous hydro with CGC-type initial conditions reproduce $dN/dy, \langle p_T \rangle, v_2$ data? If so, what are the extracted $\zeta/s,\ \eta/s$?
- Causal viscous hydro + hadronic cascade not done yet.
- There are issues related to the formalism itself: Israel-Stewart theory is not the only approach to causal hydro.
- Various numerical codes need to be compared with each other: TECHQM.

TECHQM

- Theory-Experiment Collaboration for Hot QCD Matter
- Present uncertainties limit the accuracy with which conclusions can be drawn
- Detailed, quantitative analysis of experimental data and theoretical models
- coherent, sustained, collaborative effort of experts in all stages of heavy-ion collisions
- https://wiki.bnl.gov/TECHQM

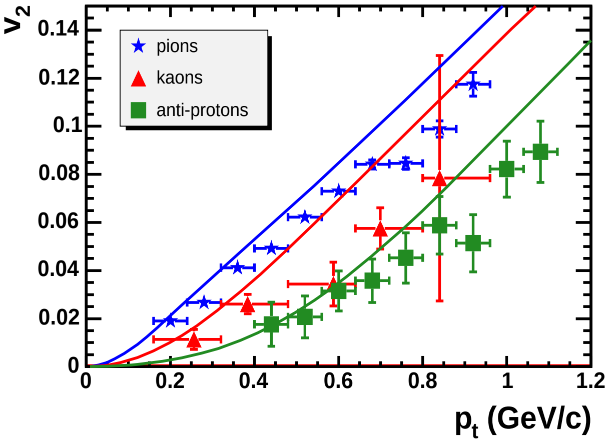
Thank You

Shortcomings of Ideal Hydro



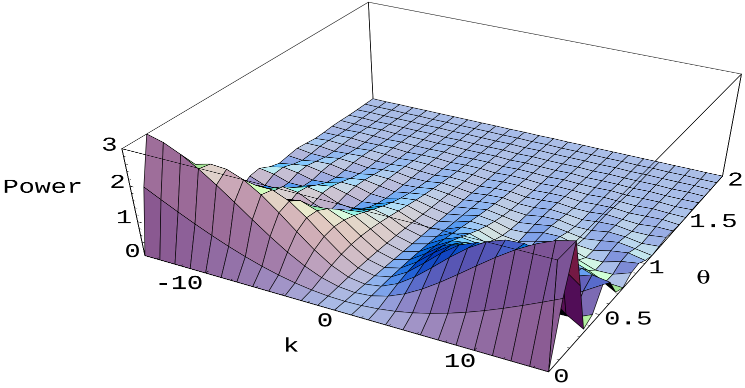
Centrality bins 40-50%, 20-30%, 0-5%. Lines: ideal hydro of Kolb & Rapp (2003). α: initial transverse velocity kick parameter. STAR nucl-ex/0409033.

Shortcomings of Ideal Hydro



STAR data: $v_2\{4\}$, 20-60% centrality, nucl-ex/0409033.
Lines: ideal hydro of Huovinen et al. (2001).

Power spectrum of fluctuations in ϵ (Ideal Fluid)



Power spectrum of fluctuations in ϵ (Simple Fluid, ELNS hydrodynamics)