Lectures on hydrodynamics - Part II: Covariant transport and the hydrodynamic limit

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Outline

- Nonrelativistic transport
- Covariant transport
- Connection to hydrodynamics, transport coefficients
- Initial conditions, hadronization
- Heavy-ion collisions - cooling, elliptic flow, hydrodynamic limit
- Radiative transport

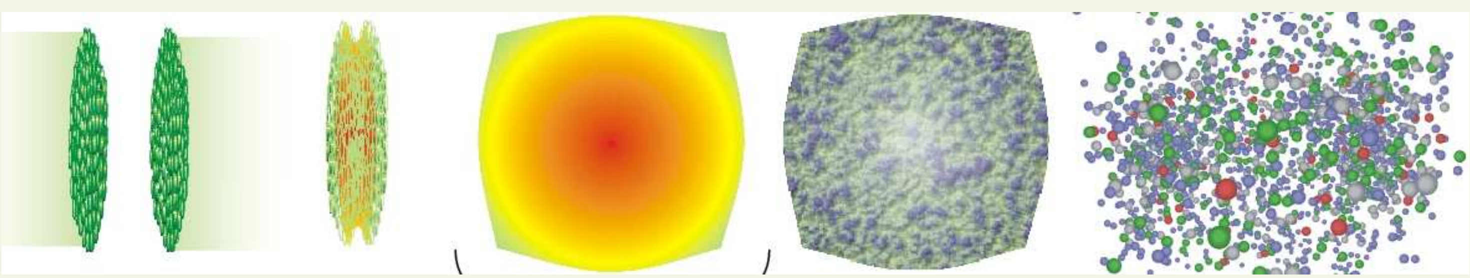
Covariant transport

- **covariant non-equilibrium description, in terms of phase space densities of (quasi-)particles**

- **dynamics driven by covariant local scattering rates, capable of equilibration**

- **in heavy-ion physics: used for the plasma and the hadron gas stages**

initial nuclei parton plasma hadronization hadron gas



↑ $\sim 10^{-14}$ **m** **= 10 fermis** ↓

<—— $\sim 10^{-23}$ **sec = 10 yochto(!)-secs** ——>

Nonrelativistic kinetic theory

Lagrangian mechanics

$$\vec{k}_i \equiv \frac{\partial L}{\partial \dot{\vec{x}}_i}, \qquad \dot{\vec{k}}_i = \frac{\partial L}{\partial \vec{x}_i} \ , \qquad L = \sum_i \frac{m\dot{\vec{x}}_i^{\,2}}{2} - V(\{\vec{x}_i\}) \ , \tag{1}$$

Klimontovich phase space distribution

$$F(\vec{x}, \vec{k}, t) \equiv \sum_i \delta^3 \left(\vec{x} - \vec{x}_i(t)\right) \delta^3 \left(\vec{k} - \vec{k}_i(t)\right) \ , \tag{2}$$

satisfies the Klimontovich equation (Problem 0)

$$\frac{d}{dt}F = \frac{\partial}{\partial t}F + \frac{\vec{k}}{m}\frac{\partial}{\partial \vec{x}}F - \vec{\mathcal{K}}[F,V]\frac{\partial}{\partial \vec{k}}F = 0 \tag{3}$$

E.g., for static two-body potentials

$$V(\{\vec{x}_i\}) = \frac{1}{2}\sum_{i \neq j} V(|\vec{x}_i - \vec{x}_j|) \ \Rightarrow \ \vec{\mathcal{K}} = \int d^3x_2 d^3k_2 \nabla_x V(|\vec{x} - \vec{x}_2|) F(\vec{x}_2, \vec{k}_2, t) \tag{4}$$

BBGKY hierarchy [Bogoliubov-Born-Green-Kirkwood-Yvon]

Define 1-, 2-, ...(n-) particle phasespace distributions via averaging over an ensemble of particles

$$\begin{aligned} f_1(t) &\equiv \langle F(1,t)\rangle \\ f_{12}(t) &\equiv \langle F(1,t)F(2,t)\rangle - \delta^6(1-2)f_1(t) \\ \ldots \end{aligned} \qquad (5)$$

where $1 \equiv (\vec{x}_1, \vec{p}_1)$**, etc.**

Equations of motions for these can be obtained via averaging the Klimontovich eqn, as needed multiplied by F**,** $F \cdot F$**, ... terms**

For 2-body potentials the eqns are not closed, lowest-order involves f_{12} **on the RHS**

$$(\partial_t + \frac{\vec{k}}{m}\vec{\nabla}_{x_1})f_1(t) = \langle \vec{\mathcal{K}}[F]\vec{\nabla}_{k_1}F(1,t)\rangle \qquad (6)$$

IGNORING correlations $f_{12} = f_1 f_2$ **gives the Vlasov eqn**

$$\left[\partial_t + \frac{\vec{p}}{m}\vec{\nabla}_x - \left(\int d^3x_2 d^3p_2 f(\vec{x}_2, \vec{p}_2, t)\vec{\nabla}_x V(|\vec{x} - \vec{x}_2|)\right)\vec{\nabla}_p\right] f(\vec{x}, \vec{p}, t) = 0 \ , \qquad (7)$$

which gives in a self-consistent field. E.g., for Coulomb $-(...) = q\vec{E}_{selfc.}$

With the Vlasov eqn, you can study dielectric properties (Problem 0b)

taking a small external field $\delta\vec{E}_{ext}(\vec{x},t) \to \delta f(\vec{x},\vec{p},t) \to \delta\vec{E}(\vec{x},t)$ **gives**

$$\epsilon_L(\vec{k},\omega) = \frac{\delta E_{ext}(\vec{k},\omega)}{\delta E_{ext}(\vec{k},\omega)} \tag{8}$$

E.g., with a static point-charge δq

$$\epsilon_L(\omega=0,\vec{k}) = 1 + \frac{4\pi q^2 n}{Tk^2} = 1 + \frac{\mu_D^2}{k^2} \quad \to \quad \phi(r) = \frac{\delta q}{r} e^{-\mu_D r} \ . \tag{9}$$

There is more to the story if you include higher correlations (quite involved). Progress can be made only if one assumes (Bogolyubov) that higher-order correlations evolve on progressively much faster scales then lower-order ones.

Next order (Lenard-Balescu-Landau) **gives so-colled collision terms, for Coulomb interactions these correspond to a screened scattering cross section**

$$v_{rel}\frac{d^3\sigma^{\rm eff}}{d^3k} = \frac{4e^4}{k^4}\frac{\delta\left(\vec{k}(\vec{v}_p - \vec{v}_{p_1})\right)}{|\epsilon_L(\vec{k},\vec{k}\cdot\vec{v}_p)|^2} \tag{10}$$

Relativistic generalization

NO-GO theorems in relativistic Hamilton dynamics Currie, Jordan, Sudarshan...
- no viable relativistic potential approach

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Alternatives

- **live with Vlasov limit**

$$p^\mu \left(\partial_\mu + qF_{\mu\nu}\frac{\partial}{\partial p_\nu}\right) f = 0$$

$$\partial_\mu F^{\mu\nu} = J^\nu = q\int \frac{d^3p}{E} p^\nu \, f$$

- **have local (in space-time) interactions/rates**

- **derive from quantum field theory (involved)** Heinz, Elze, Gyulassy, Thoma, ...

Covariant transport

(on-shell) phase-space density

$$f(x, \vec{p}) \equiv \frac{dN(\vec{x}, \vec{p}, t)}{d^3x d^3p} \qquad (p^\mu p_\mu = m^2)$$

is a Lorentz scalar (Problem 1)

Free streaming $\vec{x}(t) = \vec{x}(t_0) + \vec{v}(t - t_0)$ **implies** $f(\vec{x}_0, \vec{p}, t + \Delta t) = f(\vec{x}_0 - \vec{v}\Delta t, \vec{p}, t)$

$$\partial_t f + \vec{v}\vec{\nabla} f = 0 \qquad \Leftrightarrow \qquad p^\mu \partial_\mu f = 0$$

manifestly covariant.

Interactions are incorporated via collision term

$$p^\mu \partial_\mu f(x, \vec{p}) = C[f](x, \vec{p})$$

E.g., introduce two-body scatterings via a rate

$$\frac{dN_{sc}(x)}{dt} \equiv \sigma \cdot j_p(x)\, dA \cdot n_t(x)\, dz$$

j_p **- projectile current density,** n_t **- target density,** σ **- cross section**

Substitute $j_p \equiv n_p v_p$ **and obtain the rate per unit volume at** x

$$\frac{dN_{sc}(x)}{dV\,dt} \equiv \frac{dN_{sc}(x)}{d^4x} = \sigma\, \frac{n_p(x)}{E_p}\, \frac{n_t(x)}{E_t}\, E_p E_t v_p \ .$$

LHS is a Lorentz scalar (# of scatterings is frame independent), and n/E **is a scalar. Noticing that in the target rest frame**

$$E_p E_t v_p = \sqrt{(p_p \cdot p_t)^2 - m_p^2 m_t^2} \equiv T(p,t) \qquad \leftarrow flux\ factor$$

we DEFINE sigma to be a scalar via the manifestly covariant

$$\frac{dN_{sc}(x)}{d^4x} = \sigma\, \frac{n_p(x)}{E_p}\, \frac{n_t(x)}{E_t} T(p,t) \ .$$

[an equivalent alternative form is $T(1,2) = E_1 E_2 \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}$**]**

Differential $12 \to 34$ rate per unit incoming/outgoing momentum cell

$$\frac{dN_{sc}(x,\vec{p}_1)}{d^4x} = \left(\prod_{i=1}^{4} \frac{d^3p_i}{E_i}\right) \left(E_3 E_4 \frac{d\sigma(1,2)}{d^3p_3\, d^3p_4}\right) \frac{dn_p(x,\vec{p}_1)}{d^3p_1}\, \frac{dn_t(x,\vec{p}_2)}{d^3p_2}\, T(1,2)\ .$$

Combine rate of particles leaving from momentum cell d^3p_1 - *loss term*

$$E_1 \frac{dN_p^{loss}(x,\vec{p}_1)}{d^4x d^3p_1} = -(1 - \frac{1}{2}\delta_{pt}) \int \frac{d^3p_2}{E_2}\, \frac{d^3p_3}{E_3}\, \frac{d^3p_4}{E_4} \left(E_3 E_4 \frac{d\sigma^{p+t}(1,2)}{d^3p_3\, d^3p_4}\right) \times f_p(x,\vec{p}_1)\; f_t(x,\vec{p}_2)\; T(1,2)$$

and those entering cell d^3p_1 - *gain term* to obtain

$$E_1 \frac{df_1}{dt} = (1 - \frac{1}{2}\delta_{pt}) \int\limits_{234} (f_3^p f_4^t W_{34\to 12} - f_1^p f_2^t W_{12\to 34}) \equiv C_{2\to 2}[f](x,\vec{p}_1)$$

with shorthands

$$\int\limits_i \equiv \int \frac{d^3p_i}{E_i}\ , \qquad f_i^a \equiv f_a(x,\vec{p}_i)\ , \qquad W_{12\to 34} \equiv T(1,2) E_3 E_4 \frac{d\sigma^{p+t}(1,2)}{d^3p_3\, d^3p_4}$$

With detailed balance **(if time-reversal and parity invariance)**

$$W_{12\to34} = W_{34\to12} \qquad (W_{n\to m} = W_{m\to n})$$

we obtain the $2 \to 2$ **transport equation for a one-component system** **(**$p = t$**)**

$$p^\mu \partial_\mu f_1 = \frac{1}{2} \int\limits_{234} (f_3 f_4 - f_1 f_2) W_{12\to34}$$

Generalization to multicomponent case is straightforward

$$p^\mu \partial_\mu f_1^a = \frac{1}{2} \sum_{bcd} \int\limits_{234} (f_3^c f_4^d - f_1^a f_2^b) W_{12\to34}^{ab\to cd}$$

and arbitrary $n \to m$ **interactions can also be included**

$$p\partial f = C_{2\to2}[f] + C_{3\leftrightarrow2}[f] + \cdots$$

Connection to perturbation theory:

$$\begin{aligned} W_{12\to34} &\equiv \frac{s(s-4m^2)}{4\pi} \frac{d\sigma}{dt} \delta^4(p_1 + p_2 - p_3 - p_4) \\ &\equiv \frac{1}{64\pi^2} |\mathcal{M}_{12\to34}|^2 \delta^4(12 - 34) \end{aligned}$$

Covariant transport solutions

Very challenging to solve, integro-differential eqn. in 6+1D

- **analytic solutions for free streaming (linear problem), or linearized transport equation**

- **approximate 0+1D analytic solutions in relaxation time approximation** [e.g., Zhang & Gyulassy ('98)]

- **numerical codes (3+1D) - http://karman.physics.purdue.edu/OSCAR**

  Cartesian with $2 \to 2$**: Zhang ZPC**

  Cartesian with $2 \to 2$, $3 \leftrightarrow 2$**: Molnar MPC**

  $x{-}y{-}\eta{-}\tau$ **with** $2 \to 2$**: Cheng, Pratt & Csizmadia GROMIT (private)**

  $x{-}y{-}\eta{-}t$, $2 \to 2$, $3 \leftrightarrow 2$**: Xu & Greiner BAMS (private)**

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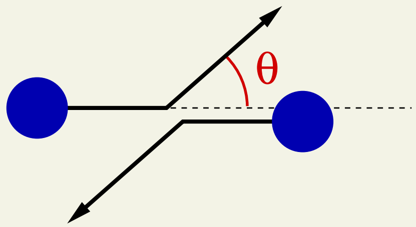
Hadronic codes: Bleicher, Bass et al **UrQMD**
Ko & Lin & Subrata **AMPT**
Nara et al **JAM**

mean free path: characterizes local conditions

$$\lambda(x) \equiv \frac{1}{\textbf{cross section} \times \textbf{density}(\mathbf{x})} \qquad \begin{cases} \lambda = 0 & - \text{ideal hydrodynamics} \\ \lambda = \infty & - \text{free streaming} \end{cases}$$

transport opacity: time-integrated, spatially averaged [DM & Gyulassy NPA 697 ('02)]

$$\chi \equiv \langle n_{coll} \rangle \langle \sin^2 \theta_{CM} \rangle \quad \sim \text{\# of collisions per parton} \times \text{mom. transfer efficiency}$$



$$\chi = \int dz\, \rho(z)\, \sigma_{transp} = \int dz\, \frac{1}{\lambda_{tr}(z)}$$

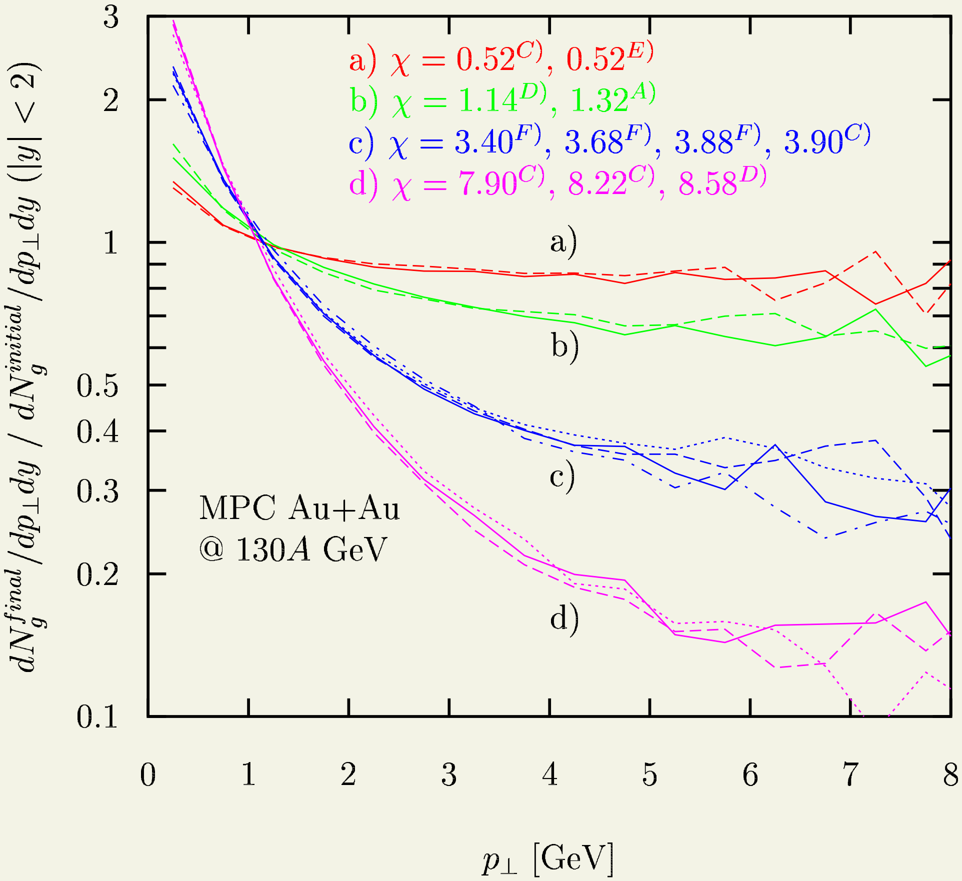
near equilibrium: related to transport coefficients (viscosity, diffusion constants)

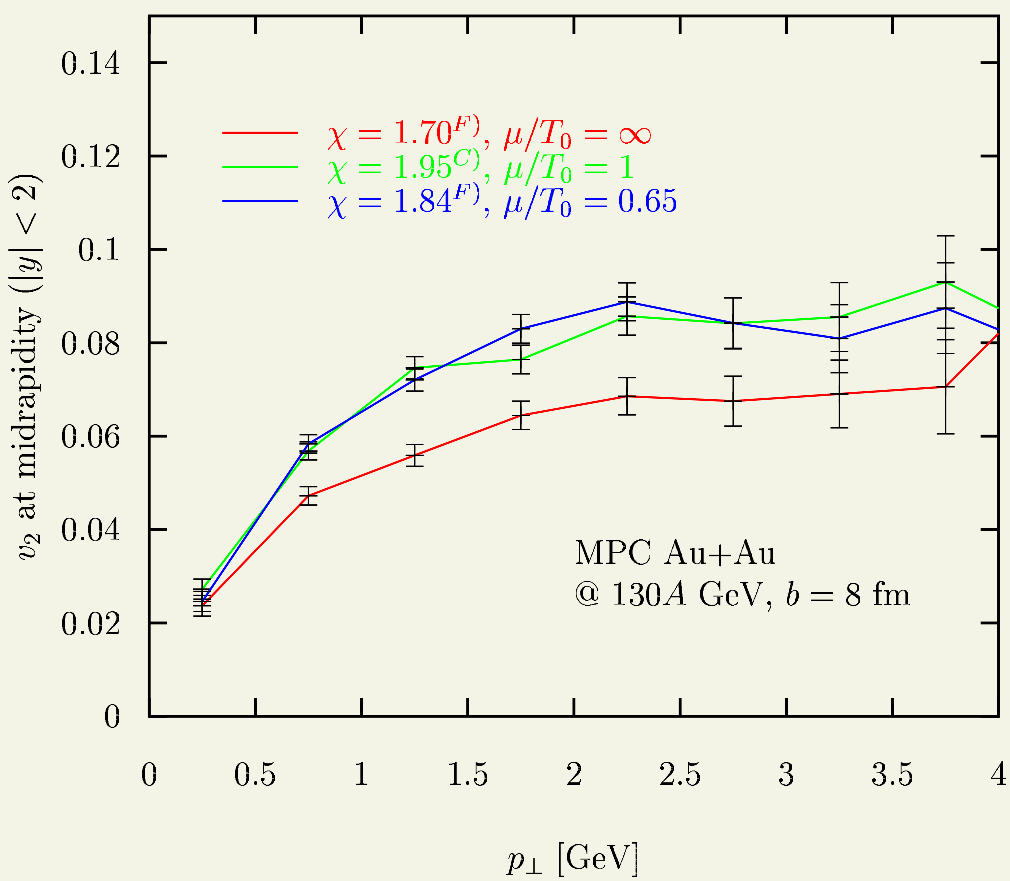
e.g., shear viscosity $\eta \approx \frac{4}{5} \frac{T}{\sigma_{tr}}$

Transport opacity scaling

to good approximation, results (from same initial condition) mainly depend on transport opacity DM & Gyulassy, NPA697 ('02)

final spectra normalized to the initial one **elliptic flow** $v_2(p_T)$





[$\mu/T_0 \to \infty$: **isotropic scattering,** $\mu/T_0 \to 0$: **forward-peaked**]

Exact scalings of solutions

[DM & Gyulassy, PRC62 ('00)]

- **extended subdivision covariance:**

$$f_i \to f_i' \equiv \ell \cdot f_i, \qquad W^{n\to m} \to W'^{n\to m} \equiv W^{n\to m}/\ell^{n-1} \qquad (\sigma \to \sigma' \equiv \sigma/\ell)$$

- rescaled problem gives same answer, provided final f is divided by ℓ

- **momentum scaling:**

$$f(x,\vec{p}) \to f'(x,\vec{p}) \equiv {\ell_p}^{-3} f\left(x, \frac{\vec{p}}{\ell_p}\right), W(\{p_i\}) \to W'(\{p_i\}) \equiv {\ell_p}^2 W\left(\left\{\frac{p_i}{\ell_p}\right\}\right)$$
$$m \to m' = m/\ell_p$$

- rescales mometa and W such that particle density is unchanged

- **coordinate scaling:**

$$f(x,\vec{p}) \to f'(x,\vec{p}) \equiv f\left(\frac{x}{\ell_x}, \vec{p}\right), \quad W \to W' \equiv \frac{W}{\ell_x}$$

- rescales space-time and W such that the particle density stays the same

Suppose you have some initial conditions with timescale τ_0, lengthscale R_0, temperature T_0, momentum/mass scale μ, cross sections σ and rapidity density $dN_0/d\eta$. The scalings imply that

$$\sigma' = l_x^{-1} l^{-1} \sigma, \qquad T_0' = l_p T_0, \qquad R_0' = l_x R_0,$$
$$\frac{dN_0{}'}{d\eta} = l_x l \frac{dN_0}{d\eta} \qquad \mu' = l_p \mu, \qquad \tau_0' = l_x \tau_0.$$

Therefore, we can scale a solution to others provided that all three ratios

$$\frac{\mu}{T_0}\,, \qquad \frac{R_0}{\tau_0}\,, \qquad \sigma \frac{dN_0}{d\eta} \sim \frac{\tau_0}{\lambda_{MFP}}$$

remain the same ($2 \to 2$ transport)

Macroscopic quantities

charge current: $N_c^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu c_i f_i(x, \vec{p})$

energy-momentum tensor: $T^{\mu\nu}(x) = \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(x, \vec{p})$

EOM + conservation laws in scatterings imply:

$$\partial_\mu N_c^\mu = 0 \qquad (c_1 + c_2 = c_3 + c_4) \ , \qquad \partial_\mu T^{\mu\nu} = 0 \qquad (p_1 + p_2 = p_3 + p_4)$$

for a local equilibrium distribution (see next slide)

$$f_{eq}(p, x) = \frac{g}{(2\pi^3)} \exp\left[\frac{\mu(x) - p_\mu u^\mu(x)}{T(x)}\right]$$

we recover (Problem 2) the ideal hydrodynamic expressions

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p\, g^{\mu\nu} \ , \qquad N_c^\mu = n_c\, u^\mu$$

Hydrodynamic limit

entropy current ($g = 1$): $S^\mu(x) = \sum_i \int \frac{d^3p}{E} p^\mu f_i(x, \vec{p}) \left\{1 - \ln[f_i(x,\vec{p}) h^3]\right\}$

H-theorem: $\partial_\mu S^\mu \geq 0 \quad \Rightarrow$ **entropy production in general**

$$\begin{aligned}
\partial_\mu S^\mu &= -\int\limits_1 \ln(h^3 f_1) C[f_1] \\
&= \frac{1}{2} \int\limits_{1234} (f_3 f_4 - f_1 f_2) W_{12\to 34} \frac{\ln f_3 + \ln f_4 - \ln f_1 - \ln f_2}{4} \\
&= \frac{1}{8} \int\limits_{1234} f_1 f_2 W_{12\to 34} \, (z-1)\ln z \quad \geq \quad 0
\end{aligned}$$

where $z \equiv f_3 f_4/(f_1 f_2) \geq 0$ **and** $(z-1)\ln z \geq 0$.

Equality (entropy maximum) requires $f_3 f_4 = f_1 f_2$ **for ANY momenta**

$$\Rightarrow \qquad f_{eq}(p,x) = e^{p_\mu A^\mu(x) + B(x)} = \frac{g}{(2\pi^3)} \exp\left[\frac{\mu(x) - p_\mu u^\mu(x)}{T(x)}\right]$$

i.e., entropy production until local equilibrium is reached

Transport coefficients

Hydrodynamic eqns come from expansion in *small gradients* near local equil

$$f(x,\vec{p}) = f_{eq}(x,\vec{p})[1+\phi(x,\vec{p})] \qquad (|\phi| \ll 1 \ , \quad |p^\mu\partial_\mu\phi| \ll |p^\mu\partial_\mu f_{eq}|/f_{eq})$$

AND substitution of the N^μ and $T^{\mu\nu}$ moments of the solution into the conservation laws.
As we saw, the 0-th order $\phi = 0$ gives ideal hydrodynamics (N_0^μ, $T_0^{\mu\nu}$). The first order solution

$$\begin{aligned} p^\mu\partial_\mu f_{eq}(x,\vec{p}) &= C[f_{eq}, f_{eq}\phi](x,\vec{p}) + C[f_{eq}\phi, f_{eq}](x,\vec{p}) \\ C[f,g] &\equiv \frac{1}{2}\int\limits_{234} (f_3 g_4 - f_1 g_2) W_{12\to 34} \end{aligned}$$

leads to the Navier-Stokes equations. Comparison to

$$\begin{aligned} T_1^{\mu\nu} &= T_0^{\mu\nu} + \eta_s(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha \\ N_1^\mu &= N_0^\mu + \kappa_q\left(\frac{nT}{\varepsilon+p}\right)^2 \nabla^\mu\left(\frac{\mu}{T}\right) \qquad (\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu \quad , \quad \Delta^\mu \equiv \Delta^{\mu\nu}\partial_\nu) \end{aligned}$$

gives the shear (η_s) and bulk (ζ) viscosities, and heat conductivity (κ_q).

Computing transport coefficients can be involved [see De Groot et al, *Relativistic kinetic theory*, or Arnold, Moore & Yaffe, JHEP 0011, 001 ('00) ...**]**

Nevertheless, because the product $\sigma\phi$ appears, it is clear that $\phi \propto 1/\sigma$ and therefore

$$\eta_s, \zeta \ \propto \ \frac{T}{\sigma} \sim nT\lambda_{MFP} \ , \qquad \kappa_q \ \propto \ \frac{1}{\sigma} \sim n\lambda_{MFP}$$

i.e., in the zero mean free path limit the transport coefficients vanish and we recover ideal hydrodynamics.

Heavy-ion applications (early stage)

- **perturbative calculations - Debye-screened perturbative cross sections, e.g.,**

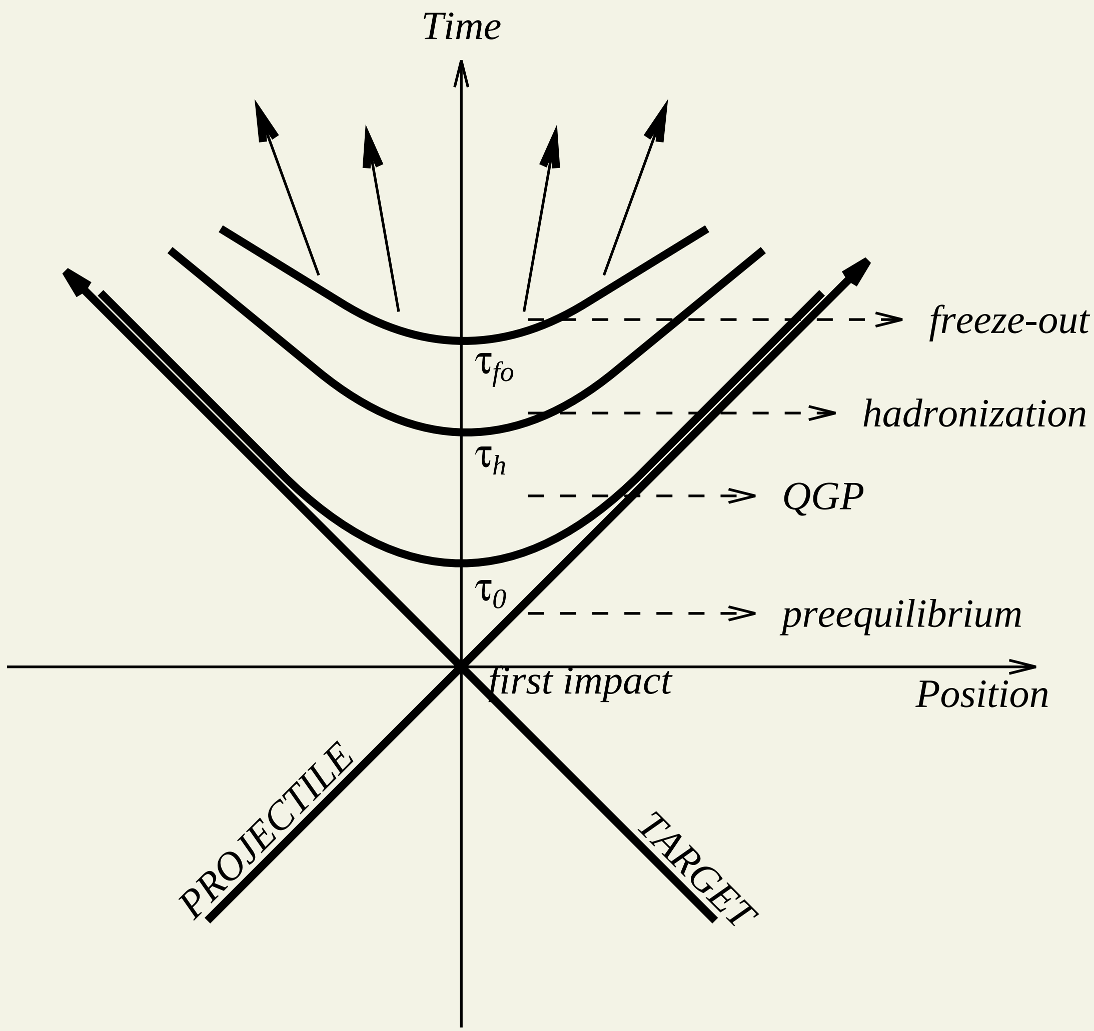
$$\frac{d\sigma^{gg\to gg}}{dt} = \frac{1}{16\pi s^2}\left|\bar{\mathcal{M}}_{gg\to gg}\right|^2 = \frac{9\pi\alpha_s^2}{2}\frac{1}{t^2} \;\to\; \frac{9\pi\alpha_s^2}{2}\frac{1}{(t-\mu_D^2)^2} \qquad (\mu_D \sim gT)$$

in radiative $ggg \leftrightarrow gg$, account for the LPM effect (need $\tau_{form} \lesssim \lambda_{MFP}$)

$$|\mathcal{M}_{gg\to ggg}|^2 = \left(\frac{9g^4}{2}\frac{s^2}{(\mathbf{q}_T^2+\mu_D^2)^2}\right)\left(\frac{12g^2\mathbf{q}_T^2}{\mathbf{k}_T^2[(\mathbf{k}_T-\mathbf{q}_T)^2+\mu_D^2]}\right)\Theta(k_T\lambda_{MFP}-\mathrm{ch}y)$$

- **studies near the hydro limit - match σ to required transport coefficients, e.g.,**

$$\eta_s \approx \frac{4T}{5\sigma_{tr}}$$



we need: **initial conditions**
boundary conditions = expansion to vacuum ("empty" outside)
and hadronization model

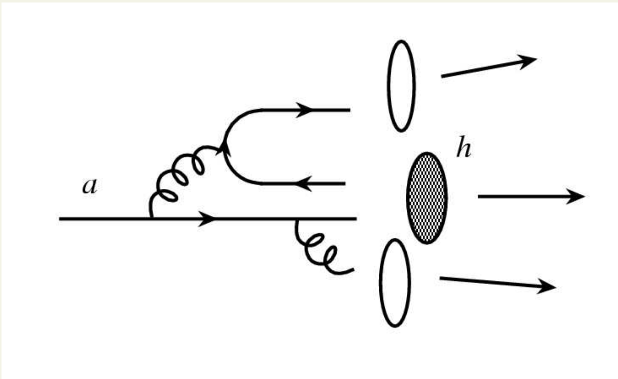
Hadronization

A poorly understood process - significant theory uncertainties.

- **local parton-hadron duality (one to one)** e.g., Eskola et al

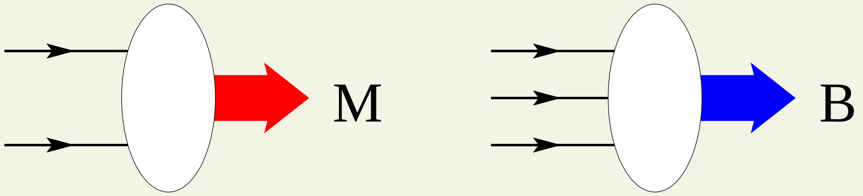
assumes quarks and gluons convert to hadrons (mainly pions), preserving momenta, especially useful at low pT

- **independent fragmentation (one to many)** Feymann, Field, ...



relevant for high-pT quarks and gluons

- **coalescence/recombination (few to one)** Hwa, Yang, Biró, Zimányi, Lévai, Csizmadia, Ko, Lin, Voloshin, DM, Greco, Fries, Müller, Nonaka, Bass, ...



relevant in A+A at intermediate $2 \lesssim p_T \lesssim 6$ GeV (needs high phasespace densities)

to lowest order $q\bar{q} \to M$, $qqq \to B$

Initial conditions

Similarities with hydrodynamics (one needs to specify initial "shapes"), however, transport also requires momentum distributions as input.

basic density profiles are similar to hydrodynamic calculations

- wounded nucleon (Glauber)
- binary collisions
- saturation model (e.g., Gribov-Levin-Rishikin approach)

momentum distributions are often based on

- at high $p_T > p_0 \sim 2$ GeV, perturbative QCD jet rates
- at low $p_T < p_0 \sim 2$ GeV, saturation physics or extrapolations (to set a certain total dN/dy)

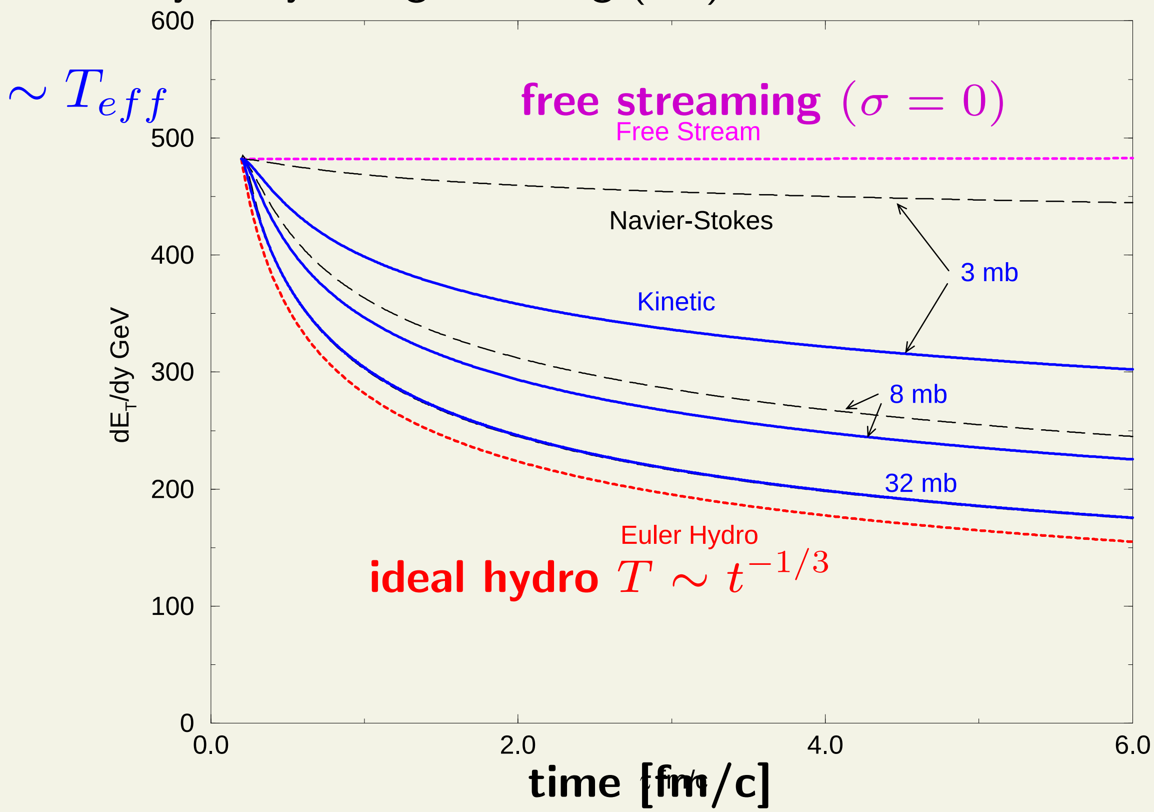
typical initialization times are $\tau_0 \sim 0.1 - 0.2$ fm

in studies at midrapidity, boost invariance can be useful to impose

Cooling

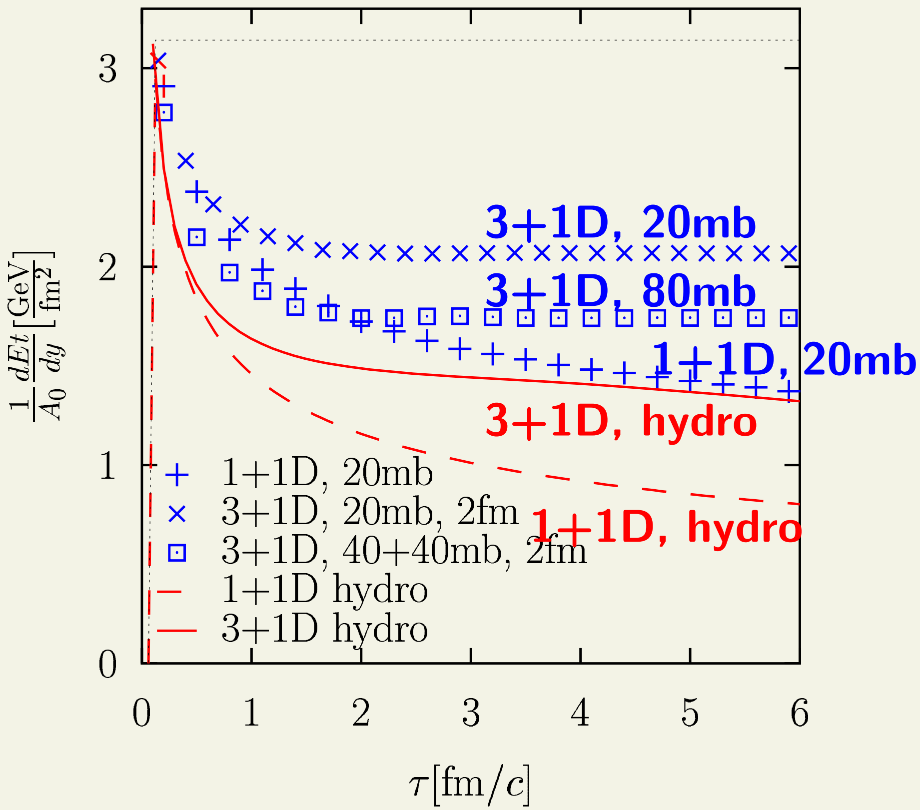
Expanding systems cool due to $p\,dV$ work

Gyulassy, Pang & Zhang ('97): 1+1D



DM & Gyulassy ('00): 3+1D ($dN/d\eta = 210$)

MPC vs hydro (1+1D and 3+1D)

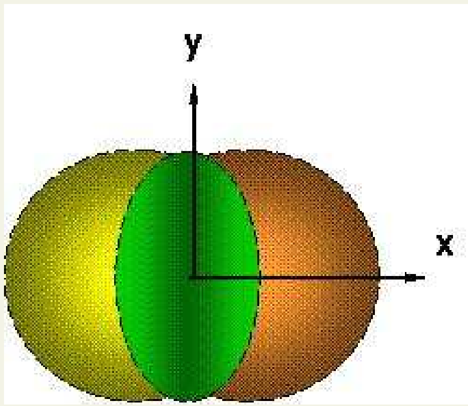


dissipation in transport slows cooling, **especially in 3+1D**

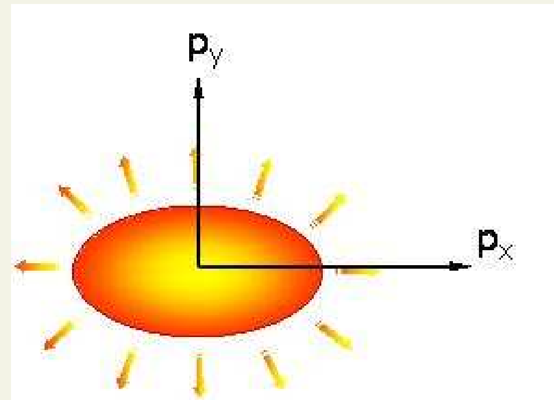
Elliptic flow

spatial anisotropy $\rightarrow$ final azimuthal momentum anisotropy

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



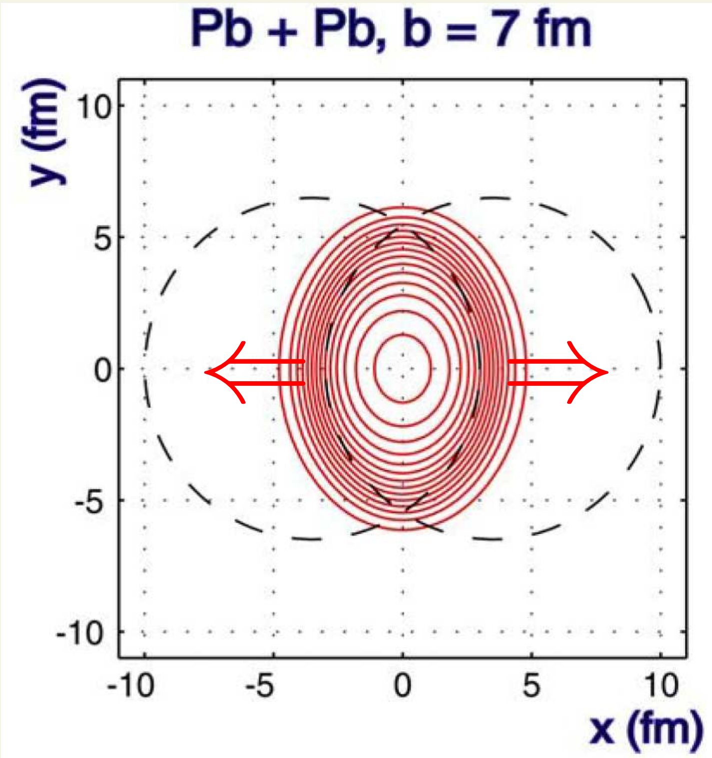
$$\longrightarrow \quad v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$



- measures strength of interactions
- self-quenching, develops at early times

macroscopically: **pressure gradients**

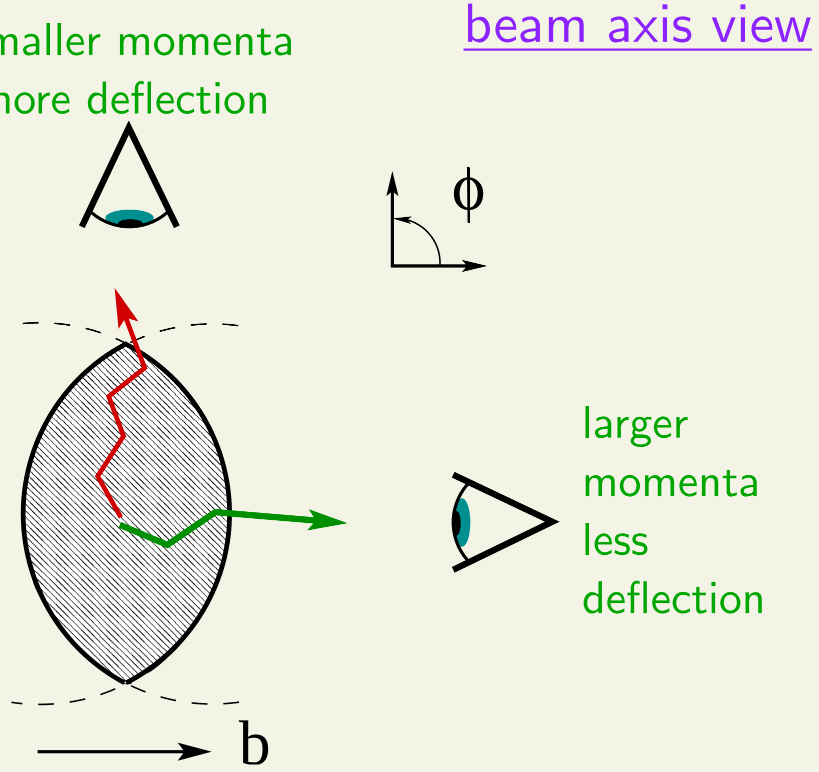
$$\Delta\vec{F}/\Delta V = -\vec{\nabla}p$$



$\Rightarrow$ **larger acceleration in impact parameter direction**

microscopically: **transport opacity**

beam axis view

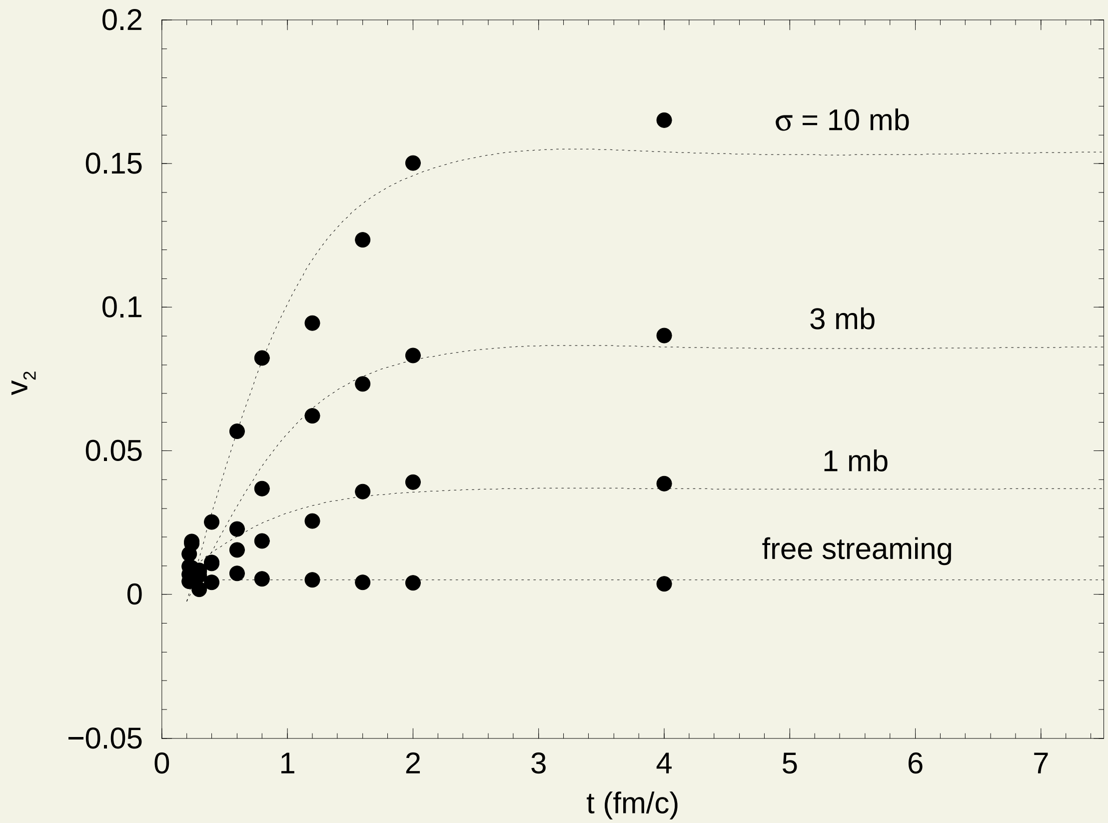


variation in pathlength

$\Rightarrow$ **momentum anisotropy** v_2

v_2 builds up early

Zhang, Gyulassy & Ko ('99): **anisotropy builds up during first ~ 2 fm/c**

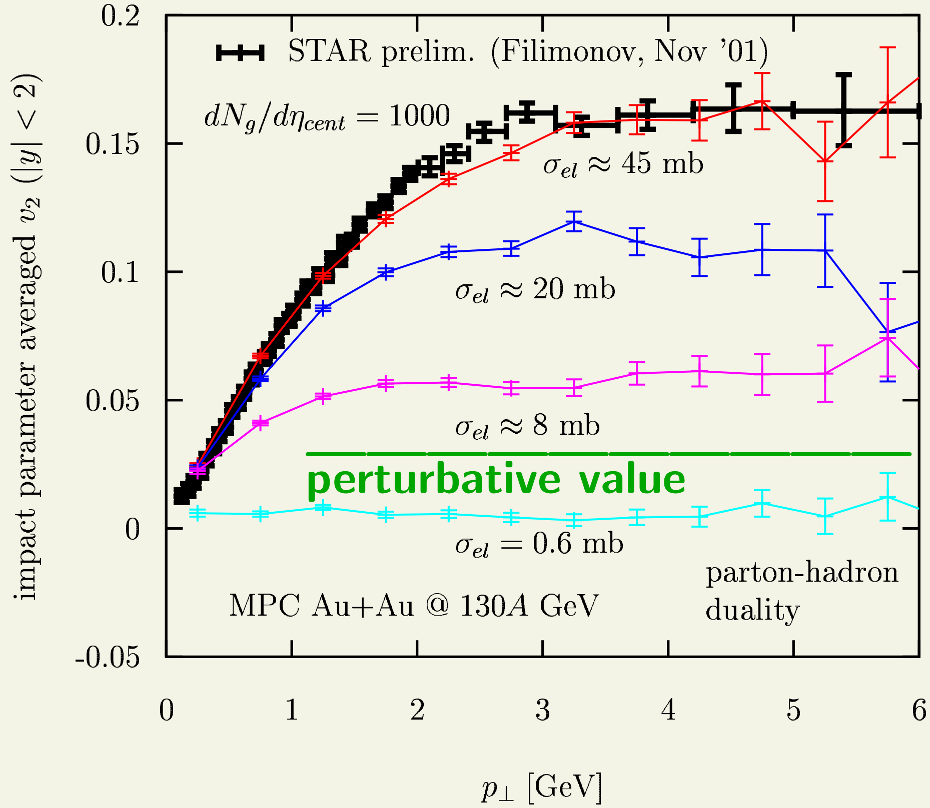


sharp cylinder $R = 5$ fm, $\tau_0 = 0.2$ fm/c, $b = 7.5$ fm, $dN^{cent}/dy = 300$

Strong interactions at RHIC

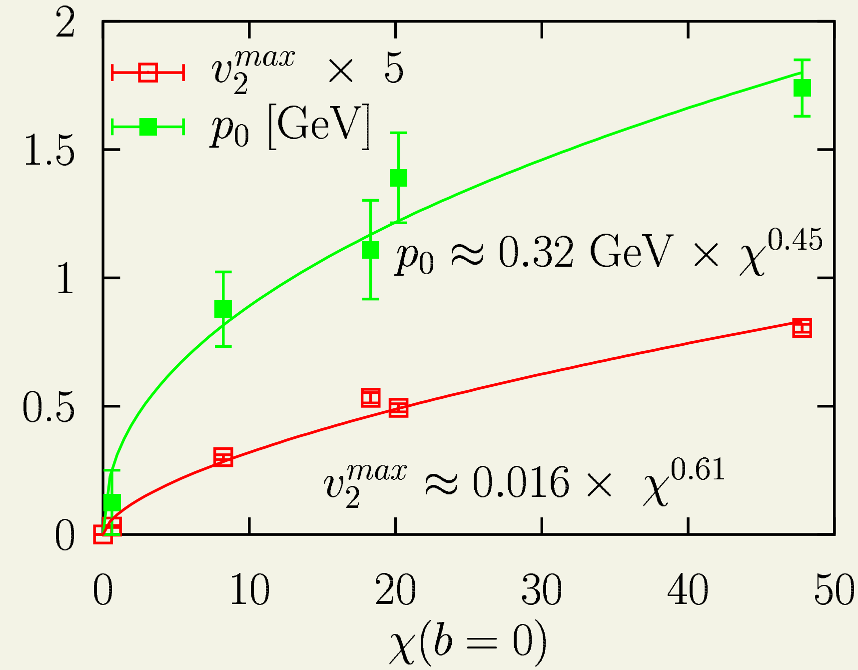
Au+Au @ 130 GeV, $b = 8$ fm

DM & Gyulassy, NPA 697 ('02): $\boldsymbol{v_2(p_T, \chi)}$



nonlinear opacity dependence

$$v_2(p_T, \chi) \approx v_2^{max}(\chi) \tanh(p_T/p_0(\chi))$$



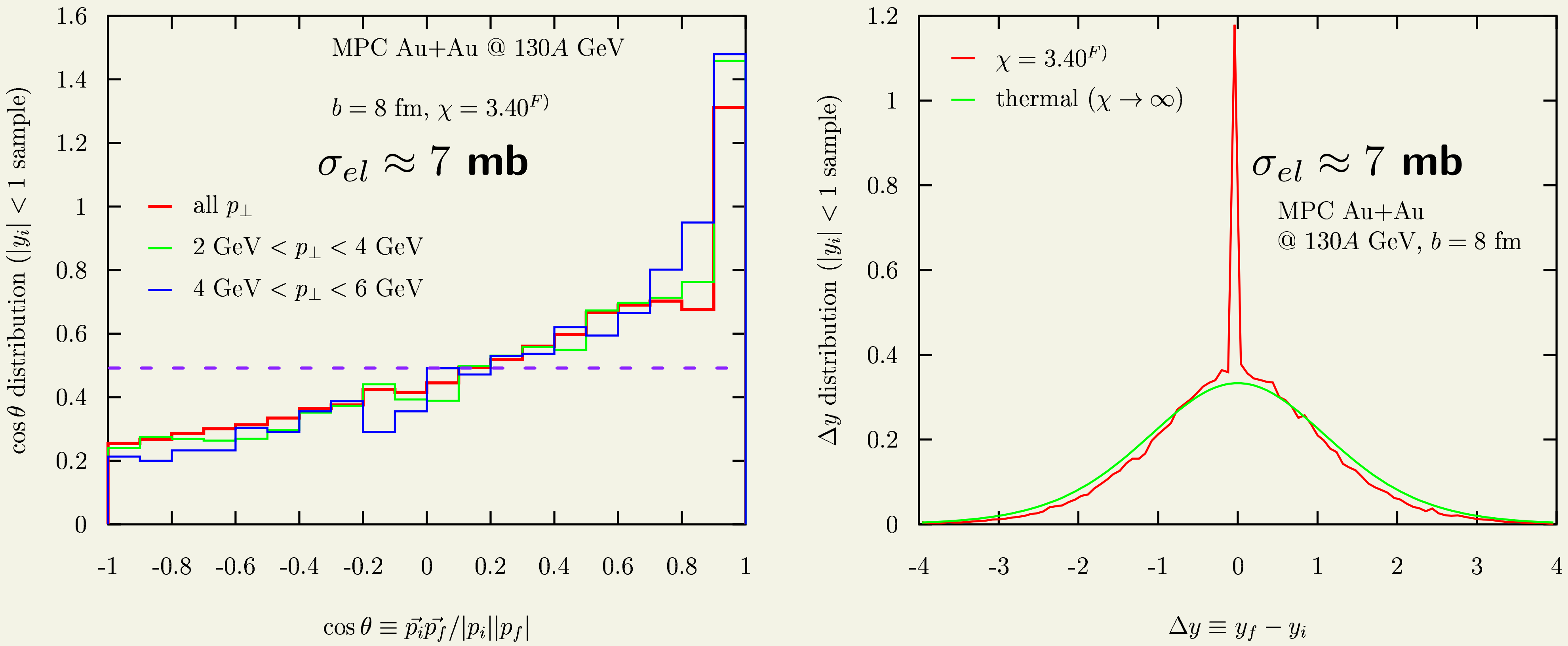
need $15\times$ **perturbative opacities -** $\sigma_{el} \times dN_g/d\eta \approx 45$ **mb** $\times 1000$

(saturated gluon $\frac{dN^{cent}}{d\eta} = 1000$, $T_{eff} \approx 0.7$ GeV, $\tau_0 = 0.1$ fm, 1 parton $\rightarrow$ 1 π hadronization)

Significant randomization

correlation between initial and final momenta

a) deflection angle $\vec{p}_i \angle \vec{p}_f$ b) rapidity shift $y_f - y_i$



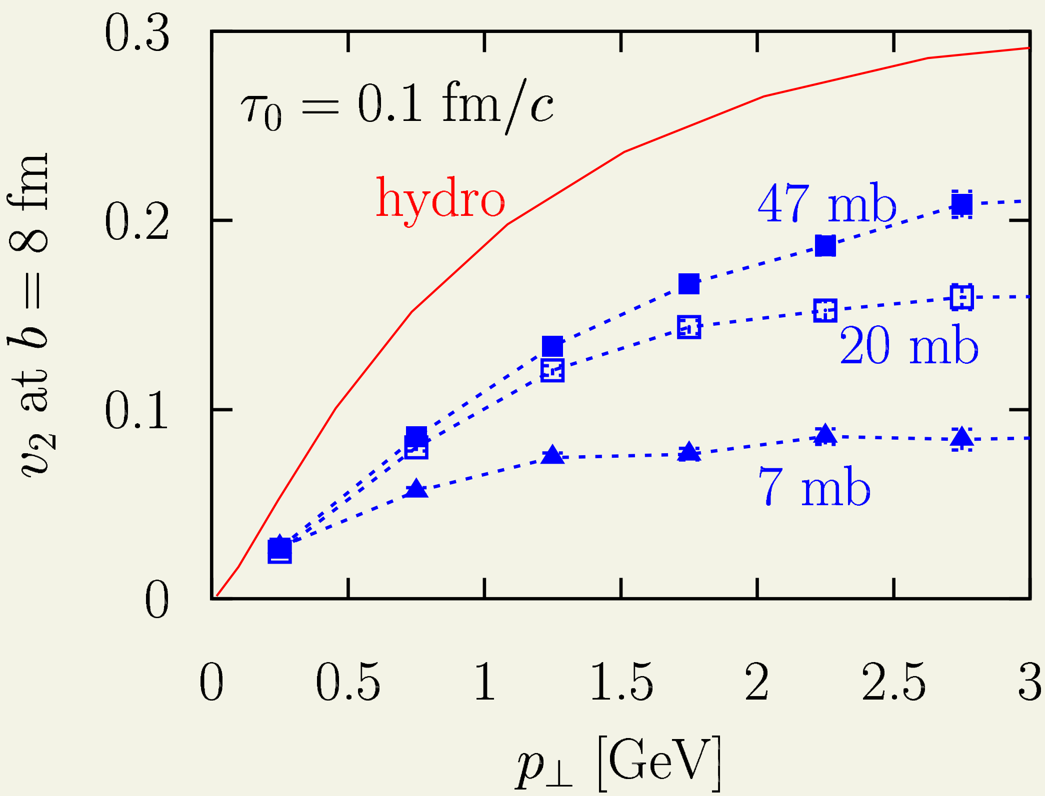
light parton momenta randomize to large degree, already for $\sigma \sim 7$ mb ($\chi \sim 7$)

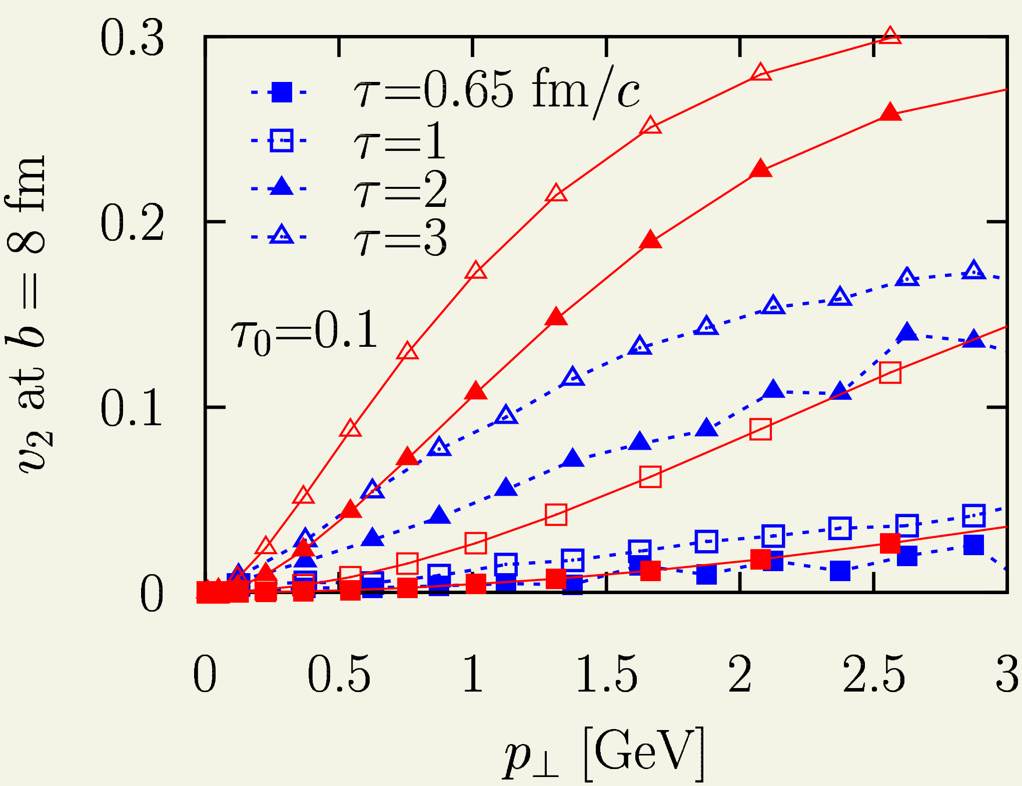
Not an ideal fluid

dissipation reduces v_2 by $30-50$% even for $\sigma_{gg\to gg} \sim 50$ mb

DM & Huovinen, PRL94 ('05): **final $v_2(p_T)$**

$v_2(\tau, p_T)$



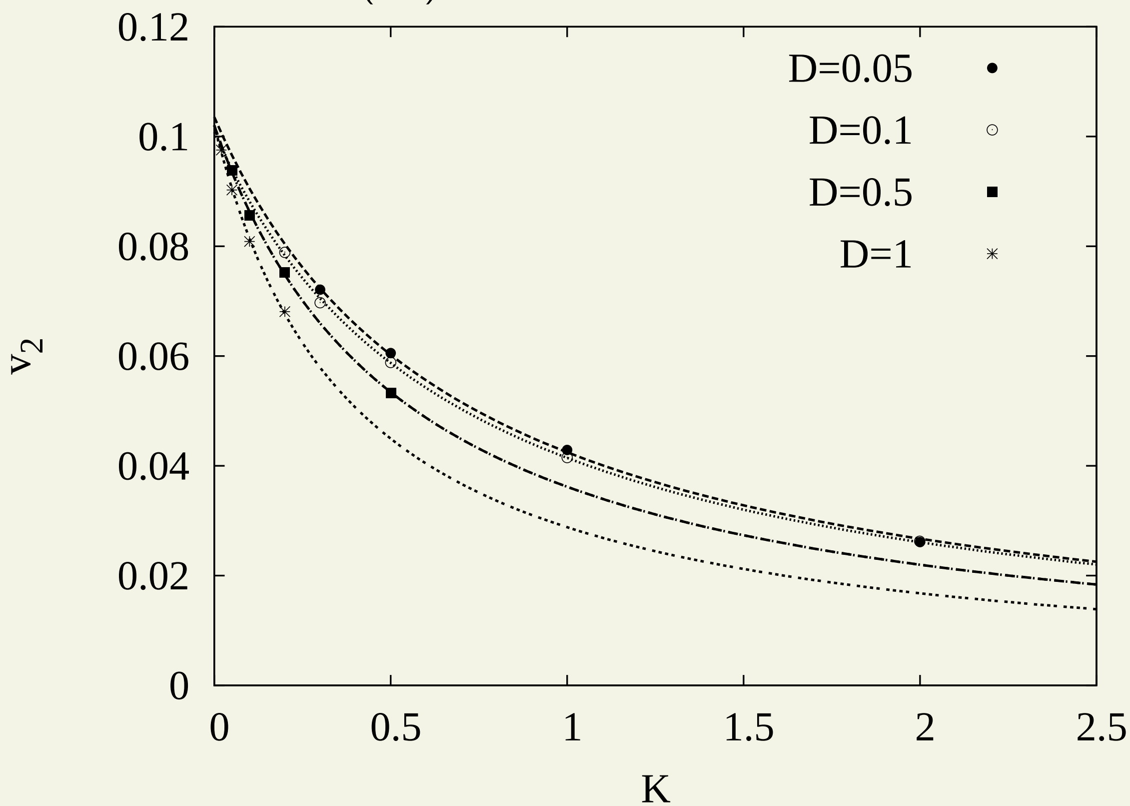


$\rightarrow$ **dense, strongly collective system, but still dissipative**

Hydrodynamic limit and v_2

2+1D calculation (2D space, 2D momenta, no longitudinal expansion)

Ollitrault & Gombeaud ('07):



$$v_2^{integrated} \sim \frac{\tilde{v_2}}{\sigma_0/\sigma + 1}$$

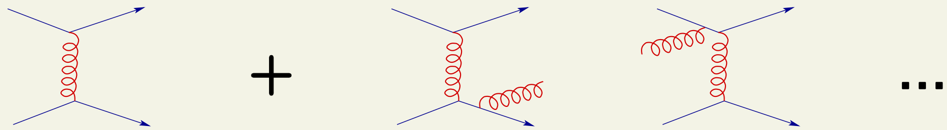
$$(\eta_{shear} \sim 1/\sigma \sim K_{Knudsen})$$

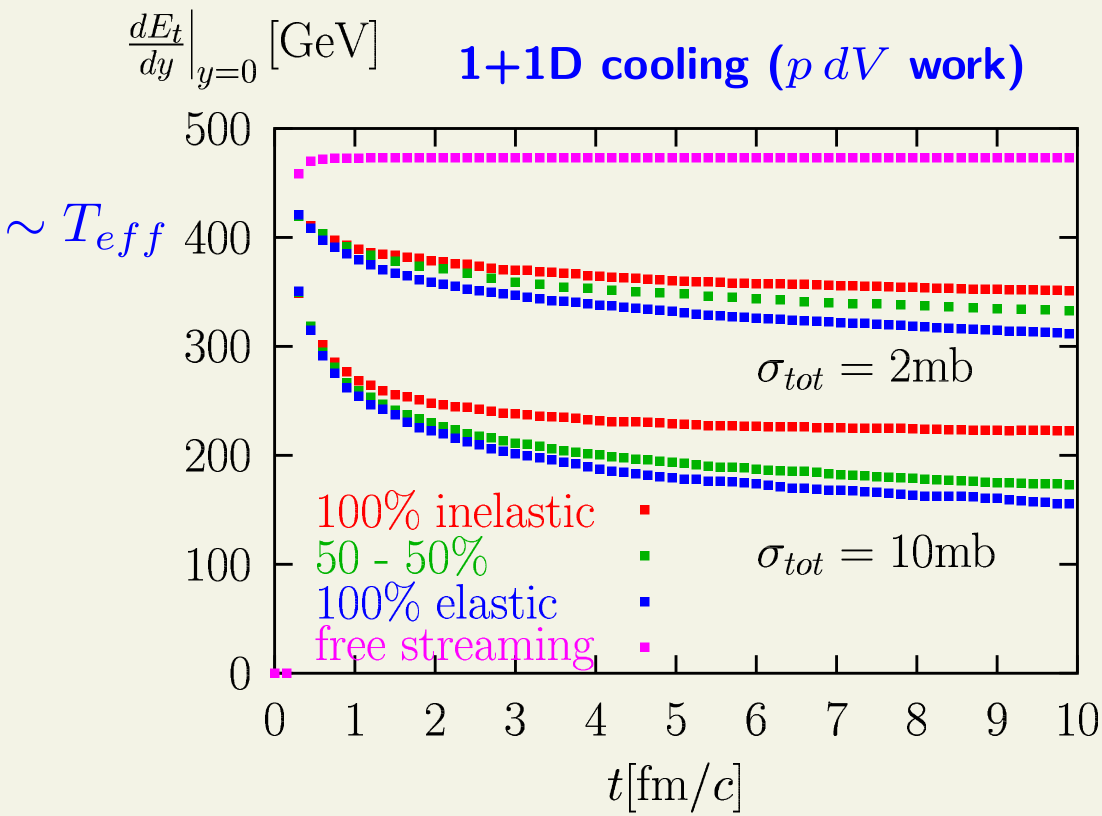
also works for 3+1D transport - $v_2(p_T, \sigma) = v_2^{max} \tanh(p_T/p_0)$ **fits to MPC results for Au+Au at RHIC,** $b = 8$ **fm**

$$v_2^{max}(\sigma) \approx \frac{0.404}{0.554\, mb/\sigma + 1}\,, \qquad p_0(\sigma) \approx \frac{2.92\, GeV}{0.187\, mb/\sigma + 1}$$

Radiative transport

higher-order processes also contribute to thermalization





DM & Gyulassy, NPA 661, 236 ('99)

fixed transport cross section

but vary degree of inelasticity

100% elastic, **100% inelastic**, **50-50%**

$2 \to 2$ $3 \leftrightarrow 2$ mixed

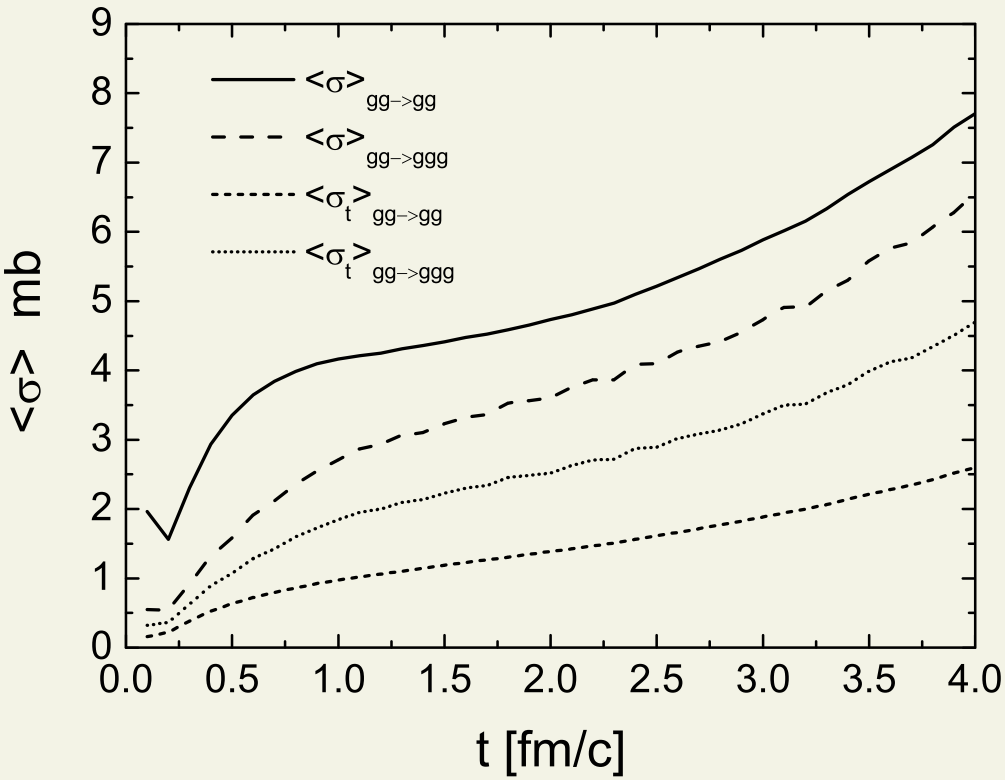
isotropic scattering(!)

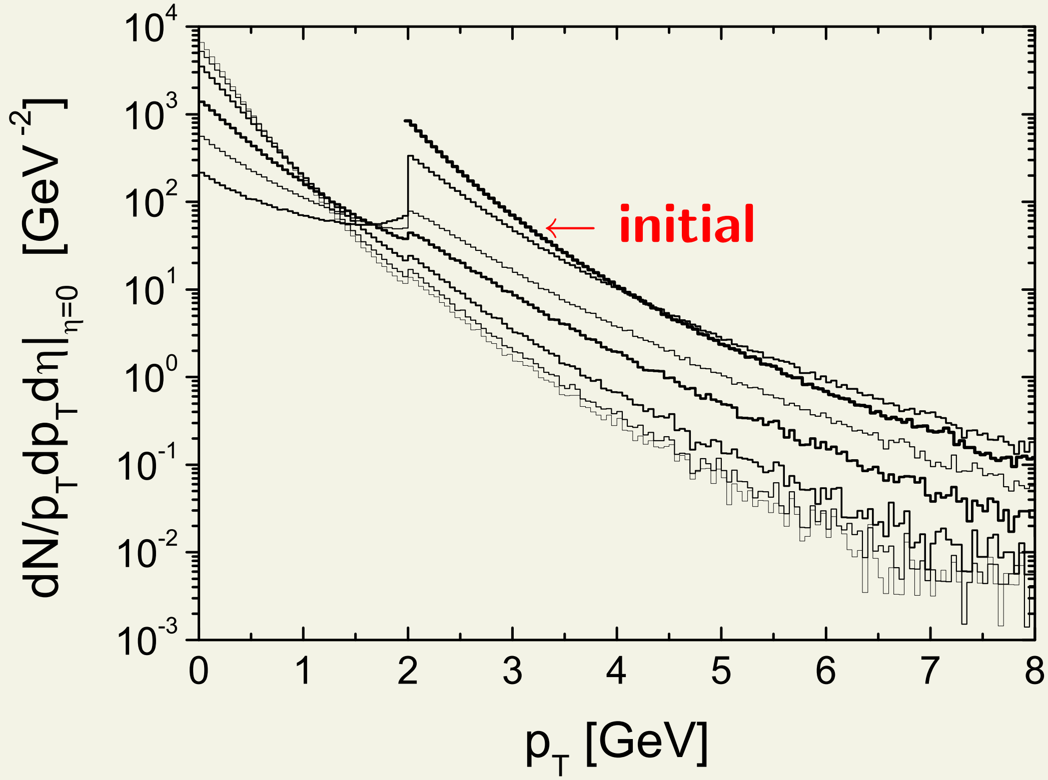
$\Rightarrow$ **inelastic $3 \leftrightarrow 2$ roughly same as elastic with same transport cross section**

more enhanced for pQCD cross sections because $3 \to 2$ allows large angles

Greiner & Xu '04: **find thermalization time-scale $\tau \sim 2 - 3$ fm/c**

$2 \to 2$, $2 \to 3$ transport cross sections | **spectra vs. time**

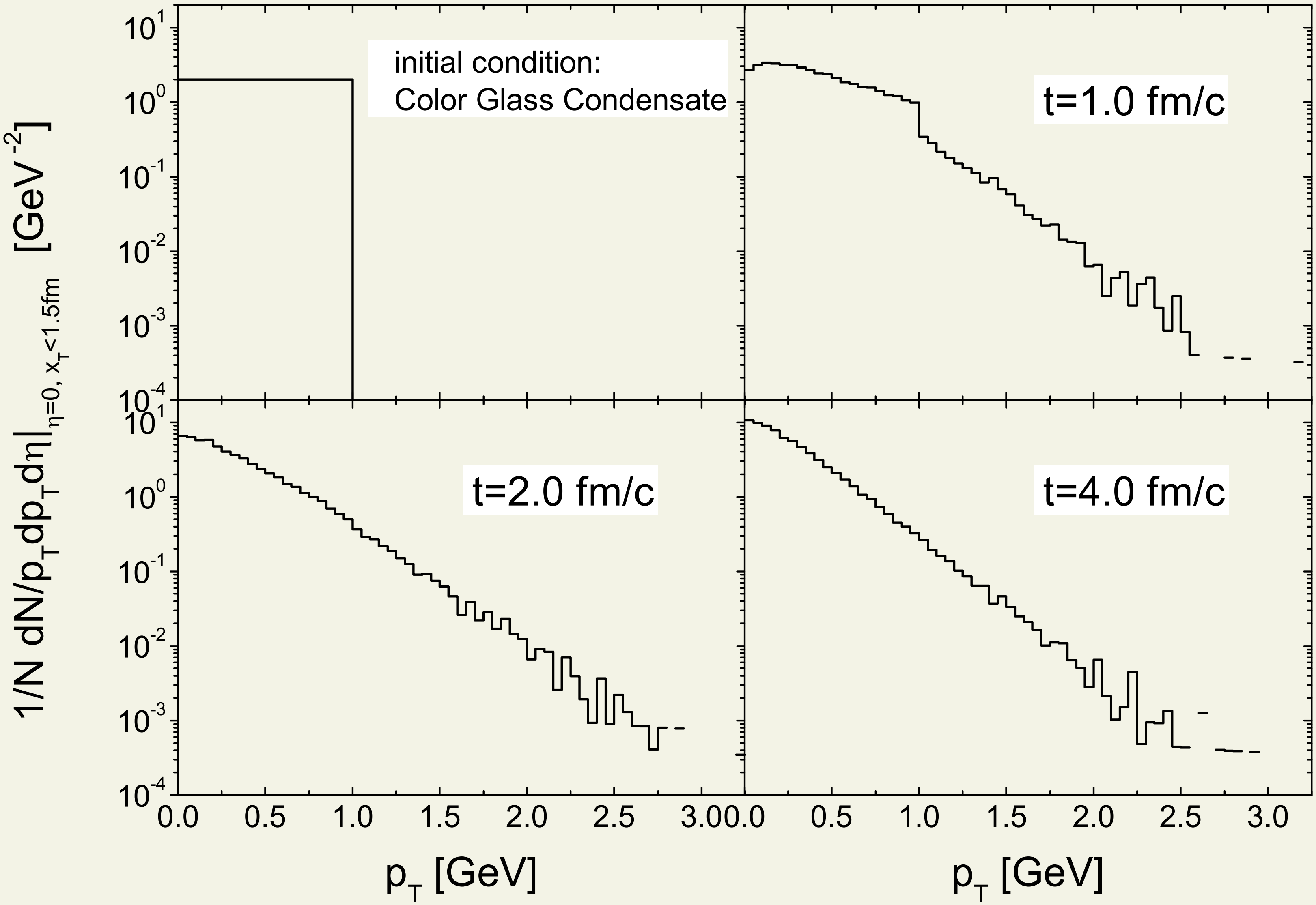




inelastic roughly doubles σ_{tr}

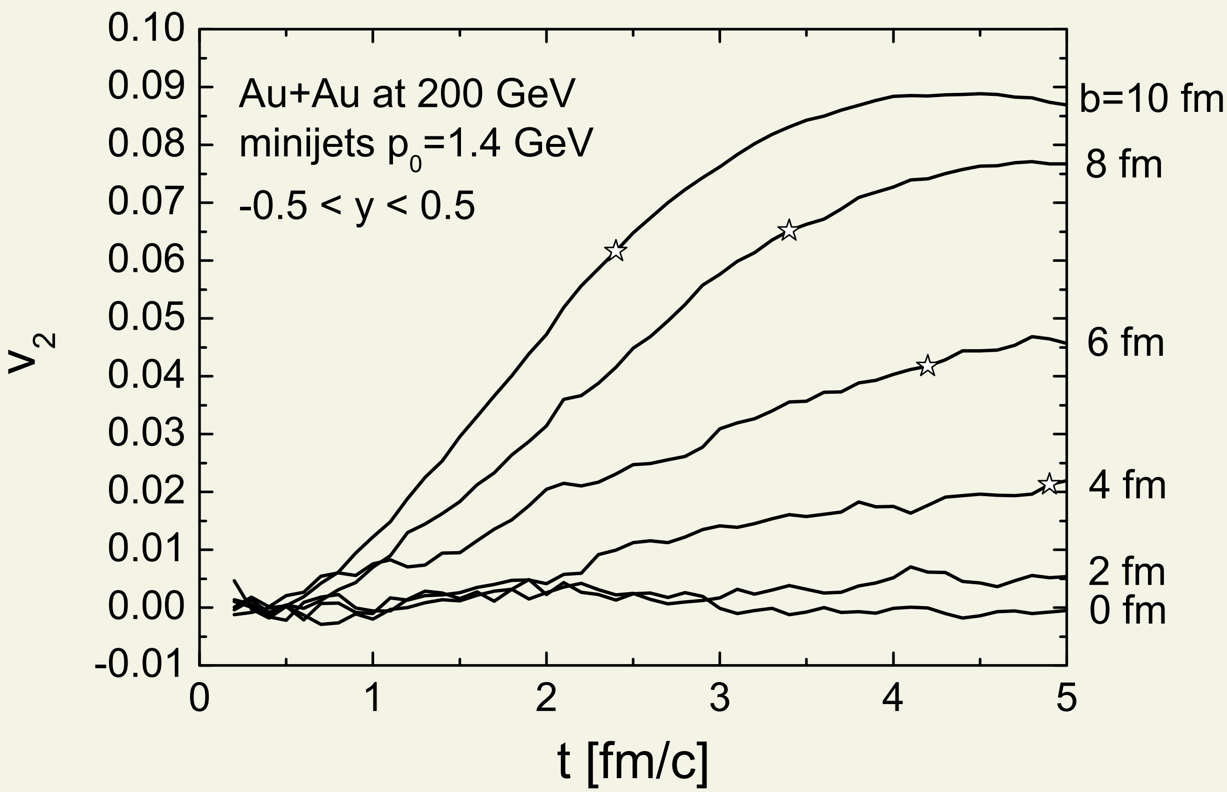
rapid cooling via $2 \to 3$, but may be because low-momentum region is empty

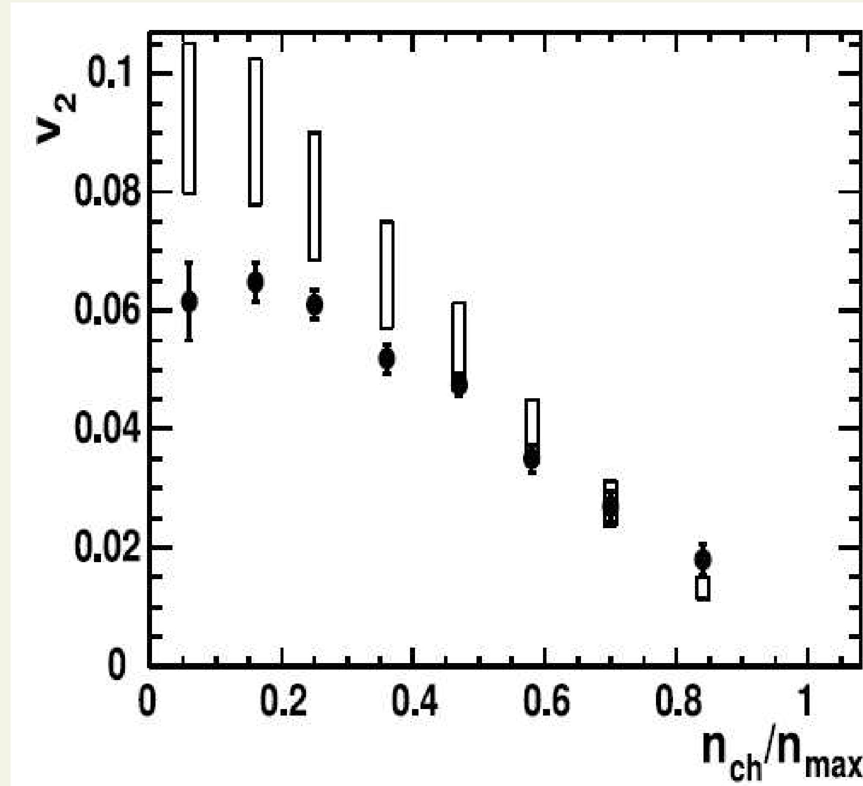
Xu & Greiner, NPA785, 132 ('07)



this other extreme also indicates short timescales ~ 2 fm

elliptic flow with $ggg \leftrightarrow gg$ (minijet initconds, $p_0 = 1.4$ GeV)





close to the experimental values at RHIC

Other interesting areas

- **heavy quarks - useful cross-check of dynamics/equilibration**

can also be done in the Fokker-Planck (small-angle) approximation, or in a Langevin approach (many random scatterings - Brownian motion)

Moore & Teaney... Gossiaux...

- **coupling to classical color fields**

Mrowczynski... Arnold, Moore, Yaffe... Dumitru, Strickland...

Wong equation: color Vlasov-Boltzmann Wong, Heinz

$$p^\mu \left(\partial_\mu + g t_a F^a_{\mu\nu} \partial_{p_\nu} + g f_{abc} A^b_\mu t^c \partial_{t_a}\right) f = C[f]$$

$$[D_\mu, F^{\mu\nu}_a] = J^\nu_a = g \int p^\nu t_a \left(f_q - \bar{f}_q + f_g\right) dP dQ$$

thermal plasma at $T \gg T_c$ would appear neutral - fast color rotations

$\tau^{-1}_{color} \sim g^2 \ln(1/g) T \gg \tau^{-1}_{mom} \sim g^4 \ln(1/g) T$ Selikhov '91 Gyulassy '92

However, anistropic particle distributions are rapidly isotropised ($\sim$ two-stream instability)

Covariant transport - summary

- covariant transport is a nonequilibrium framework to study a system of on-shell (quasi-)particles

- also useful to test formulations of hydrodynamics (transport is always causal and stable)

- elliptic flow data in Au+Au at RHIC reproduced suprisingly well with 15$\times$ enhanced perturbative $2 \to 2$ rates - not enough for ideal fluid behavior

- radiative $3 \leftrightarrow 2$ is very important for thermalization, results challenge the strongly-coupled plasma paradigm (should be verified independently)

- limited equation of state (no phase transition)

- hadronization challenging

- thermalization benefits from new ingredients such as classical fields