Lectures on hydrodynamics - Part III: Causal dissipative hydrodynamics

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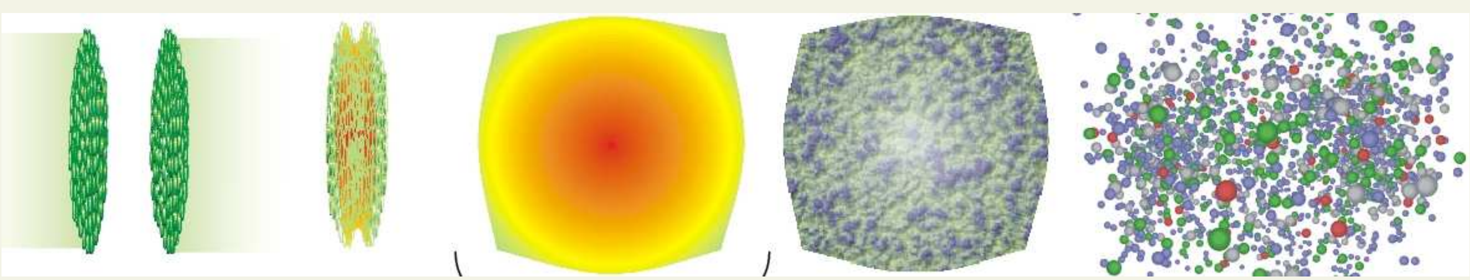
Outline

- **Why dissipative**
- **Viscosities in QCD**
- **Dissipative hydro EOMs - Navier-Stokes, Israel-Stewart theory**
- **What are the right equations? - cross-check from covariant transport**
- **Current state of art and open problems**

Hydrodynamics

- describes a system near local equilibrium
- long-wavelength, long-timescale dynamics, driven by conservation laws
- in heavy-ion physics: mainly used for the plasma stage of the collision

initial nuclei parton plasma hadronization hadron gas



$\sim 10^{-14}$ **m** = **10 fermis**

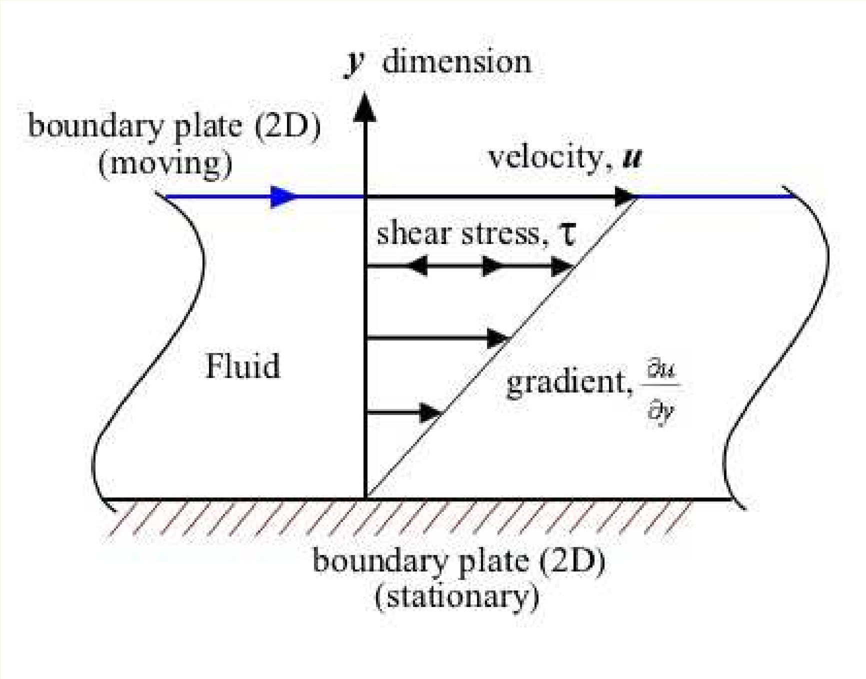
<—— $\sim 10^{-23}$ **sec = 10 yochto(!)-secs** ——>

nontrivial how hydrodynamics can be applicable at such microscopic scales

Shear viscosity

1687 - I. Newton (Principia)

$$T_{xy} = -\eta \frac{\partial u_x}{\partial y}$$



acts to reduce velocity gradients

1985 - quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

+ kinetic theory: $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz, PRD 31 ('85)

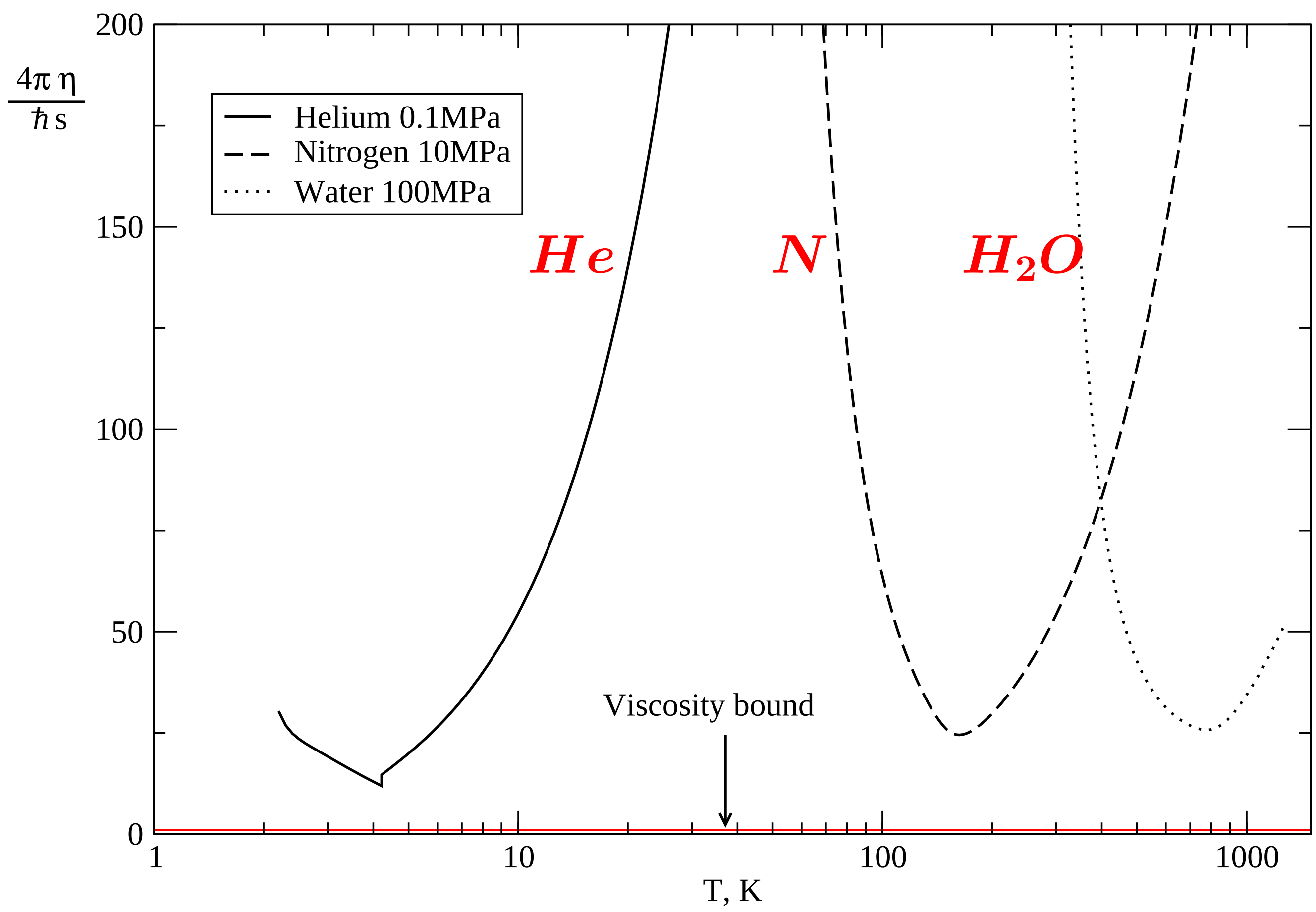
$\eta \approx 4/5 \cdot T/\sigma_{tr}$, **entropy** $s \approx 4n$

gives minimal viscosity: $\eta/s = \frac{\lambda_{tr} T}{5} \geq \hbar/15$

2004 - string theory AdS/CFT: $\eta/s \geq \hbar/4\pi$ Policastro, Son, Starinets, PRL87 ('02) Kovtun, Son, Starinets, PRL94 ('05)

revised to $4\hbar/(25\pi)$ Brigante et al, arXiv:0802.3318

$\sim 1/(4\pi)$ **bound conjectured universal - at least no other known substance comes within a factor 10** Kovtun, Son, Starinets, PRL94 ('05):



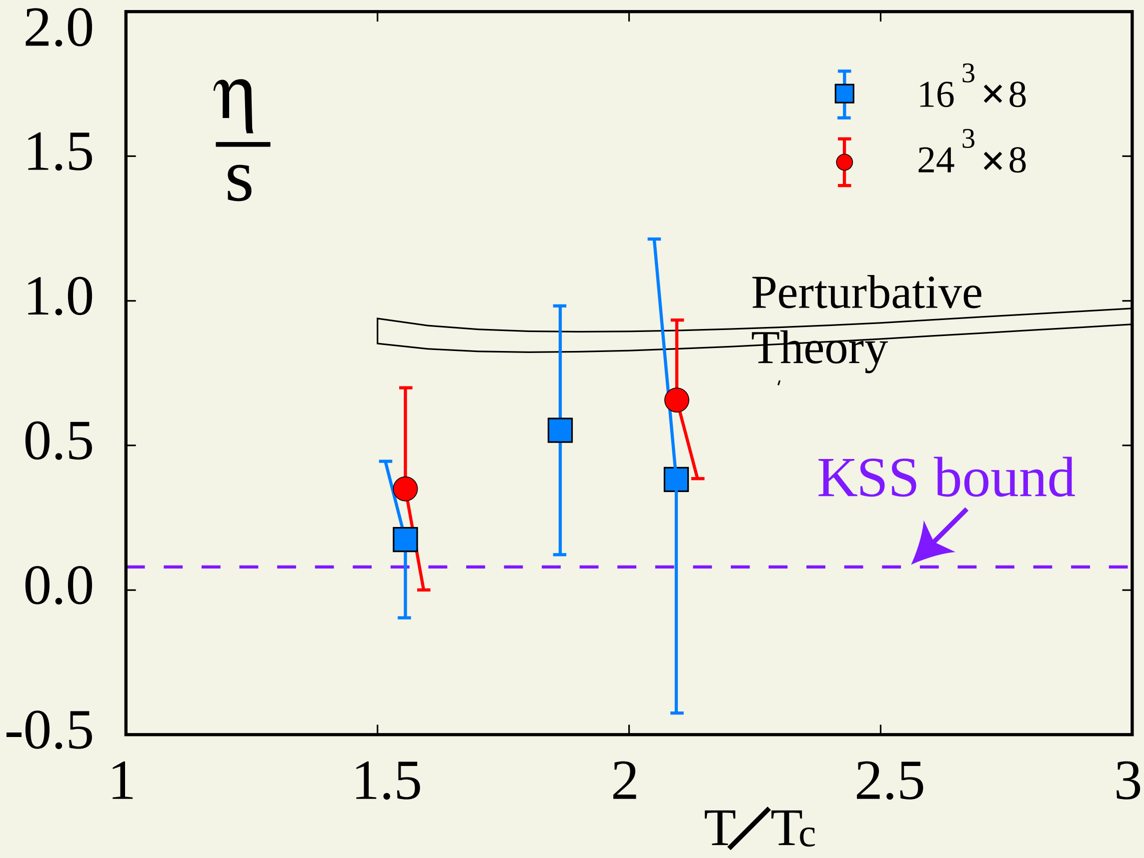
(perhaps strongly-interacting cold atoms...)

Shear viscosity in QCD

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

perturbative QCD: $\eta/s \sim 1$, **lattice QCD: correlator very noisy**

Nakamura & Sakai, NPA774, 775 ('06):



Meyer, PRD76, 101701 ('07)

upper bounds:

$\eta/s(T{=}1.65T_c) < 0.96$

$\eta/s(T{=}1.24T_c) < 1.08$

best estimate:

$\eta/s(T{=}1.65T_c) < 0.13 \pm 0.03$

$\eta/s(T{=}1.24T_c) < 0.10 \pm 0.05$

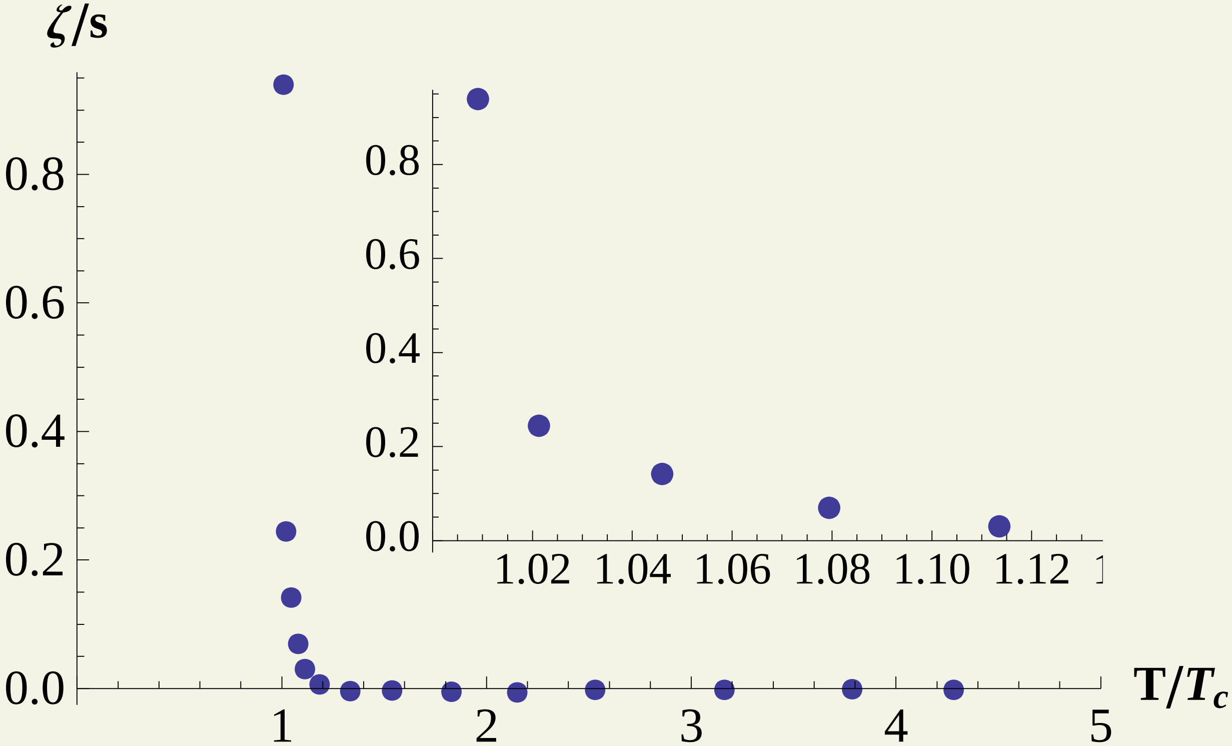
many practitioners regard these VERY preliminary

Bulk viscosity in QCD

$$\zeta = \lim_{\omega\to 0} \frac{1}{18\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T^\mu_\mu(\vec{x},t), T^\mu_\mu(0)] \rangle$$

perturbative QCD: $\zeta/s \sim 0.02\alpha_s^2$ **is tiny** Arnold, Dogan, Moore, PRD74 ('06)

from $\varepsilon - 3p > 0$**:** Kharzeev & Tuchin, arXiv:0705.4280v2



on lattice: Meyer, arXiv:0710.3717

best estimates:

$\eta/s(T{=}1.65T_c) \sim 0 - 0.015$

$\eta/s(T{=}1.24T_c) \sim 0.06 - 0.1$

$\eta/s(T{=}1.02T_c) \sim 0.2 - 2.7$

many practitioners regard these as well VERY preliminary

If we can quantify dissipative effects on heavy ion observables, we could constrain the viscosities. But cannot use ideal hydro, which has no dissipation.

Two ways to study dissipative effects in heavy-ion collisions

- causal dissipative hydrodynamics

Israel, Stewart; ... Muronga, Rischke; Teaney et al; Romatschke et al; Heinz et al, DM & Huovinen

flexible in macroscopic properties

numerically cheaper

- covariant transport

Israel, de Groot,... Zhang, Gyulassy, DM, Pratt, Xu, Greiner...

completely causal and stable

fully nonequilibrium $\longrightarrow$ interpolation to break-up stage

Several active groups

- **Paul Romatschke et al**
- **Huichao Song & Ulrich Heinz**
- **Derek Teaney & Kevin Dusling**
- **DM & Pasi Huovinen**
- **Takeshi Kodama & Tomo Koide et al**
- **...**

no public codes (yet)

State of the art is 2+1D calculations (with Bjorken boost invariance)

Relativistic dissipative hydro

Decompose energy-momentum tensor and currents using a flow field $u^\mu(x)$

In local rest frame (LR) (where $u^\mu_{LR} = (1, \vec{0})$),

$$T^{\mu\nu}_{LR} = \begin{pmatrix} \varepsilon & h_x & h_y & h_z \\ h_x & p + \pi_{xx} + \Pi & \pi_{xy} & \pi_{xz} \\ h_y & \pi_{xy} & p + \pi_{yy} + \Pi & \pi_{yz} \\ h_z & \pi_{xz} & \pi_{yz} & p + \pi_{zz} + \Pi \end{pmatrix} , \qquad N^\mu_{LR} = (n, \vec{V})$$

$\vec{h}(x)$ - **energy flow,** $\Pi(x)$ - **bulk pressure,** $\pi_{ij}(x)$ - **shear stress**

In general frame

$$T^{\mu\nu} = (\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)\, g^{\mu\nu} + (u^\mu h^\nu + u^\nu h^\mu) + \pi^{\mu\nu}$$
$$N^\mu = n u^\mu + V^\mu \qquad (u^\mu h_\mu = 0 = u^\mu V_\mu \ , \quad u^\mu \pi_{\mu\nu} = 0 = \pi_{\mu\nu} u^\nu, \quad \pi^\nu_\nu = 0)$$

So far $u^\mu(x)$ is arbitrary. Most common choices:

- **Eckart: no particle flow in LR** $\rightarrow$ $\vec{V} = 0$, $u^\mu = N^\mu / \sqrt{N_\alpha N^\alpha}$
- **Landau: no energy flow in LR** $\rightarrow$ $\vec{h} = 0$, $u^\mu = u^\nu T^{\mu\nu} / \sqrt{u_\alpha T^{\alpha\beta} T_{\beta\gamma} u^\gamma}$

Dissipative hydrodynamics

relativistic Navier-Stokes hydro: **small corrections linear in gradients** [Landau]

$$T^{\mu\nu}_{NS} = T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha$$

$$N^\nu_{NS} = N^\nu_{ideal} + \kappa\left(\frac{n}{\varepsilon + p}\right)^2 \nabla^\nu\left(\frac{\mu}{T}\right)$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ **shear and bulk viscosities,** κ **heat conductivity**

Equation of motion: $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu N^\mu = 0$

two problems:

parabolic equations $\rightarrow$ **acausal** Müller ('76), Israel & Stewart ('79) ...

instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

As an illustration, consider heat flow in a static, incompressible fluid (Fourier)

$$\partial_t T = \kappa \Delta T$$

parabolic eqns.

Greens function is acausal (allows $\Delta x > \Delta t$)

$$G(\vec{x}, t; \vec{x}_0, t_0) = \frac{1}{[4\pi\kappa(t - t_0)]^{3/2}} \exp\left[-\frac{(\vec{x} - \vec{x}_0)^2}{4\kappa(t - t_0)}\right]$$

—–

Adding a second-order time derivative makes it hyperbolic

$$\tau \partial_t^2 T + \partial_t T = \kappa \Delta T$$

Note, this is equivalent to a relaxing heat current

$$\partial_\tau T = \vec{\nabla} \vec{j} \;, \qquad \partial_t \vec{j} = -\frac{\vec{j} - \kappa \vec{\nabla} T}{\tau}$$

The wave dispersion relation is $\omega^2 + i\omega/\tau = \kappa k^2/\tau$, i.e., now signals propagate at speeds $c_s = \sqrt{\kappa/\tau}$ (at low frequencies), causal for large enough τ.

Causal dissipative hydro

Bulk pressure Π, shear stress $\pi^{\mu\nu}$, heat flow q^μ are dynamical quantities

$$T^{\mu\nu} \equiv T^{\mu\nu}_{ideal} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu} \ , \qquad N^\mu \equiv N^\mu_{ideal} - \frac{n}{e+p} q^\mu$$

Israel-Stewart: truncate entropy current at quadratic order [Ann.Phys 100 & 118]

$$S^\mu = u^\mu \left[s - \frac{1}{2T} \left(\beta_0 \Pi^2 - \beta_1 q_\nu q^\nu + \beta_2 \pi_{\nu\lambda}\pi^{\nu\lambda} \right) \right] + \frac{q^\mu}{T} \left(\frac{\mu n}{\varepsilon + p} + \alpha_0 \Pi \right) - \frac{\alpha_1}{T} \pi^{\mu\nu} q_\nu$$

in Landau frame. Note, $\alpha_0 = \alpha_1 = \beta_0 = \beta_1 = \beta_2 = 0$ gives Navier-Stokes.

Impose $\partial_\mu S^\mu \geq 0$ via a quadratic ansatz

$$T\partial_\mu S^\mu = \frac{\Pi^2}{\zeta} - \frac{q_\mu q^\mu}{\kappa_q T} + \frac{\pi_{\mu\nu}\pi^{\mu\nu}}{2\eta_s} \geq 0$$

E.g.,

$$T\partial_\mu S^\mu = \Pi X - q^\mu X_\mu + \pi^{\mu\nu} X_{\mu\nu}$$

will lead to equations of motion

$$\Pi = \zeta X \ , \quad q^\mu = \kappa T \Delta^{\mu\nu} X_\nu \ , \quad \pi^{\mu\nu} = 2\eta_s X^{\langle\mu\nu\rangle}$$

The resulting equations relax dissipative quantities on time scales

$$\tau_\Pi(e,n) = \beta_0 \zeta \ , \qquad \tau_\pi(e,n) = 2\beta_2 \eta \ , \qquad \tau_q(e,n) = \beta_1 \kappa T$$

toward values set by gradients - because not only first but also certain second derivatives are kept.

<u>**schematically**</u>

$$\dot{X} = -\frac{X - X_0}{\tau_X} + X_c$$

restores causality (for not too small τ_X) telegraph eqn

Splitting $q\Pi$ and $q\pi$ terms between heat and bulk, and heat and shear equations is ambiguous, and requires additional matter parameters $a_0(\varepsilon, n)$, $a_1(\varepsilon, n)$ to specify Israel, Stewart... Huovinen & DM, arXiv:0808.0953

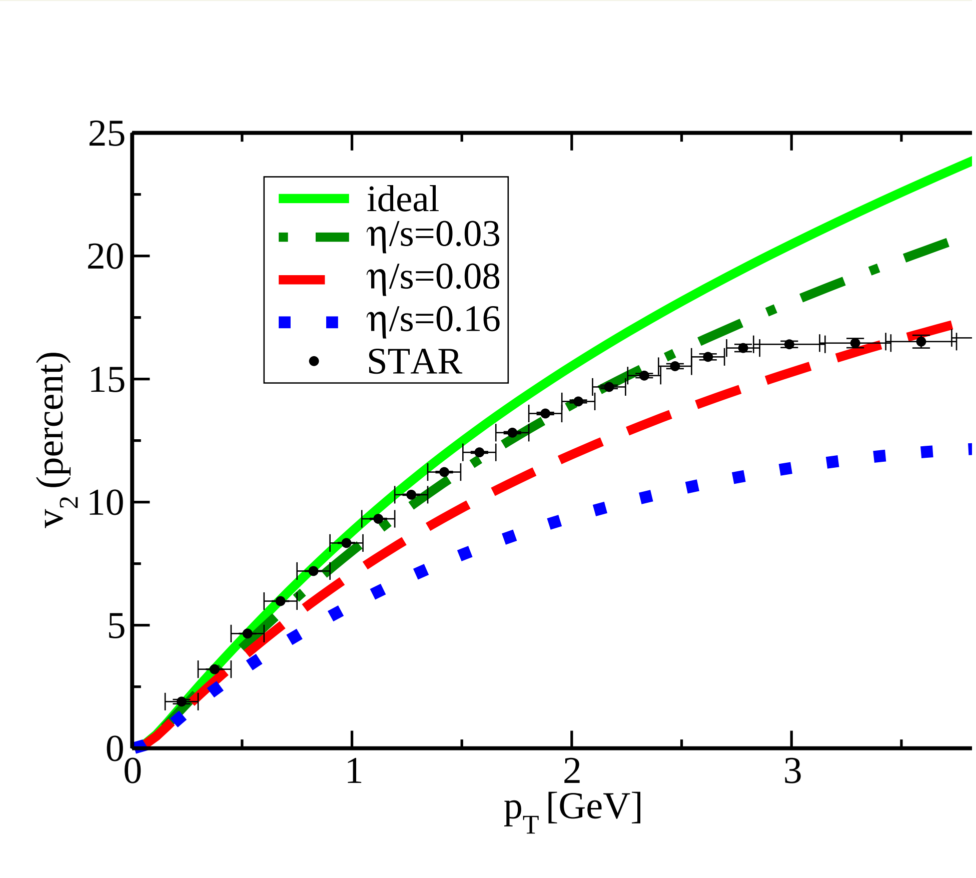
Moreover, further terms that produce no entropy can be added, which are missed by the Israel-Stewart procedure.

Complete set of Israel-Stewart equations of motion

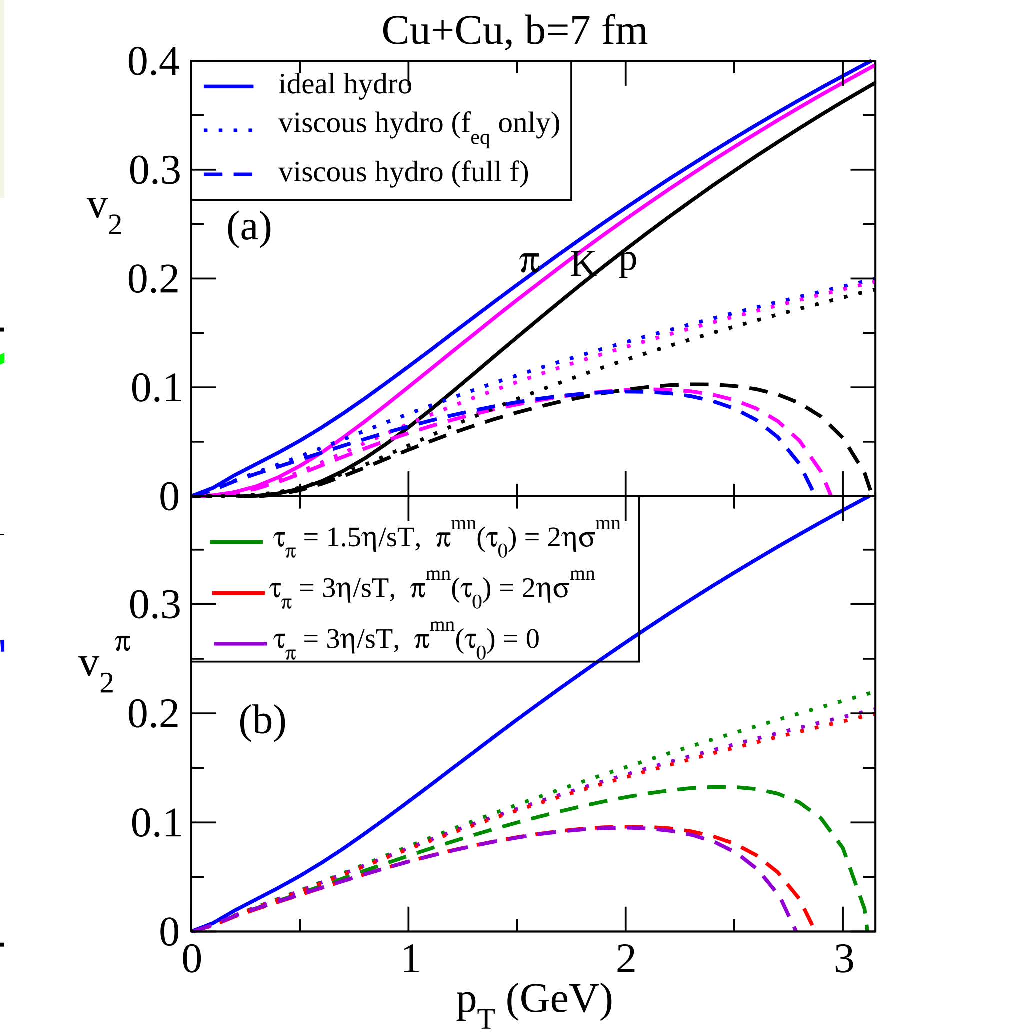
$$
\begin{aligned}
D\Pi &= -\frac{1}{\tau_\Pi}(\Pi + \zeta\nabla_\mu u^\mu) \\
&\quad -\frac{1}{2}\Pi\left(\nabla_\mu u^\mu + D\ln\frac{\beta_0}{T}\right) \\
&\quad +\frac{\alpha_0}{\beta_0}\partial_\mu q^\mu - \frac{a_0'}{\beta_0} q^\mu D u_\mu \qquad (1)\\
Dq^\mu &= -\frac{1}{\tau_q}\left[q^\mu + \kappa_q \frac{T^2 n}{\varepsilon + p}\nabla^\mu\left(\frac{\mu}{T}\right)\right] - u^\mu q_\nu D u^\nu \\
&\quad -\frac{1}{2} q^\mu\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_1}{T}\right) - \omega^{\mu\lambda} q_\lambda \\
&\quad -\frac{\alpha_0}{\beta_1}\nabla^\mu\Pi + \frac{\alpha_1}{\beta_1}(\partial_\lambda \pi^{\lambda\mu} + u^\mu \pi^{\lambda\nu}\partial_\lambda u_\nu) + \frac{a_0}{\beta_1}\Pi D u^\mu - \frac{a_1}{\beta_1}\pi^{\lambda\mu} D u_\lambda \qquad (2)\\
D\pi^{\mu\nu} &= -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu) D u_\lambda \\
&\quad -\frac{1}{2}\pi^{\mu\nu}\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_2}{T}\right) - 2\pi_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} \\
&\quad -\frac{\alpha_1}{\beta_2}\nabla^{\langle\mu} q^{\nu\rangle} + \frac{a_1'}{\beta_2} q^{\langle\mu} D u^{\nu\rangle} \ . \qquad (3)
\end{aligned}
$$

where $A^{\langle\mu\nu\rangle} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(A_{\alpha\beta}+A_{\beta\alpha}) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}A^{\alpha\beta}$, $\qquad \omega^{\mu\nu} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\beta u_\alpha - \partial_\alpha u_\beta)$

Recent viscous hydro calculations disagreed



Romatschke & Romatschke, arxiv:0706.1522



Song & Heinz, arxiv:0709.0742

for $\eta/s = 1/(4\pi)$, $\sim$20% or 50+% elliptic flow reduction??

Origin of difference is in WHICH TERMS are kept in Israel-Stewart eqns:

$$\begin{aligned}\dot{\pi}^{\mu\nu} &= -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}) - (u^\mu\pi^{\nu\alpha} + u^\nu\pi^{\alpha\mu})\dot{u}_\alpha \\ &- \frac{1}{2}\pi^{\mu\nu}D_\alpha u^\alpha - \frac{1}{2}\pi^{\mu\nu}[\ln\dot{\frac{\beta_2}{T}}] + 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} \qquad (4)\end{aligned}$$

Heinz et al neglected terms in green.

Which terms to keep?? Can only tell via comparing to a nonequilibrium theory.

IS hydro and covariant transport

Israel-Stewart hydro can be derived from covariant transport through Grad's 14-moment approximation

$$f(x,p) \approx [1 + \tilde{C}_\alpha p^\alpha + C_{\alpha\beta} p^\alpha p^\beta] f_{eq}(x,p)$$

via taking the "1", p^ν, and $p^\nu p^\alpha$ moments of the transport equation.

However, whereas Navier-Stokes came from a rigorous expansion in small deviations near local equilibrium retaining all powers of momentum (recall integral eqn from Part II), the quadratic truncation in Grad's approach has no small control parameter.

If relaxation effects important, NS and IS are different

$\Rightarrow$ **control against a nonequilibrium theory is crucial**

Applicability of IS hydro

Important to realize - in heavy ion physics applications, gradients $\partial^\mu u^\nu / T$, $|\partial^\mu e|/(Te)$, $|\partial^\mu n|/(Tn)$ at early times $\tau \sim 1$ fm are large $\sim \mathcal{O}(1)$, and therefore cannot be ignored.

Hydrodynamics may still apply, if viscosities are unusually small $\eta/s \sim 0.1$, $\zeta/s \sim 0.1$, where s is the entropy density in local equilibrium. In that case, pressure corrections from Navier-Stokes theory still moderate

$$\frac{\delta T^{\mu\nu}_{NS}}{p} \approx \left(2\frac{\eta_s}{s}\frac{\nabla^{\langle\mu}u^{\nu\rangle}}{T} + \frac{\zeta}{s}\frac{\nabla_\alpha u^\alpha}{T}\right)\frac{\varepsilon + p}{p} \sim \mathcal{O}\left(\frac{8\eta_s}{s}, \frac{4\zeta}{s}\right) \quad . \tag{5}$$

Heat flow effects can also be estimated based on

$$\frac{\delta N^\mu_{NS}}{n} \approx \frac{\kappa_q T}{s}\frac{n}{s}\frac{\nabla^\mu(\mu/T)}{T} \tag{6}$$

and should be very small at RHIC because $\mu/T \sim 0.2$, $n_B/s \sim \mathcal{O}(10^{-3})$

Consider Bjorken scenario, NO transverse expansion, $u^\mu(x) = (t,0,0,z)/\sqrt{t^2-z^2}$, which approximates well the initial evolution in a heavy ion collision, and follow shear stress only.

$$T^{\mu\nu}_{LR} = \begin{pmatrix} e & & & \\ & p-\frac{\pi_L}{2} & & \\ & & p-\frac{\pi_L}{2} & \\ & & & p+\pi_L \end{pmatrix} = \begin{pmatrix} e & & & \\ & p_T & & \\ & & p_T & \\ & & & p_L \end{pmatrix}$$

importance of dissipation can be gauged via the pressure anisotropy

$$R \equiv \frac{p_L}{p_T} = \frac{p+\pi_L}{p-\pi_L/2} \qquad \text{(typically } \pi_L < 0 \Rightarrow R < 1)$$

study R as a function of the initial <u>inverse</u> mean free path $K_0 \equiv \tau_0/\lambda_{tr,0}$

if Grad's approximation is valid, IS should apply at large enough K_0

take simplest of all cases - 1D Bjorken, massless $e = 3p$ EOS, $2 \to 2$

$\pi^{\mu\nu}_{LR} = diag(0, -\frac{\pi_L}{2}, -\frac{\pi_L}{2}, \pi_L)$, $\Pi \equiv 0$, $q^\mu \equiv 0$ **(reflection symmetry)**

$$\dot{p} + \frac{4p}{3\tau} = -\frac{\pi_L}{3\tau} \tag{7}$$

$$\dot{\pi}_L + \frac{\pi_L}{\tau}\left(\frac{2K(\tau)}{3C} + \frac{4}{3} + \frac{\pi_L}{3p}\right) = -\frac{8p}{9\tau}\ , \tag{8}$$

where

$$K(\tau) \equiv \frac{\tau}{\lambda_{tr}(\tau)}\ , \quad C \approx \frac{4}{5}\ . \tag{9}$$

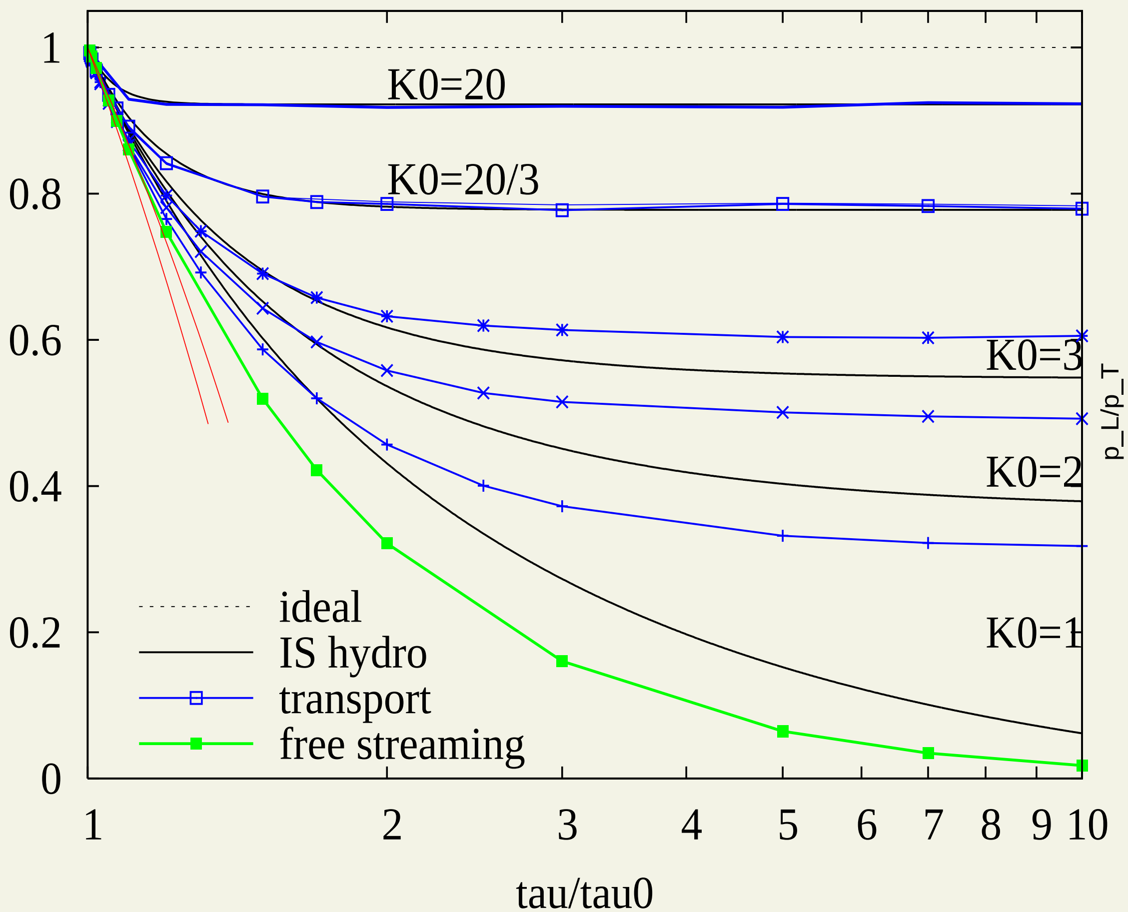
For $\sigma = const$: $\lambda_{tr} = 1/n\sigma_{tr} \propto \tau \Rightarrow K = K_0 = const$, $\eta/s \sim T\lambda_{tr} \sim \tau^{2/3}$

for $\eta/s \approx const$: $K = K_0(\tau/\tau_0)^{\sim 2/3} \propto \tau^{\sim 2/3}$

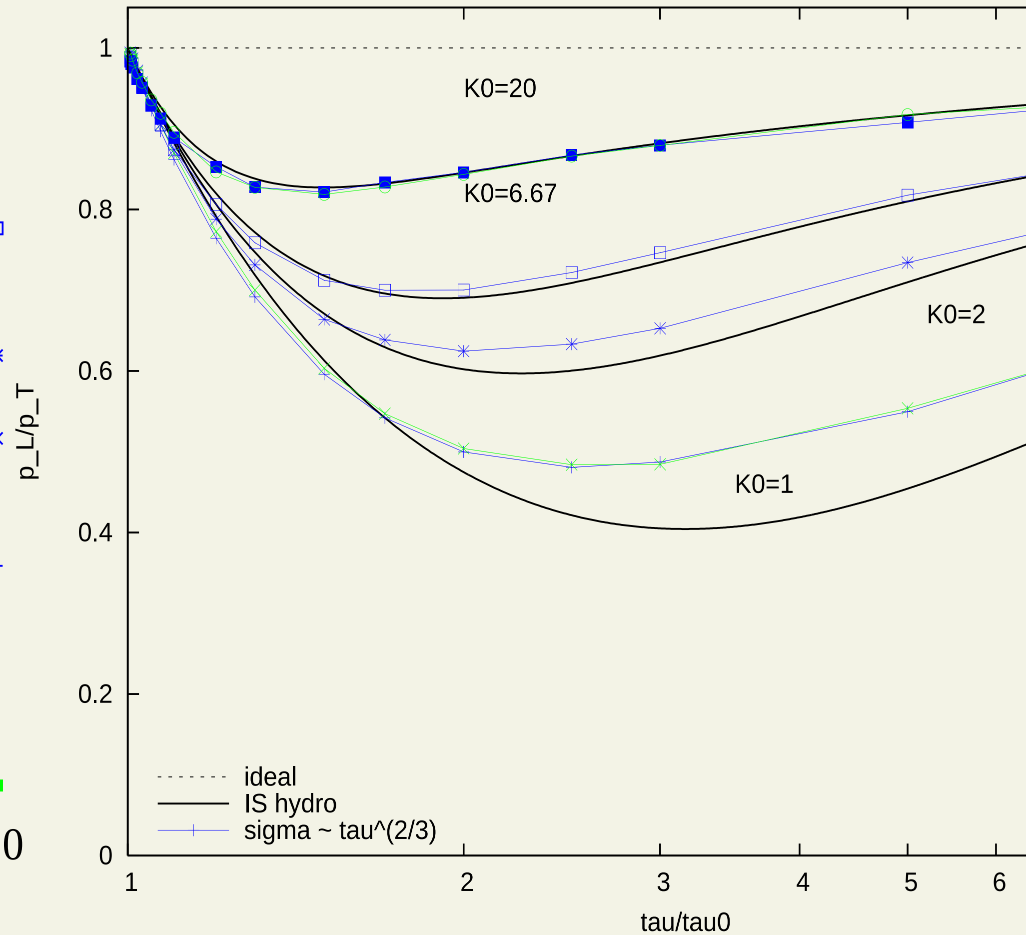
And we kept the COMPLETE Israel-Stewart equations (every term)

Huovinen & DM, arXiv:0808.0953: **pressure anisotropy T_{zz}/T_{xx} for $e = 3p$ EOS**

$\sigma = const$



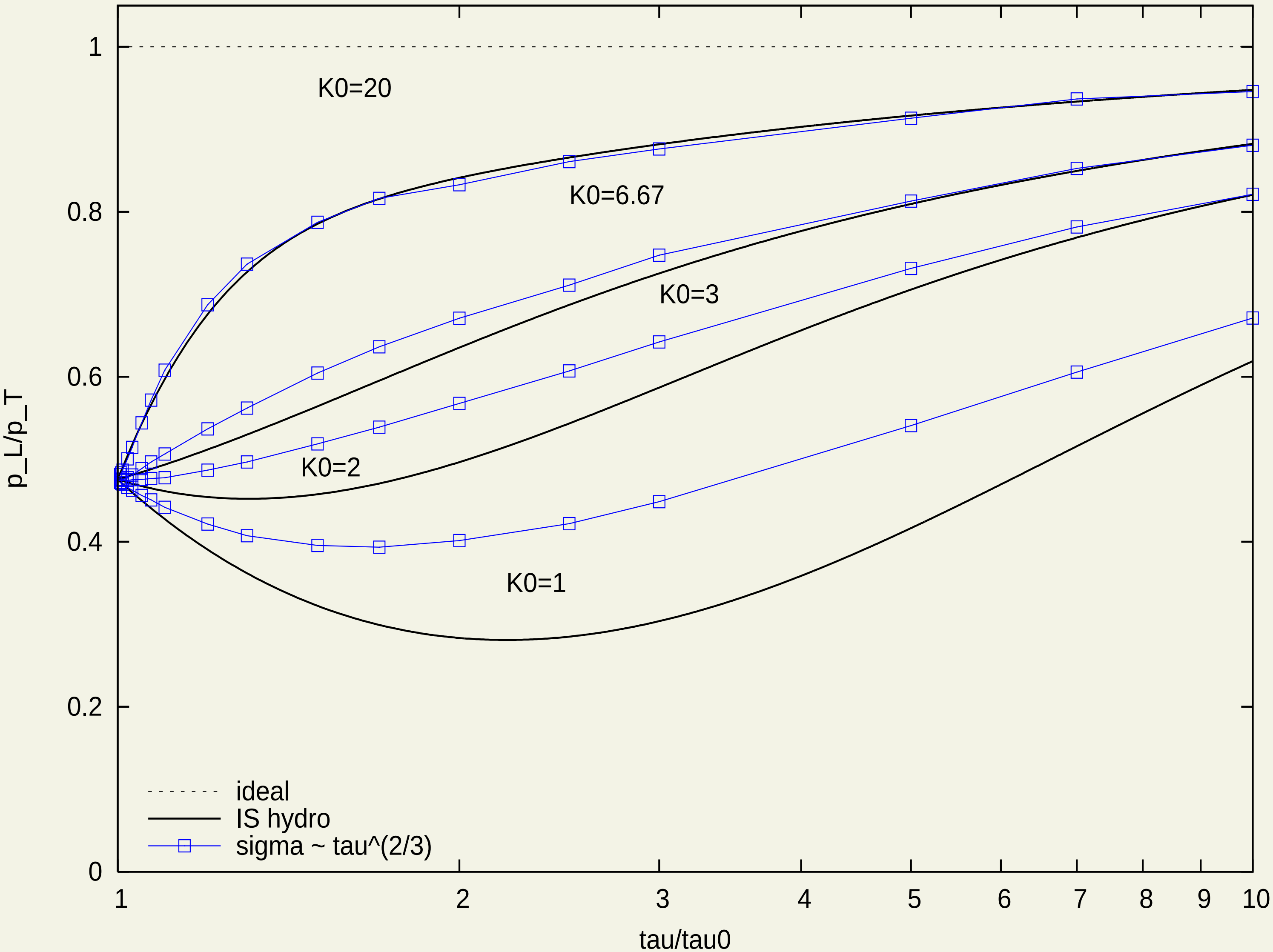
$\eta/s \approx const$



Need $K_0 \gtrsim 2-3$ for IS hydro to apply, i.e., $\lambda_{tr} \lesssim 0.3 - 0.5\ \tau_0$

In very center of Au+Au at RHIC: $K_0 \approx 10 - 20$ **if** $\eta/s = 1/(4\pi)$

Same conclusion even if we start from a LARGE initial anisotropy $R \approx 0.3$, well outside the Navier-Stokes regime.



Viscous IS hydro in 2D

We solve the full Israel-Stewart equations, including vorticity terms from kinetic theory, in a 2+1D boost-invariant scenario. **Shear stress only.**

$$\dot{\pi}^{\mu\nu} + \frac{1}{\tau_\pi}\pi^{\mu\nu} = \frac{1}{\beta_2}\nabla^{\langle\mu}u^{\nu\rangle} - \frac{1}{2}\pi^{\mu\nu}D_\alpha u^\alpha - \frac{1}{2}\pi^{\mu\nu}[\ln \dot{\frac{\beta_2}{T}}] + 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - (u^\mu\pi^{\nu\alpha} + u^\nu\pi^{\alpha\mu})\dot{u}_\alpha$$

Mimic a known reliable transport model:

- **massless Boltzmann particles** $\Rightarrow \epsilon = 3P$
- **only** $2 \leftrightarrow 2$ **processes, i.e. conserved particle number**
- $\eta = 4T/(5\sigma_{\rm tr})$, $\beta_2 = 3/(4p)$
- **either** σ **= const.** $= 47$ **mb (**$\sigma_{tr} = 14$ **mb)** $\leftarrow$ **the simplest in transport**
  or $\sigma \propto \tau^{2/3}$ $\Rightarrow \eta/s \approx 1/(4\pi)$

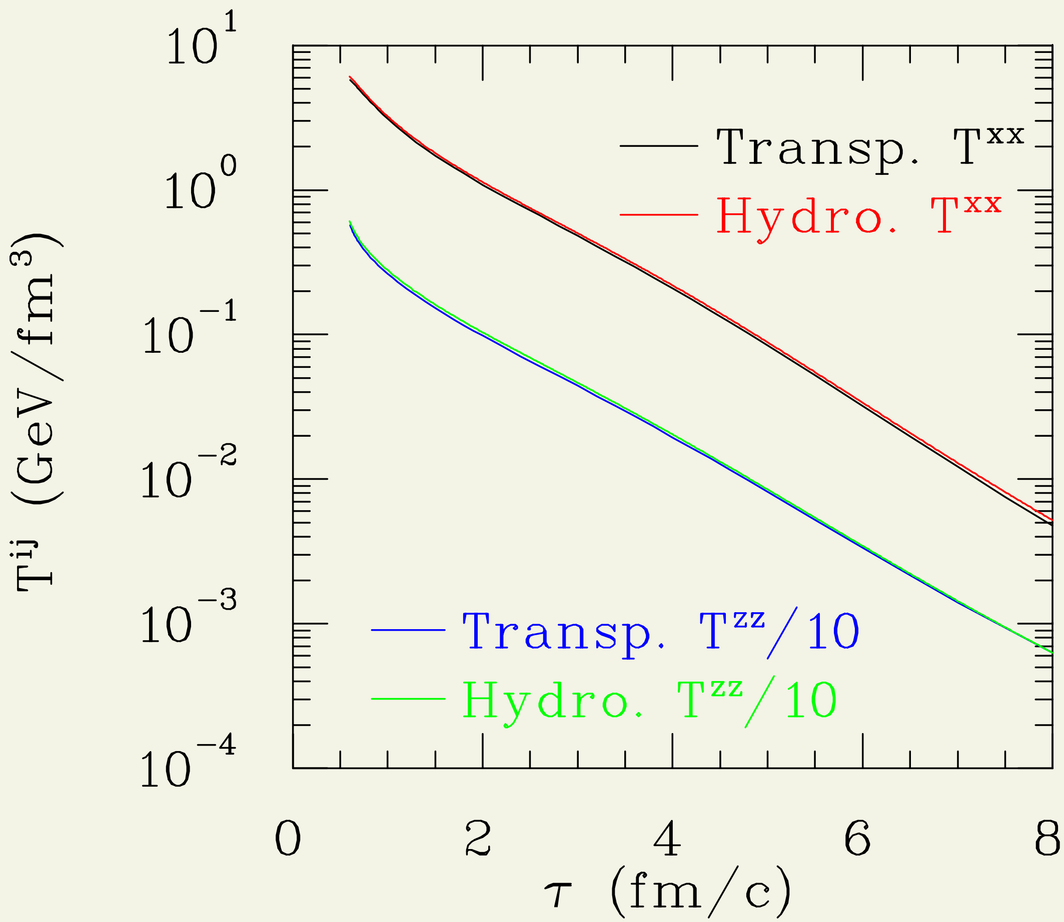
"RHIC-like" initialization:

- $\tau_0 = 0.6$ **fm**$/c$
- $b = 8$ **fm**
- $T_0 = 385$ **MeV and** $dN/d\eta|_{b=0} = 1000$
- **freeze-out at constant** $n = 0.365$ **fm**$^{-3}$

Pressure evolution in the core

T^{xx} **and** T^{zz} **averaged over the core of the system,** $r_\perp < 1$ **fm:**

$\eta/s \approx 1/(4\pi)$ $\quad (\sigma \propto \tau^{2/3})$



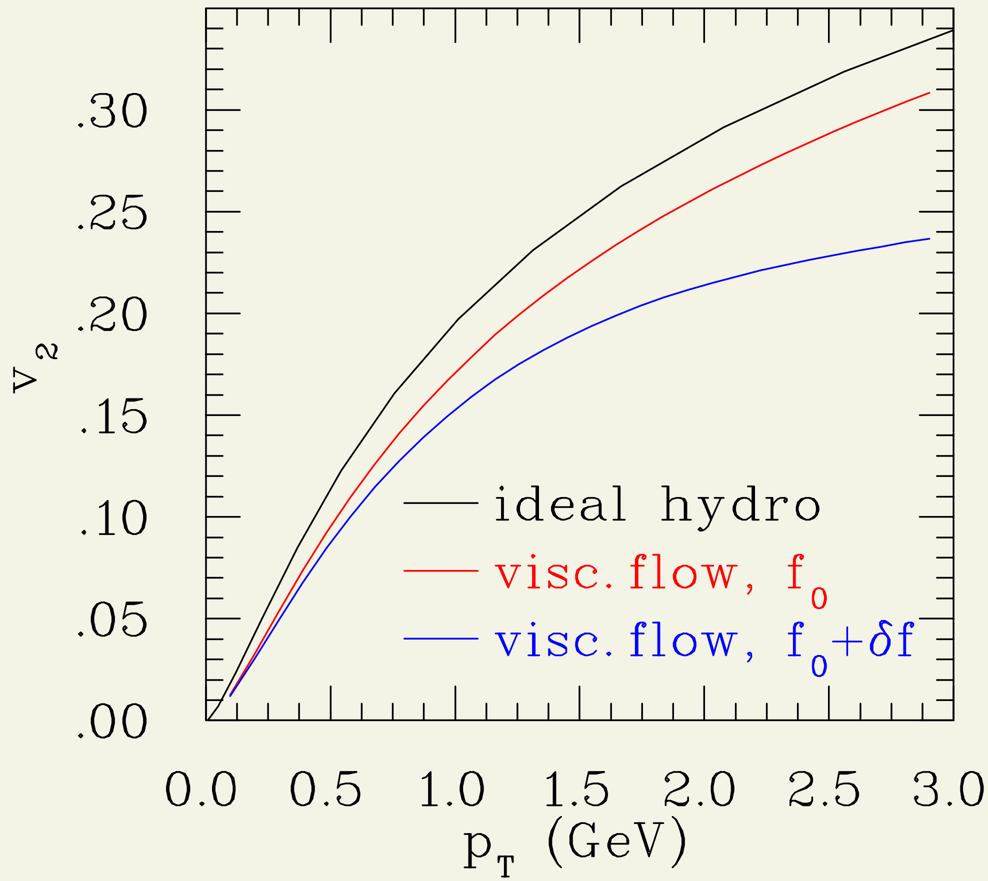
Huovinen & DM, arXiv:0806.1367

remarkable similarity!

Viscous hydro elliptic flow

TWO effects: **- dissipative corrections to hydro fields** u^μ, T, n
- dissipative corrections in Cooper-Frye freezeout $f \to f_0 + \delta f$

$\boldsymbol{\eta/s \approx 1/(4\pi)}$ $(\sigma \propto \tau^{2/3})$



Must use Grad's quadratic correction in Cooper-Frye formula

$$E\frac{dN}{d^3p} = \int p^\mu d\sigma_\mu (f_0 + \delta f)$$

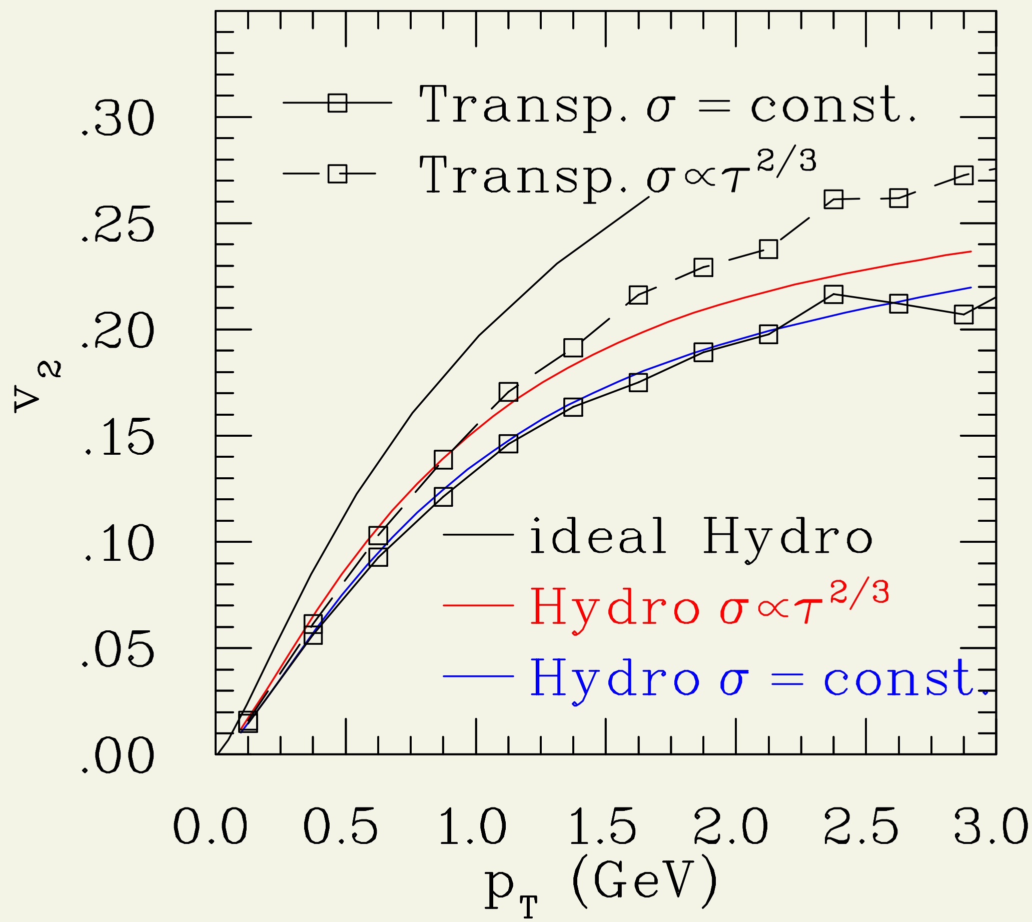
for massless $\varepsilon = 3p$**, shear only**

$$\delta f = f_0 \left[1 + \frac{p^\mu p^\nu \pi_{\mu\nu}}{8nT^3}\right]$$

calculation for $\sigma_{\rm tr} = const \sim 15mb$ **shows similar behavior**

Viscous hydro vs transport v_2

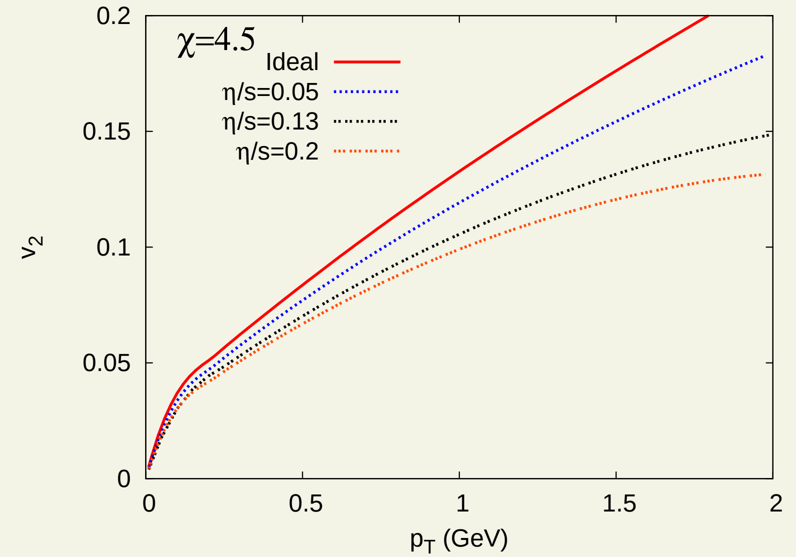
Huovinen & DM, arXiv:0806.1367



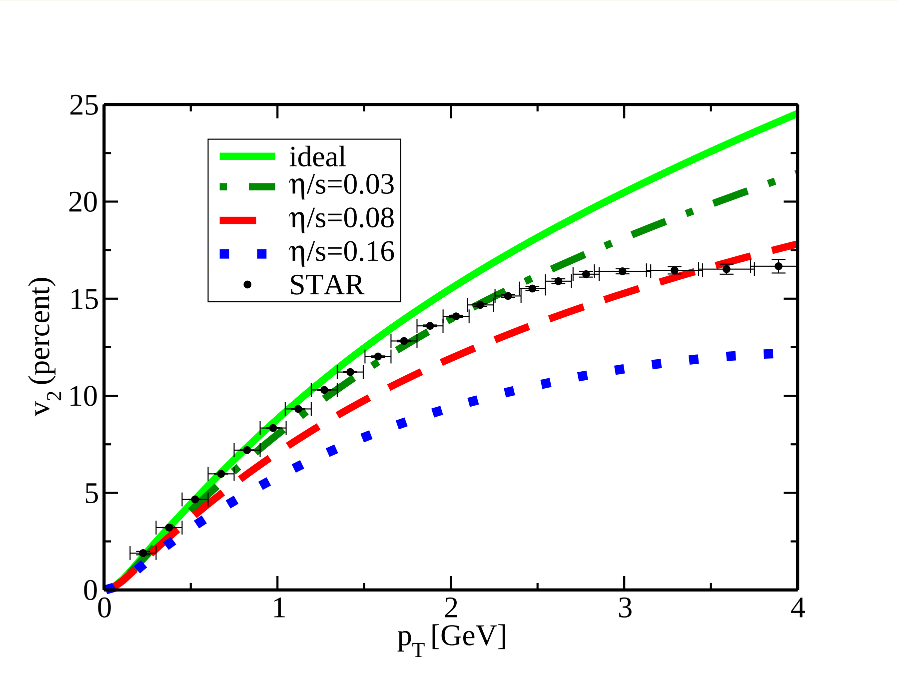
- **excellent agreement when $\sigma =$ const $\sim 47mb$**
- **good agreement for $\eta/s \approx 1/(4\pi)$, i.e., $\sigma \propto \tau^{2/3}$**

This means that now all groups agree that viscous corrections to elliptic flow in Au+Au at RHIC are modest $\sim 20\%$ if $\eta/s \sim 1/(4\pi)$

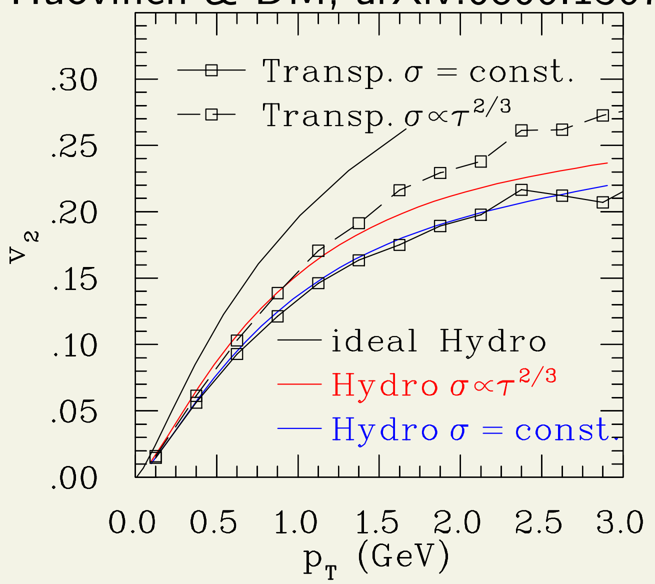
Dusling & Teaney, PRC77



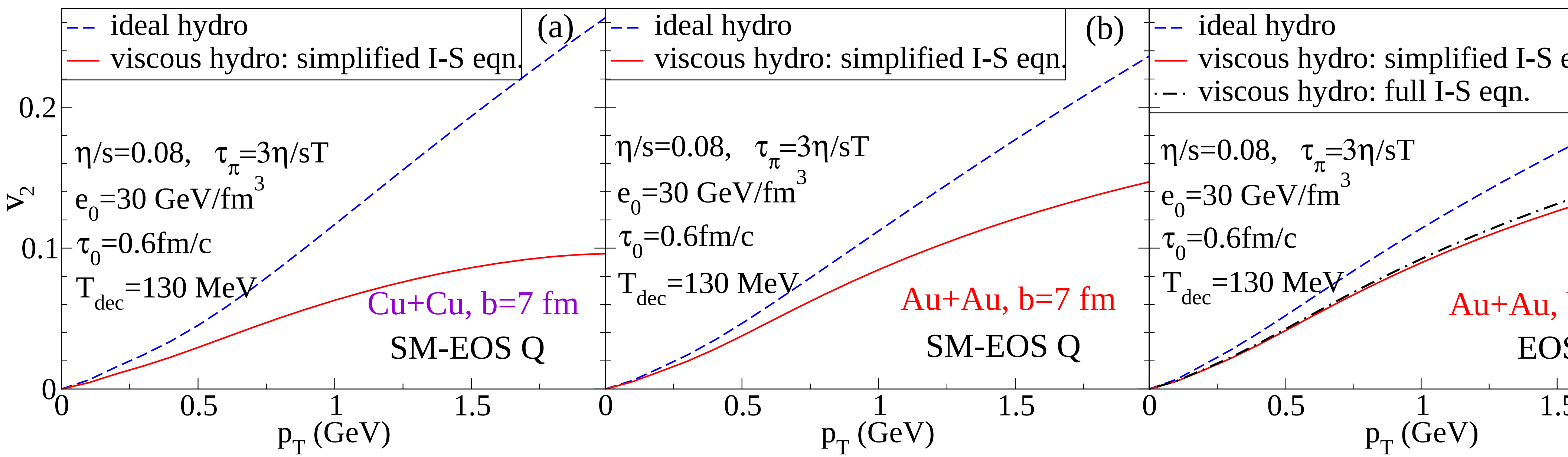
Romatschke & Romatschke, arxiv:0706.1522



Huovinen & DM, arXiv:0806.1367



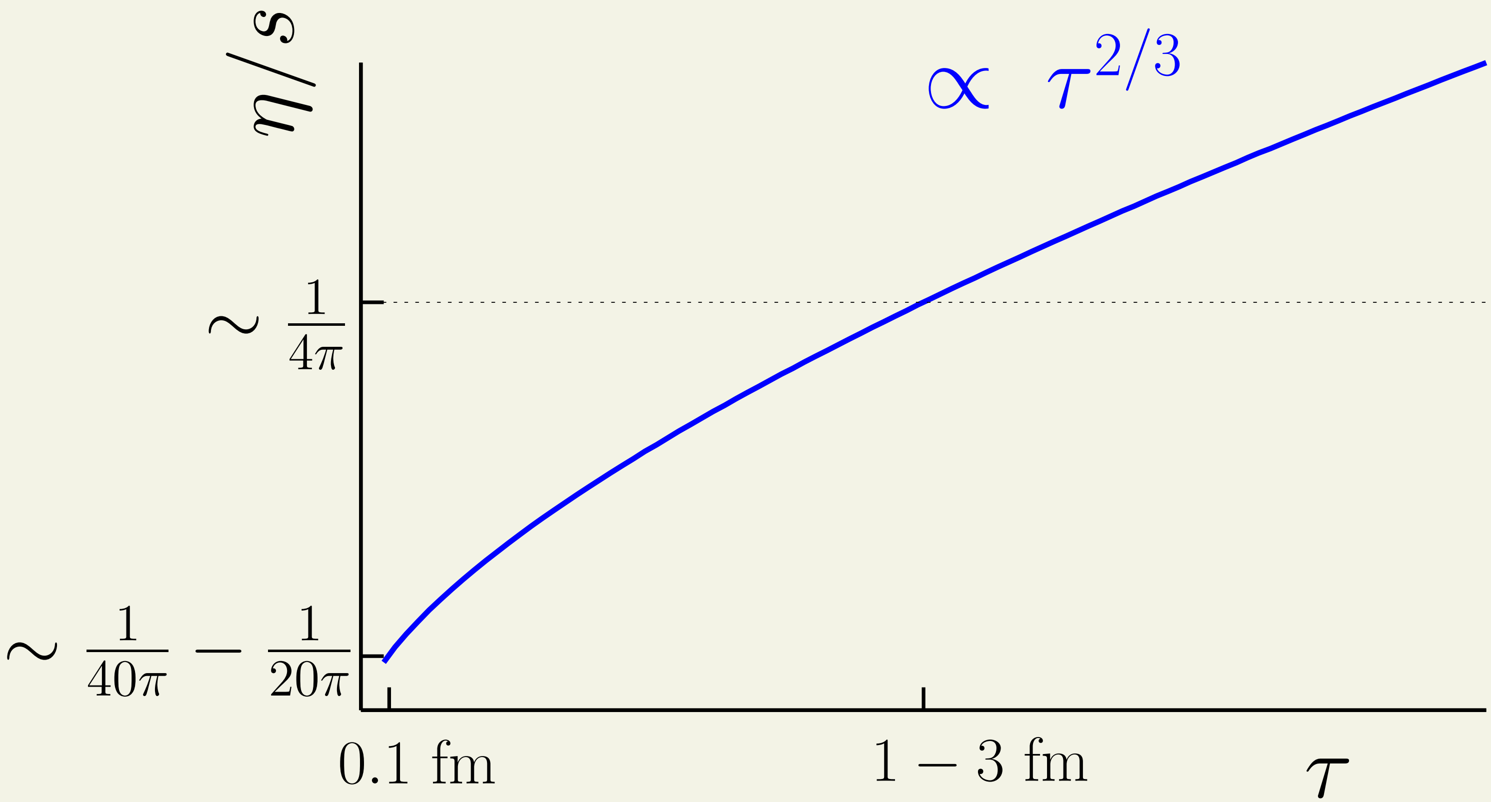
Song & Heinz, PRC78



$\sigma \approx 45$ **mb result for RHIC corresponds to**

$\eta/s \sim \lambda_{tr} T \sim 1/(\sigma T^2)$

$T \sim \tau^{-1/3}$ **cooling**



at early times, violates conjectured viscosity bound DM, arXiv:0806.0026

Yet more hydro terms?

If we do not start from Israel-Stewart procedure but instead impose conformal invariance (implies $\varepsilon = 3p$), even further terms are possible in the shear stress equation Baier, Romatschke, Son, JHEP04, 100 ('08)

$$\dot{\pi}^{\mu\nu} = \cdots + \frac{\lambda_1}{\eta^2}\pi^{\alpha\langle\mu}\pi_\alpha{}^{\nu\rangle} + \lambda_3\omega^{\alpha\langle\mu}\omega_\alpha{}^{\nu\rangle} \tag{10}$$

In the calculations shown so far, ω is rather small, while π should not be very large for hydro to apply. Nevertheless, the importance of these terms depends on the magnitude of matter coefficients in front.

Based on the recent successful hydro-transport comparisons, which did not include the new terms in the hydro, these extra terms are expected to have negligible influence. They matter more, however, for a nonequilibrium theory other than covariant transport.

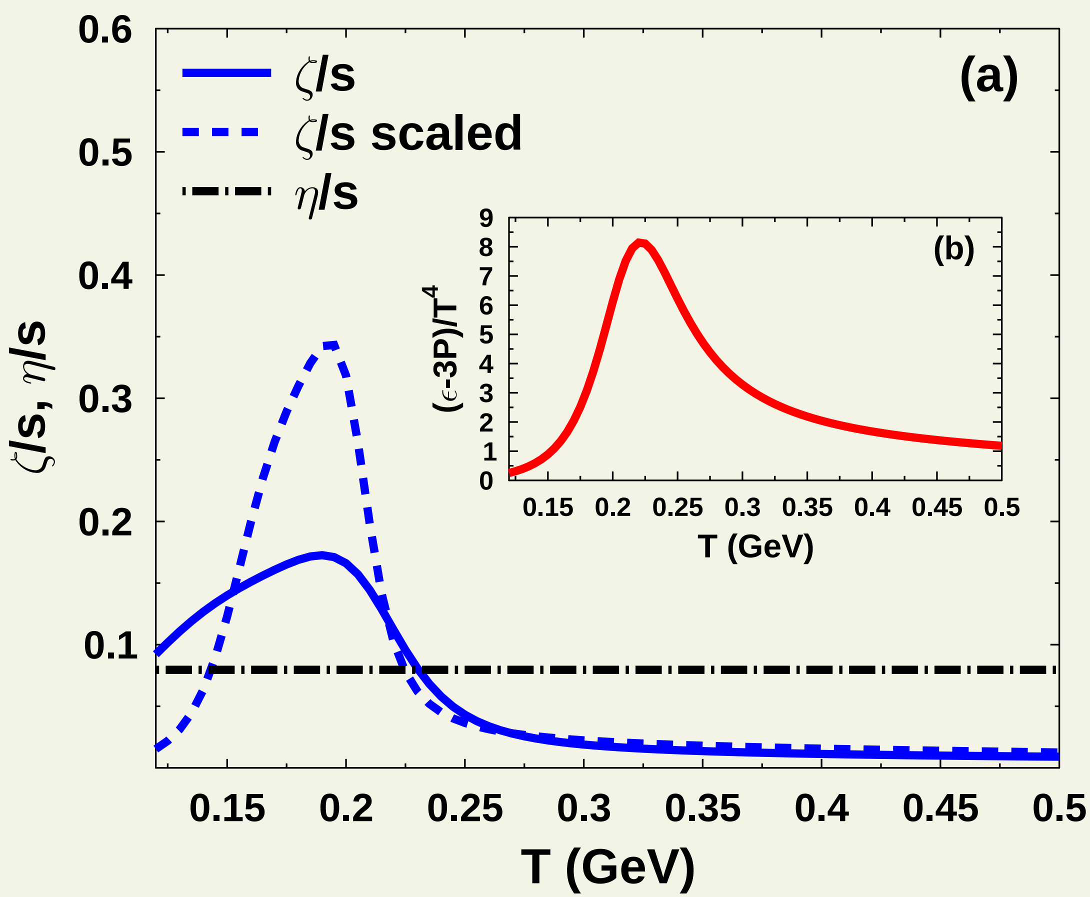
Note, if we relax conformal invariance, the numerous other terms become possible.

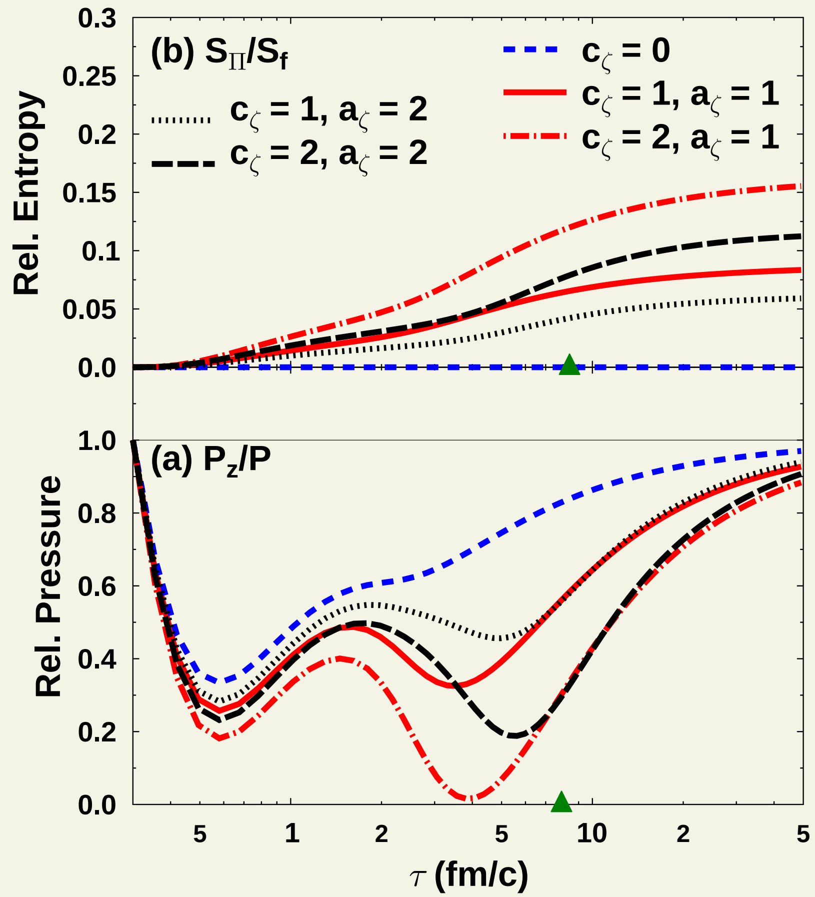
Bulk viscosity

Recent 0+1D explorations Fries et al, arxiv:0807.4333 **based on relaxation eqn**

$$\dot{\Pi} = -\frac{1}{\tau_\Pi}(\Pi - \Pi_{NS})$$

find significant entropy production: $\zeta/s_{max} \sim 0.4$ **similar to** $\eta/s = 1/4\pi$





Dissipative hydro - summary

• dissipative hydro describes the evolution of a system near local equilibrium, in terms of a few more macroscopic parameters

• causality requires abandoning Navier-Stokes, in favor of second-order formulation, such as Israel-Stewart. This can be motivated both from thermodynamic principles, and from Grad's 14-moment approximation in kinetic theory.

• recent comparison between IS hydro and covariant transport in 0+1D and 2+1D Bjorken geometry shows that the Israel-Stewart (i.e., Grad's 14-moment) approximation, though uncontrolled, is quite accurate in practice

• lot more work ahead - e.g., latest lattice EOS, coupling to hadron transport

• difficult to determine transport coefficients and relaxation times from first principles

• conceptual problems with freezeout remain

• weakest link, as always, initial conditions - thermalization mechanism needs to be understood