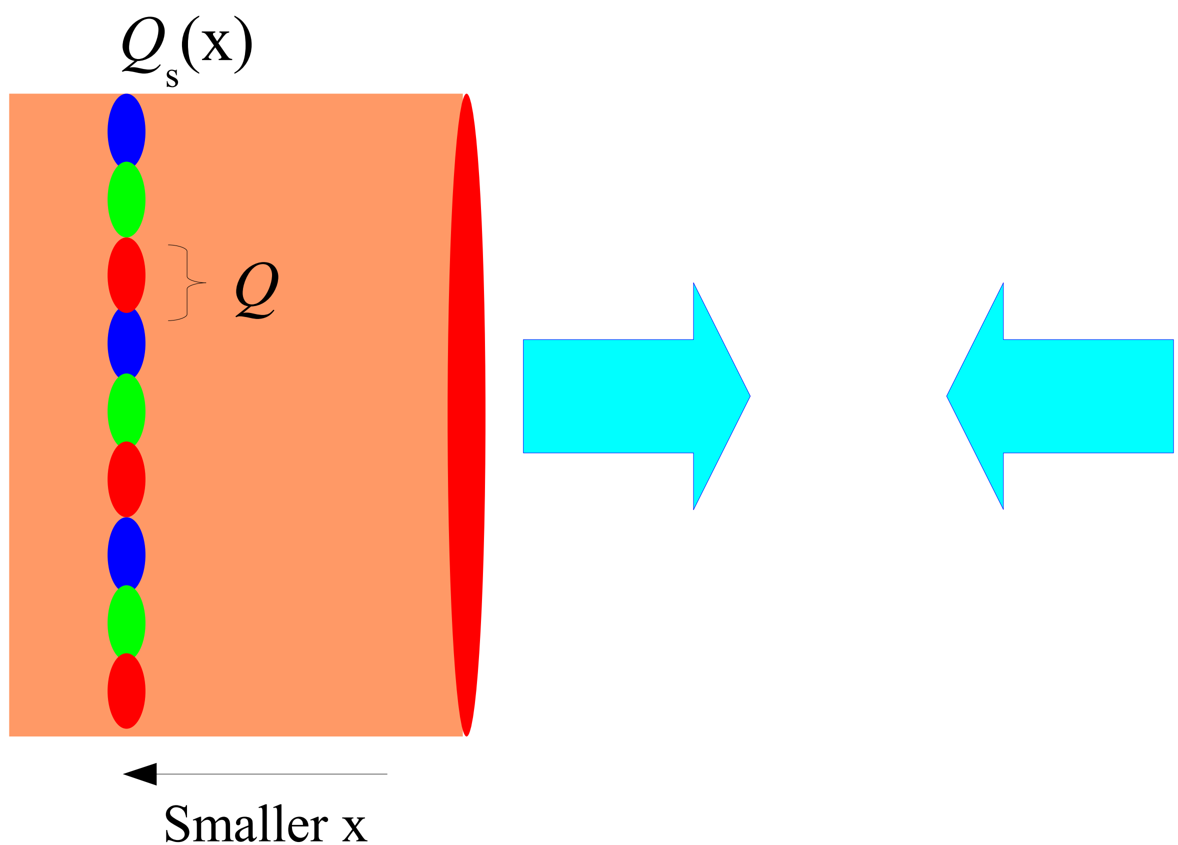
Initial Energy and Gluon Distribution in the Heavy-Ion Collisions from the Color Glass Condensate

Kenji Fukushima
Yukawa Institute for Theoretical Physics
Kyoto University

September 2008 at Goa

Physics Picture

- Classical collisions from quantum radiations



Color Dipole
= DIS

Heavy Nucleus
= HIC

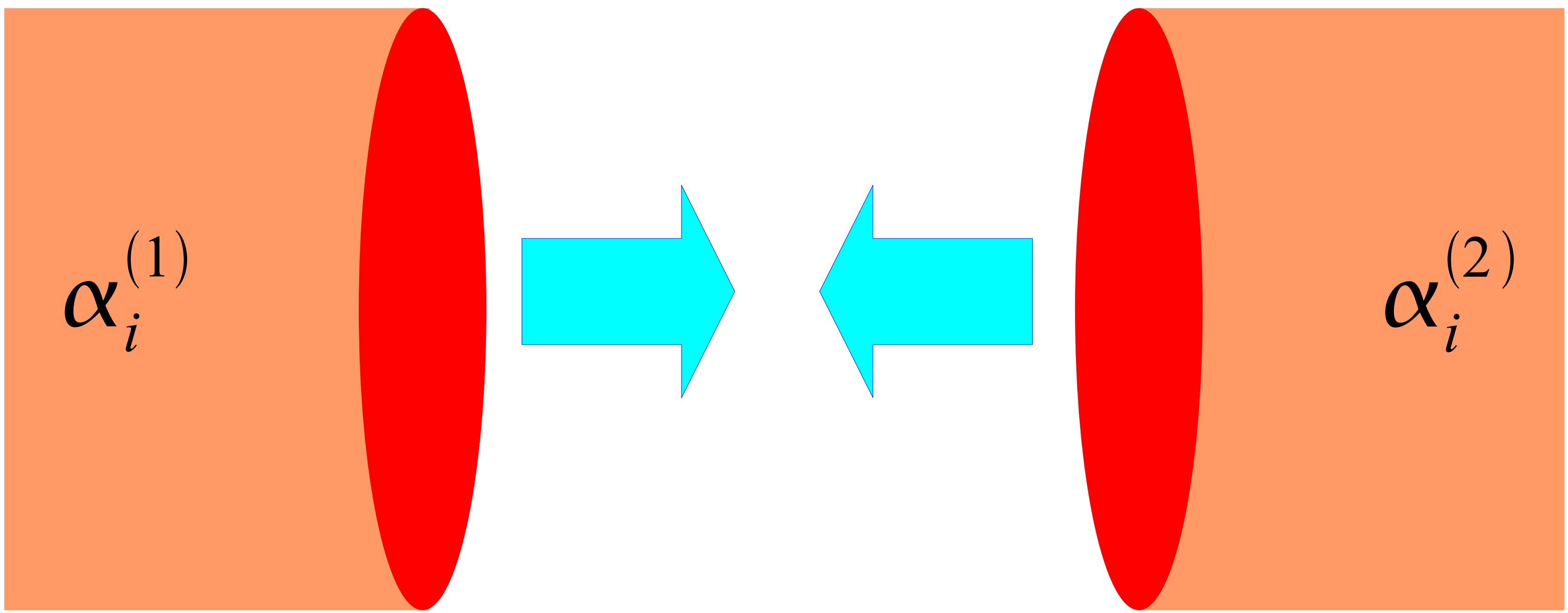
Softer and Denser due to quantum radiations $\rightarrow$ **Classical Fields**

Q_s is the only scale

Model Picture

- Nucleus-Nucleus Collisions

$$x \sim p_t / \sqrt{s} \sim 1\,\mathrm{GeV} / 200\,\mathrm{GeV} \sim 10^{-2}$$



$$\rho_1(\boldsymbol{x})\,\delta(x^-) \qquad \rho_2(\boldsymbol{x})\,\delta(x^+)$$

$$\sim g\mu \qquad \sim g\mu$$

MV Model

- Equations of motion

$$E^i = \frac{\delta(\tau\mathcal{L})}{\delta(\partial_\tau A_i)} = \tau\partial_\tau A_i\,, \qquad \partial_\tau E^i = -\frac{\delta(\tau\mathcal{H})}{\delta A_i} = \frac{1}{\tau}D_\eta F_{\eta i} + \tau D_j F_{ji}\,,$$

$$E^\eta = \frac{\delta(\tau\mathcal{L})}{\delta(\partial_\tau A_\eta)} = \frac{1}{\tau}\partial_\tau A_\eta\,. \qquad \partial_\tau E^\eta = -\frac{\delta(\tau\mathcal{H})}{\delta A_\eta} = \frac{1}{\tau}D_j F_{j\eta}\,.$$

- Initial condition

$$A_{i(0)} = \alpha_i^{(1)} + \alpha_i^{(2)}\,, \quad A_{\eta(0)} = 0\,,$$

$$E^i_{(0)} = 0\,, \quad E^\eta_{(0)} = ig\left[\alpha_i^{(1)}, \alpha_i^{(2)}\right],$$

$$\alpha_i^{(1)}(\boldsymbol{x}_\perp) = -\frac{1}{ig}V_\infty(\boldsymbol{x}_\perp)\,\partial_i\,V_\infty^\dagger(\boldsymbol{x}_\perp)\,,$$

$$\alpha_i^{(2)}(\boldsymbol{x}_\perp) = -\frac{1}{ig}W_\infty(\boldsymbol{x}_\perp)\,\partial_i\,W_\infty^\dagger(\boldsymbol{x}_\perp)$$

- Wilson lines

$$V_{x^-}^\dagger(\boldsymbol{x}_\perp) = \mathcal{P}_-\exp\left[-ig\int_{-\infty}^{x^-} dz^-\,\frac{\rho^{(1)}(\boldsymbol{x}_\perp, z^-)}{\boldsymbol{\partial}^2}\right]$$

$$W_{x^+}^\dagger(\boldsymbol{x}_\perp) = \mathcal{P}_+\exp\left[-ig\int_{-\infty}^{x^+} dz^+\,\frac{\rho^{(2)}(\boldsymbol{x}_\perp, z^+)}{\boldsymbol{\partial}^2}\right]$$

Observable

- Expectation value

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\rho^{(1)} \mathcal{D}\rho^{(2)}\, \mathcal{W}[\rho^{(1)}, \rho^{(2)}]\, \mathcal{O}[V, W]$$

- Gaussian weight

$$\left\langle \rho^{(m)a}(\boldsymbol{x}_\perp, z) \rho^{(n)b}(\boldsymbol{y}_\perp, z') \right\rangle = g^2 \mu^2(z)\, \delta^{mn}\, \delta^{ab}\, \delta(z - z')\, \delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$

- Example

$$\lim_{\epsilon \to 0} \lim_{\zeta \to 0} \langle V_\epsilon^\dagger \rangle_\zeta = \exp\left[-g^4 \bar{\mu}^2 \frac{N_c^2 - 1}{4N_c} L(0,0) \right]$$

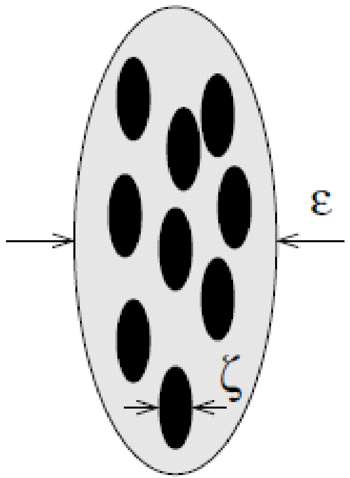
Blaizot-Gelis-Venugopalan

$$L(\boldsymbol{x}_\perp) = \frac{1}{(\partial^2)^2} \delta^{(2)}(\boldsymbol{x}_\perp) = \int \frac{d^2 \boldsymbol{k}_\perp}{(2\pi)^2} \frac{e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}}{(\boldsymbol{k}_\perp^2 + m^2)^2}$$

Why Analytical Calculations???

- Successfully solved numerically...
  Venugopalan, Krasnitz, Nara, Lappi, Romatschke, etc

- Approximation (scheme???)
  - Two Dirac delta functions



$$\zeta \qquad \langle \rho^{(m)a}(\boldsymbol{x}_\perp, z)\rho^{(n)b}(\boldsymbol{y}_\perp, z')\rangle = g^2\mu^2(z)\,\delta^{mn}\,\delta^{ab}\boxed{\delta(z-z')}\delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$

$$\varepsilon \qquad \rho^{(1)}(\boldsymbol{x}_\perp, x^-) \to \bar{\rho}^{(1)}(\boldsymbol{x}_\perp)\boxed{\delta(x^-)}, \qquad \rho^{(2)}(\boldsymbol{x}_\perp, x^+) \to \bar{\rho}^{(2)}(\boldsymbol{x}_\perp)\boxed{\delta(x^+)}$$

$$\lim_{\zeta\to 0}\lim_{\epsilon\to 0}\langle \mathcal{O}[V_\epsilon]\rangle_\zeta \stackrel{?}{=} \lim_{\epsilon\to 0}\lim_{\zeta\to 0}\langle \mathcal{O}[V_\epsilon]\rangle_\zeta$$

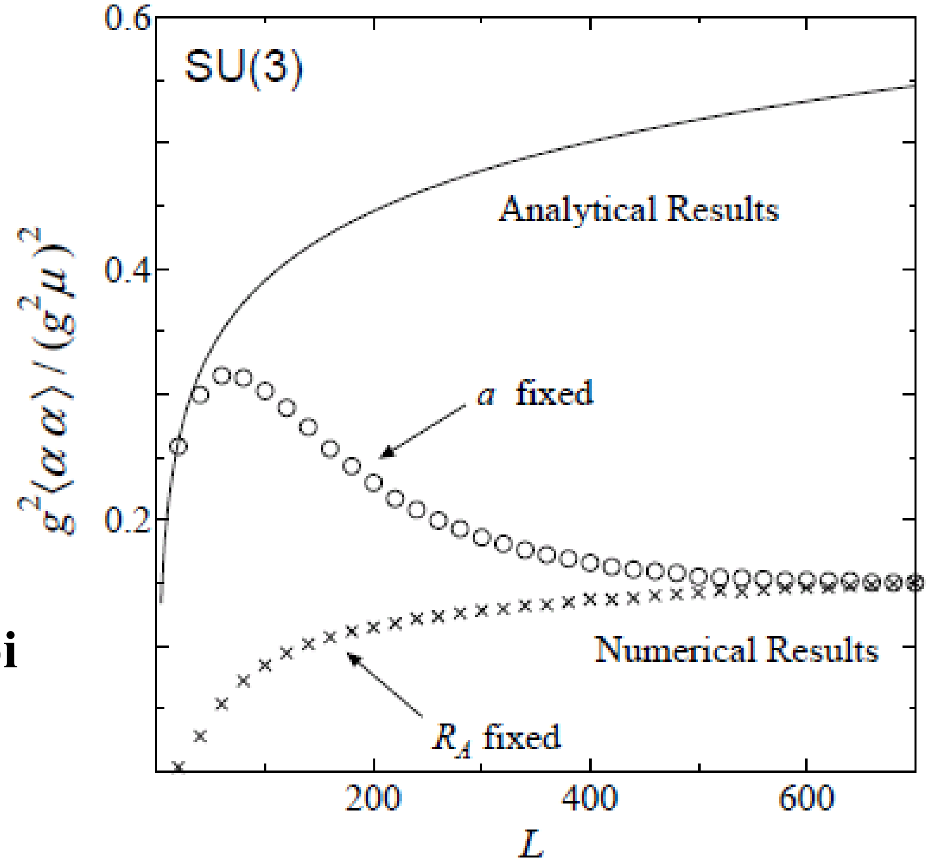
Numerical Calculation **Analytical Calculation**

Large Difference in IR

- Wilson line correlator
  - R – size; a – lattice spacing
  - Analytical Results (exact)

$$\sim \mu^2 \ln\left(\frac{R}{a}\right) = \mu^2 \ln L$$

  - R fixed → a dependence
    UV property
    absorbed by $\mu \to 2\mu$ **Lappi**
  - a fixed → R dependence
    IR property
    cannot be absorbed by μ



Expansion in τ

- Solve the YM equations in power of τ **Fries-Kapusta-Li**

$$\mathcal{O}(\tau) = \sum_{n=0}^{\infty} \mathcal{O}_{(n)} \tau^n$$

- Zeroth Order

$$E^i_{(0)} = 0\,, \qquad B^i_{(0)} = 0\,, \quad B^\eta_{(0)} = F_{12(0)}$$

$$E^\eta_{(0)} = ig\Big(\big[\alpha_1^{(1)}, \alpha_1^{(2)}\big] + \big[\alpha_2^{(1)}, \alpha_2^{(2)}\big]\Big) \qquad F_{ij(0)} = -ig\Big(\big[\alpha_i^{(1)}, \alpha_j^{(2)}\big] + \big[\alpha_i^{(2)}, \alpha_j^{(1)}\big]\Big)$$

Glasma – McLerran-Lappi

- Second Order

$$E^i_{(2)} = \tfrac{1}{2} D_{j(0)} F_{ji(0)} = -\epsilon^{ij} \tfrac{1}{2} D_{j(0)} B^\eta_{(0)}\,, \qquad B^i_{(2)} = \epsilon^{ij} F_{j\eta(2)} = \epsilon^{ij} \tfrac{1}{2} D_{j(0)} E^\eta_{(0)}\,,$$

$$E^\eta_{(2)} = \tfrac{1}{2} D_{j(0)} F_{j\eta(2)} = \tfrac{1}{4} D_{j(0)} D_{j(0)} E^\eta_{(0)}\,, \qquad B^\eta_{(2)} = F_{12(2)} = \tfrac{1}{4} D_{j(0)} D_{j(0)} B^\eta_{(0)}\,.$$

Energy Density

- Definition

$$\mathcal{H} = \mathrm{tr}\left[\frac{1}{\tau^2}E^i E^i + E^\eta E^\eta + \frac{1}{\tau^2}B^i B^i + B^\eta B^\eta\right]$$

- Zeroth Order

$$\begin{aligned}\varepsilon_{(0)} &= 2N_c(N_c^2-1)g^2\langle\alpha\alpha\rangle^2 \\ &= g^6\mu_A^4\cdot\frac{3}{\pi^2}\left[\ln\frac{\Lambda}{m}\right]^2,\end{aligned}$$

- Second Order

$$\varepsilon_{(2)}^L = -g^6\mu_A^4\cdot\frac{3}{2\pi^2}\Lambda^2\ln\frac{\Lambda}{m} - g^{10}\mu_A^6\cdot\frac{45}{8\pi^3}\left[\ln\frac{\Lambda}{m}\right]^3$$

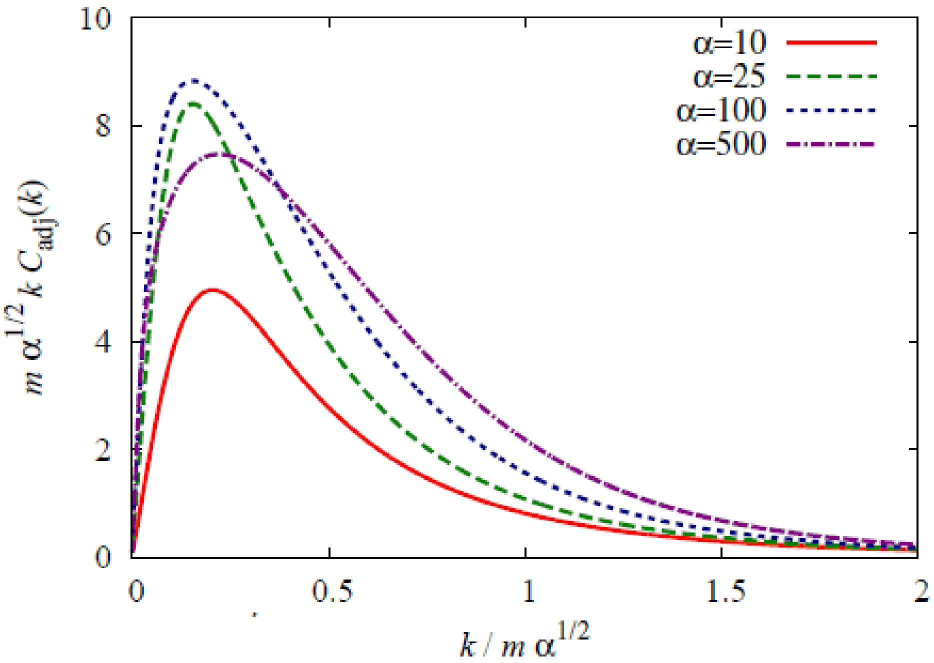
$$\varepsilon_{(2)}^T = -\frac{1}{2}\varepsilon_{(2)}^L$$

UV divergence!
But UV is not bad!!

Zeroth Order

- Fourier decomposition

$$\varepsilon_{(0)}(\boldsymbol{k}_\perp) = \frac{1}{V}\Big\langle \mathrm{tr}\big[E^\eta_{(0)}(-\boldsymbol{k}_\perp)E^\eta_{(0)}(\boldsymbol{k}_\perp) + B^\eta_{(0)}(-\boldsymbol{k}_\perp)B^\eta_{(0)}(\boldsymbol{k}_\perp)\big]\Big\rangle$$



$$= \frac{1}{2}N_c(N_c^2-1)g^6\mu_A^4\int\frac{d\theta_k}{2\pi}$$

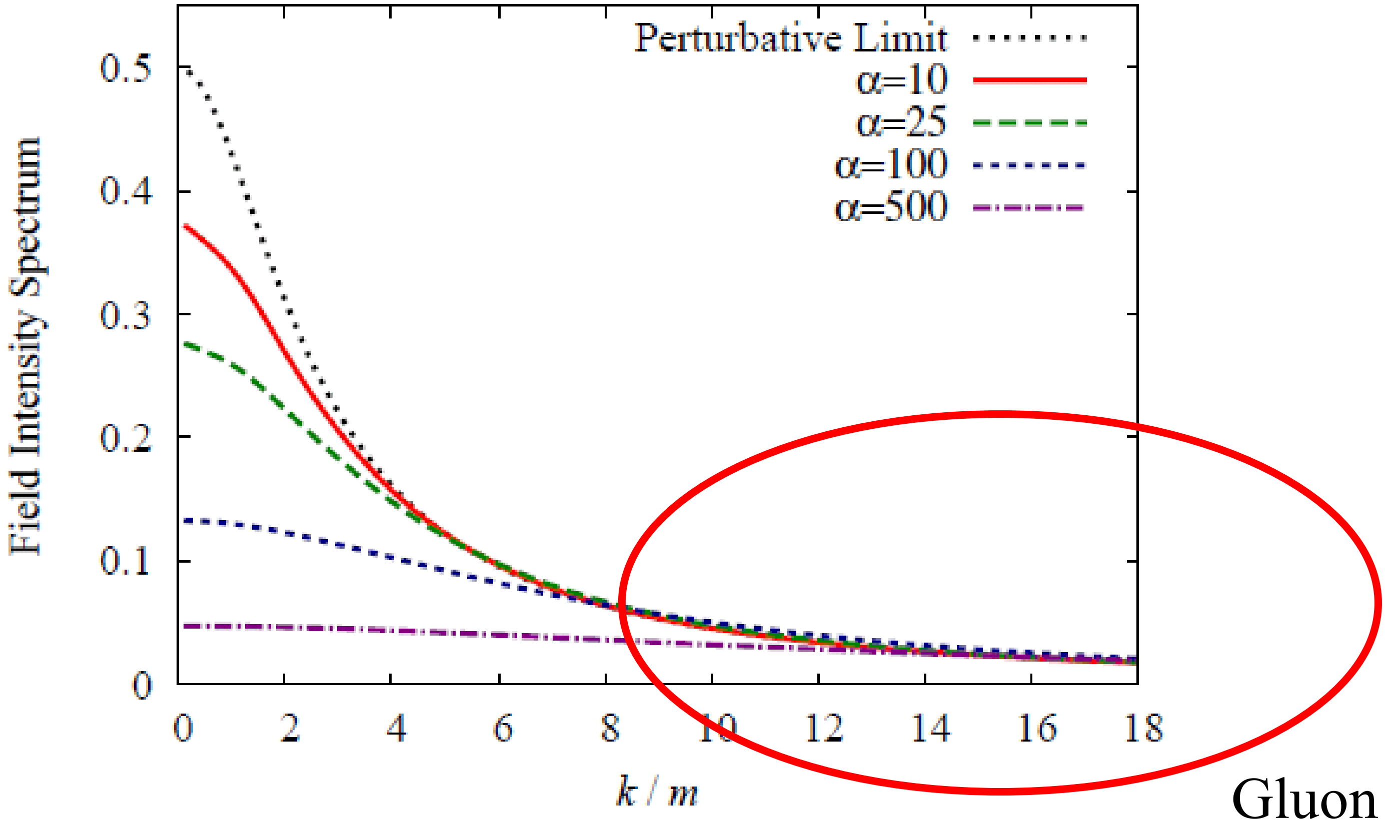
$$\times\int\frac{d^2q_1}{(2\pi)^2}\frac{d^2q_2}{(2\pi)^2}\frac{d^2q_3}{(2\pi)^2}\frac{\bar{C}_{\mathrm{adj}}(\boldsymbol{q}_{1\perp})\bar{C}_{\mathrm{adj}}(\boldsymbol{q}_{2\perp})}{(q_{3\perp}^2+m^2)[(\boldsymbol{k}'_\perp-\boldsymbol{q}_{3\perp})^2+m^2]}$$

$$= \frac{1}{4\pi m^2}N_c(N_c^2-1)g^6\mu_A^4\,\mathcal{T}(\alpha;k_\perp/m)$$

$$\alpha = \frac{N_c(g^2\mu_A)^2}{2m^2}$$

Zeroth Order

- Momentum spectrum



Dominant at $\tau = 0$ leading to

$$\int \frac{d^2 k_t}{(2\pi)^2} \frac{1}{k_t^2}$$

Gluon Distribution

$$n(\tau, \boldsymbol{k}_\perp) = \frac{1}{\sqrt{k_\perp + m^2}} \varepsilon(\tau, \boldsymbol{k}_\perp)$$

Gunion-Bertsch

Perturbative Resummation

- UV divergences = Perturbative contributions

$$\partial_\tau E^i = \tau\partial_j\big(\partial_j A_i - \partial_i A_j\big) = \tau\partial^2 P^T_{ij} A_j$$

$$\partial_\tau E^\eta = \frac{1}{\tau}\partial^2 A_\eta \,.$$

$$A_i(\tau, \boldsymbol{k}_\perp) = A_{i(0)} J_0(k_\perp \tau)\,,$$

$$A_\eta(\tau, \boldsymbol{k}_\perp) = A_{\eta(2)} \frac{2\tau}{k_\perp} J_1(k_\perp \tau)$$

$$E^\eta(\tau, \boldsymbol{k}_\perp) = E^\eta_{(0)}(\boldsymbol{k}_\perp) J_0(k_\perp \tau)\,,$$

$$E^i(\tau, \boldsymbol{k}_\perp) = E^i_{(2)}(\boldsymbol{k}_\perp) \frac{2\tau}{k_\perp} J_1(k_\perp \tau)\,,$$

$$B^\eta(\tau, \boldsymbol{k}_\perp) = B^\eta_{(0)}(\boldsymbol{k}_\perp) J_0(k_\perp \tau)\,,$$

$$B^i(\tau, \boldsymbol{k}_\perp) = B^i_{(2)}(\boldsymbol{k}_\perp) \frac{2\tau}{k_\perp} J_1(k_\perp \tau)\,.$$

$$\varepsilon_{(0)}(k_\perp) = \frac{1}{4\pi m^2} N_c(N_c^2 - 1) g^6 \mu_A^4\, \mathcal{T}(\alpha; k_\perp/m)$$

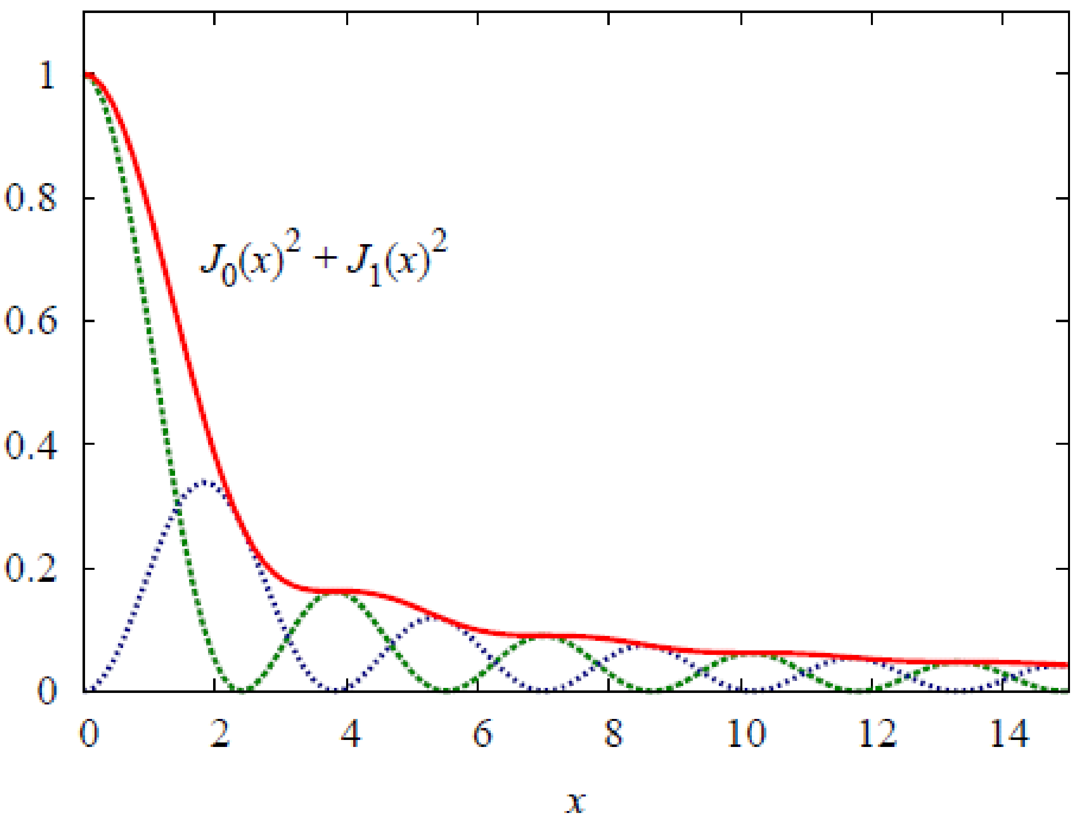
$$\boxed{\begin{aligned} \varepsilon^L(\tau, \boldsymbol{k}_\perp) &= \frac{6}{\pi m^2} g^6 \mu_A^4\, \mathcal{T}(k_\perp/m) \big[J_0(k_\perp\tau)\big]^2 \\ \varepsilon^T(\tau, \boldsymbol{k}_\perp) &= \frac{6}{\pi m^2} g^6 \mu_A^4\, \mathcal{T}(k_\perp/m) \big[J_1(k_\perp\tau)\big]^2 \end{aligned}}$$

Kovchegov

$$\varepsilon^T_{(2)} = -\frac{1}{2}\varepsilon^L_{(2)}$$

Time Evolution

- UV divergence free



$$\left[J_0(k_\perp \tau)\right]^2 + \left[J_1(k_\perp \tau)\right]^2 \to \frac{2}{\pi k_\perp \tau}$$

τ is a "**UV cutoff**" in the early time.

Up to some cutoff τ, the perturbative time evolution is dominant.

UV Finite Formulae

- Energy density

$$\varepsilon(\tau) = \frac{3}{\pi^2 m^2} \cdot \frac{1}{g^2} (g^2 \mu_A)^4$$

Full Nonlinear Effect at τ=0

$$\times \int dk_\perp \, k_\perp \, \mathcal{T}(k_\perp / m) \left\{ \left[J_0(k_\perp \tau) \right]^2 + \left[J_1(k_\perp \tau) \right]^2 \right\}$$

$$= \frac{3}{\pi^2} \cdot \frac{1}{g^2} (g^2 \mu_A)^4 I_E(\alpha; m\tau) \,,$$

Perturbative Time Evolution

- Gluon Distribution

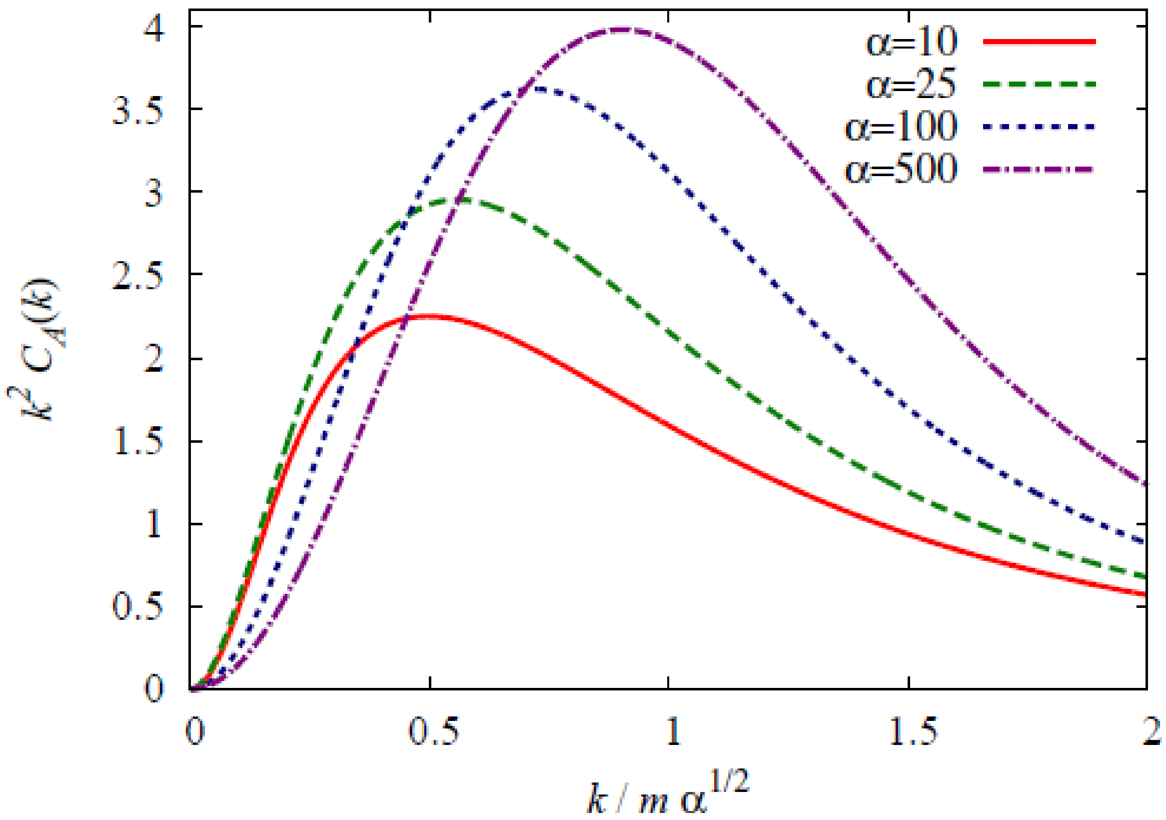
$$n(\tau) = \frac{3}{\pi^2 m^2} \cdot \frac{1}{g^2} (g^2 \mu_A)^4$$

$$\times \int dk_\perp \frac{k_\perp \mathcal{T}(k_\perp / m)}{\sqrt{k_\perp^2 + m^2}} \left\{ \left[J_0(k_\perp \tau) \right]^2 + \left[J_1(k_\perp \tau) \right]^2 \right\}$$

$$= \frac{3}{\pi^2 m} \cdot \frac{1}{g^2} (g^2 \mu_A)^4 I_N(\alpha; m\tau) \,,$$

Numerics

- Determination of $g^2\mu$ and m



$$\alpha = \frac{N_c (g^2 \mu_A)^2}{2m^2}$$

Analytical

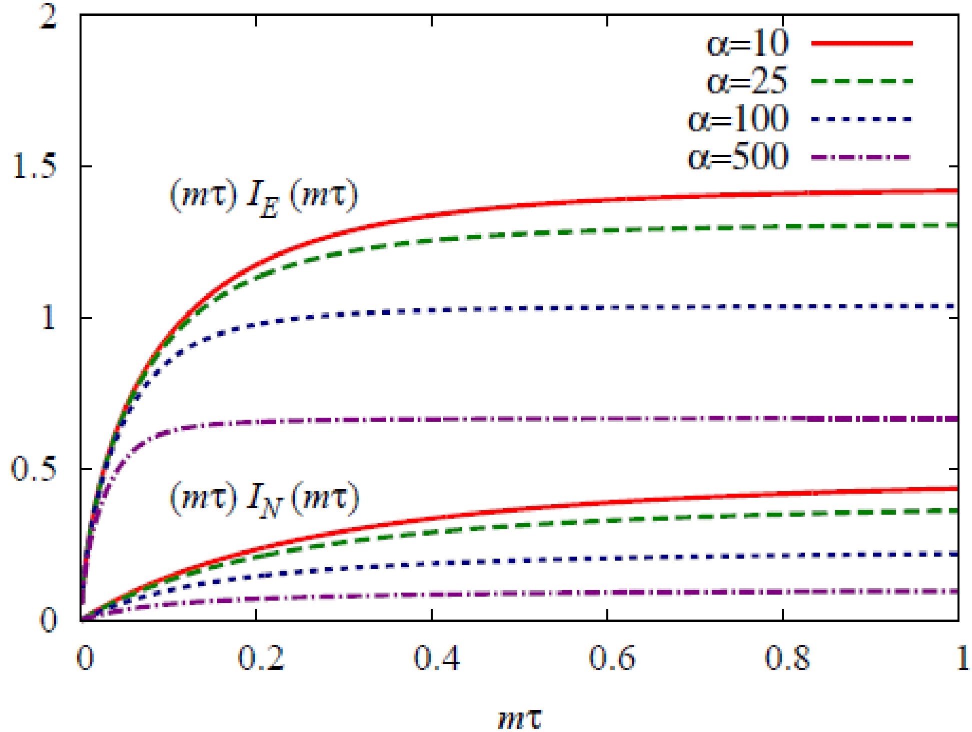
α	m	$g^2\mu_A$
10	$0.64Q_s$	$1.65Q_s$
25	$0.36Q_s$	$1.46Q_s$
100	$0.14Q_s$	$1.13Q_s$
500	$0.050Q_s$	$0.90Q_s$

Numerical

Gelis-Peshier
Lappi

Asymptotic Values

- Time evolution → Free streaming



Too big numbers!
α dependence!

α	$dE/d\eta$	$dN/d\eta$
10	$0.49 - 1.4 \times 10^4$ GeV	$2.5 - 5.0 \times 10^3$
25	$0.49 - 1.4 \times 10^4$ GeV	$4.0 - 8.1 \times 10^3$
100	$0.35 - 1.0 \times 10^4$ GeV	$6.0 - 12 \times 10^3$
500	$0.27 - 0.75 \times 10^4$ GeV	$8.0 - 16 \times 10^3$

$Q_s^2 = $ 1-2 GeV^2

Beyond Perturbative Evolution

- Nonlinear terms important when away from $\tau=0$

$$\partial_\tau E^i = \tau \partial_j(\partial_j A_i - \partial_i A_j) = \tau \partial^2 P^T_{ij} A_j$$

$$\partial_\tau E^\eta = \frac{1}{\tau}\partial^2 A_\eta \,.$$

$A_j A_j A_i \rightarrow \langle A_j A_j \rangle \bar{A}_i$ Gaussian approximation

$$\partial_\tau E^i = \tau\left(\partial^2 - 2g^2 N_c \langle\alpha\alpha\rangle\right) A^T_i$$

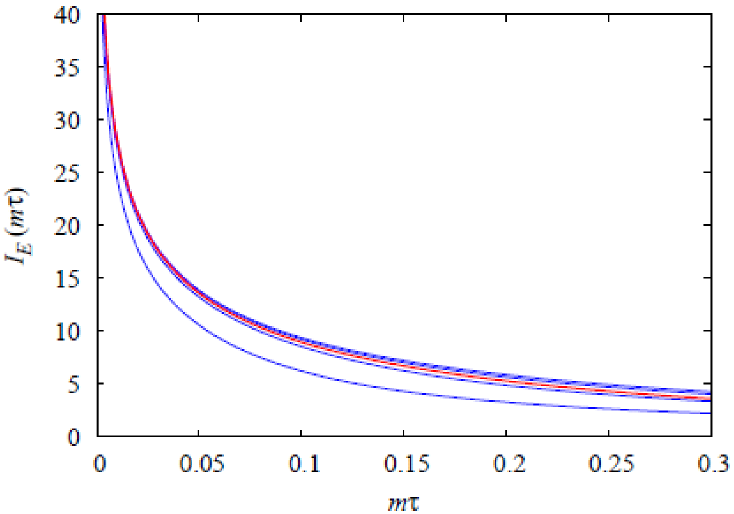
$$\partial_\tau E^\eta = \frac{1}{\tau}\left(\partial^2 - 2g^2 N_c \langle\alpha\alpha\rangle\right) A_\eta$$

$$g^2 N_c \langle\alpha\alpha\rangle \;=\; (m^2\alpha)/(2\pi)\ln[\Lambda/m]$$

$\ln[\Lambda/m] \rightarrow \ln[2/m\tau]$

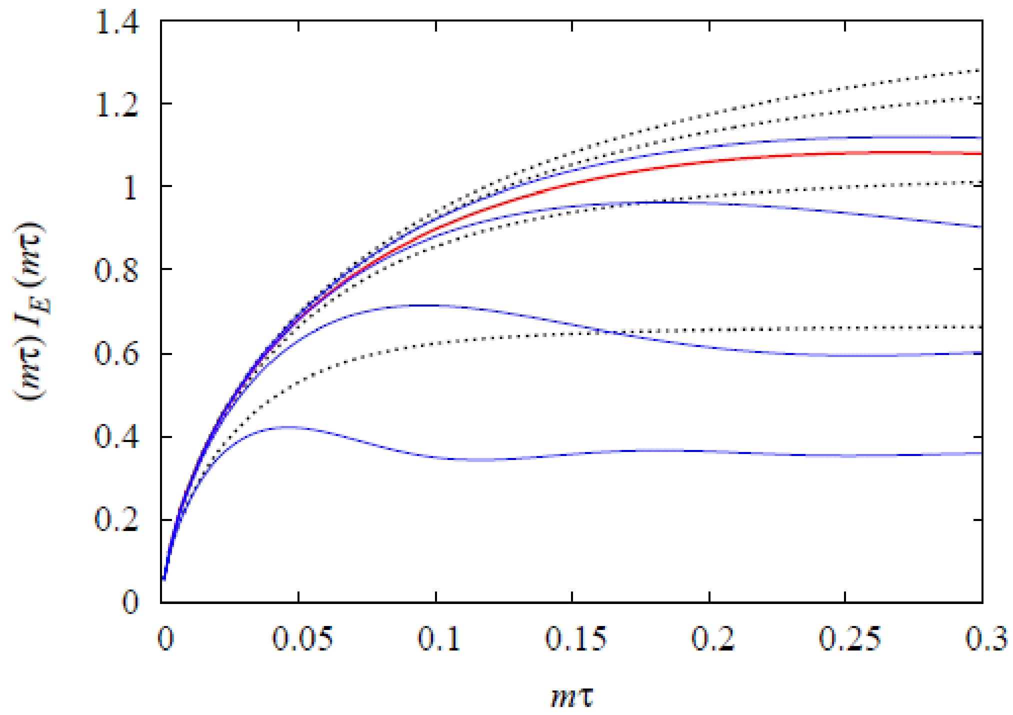
$\ln[20] \simeq 3$ and $\ln[200] \simeq 5.3.$

$k_\perp^2 \rightarrow k_\perp^2 + m^2\alpha$



Modified Time Evolution

- Effect of α



α	$dE/d\eta$	$dN/d\eta$
10	$3.4 - 9.7 \times 10^3$ GeV	$1.0 - 2.1 \times 10^3$
25	$3.2 - 9.0 \times 10^3$ GeV	$1.4 - 2.8 \times 10^3$
100	$2.1 - 5.9 \times 10^3$ GeV	$1.6 - 3.3 \times 10^3$
500	$1.4 - 4.0 \times 10^3$ GeV	$1.6 - 3.2 \times 10^3$

Close to empirical numbers!
α dependence reduced!

Summary

- Analytical estimate in the MV model
  - Different treatment for the longitudinal extent.
  - Check the consistency to the numerical simulation.
- Full nonlinear evaluation at $\tau=0$ combined with the perturbative time evolution.
  - UV divergence free
  - Overestimate the energy and the multiplicity
  - IR sensitive...
- Nonlinear time evolution in the Gaussian approx.
  - IR stable relatively
  - Close to the empirical values