New results in QCD at finite μ

Rajiv Gavai and Sourendu Gupta

ILGTI: TIFR

Initial Conditions in Heavy-Ion Collisions
Dona Paula, Goa
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1. Quark Number Susceptibilities
2. The Critical End Point
3. Series sums and Padé resummations
4. Summary

Evading the sign problem

Quark determinant for QCD not real and positive for $\mu_B \neq 0$; direct lattice simulations ruled out. Methods developed in the past include

1. Expand the determinant for heavy-quarks (quenched limit: Touissant *et al.*); series does not converge for small masses.
2. Numerically reweight the integrand and hope to avoid the sign problem (Glasgow). Does not work at $T = 0$; for $T > 0$ less difficult (Fodor and Katz), but effort increases exponentially with volume.
3. Expand the determinant in a Taylor series (Bielefeld Swansea). Differential form of reweighting.
4. Simulate the theory at imaginary chemical potential (Lombardo; de Forcrand and Philipsen) and make analytical continuation. Numerically hard.
5. Simulate the theory at zero chemical potential and extract the Taylor coefficients of the free energy (Gavai and SG).

Crawling towards the continuum

- Before this year: state of the art lattice computations of physics at finite chemical potential used lattice cutoff $\Lambda = 4T \simeq 800$ MeV near T_c.
- Our earlier computation used $m_\pi \simeq 230$ MeV and spatial sizes with $LT = 2$, 3, 4 and 6. This enabled extrapolation to the thermodynamic limit, *i.e.*, $L \to \infty$.
- Now: new computations with $\Lambda = 6T \simeq 1200$ MeV near T_c.
- m_π remains unchanged (230 MeV), but spatial volumes are somewhat smaller ($LT = 2$, 3 and 4). No extrapolation to $L \to \infty$ yet.
- 20000–50000 configurations at each coupling; stochastic determination of traces with 500 random vectors on each configuration. (Gavai and SG, Phys. Rev. D 68, 2003, 034506.)

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What is a QNS?

Taylor coefficient of the pressure in $N_f = 2$ QCD is

$$P(T,\mu_u,\mu_d) = \sum_{n_u,n_d} \frac{1}{n_u!n_d!} \chi_{n_u,n_d}(T) \mu_u^{n_u} \mu_d^{n_d},$$

and, since the two quark flavours are degenerate, $\chi_{n_u,n_d} = \chi_{n_d,n_u}$.
Diagonal QNS have either $n_u = 0$ or $n_d = 0$.
In two flavour QCD trade $\mu_{u,d}$ for $\mu_{B,Q}$. Then

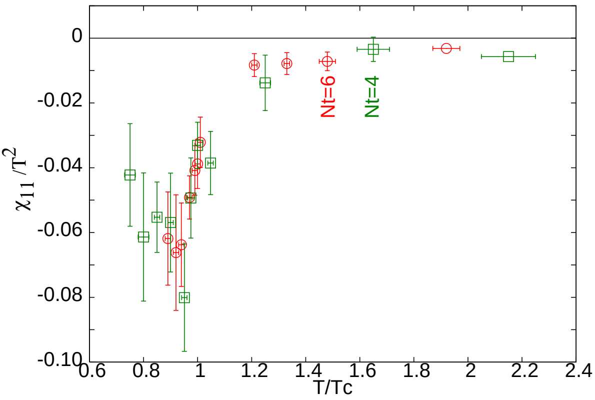
$$\chi_B = \frac{2}{9}(\chi_{20} + \chi_{11}) = 2\chi_{BQ}$$

$$\chi_Q = \frac{1}{9}(5\chi_{20} - 4\chi_{11}).$$

Transforming to μ_{B,I_3}, one has

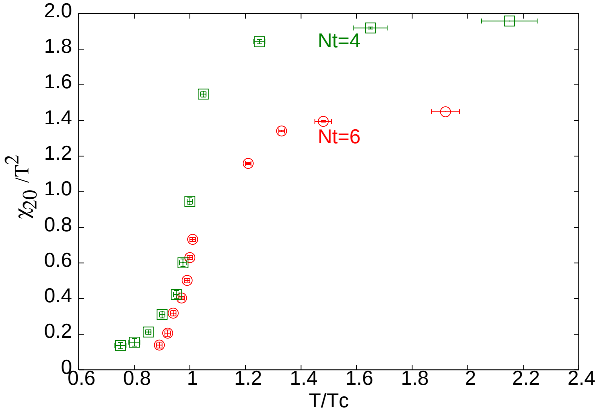
$$\chi_{BI_3} = 0, \qquad \chi_{I_3} = \frac{1}{2}(\chi_{20} - \chi_{11}).$$

Off-diagonal QNS



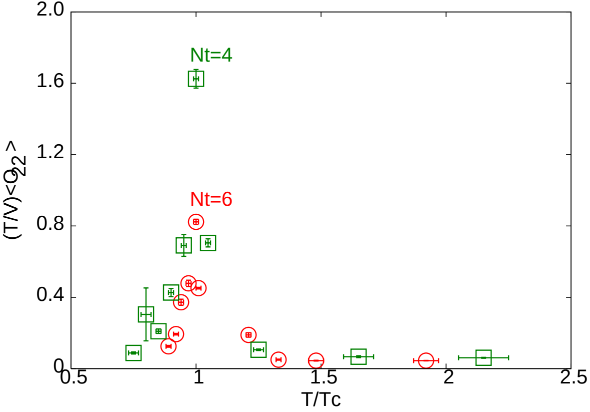
Sees only $\langle O_{11} \rangle$. No evidence for lattice spacing effects.

Diagonal QNS



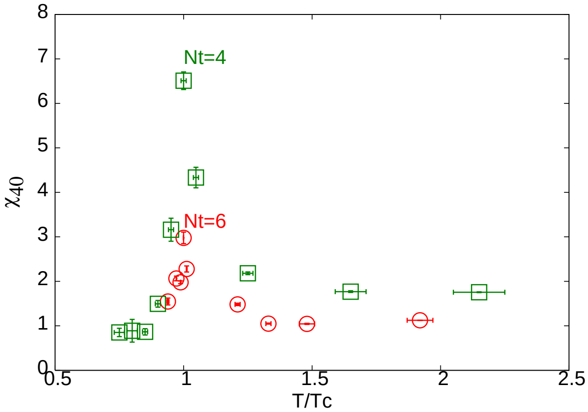
Sees $\langle O_{11} \rangle$ and $\langle O_2 \rangle$. Second expectation value is cutoff dependent. Also, has a cross over. We look at its susceptibility $\langle O_{22} \rangle_c$ to identify T_c.

"Susceptibility" of QNS: $\langle O_{22}\rangle_c$— 4th order QNS



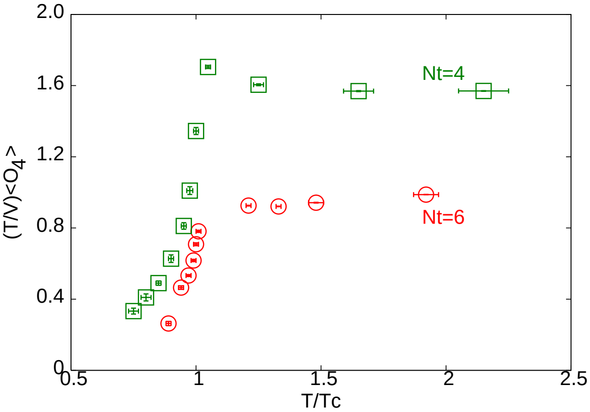
Peak at the same coupling as peak of χ_L. Within the 1% precision of T/T_c, the two quantities peak at the same coupling. See Gavai and Gupta, PR D72 (2005) 054006.

Diagonal fourth-order QNS



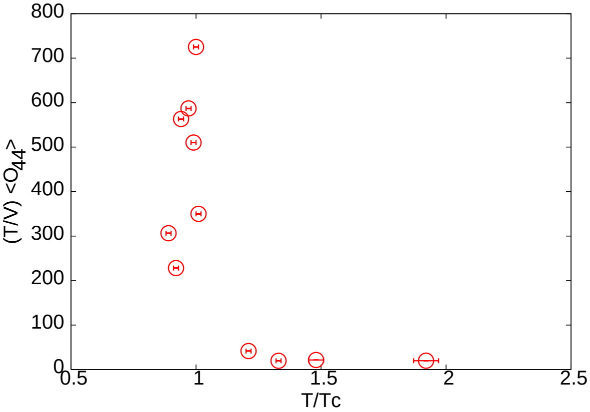
Non-zero for $T > T_c$. Has contribution from $\langle O_4 \rangle$, which has non-vanishing value for the ideal gas.

The operator O_4



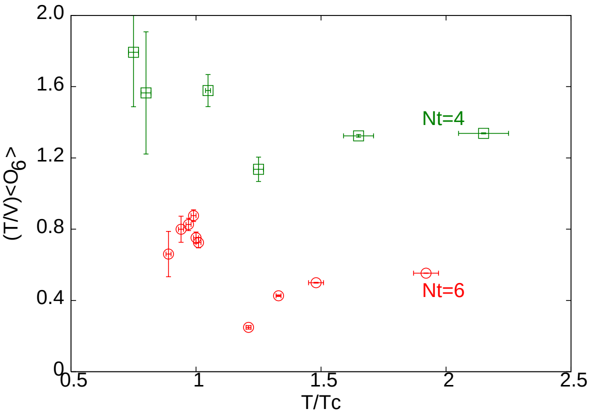
Rapid cross over from a small value in the hadronic phase to a non-vanishing value for the ideal gas.

"Susceptibility" of O_4: $\langle O_{44} \rangle_c$— 8th order QNS



This quantity peaks at the same coupling as χ_L and $\langle O_{22} \rangle_c$. Within the precision of our measurement there is no dependence of the cross over coupling on these observables.

The operator O_6— 6th order QNS



The operator expectation value $\langle O_6 \rangle$ has structure below T_c and hence its "susceptibility" cannot be used to probe the cross over coupling. Similar observation for $\langle O_8 \rangle$.

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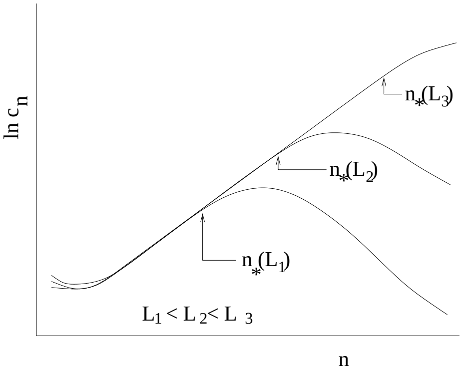
Finite size effects

- At critical point correlation length becomes infinite, appropriate susceptibilities diverge and free energy becomes singular ... in the infinite volume limit (van Hove's theorem).
- No numerical computation ever performed on infinite volumes.
- Deduce the existence of a critical point through extrapolations: finite size scaling (FSS) well developed for direct simulations.
- Example: peak of susceptibility scales as power of volume. Smaller effect: position of peak shifts from its infinite volume position by a different power of volume—

$$\chi_{max}(L) \propto L^p, \qquad T_c(L) = T_c - a/L^q, \qquad (p, q > 0).$$

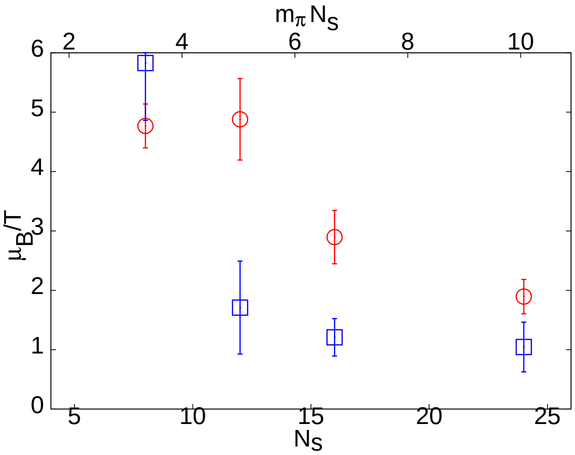
- FSS not well developed for series expansions; some aspects are known.

Series expansions



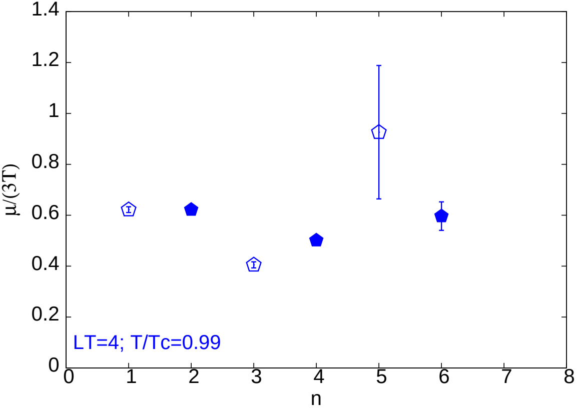
For a divergent quantity: $\chi(T,\mu_B)=\sum_n c_n(T)\mu_B^n$, the leading finite volume effects in the series coefficients.

$N_t = 4$



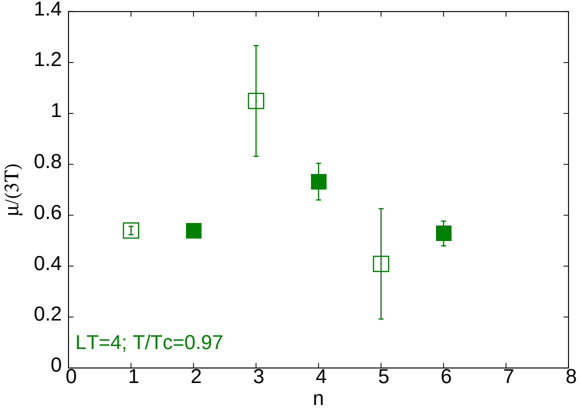
At fixed $T/T_c \simeq 0.95$. Circles: ratio of order 0 and 2; boxes: ratio of order 2 and 4. Gavai and SG, Phys. Rev. D 71, 2005, 114014.

$N_t = 6$: Radius of convergence



Filled symbols: $(\chi^{(0)}/\chi^{(n)})^{1/n}$. Open symbols: $\sqrt{\chi^{(n-1)}/\chi^{(n+1)}}$.

$N_t = 6$: Radius of convergence



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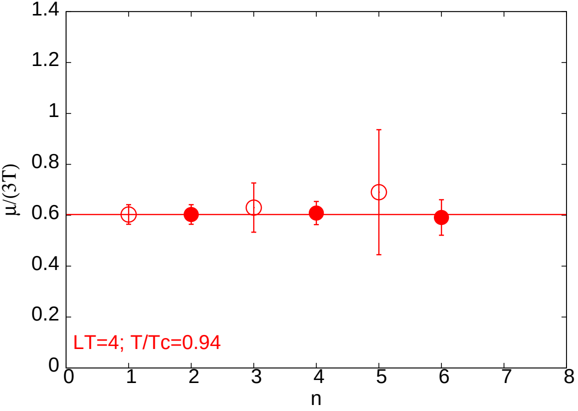

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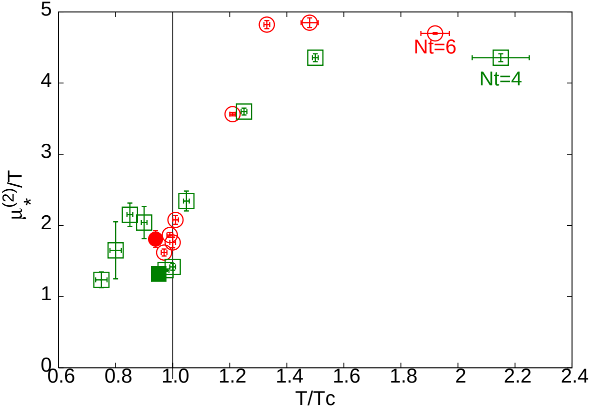
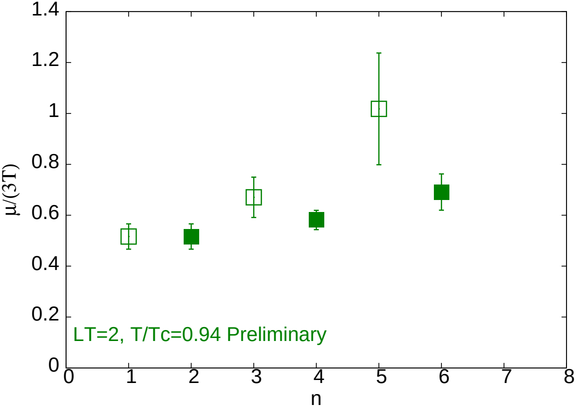
Radius of convergence



Lattice spacing dependence quantifies possible systematic errors.

$N_t = 6$: Finite size scaling



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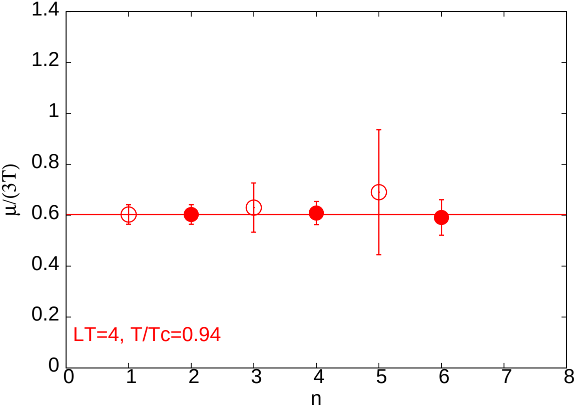
$N_t = 6$: Finite size scaling



Filled symbols: $(\chi^{(0)}/\chi^{(n)})^{1/n}$. Open symbols: $\sqrt{\chi^{(n-1)}/\chi^{(n+1)}}$.

Critical end point

- Multiple criteria agree:
  - Stability of radius of convergence with order and estimator
  - Pinching of the radius of convergence with T.
  - Smallest T where all the coefficients are positive.
  - Finite size effects roughly correct; more planned for the future.
- This gives

$$\frac{\mathbf{T^E}}{\mathbf{T_c}} = \mathbf{0.94 \pm 0.01} \qquad \text{and} \qquad \frac{\mu_{\mathbf{B}}^{\mathbf{E}}}{\mathbf{T^E}} = \mathbf{1.8 \pm 0.1}$$

with $Nf = 2$ when $m_\pi/m_\rho \simeq 0.3$ at a finite volume with $LT = 4$ and lattice cutoff of $a = 1/6T^E$.

- For a lattice cutoff of $a = 1/4T^E$ at the same renormalized quark mass and on the same volume we had found a similar value for T^E/T_c and $\mu_B^E/T^E = 1.3 \pm 0.3$. Extrapolation to $L \to \infty$ reduced this to 1.1 ± 0.1.

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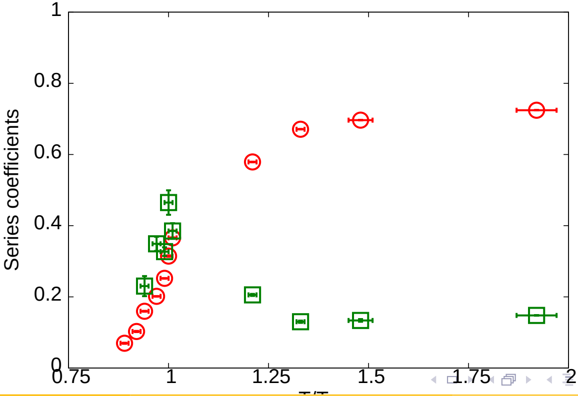
Fluctuations of Baryon number

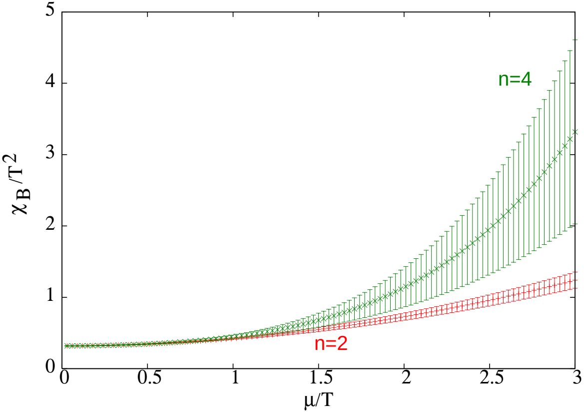
Suggestion by Stephanov, Rajagopal, Shuryak; Asakawa, Heinz, Muller; Jeon, Koch

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right).$$

Extrapolate χ_B to finite chemical potential: peak at T_c?

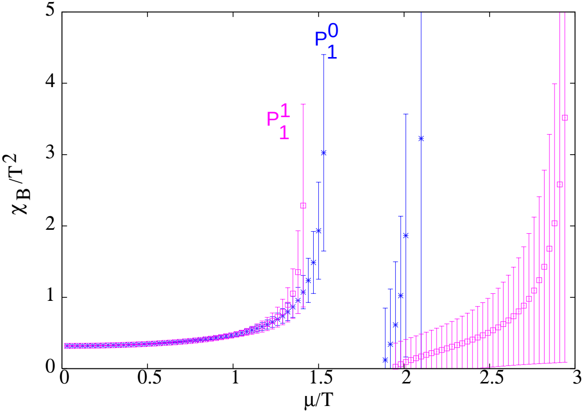


Sum the series



Summing the series never shows critical behaviour: sum is a polynomial and smoothly behaved. The sum peaks at T_c: incorrect (see SG, SEWM 2006).

Critical fluctuations



Use Padé approximants for the extrapolations: divergence at the critical end point (see Lombardo, Mumbai 2005). Error propagation requires care: see arXiv:0806.2233 [hep-lat].

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Summary 1: finite temperature

1. Simulations of $N_f = 2$ QCD (staggered quarks, Wilson action) with renormalized quark mass $m_\pi \simeq 230$ MeV with $1/a \simeq 1200$ MeV and $LT = 2$, 3 and 4.
2. Finite temperature cross over located at $\beta_c = 5.425(5)$, consistent with previous computations at neighbouring masses. Consistent measurements obtained with χ_L, $(T/V)\langle O_{22}\rangle_c$ and $(T/V)\langle O_{44}\rangle_c$ within precision of this computation.
3. Cutoff artifacts seen in many QNS. Surprisingly, measurements are more well-behaved at smaller lattice spacing (see χ_{60} and χ_{80}, for example).

Summary 2: finite chemical potential

1. Very stable estimate of the critical end point: three criteria agree.

$$\frac{T^E}{T_c} = 0.94 \pm 0.01 \qquad \text{and} \qquad \frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$$

   with lattice cutoff of $a = 1/6T^E \simeq 1100$ MeV, compared to $\mu_B^E/T^E = 1.3 \pm 0.3$ at $a = 1/4T^E \simeq 750$ MeV on the same spatial volume.

2. Series extrapolation needs resummation: Padé approximants are one possible resummation.
3. Linkage between u and d quantum numbers disappears at $T \simeq T_c$ when $\mu_B = 0$. How abrupt? Requires finite size scaling.