Open issues in hydro and transport

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TIFR Program on "Initial conditions in heavy ion collisions"

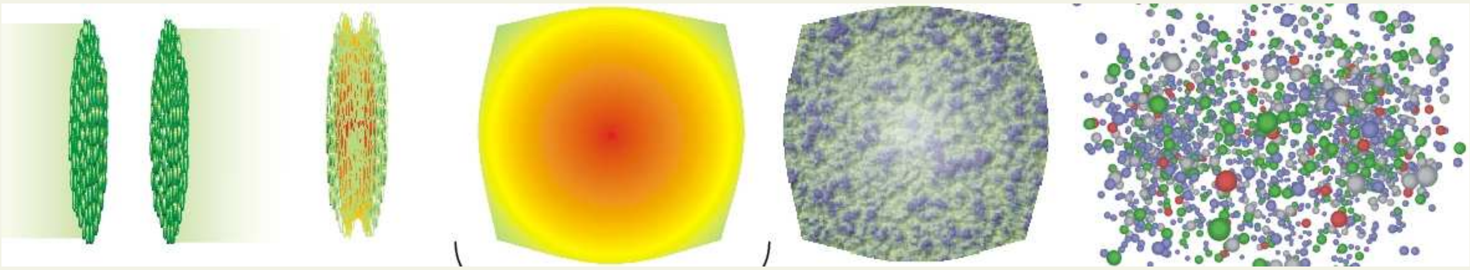
September 16, 2008, International Centre, Dona Paula, Goa, India

Outline

- Region of validity of second-order schemes
- Properties of hot QCD matter
- Initial conditions / thermalization
- Decoupling (freezeout)
- Perturbative or s-QGP?

Hydrodynamics

- describes a system near local equilibrium
- long-wavelength, long-timescale dynamics, driven by conservation laws
- in heavy-ion physics: mainly used for the plasma stage of the collision



$\sim 10^{-14}$ **m** **= 10 fermis**

<—— $\sim 10^{-23}$ **sec = 10 yochto(!)-secs** ——>

nontrivial how hydrodynamics can be applicable at such microscopic scales

In heavy-ion collisions, timescales are short, gradients are large - corrections to ideal (Euler) hydrodynamics are relevant.

Two ways to study dissipative effects in heavy-ion collisions

- causal dissipative hydrodynamics

Israel, Stewart; ... Muronga, Rischke; Teaney et al; Romatschke et al; Heinz et al, DM & Huovinen

flexible in macroscopic properties

numerically cheaper

- covariant transport

Israel, de Groot,... Zhang, Gyulassy, DM, Pratt, Xu, Greiner...

completely causal and stable

fully nonequilibrium $\longrightarrow$ interpolation to break-up stage

Dissipative hydrodynamics

relativistic Navier-Stokes hydro: small corrections linear in gradients [Landau]

$$
\begin{aligned}
T^{\mu\nu}_{NS} &= T^{\mu\nu}_{ideal} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3}\Delta^{\mu\nu}\partial^\alpha u_\alpha) + \zeta\Delta^{\mu\nu}\partial^\alpha u_\alpha \\
N^\nu_{NS} &= N^\nu_{ideal} + \kappa\left(\frac{n}{\varepsilon + p}\right)^2 \nabla^\nu\left(\frac{\mu}{T}\right)
\end{aligned}
$$

where $\Delta^{\mu\nu} \equiv u^\mu u^\nu - g^{\mu\nu}$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

η, ζ **shear and bulk viscosities,** κ **heat conductivity**

Equation of motion: $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu N^\mu = 0$

two problems:

parabolic equations $\rightarrow$ **acausal** Müller ('76), Israel & Stewart ('79) ...

instabilities Hiscock & Lindblom, PRD31, 725 (1985) ...

As an illustration, consider heat flow in a static, incompressible fluid (Fourier)

$$\partial_t T = \kappa \Delta T$$

parabolic eqns.

Greens function is acausal (allows $\Delta x > \Delta t$)

$$G(\vec{x}, t; \vec{x}_0, t_0) = \frac{1}{[4\pi\kappa(t-t_0)]^{3/2}} \exp\left[-\frac{(\vec{x}-\vec{x}_0)^2}{4\kappa(t-t_0)}\right]$$

—–

Adding a second-order time derivative makes it hyperbolic

$$\tau \partial_t^2 T + \partial_t T = \kappa \Delta T$$

Note, this is equivalent to a relaxing heat current

$$\partial_\tau T = \vec{\nabla}\,\vec{j}\ , \qquad \partial_t \vec{j} = -\frac{\vec{j} - \kappa \vec{\nabla} T}{\tau}$$

The wave dispersion relation is $\omega^2 + i\omega/\tau = \kappa k^2/\tau$, i.e., now signals propagate at speeds $c_s = \sqrt{\kappa/\tau}$ (at low frequencies), causal for not too small τ.

Causal dissipative hydro

Bulk pressure Π, shear stress $\pi^{\mu\nu}$, heat flow q^μ are dynamical quantities

$$T^{\mu\nu} \equiv T^{\mu\nu}_{ideal} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu} \ , \qquad N^\mu \equiv N^\mu_{ideal} - \frac{n}{e+p} q^\mu$$

Israel-Stewart: truncate entropy current at quadratic order [Ann.Phys 100 & 118]

$$S^\mu = u^\mu \left[s - \frac{1}{2T} \left(\beta_0 \Pi^2 - \beta_1 q_\nu q^\nu + \beta_2 \pi_{\nu\lambda}\pi^{\nu\lambda} \right) \right] + \frac{q^\mu}{T} \left(\frac{\mu n}{\varepsilon + p} + \alpha_0 \Pi \right) - \frac{\alpha_1}{T} \pi^{\mu\nu} q_\nu$$

(Landau frame). Imposing $\partial_\mu S^\mu \geq 0$ via a quadratic ansatz

$$T\partial_\mu S^\mu = \frac{\Pi^2}{\zeta} - \frac{q_\mu q^\mu}{\kappa_q T} + \frac{\pi_{\mu\nu}\pi^{\mu\nu}}{2\eta_s} \geq 0$$

gives equations of motion for Π, q^μ, $\pi^{\mu\nu}$.

Also follows from covariant transport using Grad's 14-moment approximation

$$f(x,p) \approx [1 + \tilde{C}_\alpha p^\alpha + C_{\alpha\beta} p^\alpha p^\beta] f_{eq}(x,p)$$

and taking the "1", p^ν, and $p^\nu p^\alpha$ moments of the Boltzmann equation.

Complete set of Israel-Stewart equations of motion

$$
\begin{aligned}
D\Pi &= -\frac{1}{\tau_\Pi}(\Pi + \zeta\nabla_\mu u^\mu) \\
&\quad -\frac{1}{2}\Pi\left(\nabla_\mu u^\mu + D\ln\frac{\beta_0}{T}\right) \\
&\quad +\frac{\alpha_0}{\beta_0}\partial_\mu q^\mu - \frac{a_0'}{\beta_0}q^\mu D u_\mu
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
Dq^\mu &= -\frac{1}{\tau_q}\left[q^\mu + \kappa_q\frac{T^2 n}{\varepsilon + p}\nabla^\mu\left(\frac{\mu}{T}\right)\right] - u^\mu q_\nu D u^\nu \\
&\quad -\frac{1}{2}q^\mu\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_1}{T}\right) - \omega^{\mu\lambda}q_\lambda \\
&\quad -\frac{\alpha_0}{\beta_1}\nabla^\mu\Pi + \frac{\alpha_1}{\beta_1}(\partial_\lambda\pi^{\lambda\mu} + u^\mu\pi^{\lambda\nu}\partial_\lambda u_\nu) + \frac{a_0}{\beta_1}\Pi D u^\mu - \frac{a_1}{\beta_1}\pi^{\lambda\mu}D u_\lambda
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
D\pi^{\mu\nu} &= -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) - (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)D u_\lambda \\
&\quad -\frac{1}{2}\pi^{\mu\nu}\left(\nabla_\lambda u^\lambda + D\ln\frac{\beta_2}{T}\right) - 2\pi_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} \\
&\quad -\frac{\alpha_1}{\beta_2}\nabla^{\langle\mu}q^{\nu\rangle} + \frac{a_1'}{\beta_2}q^{\langle\mu}D u^{\nu\rangle}\ .
\end{aligned}
\tag{3}
$$

where $A^{\langle\mu\nu\rangle} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(A_{\alpha\beta}+A_{\beta\alpha}) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}A^{\alpha\beta}$, $\qquad \omega^{\mu\nu} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\partial_\beta u_\alpha - \partial_\alpha u_\beta)$

We can solve these in 2+1D at present, 3+1D should also be doable.

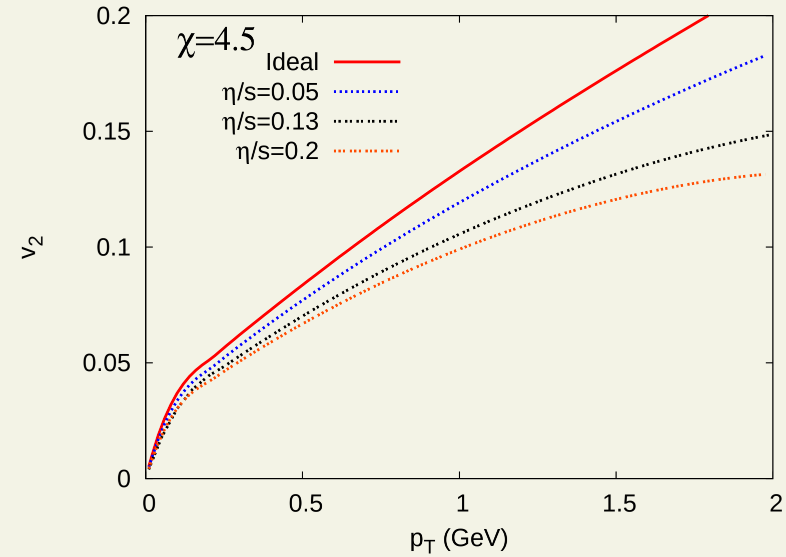
There may be more terms - e.g., imposing ONLY conformal invariance Baier, Romatschke, Son, JHEP04, 100 ('08)

$$\dot{\pi}^{\mu\nu} = \cdots + \frac{\lambda_1}{\eta^2}\pi^{\alpha\langle\mu}\pi_\alpha{}^{\nu\rangle} + \lambda_3\omega^{\alpha\langle\mu}\omega_\alpha{}^{\nu\rangle} \tag{4}$$

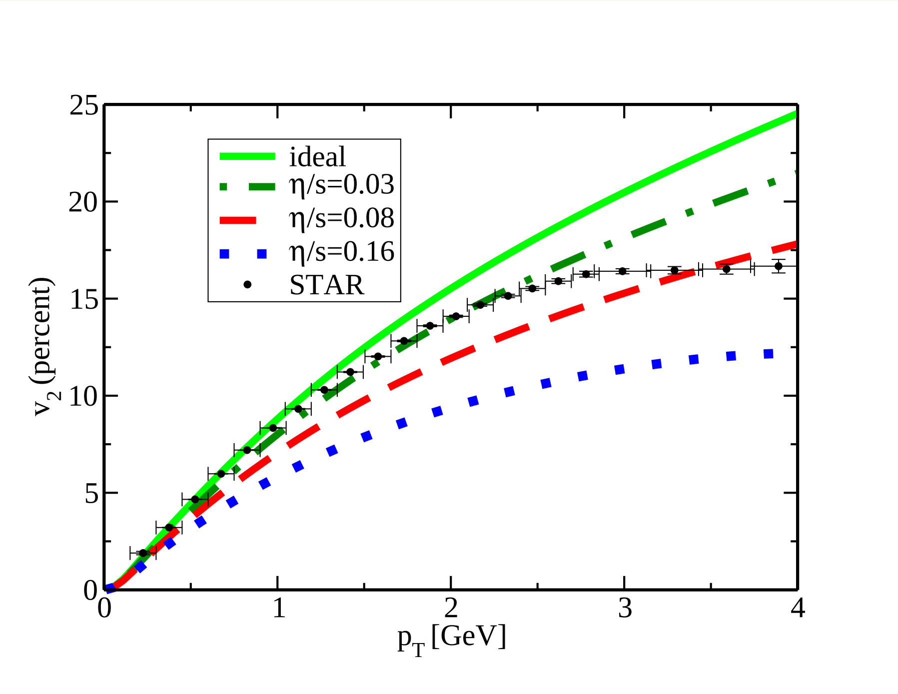
No problem, we can solve with these too.

e.g.

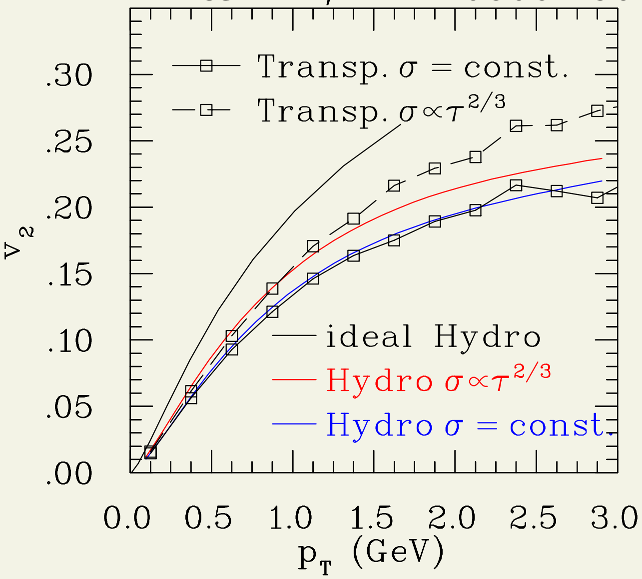
Dusling & Teaney, PRC77



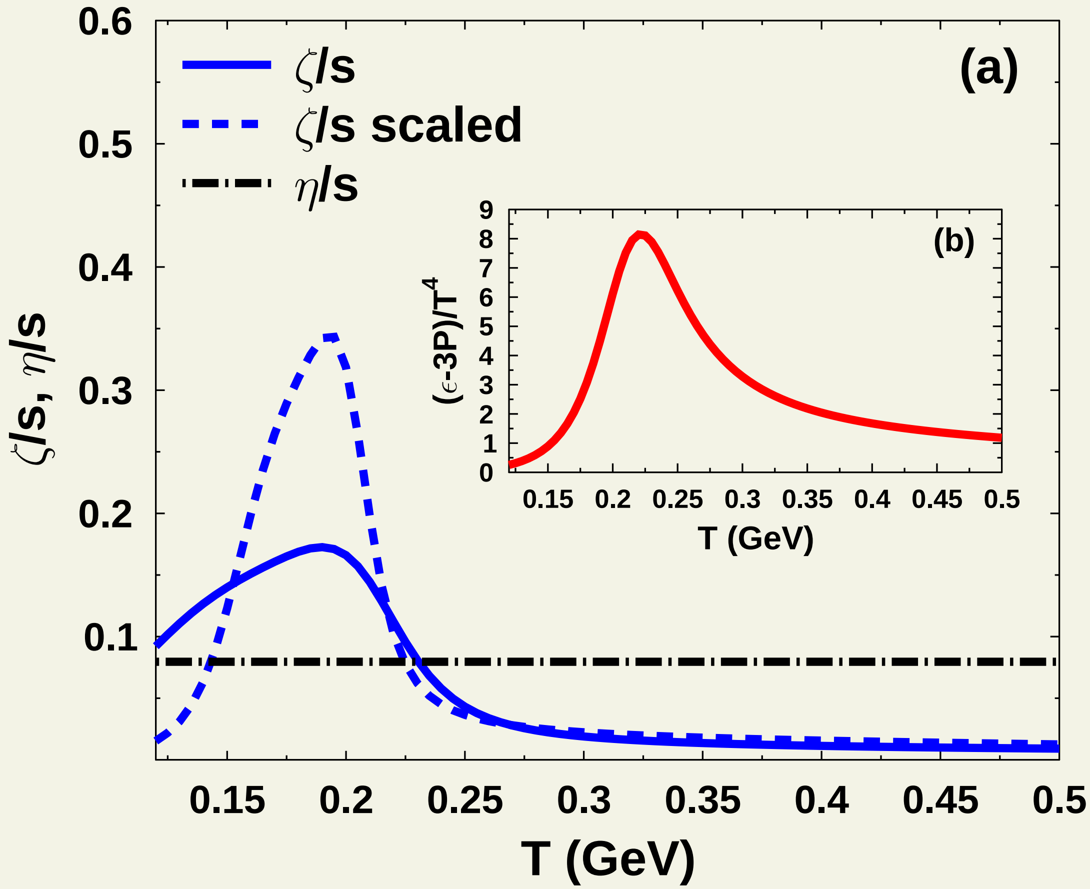
Romatschke & Romatschke, arxiv:0706.1522

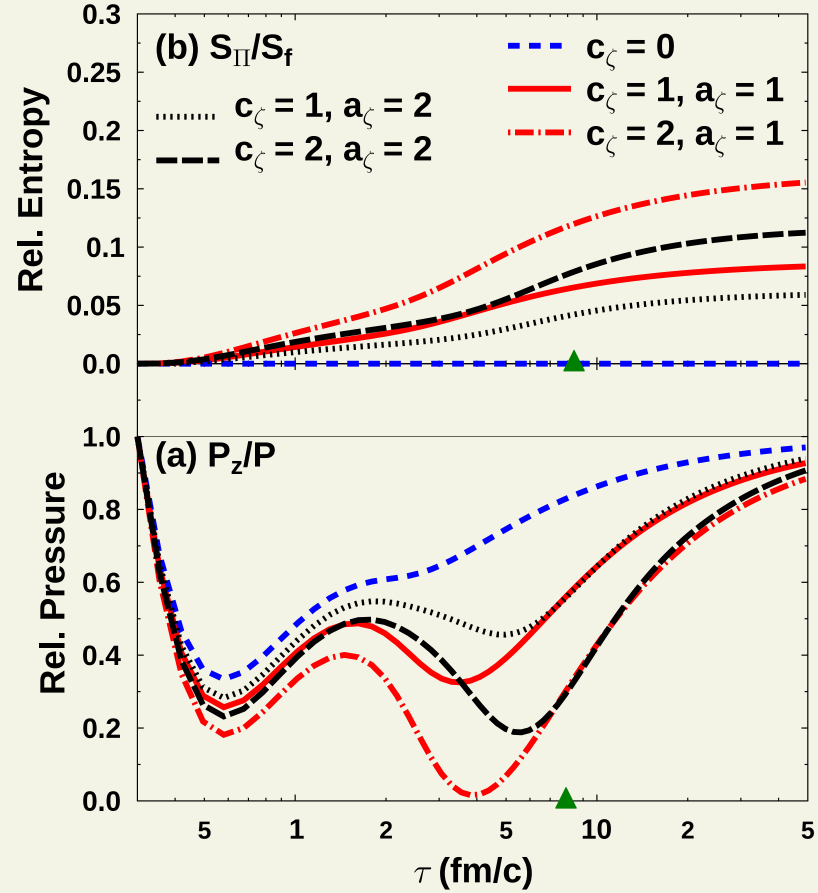


Huovinen & DM, arXiv:0806.1367



Fries et al, arxiv:0807.4333 **in 0+1D**





Applicability of IS hydro

In heavy ion physics applications, gradients $\partial^\mu u^\nu/T$, $|\partial^\mu e|/(Te)$, $|\partial^\mu n|/(Tn)$ at early times $\tau \sim 1$ fm are large $\sim \mathcal{O}(1)$, and therefore cannot be ignored.

Hydrodynamics may still apply, if viscosities are unusually small $\eta/s \sim 0.1$, $\zeta/s \sim 0.1$, where s is the entropy density in local equilibrium. In that case, pressure corrections from Navier-Stokes theory still moderate

$$\frac{\delta T^{\mu\nu}_{NS}}{p} \approx \left(2\frac{\eta_s}{s}\frac{\nabla^{\langle\mu}u^{\nu\rangle}}{T} + \frac{\zeta}{s}\frac{\nabla_\alpha u^\alpha}{T}\right)\frac{\varepsilon + p}{p} \sim \mathcal{O}\left(\frac{8\eta_s}{s}, \frac{4\zeta}{s}\right) \quad . \tag{5}$$

Heat flow effects can also be estimated based on

$$\frac{\delta N^\mu_{NS}}{n} \approx \frac{\kappa_q T}{s}\frac{n}{s}\frac{\nabla^\mu(\mu/T)}{T} \tag{6}$$

and should be very small at RHIC because $\mu/T \sim 0.2$, $n_B/s \sim \mathcal{O}(10^{-3})$

Whereas Navier-Stokes comes from a rigorous expansion in small deviations near local equilibrium retaining all powers of momentum

$$f(x,\vec{p}) = f_{eq}(x,\vec{p})[1 + \phi(x,\vec{p})] \qquad (|\phi| \ll 1\ , \quad |p^\mu \partial_\mu \phi| \ll |p^\mu \partial_\mu f_{eq}|/f_{eq})$$

the quadratic truncation in Grad's approach has no small control parameter.

In general, Navier-Stokes and Israel-Stewart are different

$\Rightarrow$ control against a nonequilibrium theory is crucial

IS hydro vs transport [Huovinen & DM, arXiv:0808.0953]

0+1D Bjorken scenario with massless $e = 3p$ EOS, $2 \to 2$ interactions

$\pi_{LR}^{\mu\nu} = diag(0, -\frac{\pi_L}{2}, -\frac{\pi_L}{2}, \pi_L)$, $\Pi \equiv 0$, $q^\mu \equiv 0$ **(reflection symmetry)**

$$\dot{p} + \frac{4p}{3\tau} = -\frac{\pi_L}{3\tau} \tag{7}$$

$$\dot{\pi}_L + \frac{\pi_L}{\tau}\left(\frac{2K(\tau)}{3C} + \frac{4}{3} + \frac{\pi_L}{3p}\right) = -\frac{8p}{9\tau} , \tag{8}$$

where

$$K(\tau) \equiv \frac{\tau}{\lambda_{tr}(\tau)} , \quad C \approx \frac{4}{5} . \tag{9}$$

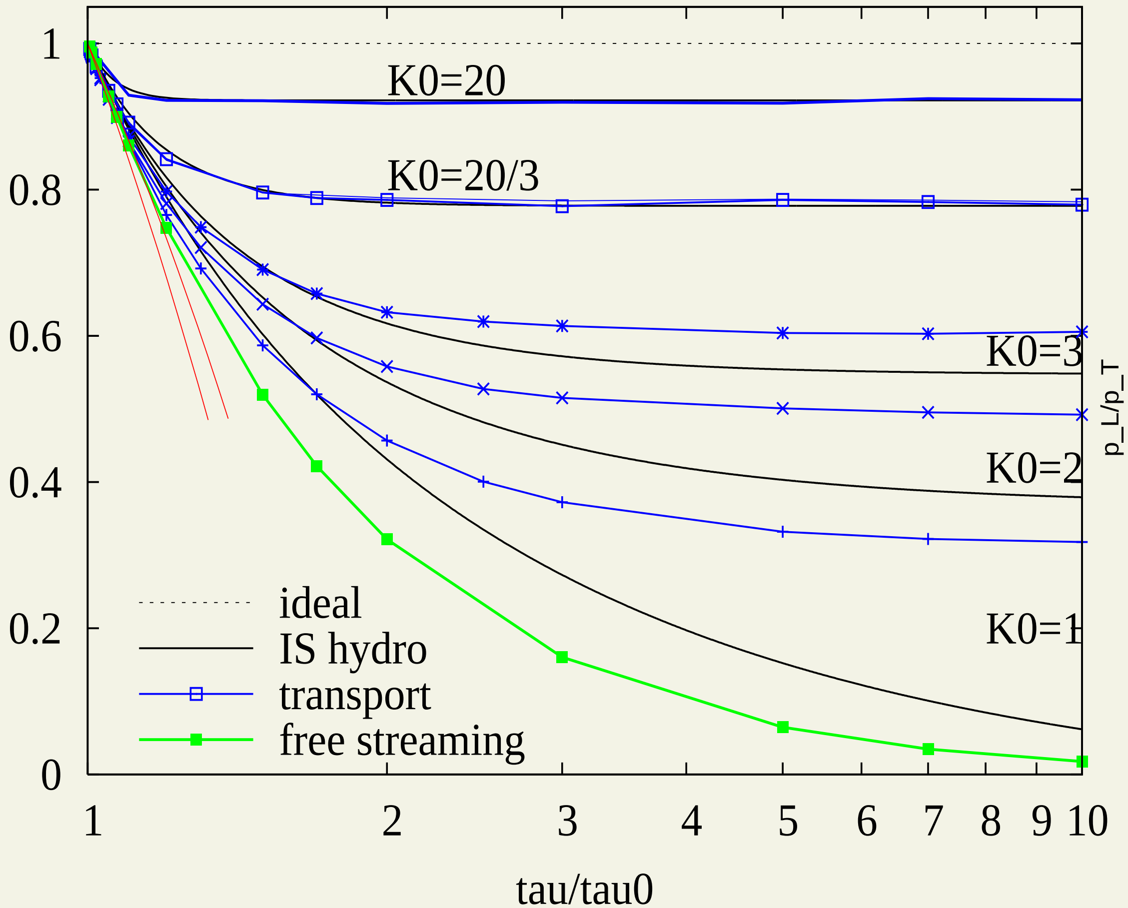
For $\sigma = const$: $\lambda_{tr} = 1/n\sigma_{tr} \propto \tau \Rightarrow K = K_0 = const$, $\eta/s \sim T\lambda_{tr} \sim \tau^{2/3}$

for $\eta/s \approx const$: $K = K_0(\tau/\tau_0)^{\sim 2/3} \propto \tau^{\sim 2/3}$

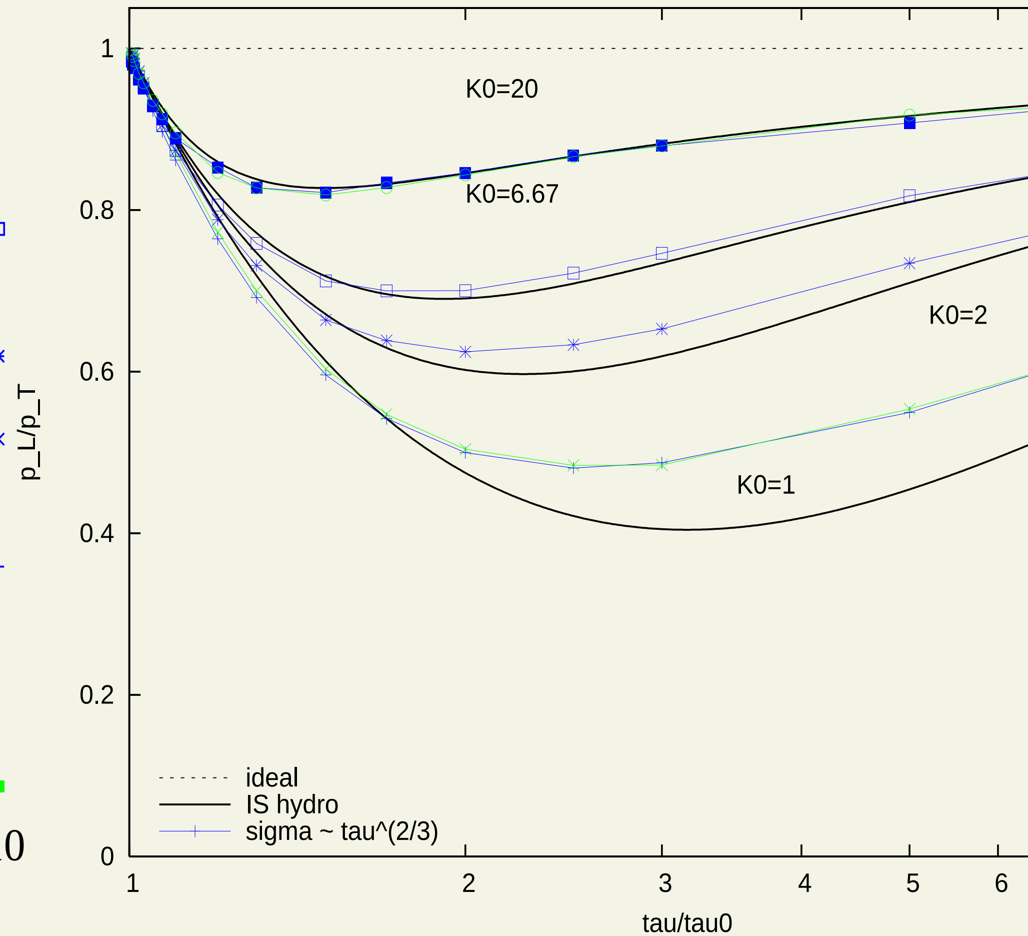
we kept the COMPLETE Israel-Stewart equations (every term)

Huovinen & DM, arXiv:0808.0953 **pressure anisotropy T_{zz}/T_{xx} for $e = 3p$ EOS**

$\sigma = const$

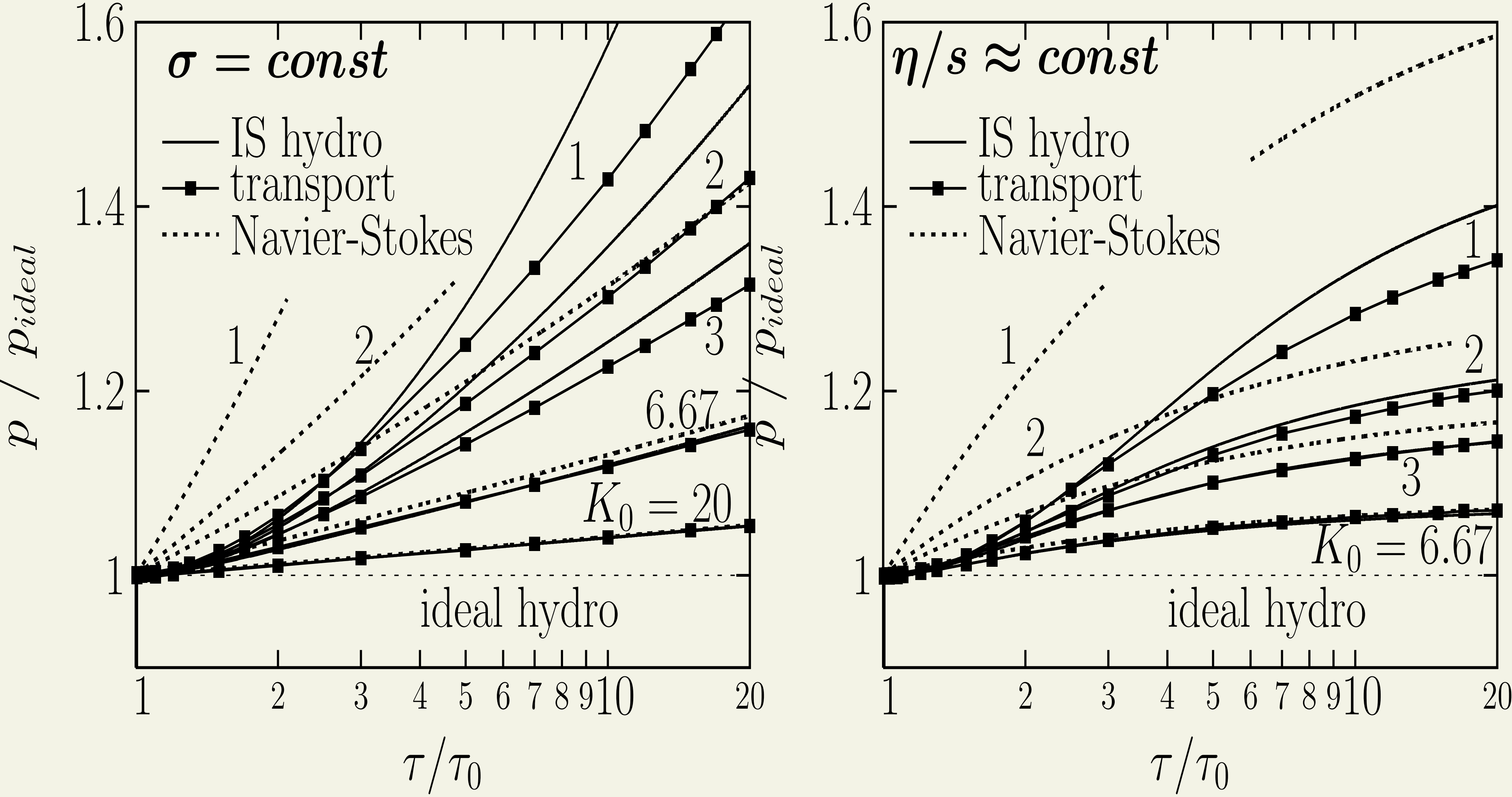


$\eta/s \approx const$



IS hydro applicable when $K_0 \gtrsim 2-3$, i.e., $\lambda_{tr} \lesssim 0.3 - 0.5\ \tau_0$

pressure evolution relative to ideal hydro



Connection to viscosity

$$K_0 \approx \frac{T_0 \tau_0}{5} \frac{s_0}{\eta_{s,0}} \approx 12.8 \times \left(\frac{T_0}{1 \text{ GeV}}\right) \left(\frac{\tau_0}{1 \text{ fm}}\right) \left(\frac{1/(4\pi)}{\eta_{s_0}/s_0}\right) \quad (10)$$

For typical RHIC hydro initconds $T_0 \tau_0 \sim 1$, therefore

$$K_0 \gtrsim 2 - 3 \qquad \Rightarrow \qquad \frac{\eta}{s} \lesssim \frac{1-2}{4\pi} \quad (11)$$

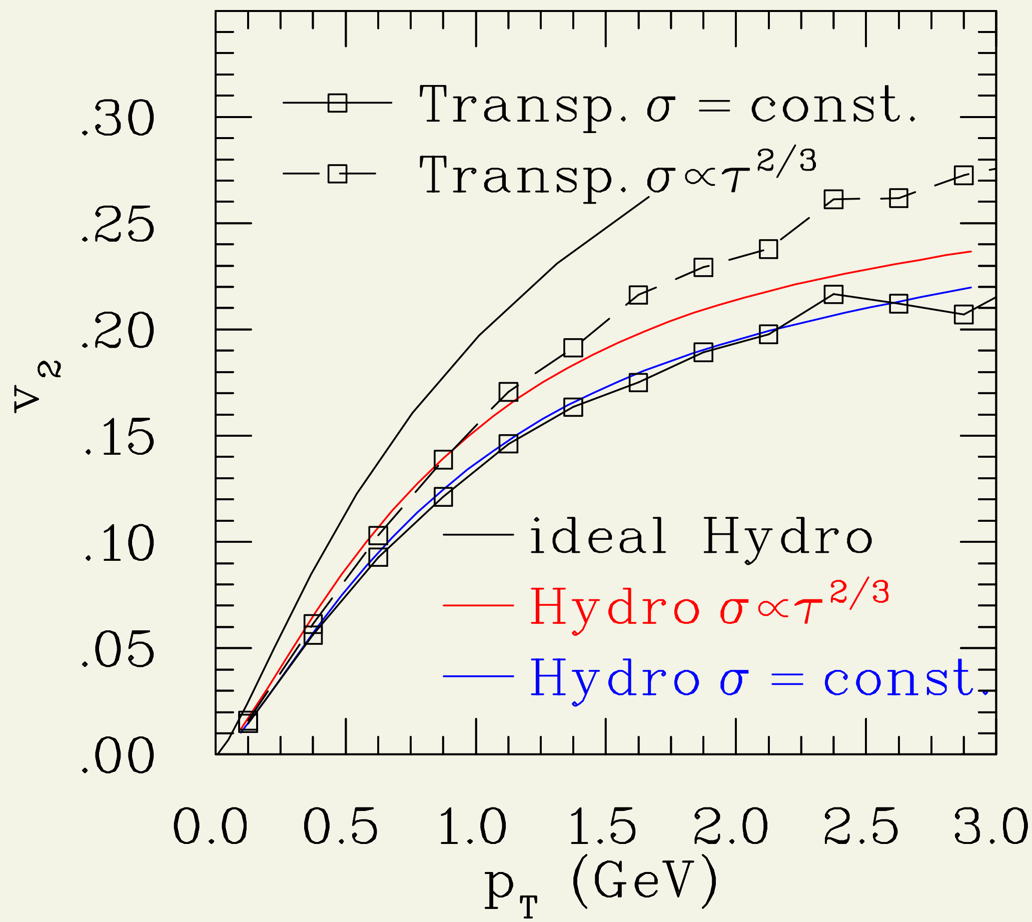
I.e., if the conjectured bound is accurate, hydro is marginally applicable.

Q1: is this true for a nonequilibrium theory other than covariant transport?

Q2: what is the viscosity of QCD matter at $T \sim 150 - 400$ MeV?

Viscous hydro vs transport in 2+1D

Huovinen & DM, arXiv:0806.1367

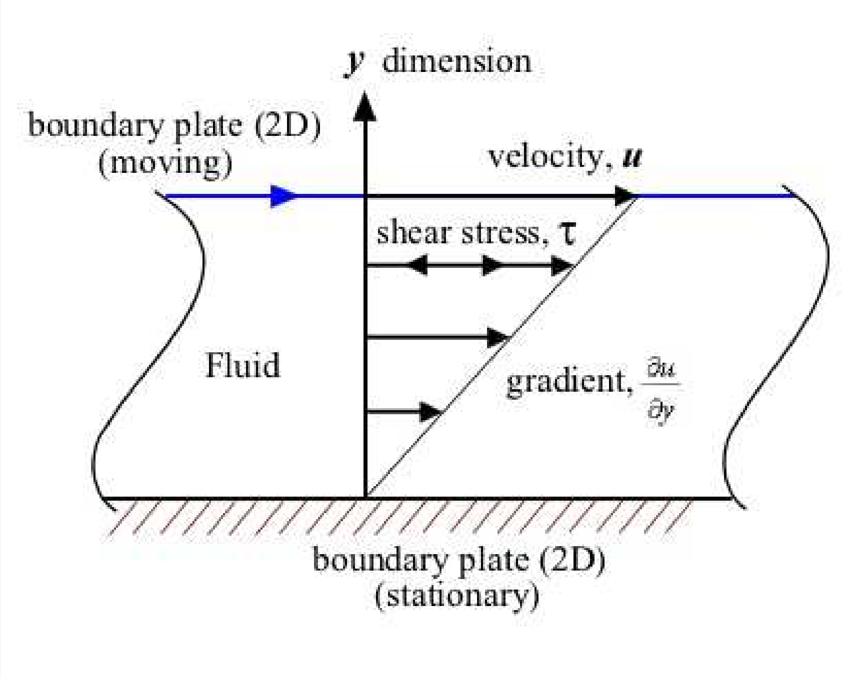


- **excellent agreement when** $\sigma =$ **const** $\sim 47mb$
- **good agreement for** $\eta/s \approx 1/(4\pi)$**, i.e.,** $\sigma \propto \tau^{2/3}$

Shear viscosity

1687 - I. Newton (Principia)

$$T_{xy} = -\eta \frac{\partial u_x}{\partial y}$$



acts to reduce velocity gradients

1985 - quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

+ kinetic theory: $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz, PRD 31 ('85)

$\eta \approx 4/5 \cdot T/\sigma_{tr}$, **entropy** $s \approx 4n$

gives minimal viscosity: $\eta/s = \frac{\lambda_{tr} T}{5} \geq \hbar/15$

2004 - string theory AdS/CFT: $\eta/s \geq \hbar/4\pi$ Policastro, Son, Starinets, PRL87 ('02) Kovtun, Son, Starinets, PRL94 ('05)

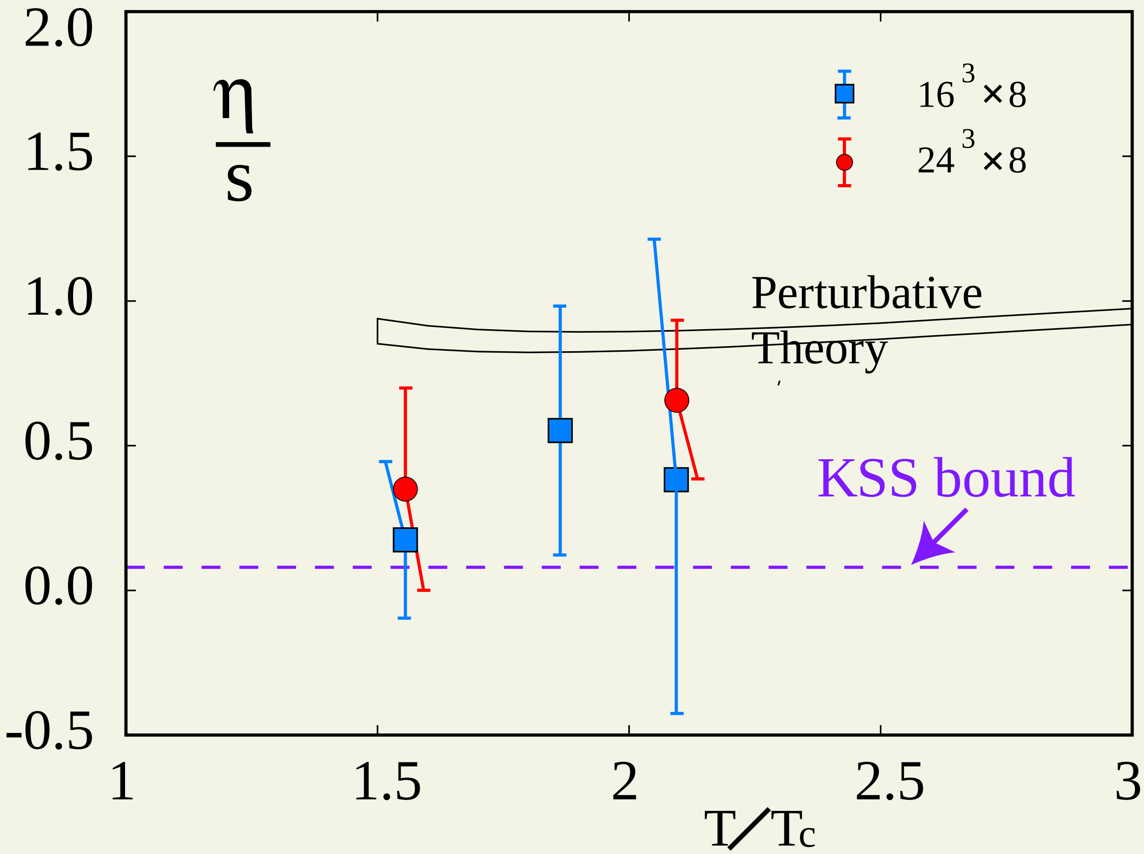
revised to $4\hbar/(25\pi)$ Brigante et al, arXiv:0802.3318

Shear viscosity in QCD

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

perturbative QCD: $\eta/s \sim 1$, **lattice QCD: correlator very noisy**

Nakamura & Sakai, NPA774, 775 ('06):



Meyer, PRD76, 101701 ('07)

upper bounds:

$\eta/s(T{=}1.65T_c) < 0.96$

$\eta/s(T{=}1.24T_c) < 1.08$

best estimate:

$\eta/s(T{=}1.65T_c) < 0.13 \pm 0.03$

$\eta/s(T{=}1.24T_c) < 0.10 \pm 0.05$

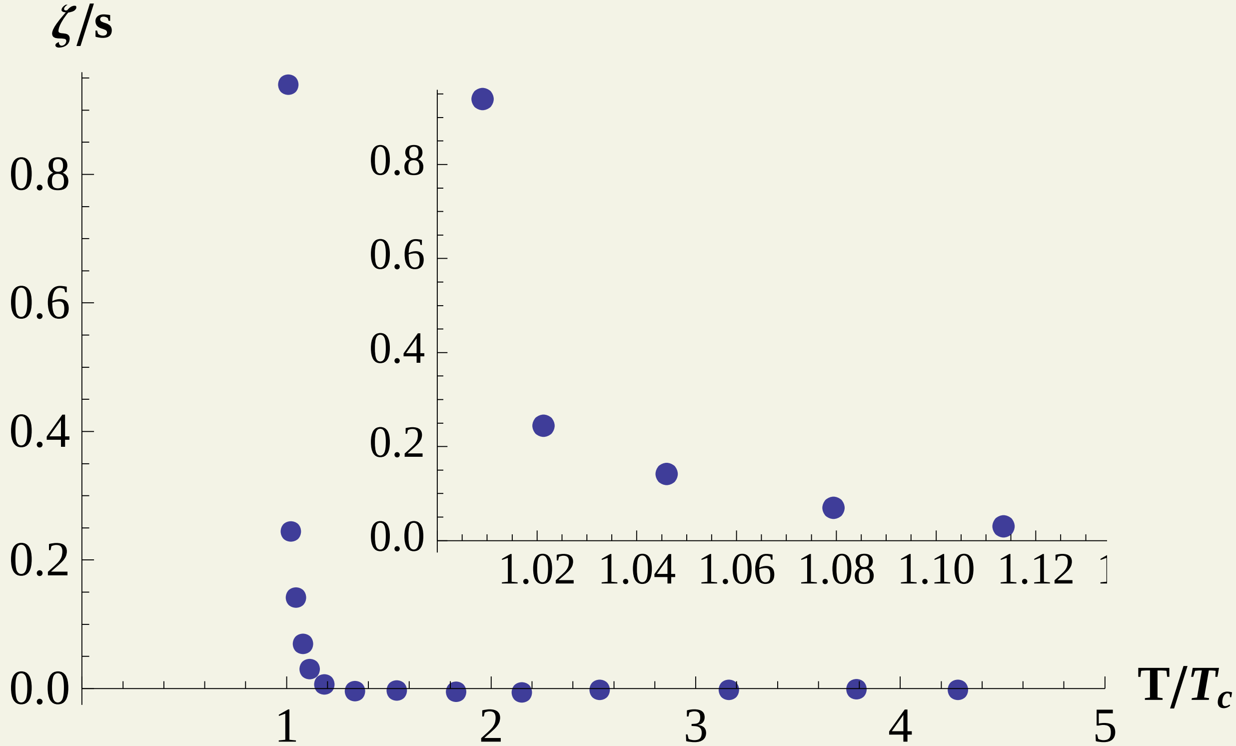
many practitioners regard these VERY preliminary

Bulk viscosity in QCD

$$\zeta = \lim_{\omega\to 0} \frac{1}{18\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T^\mu_\mu(\vec{x},t), T^\mu_\mu(0)] \rangle$$

perturbative QCD: $\zeta/s \sim 0.02\alpha_s^2$ **is tiny** Arnold, Dogan, Moore, PRD74 ('06)

from $\varepsilon - 3p > 0$**:** Kharzeev & Tuchin, arXiv:0705.4280v2



on lattice: Meyer, arXiv:0710.3717

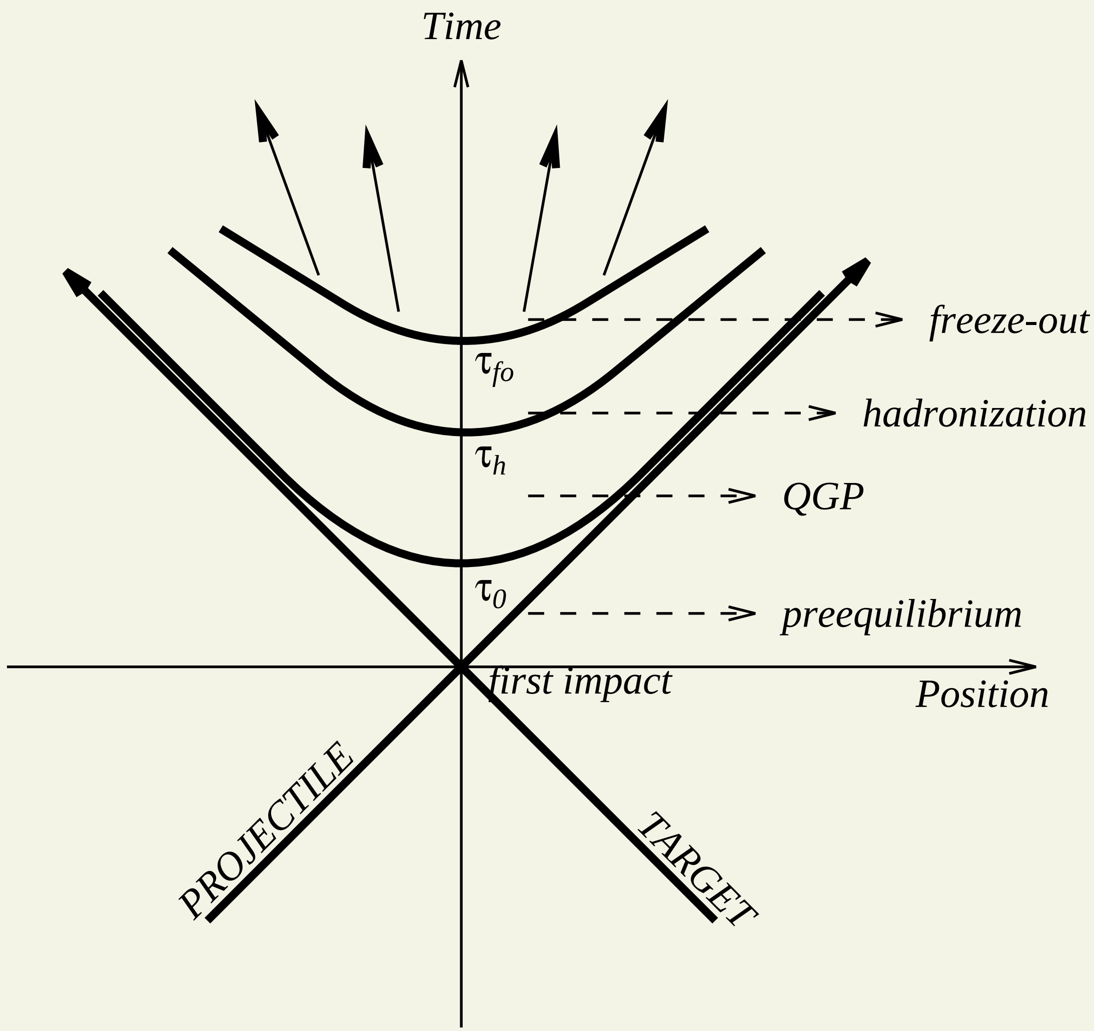
best estimates:

$\eta/s(T{=}1.65T_c) \sim 0 - 0.015$

$\eta/s(T{=}1.24T_c) \sim 0.06 - 0.1$

$\eta/s(T{=}1.02T_c) \sim 0.2 - 2.7$

many practitioners regard these as well VERY preliminary



hydro needs: **initial conditions**
boundary conditions = expansion to vacuum ($\varepsilon = p = 0$ **outside)**
and a decoupling model (transition to free-streaming gas)

Initial conditions

Hydro initial conditions should come from the very early collision dynamics.

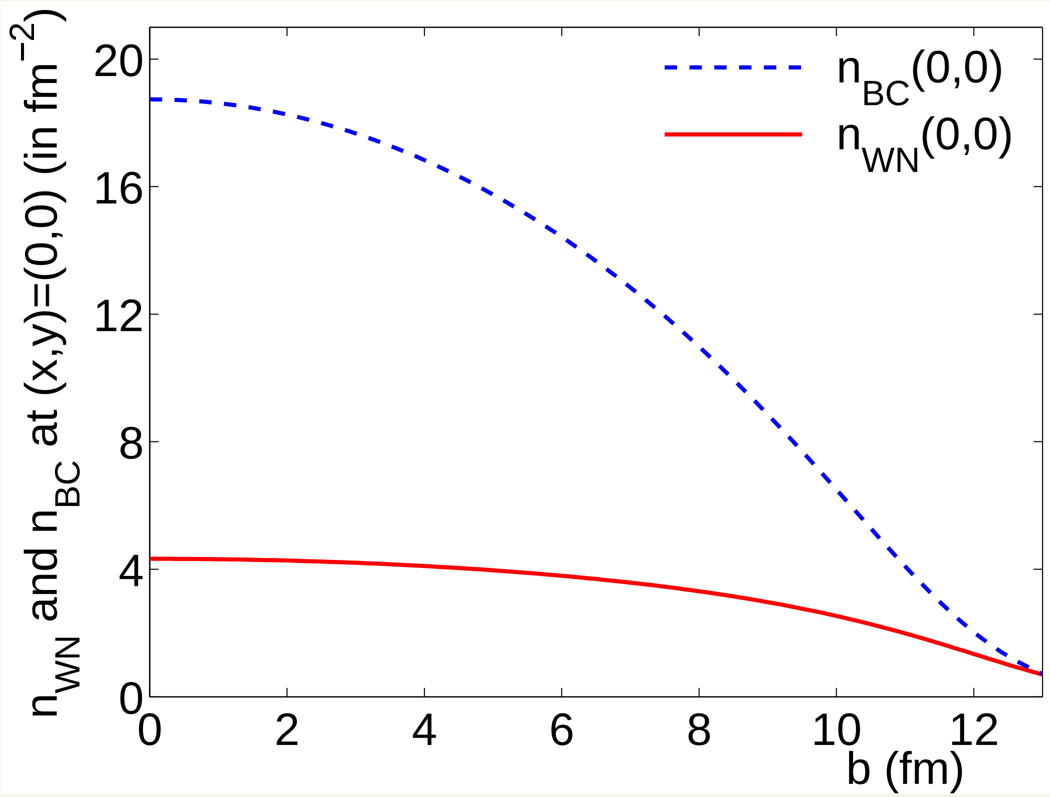
Q3: when and how (if at all) does the matter thermalize in the collision?

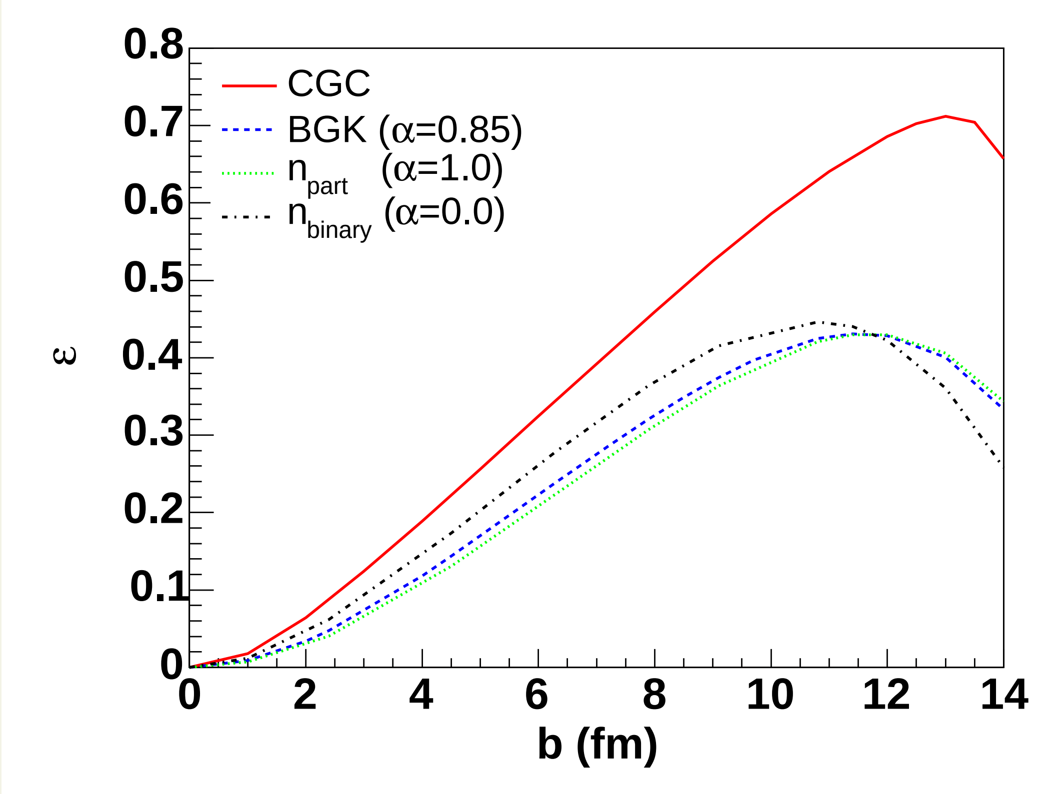
In the meantime, resort to parameterizations:

- **thermalization time τ_0**
- **some shape of initial entropy or energy density profile (e.g. Glauber, binary, saturation model)**
- **assumptions about pressure anisotropies (need the full $\pi^{\mu\nu}$, also Π, and in principle q^μ too)**
- **baryon density profiles (e.g., through entropy ratio n_B/s)**
- **Q4: any initial radial and/or elliptic flow? - typically set to zero**

Initial condition parameters, and also freezeout parameters T_{fo} (ε_{fo}), are fit to data for central collisions. Noncentral collisions can then be "predicted".

The main difference between the various profiles is in the centrality dependence (mainly affects $dN(b)/dy$) and the spatial eccentricity (mainly affects $v_2(b)$)

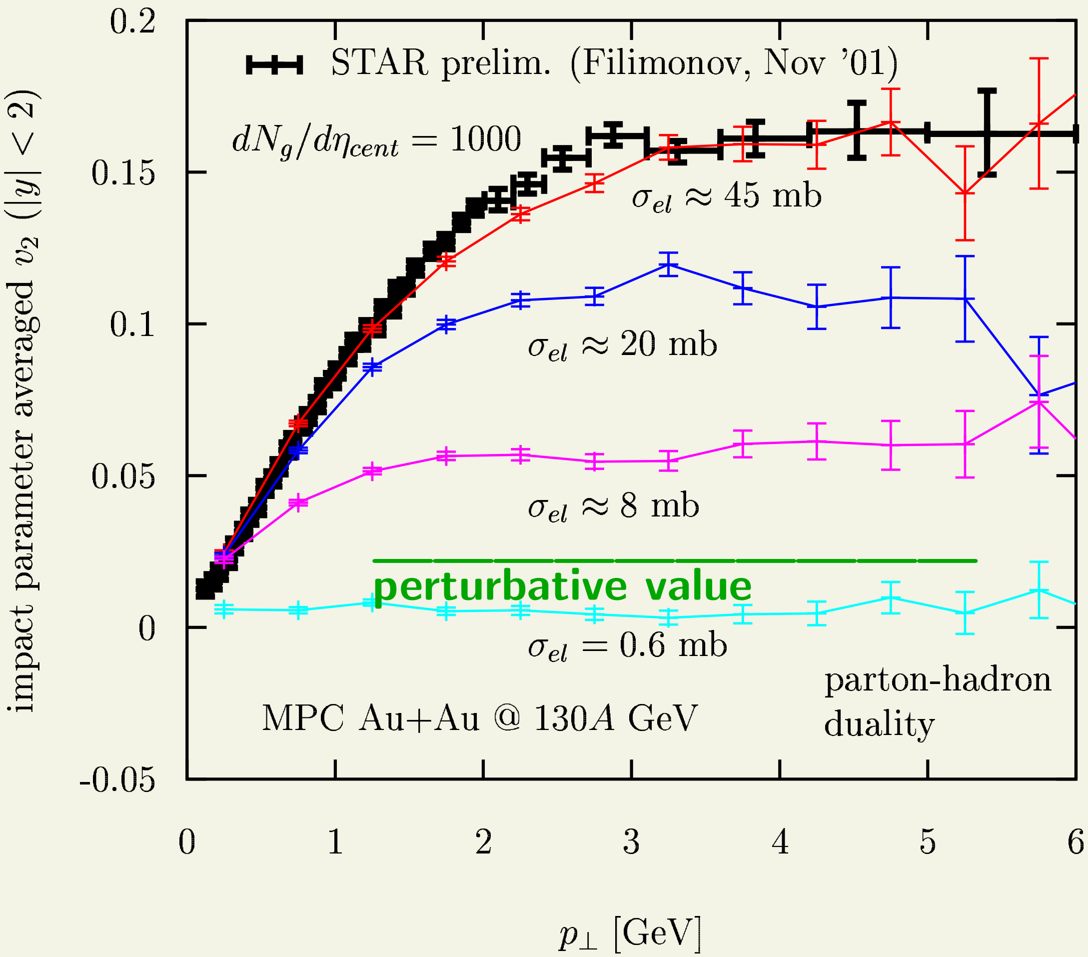




Note that naive CGC results are shown - more realistic calculations give eccentricities close to binary but results sensitive to how the edges are handled (theory breaks down because Q_s too low)

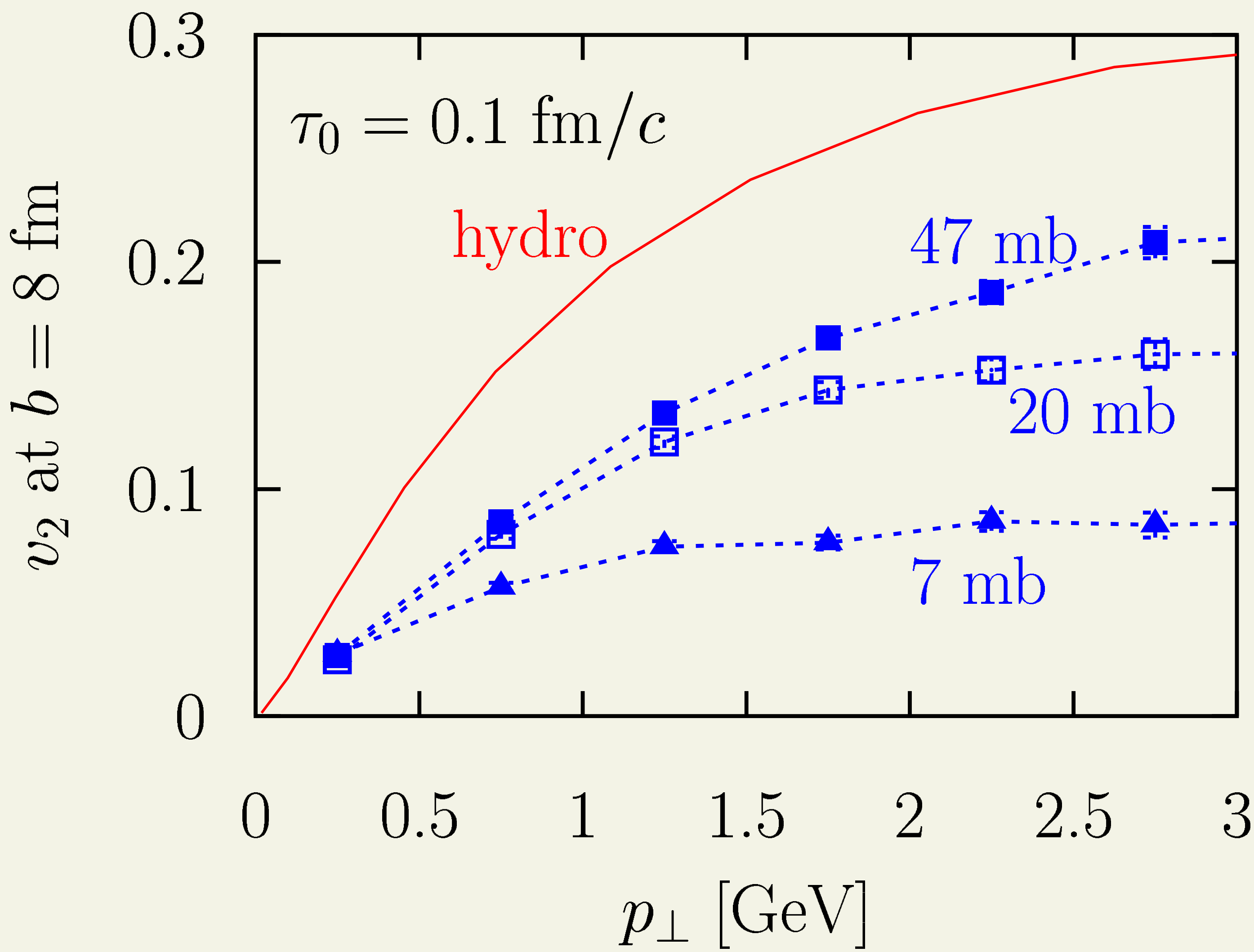
Thermalization and $2 \to 2$ transport

DM & Gyulassy, NPA 697 ('02): $v_2(p_T, \chi)$ in Au+Au @ 130 GeV, $b = 8$ fm



perturbative $2 \to 2$ transport is lousy at maintaining local equilibrium

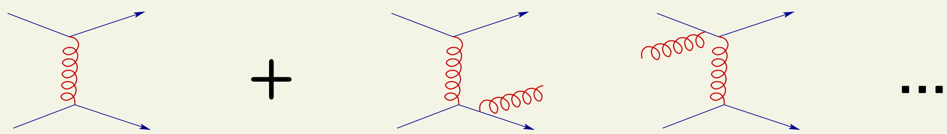
large v_2 needs $15\times$ perturbative opacities - $\sigma_{el} \times dN_g/d\eta \approx 45$ mb $\times 1000$



even 50 mb does not give an ideal fluid

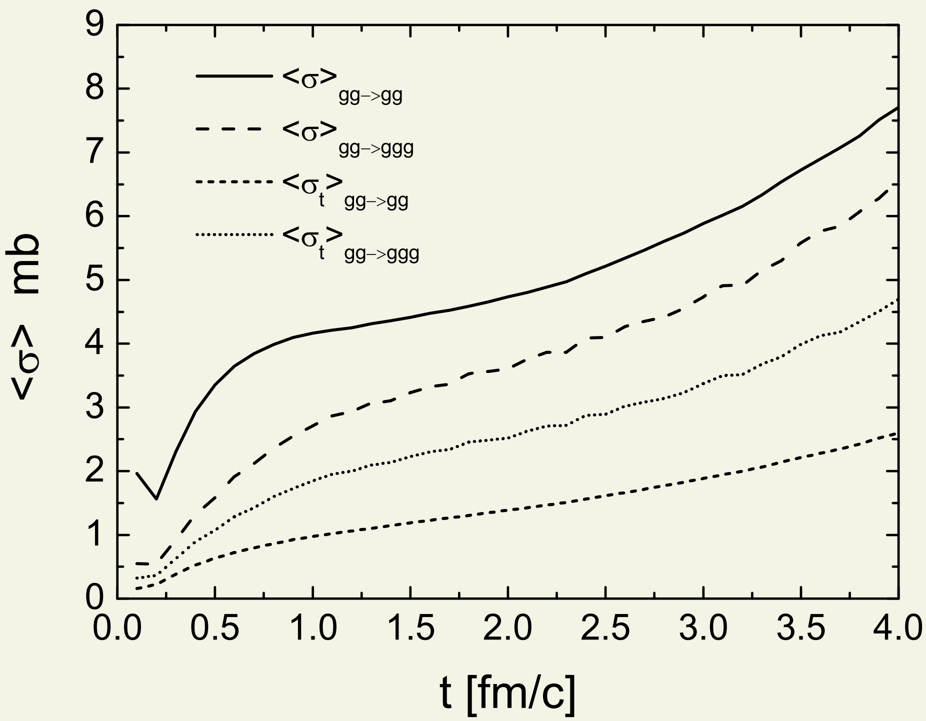
Radiative $3 \leftrightarrow 2$ transport

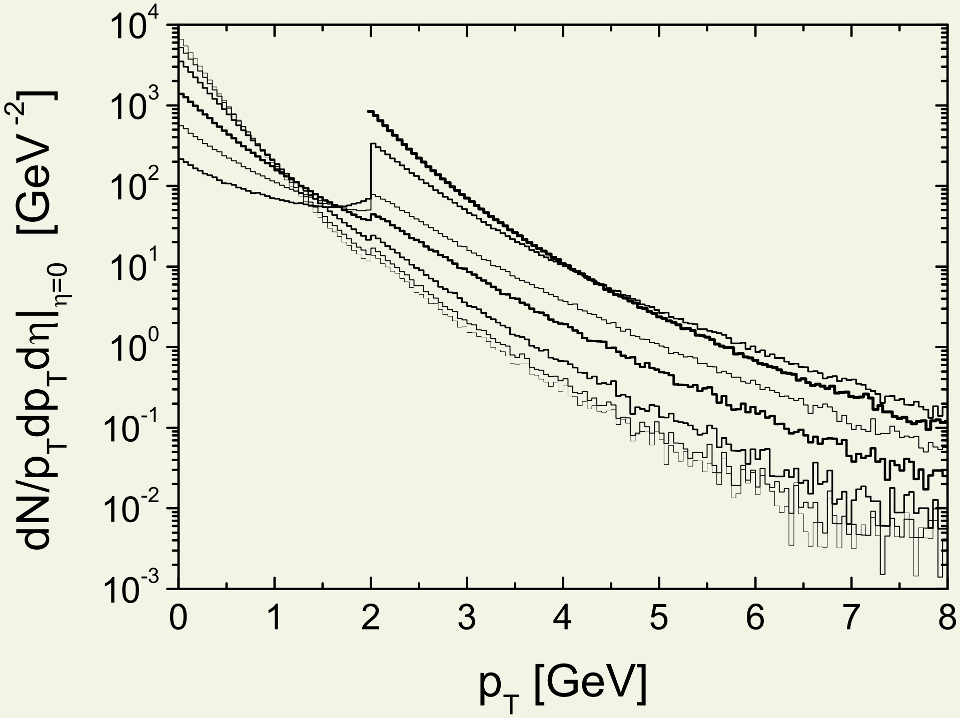
higher-order processes also contribute to thermalization



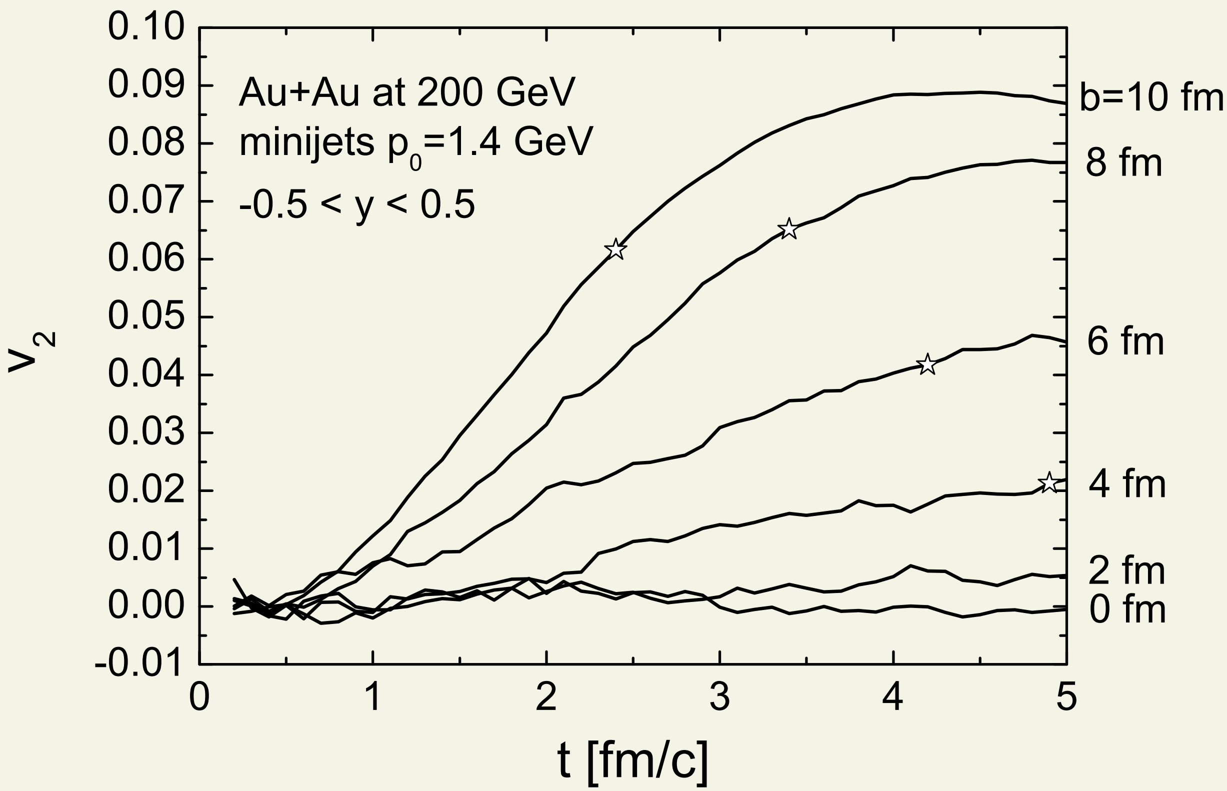
Greiner & Xu '04: **find thermalization time-scale $\tau \sim 2-3$ fm/c**

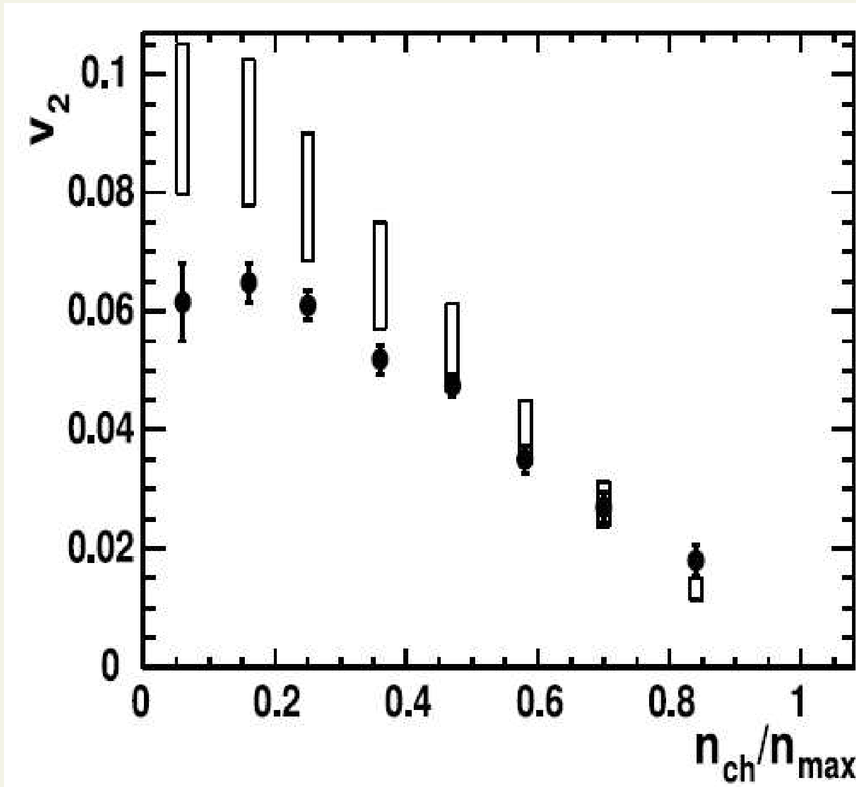
$2 \to 2$, $2 \to 3$ transport cross sections **spectra vs. time**





elliptic flow with $ggg \leftrightarrow gg$ (minijet initconds, $p_0 = 1.4$ GeV)





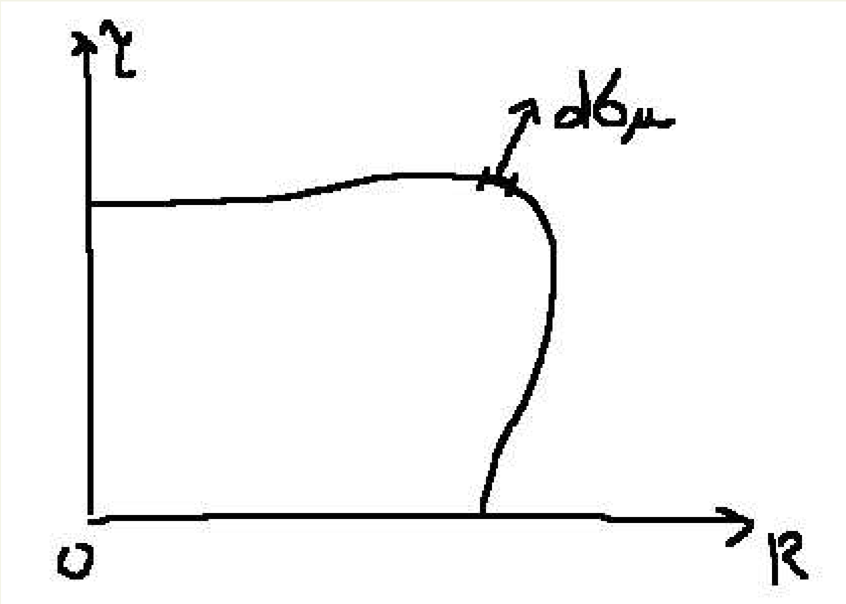
Q5: is this a perturbative alternative to the sQGP paradigm?

Q6: if yes, how do you hadronize out of equilibrium?

Decoupling

Best state of the art: conversion to a gas of hadrons, which are then evolved in a hadron transport model.

- **Cooper-Frye formula: sudden transition from fluid to a gas on a 3D hypersurface (typically $T = const$ or $\varepsilon = const$)**



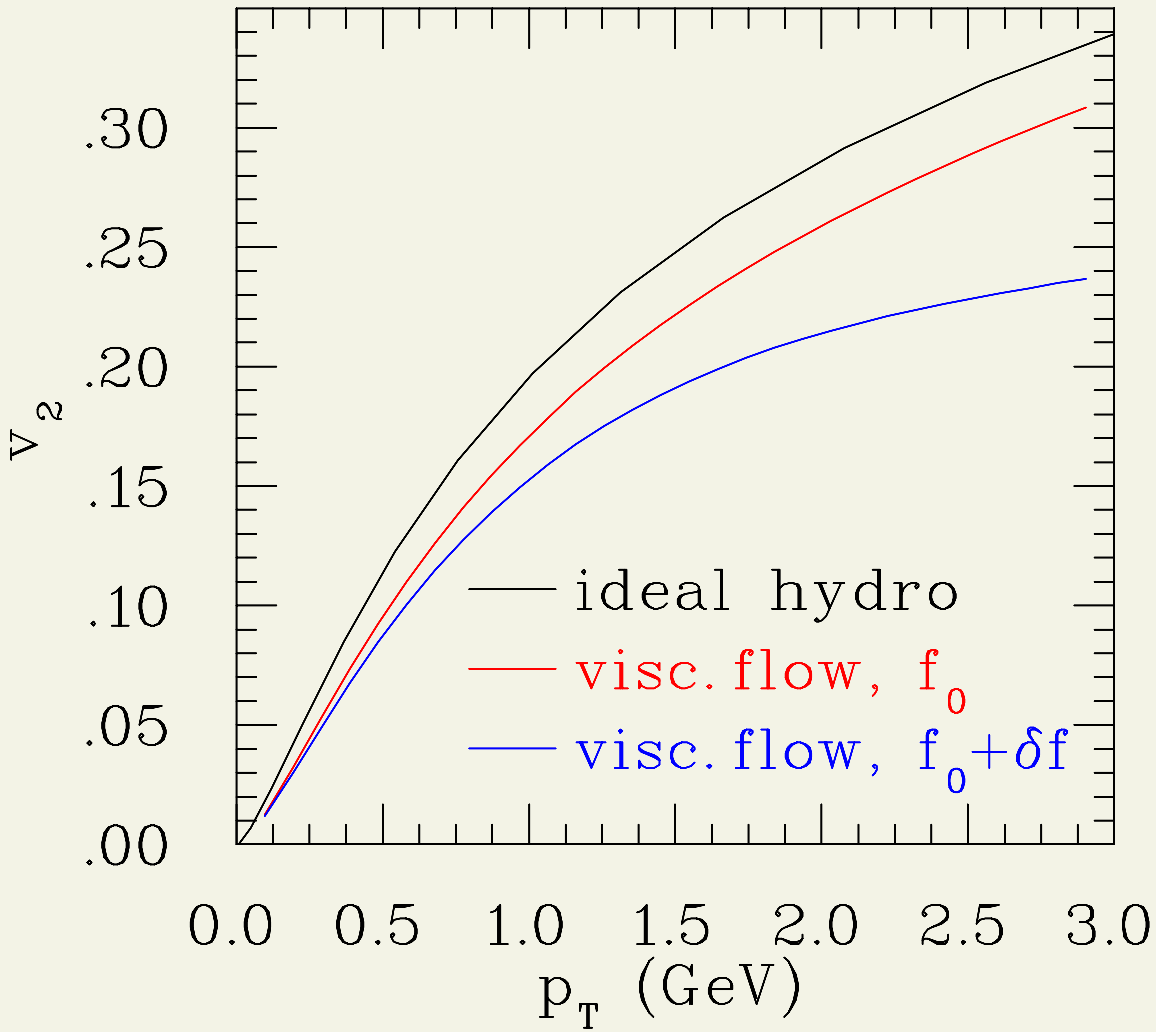
$$E\,dN = p^\mu d\sigma_\mu(x)\,d^3p\,f_{gas}(x,\vec{p})$$

where $d\sigma_\mu$ is the hypersurface normal at x

Dissipative case - must include dissipative corrections to f

$$f \to f_{eq} + \delta f \;, \qquad \delta f^{Grad}_{massless} = f_{eq}\frac{\pi_{\mu\nu}p^\mu p^\nu}{8nT^3}$$

RHIC Au+Au, $b = 8$ **fm,** $\boldsymbol{\eta/s \approx 1/(4\pi)}$ $(\sigma \propto \tau^{2/3})$



Thermodynamic consistency requires that matter properties - equation of state, transport coefficients - are the SAME for the fluid and the hadron gas at the switching point.

Q7: are hadron cascades, such as UrQMD and JAM, thermodynamically consistent with QCD (e.g., the latest lattice EOS with $T_c \approx 190$ MeV)?

Also conceptual problems:

a) for spacelike normal, $p^\mu d\sigma_\mu < 0$ possible, which gives NEGATIVE yields

ignored in practice because such contributions are small (less than a few percent) - slight violation of energy-momentum and charge conservation

b) hypersurface ($T = const$ or $\varepsilon = const$) determined running hydro until the very end, disregarding that parts of the system have frozen out already

Q8: how to dynamical interface hydro and transport?