

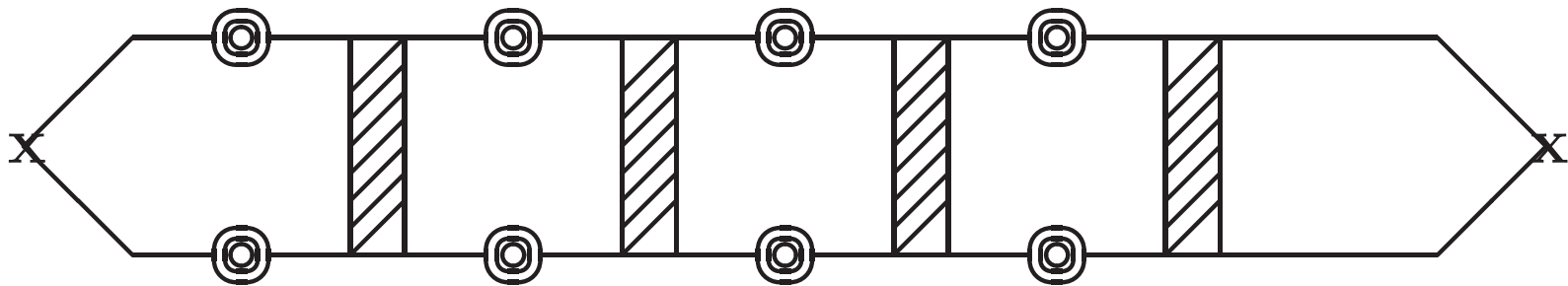
# Some remarks on transport

- Boltzmann, transport, diagrams
- Higher order in couplings—what will happen?
- Higher order in gradients—2'nd order hydro
- Nonperturbative regime; story of Bulk Viscosity

## Weak coupling

Weak coupling means long-lived quasiparticles.

Diagrammatically, leads to domination by ladder graphs:



Pairs of propagators act like propagating particles.

Once you know this you should never talk about graphs again.

## Boltzmann equation

Leading order hydro described by distrib func  $f(p, x)$ .

Consider  $f(p, x) = f(\beta, u^\mu, p_\mu)$  and derivatives.

Write as series expansion in number of derivatives:

$$f(\beta, u^\mu, p_\mu) = f_0(-\beta u^\mu p_\mu) + f_1(\beta, u, \partial u, p) + f_2(\beta, u, \partial^2(u, \beta), p)$$

Put in Boltzmann equation (written here with  $2 \leftrightarrow 2$  only)  
and expand order by order in total # of deriv's

$$2p^\mu \partial_\mu (f_0(p) + f_1 + \dots) = -\mathcal{C}[p; f_0, f_1, f_2],$$

$$-\mathcal{C}[p; f] = \int_{kp'k'} (2\pi)^4 \delta^4(P+K-P'-K') |\mathcal{M}|^2 [p, k; p', k'] \times$$

$$\left( f(p) f(k) [1 \pm f(p')] [1 \pm f(k')] - (p, k \leftrightarrow p', k') \right)$$

First order: defining ( $\Delta_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$ )

$$\sigma_{\mu\nu} = \Delta_{\mu\alpha} \Delta_{\nu\beta} \left( \partial^\alpha u^\beta + \partial^\beta u^\alpha - \frac{2}{3} g^{\alpha\beta} \partial_\gamma u^\gamma \right),$$

$$p^\mu \partial_\mu f_0(-\beta u^\nu p_\nu) = \beta f'_0 \times \left( \frac{1}{2} p^i p^j \sigma_{ij} \beta f'_0 + E^2 (\partial_t \ln \beta - \partial_i u_i / 3) \right. \\ \left. + E p_i (\partial_i \ln \beta - \partial_t u_i) \right)$$

Last two lines determine  $\partial_t(\beta, u_i)$ , then cancel.

Expanding collision term to first order,

$$\frac{p_i p_j \sigma_{ij} \beta}{2} f'_0 = \int_{kp'k'} \dots f_0(p) f_0(k) [1 \pm f_0(p')] [1 \pm f_0(k')] \times \\ \left( \frac{f_1(p)}{f_0(p) [1 \pm f_0(p)]} + (\dots k) - (\dots p') - (\dots k') \right)$$

Solution guaranteed to be of form

$$f_1(p) \equiv C_1^{-1} \frac{p_i p_j \sigma_{ij} \beta}{2} f'_0 = p_i p_j \sigma_{ij} \beta^3 \chi(-\beta p_\mu u^\mu)$$

Stress tensor

$$\begin{aligned} T_{ij} &= \int_p 2p_i p_j f(p, \beta, u) = T_{ij,eq} + \Pi_{ij} , \\ \Pi_{ij} &= \int_p 2p_i p_j f_1 = \sigma_{lm} \int_p 2p_i p_j p_l p_m \beta^3 \chi \\ &= \frac{2\sigma_{ij}}{15} \int_p 2p^4 \beta^3 \chi \end{aligned}$$

is proportional to  $\sigma_{ij}$ . Proportionality is  $\eta$ ,

$$\eta = -\frac{2}{15} \int_p 2p^4 \beta^3 \chi(\beta E_p)$$

## Next order?

Second-order in gradients terms in Boltzmann are

$$p^\mu \partial_\mu f_1(p, \beta, u, \partial) + \mathcal{C}_{11} = -\mathcal{C}_2$$

with  $\mathcal{C}_{11}$  the collision term to quad order in  $f_1$  and  $\mathcal{C}_2$  the term to linear order in  $f_2$ .

We only need parts of  $f_2$  contributing to  $\int_p p_i p_j f_2$ , that is,  $\ell = 2$  spherical harmonic terms.

Derivative term has several structures but straightforward:

$$p^\mu \partial_\mu f_1 = p_i p_j \beta^3 \chi \left( \partial_t \sigma_{ij} + \frac{1}{3} \sigma_{ij} \partial_k u_k - \sigma_{ik} \sigma_{jk} - 2 \sigma_{ik} \Omega_{ik} \right) \\ + p_i p_j p_k p_l \sigma_{ij} \sigma_{kl} \beta^4 \chi'$$

## General structure of terms

The departure  $f_2$  will involve the same tensor structures; so

$$\begin{aligned}\Pi_{ij,2} = & \tau_{\Pi}\eta \left( \partial_t \sigma_{ij} + \frac{1}{3} \sigma_{ij} \partial_k u_k \right) \\ & + \lambda_1 (\sigma_{ik} \sigma_{jk} - \text{trace}) + \lambda_2 (\sigma_{ik} \Omega_{jk} - \text{trace})\end{aligned}$$

Most general allowed form except missing  $\Omega^2$  term.

All coeff's but  $\lambda_1$  determined by  $p^\mu \partial_\mu f_1$ .

These are easily computed with known techniques: obey

$$\lambda_2 = -2\tau_{\Pi}\eta$$

$C_{11}$  terms structurally simpler, harder to analyze:

$$C_{11}[p, f] \propto p_i p_j \left( \sigma_{ik} \sigma_{jk} - \frac{1}{3} \delta_{ij} \sigma_{lm} \sigma_{lm} \right) \beta^4 \chi_{(2)}(-\beta p^\mu u_\mu)$$

Need to do  $\int_{pp'kk'} |\mathcal{M}|^2$  integrals to find it!

Worse: matrix element depends on  $f_1$ !

$$\mathcal{M} \sim p^\mu (G_0^{-1\mu\nu} + \Pi^{\mu\nu}[f])^{-1} k^\nu$$

One contribution to  $C_{11}$  is from

$$\mathcal{M} \supset p^\mu \frac{1}{Q^2 + \Pi} \Pi_1[f_1] \frac{1}{Q^2 + \Pi} k^\nu$$

Lets in interesting physics, eg,  $1/Q^2 + \Pi$  unscreened for  $\omega \rightarrow 0$  magnetic exchange but  $\Pi_1$  nonzero there.

## Sizes and signs

Don't think  $\eta/s$ , think

$$\frac{\eta}{\epsilon + P} \equiv l_\eta \quad \text{a length [time?] scale}$$

roughly  $t$  for local relax to equil. ( $\Gamma_{\text{sound}} = 1/l_\eta k^2$ .)

$$\frac{\tau_\Pi \eta}{\epsilon + P} = l_{\tau_\Pi}^2, \quad \frac{\lambda_1}{\epsilon + P} = l_{\lambda_1}^2$$

Expect dimensionless double ratio

$$\frac{(\epsilon + P)\lambda_1}{\eta^2} = \mathcal{O}(1)$$

If it's numerically large, convergence worse than expected.

## First order in $\partial$ , NLO in $g$

What can we say about higher order in  $g$  effects?

$\mathcal{O}(g)$  corrections can arise from  $gT$  momentum propagators.

Basically,  $\mathcal{O}(g)$  soft corrections to Hard thermal loops.

Example: Fraction  $g$  of scatt. is from  $k \sim gT$  particles.

Generic form of corrections:

$$(\eta^{-1}, \kappa, \dots) = Ag^n \left( \ln(T/m_D) + B + C \frac{N_c g^2 T}{m_D} \right)$$

Where computed, NLO coeff  $C \sim 1$ .

## Computed $\mathcal{O}(g)$ corrections

- Heavy quark diffusion ( $C = 1.4$ ) [Caron-Huot and Moore]
- Asymptotic mass  $m_\infty^2$ :  $C = 1$  [Caron-Huot unpublished]
- Heavy quark thermal mass shift [Chesler Vuorinen Caron-Huot]
- Transverse mom diff  $\hat{q}$  [Caron-Huot unpublished]

Notice a pattern?

## What $\mathcal{O}(g)$ corrections seem to imply

- Corrections are large: pert theory doesn't work well.
- Problem is strong coupling at  $gT$  scale, eg,  $gT, g^2T$  not well separated.
- Generally increase rate of scattering, lowering  $\eta$ ...
- Looks like we need a resummation technique!