Inclusive gluon production at NLO in A+A collisions:
the CGC, factorization & the Glasma

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ICHEC, Goa, September 2nd 2008

Multiparticle production in A+A collisions

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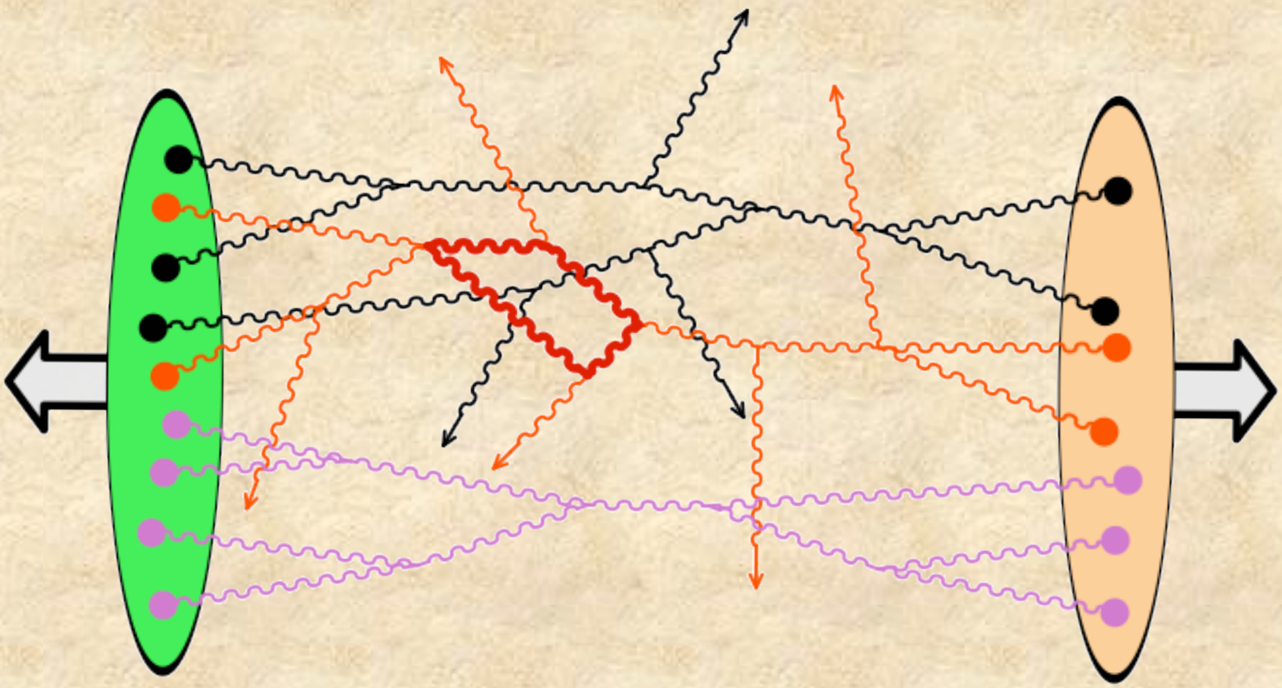
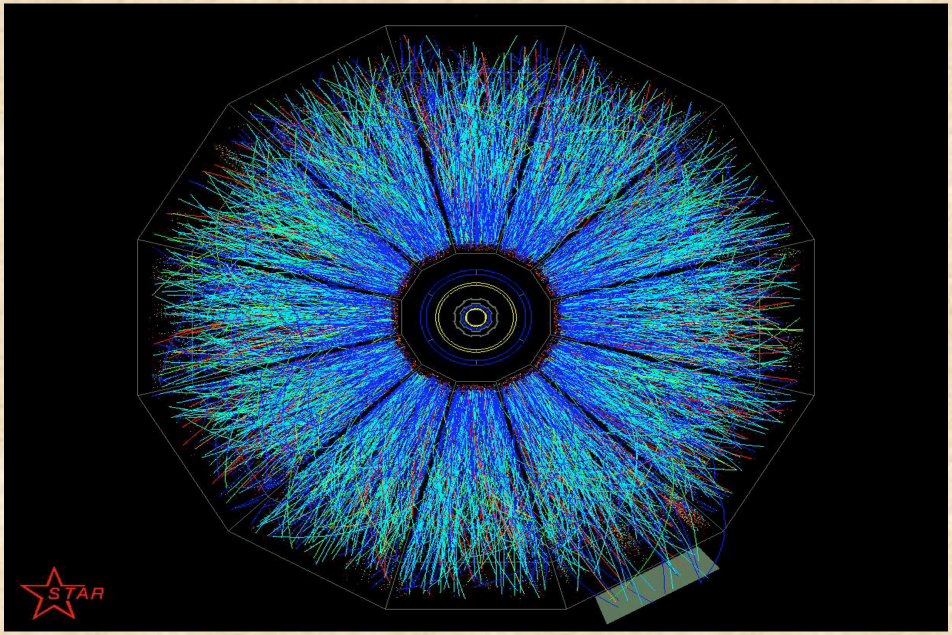


Multiparticle production in A+A collisions

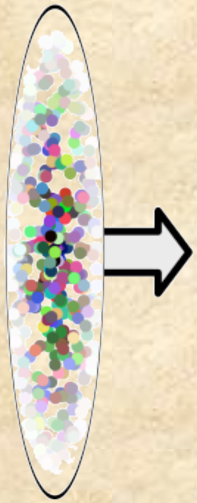


- Can we describe these rich phenomena *ab initio*?
  - Collinear pQCD: ideal for high Q^2 processes
  - HI collisions: ultimate machine for multiparticle production
- Approach: Compute particle production in field theories with *strong time dependent* sources

Talk Outline

- The high energy nuclear wavefunction: the Color Glass Condensate (CGC)
- How the wavefunction decoheres: high energy factorization -- from CGC to Glasma
- Imagining the Glasma via the near side "ridge" in heavy ion collisions at RHIC (and the LHC)

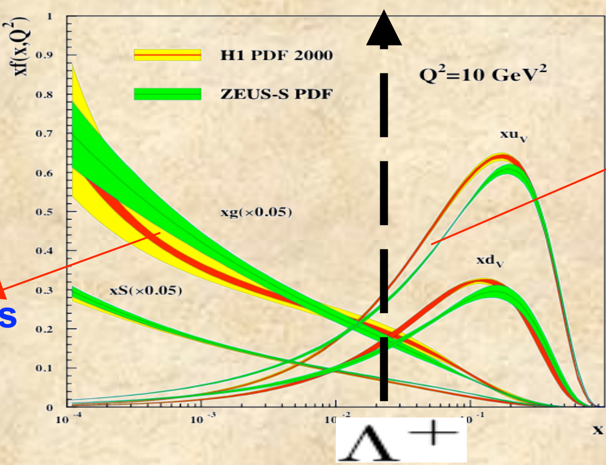
The nuclear wavefunction at high energies



$$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\,gg\ldots gq\bar{q}\rangle$$

Higher Fock components dominate multiparticle production- construct Effective Field Theory

Dynamical Wee modes



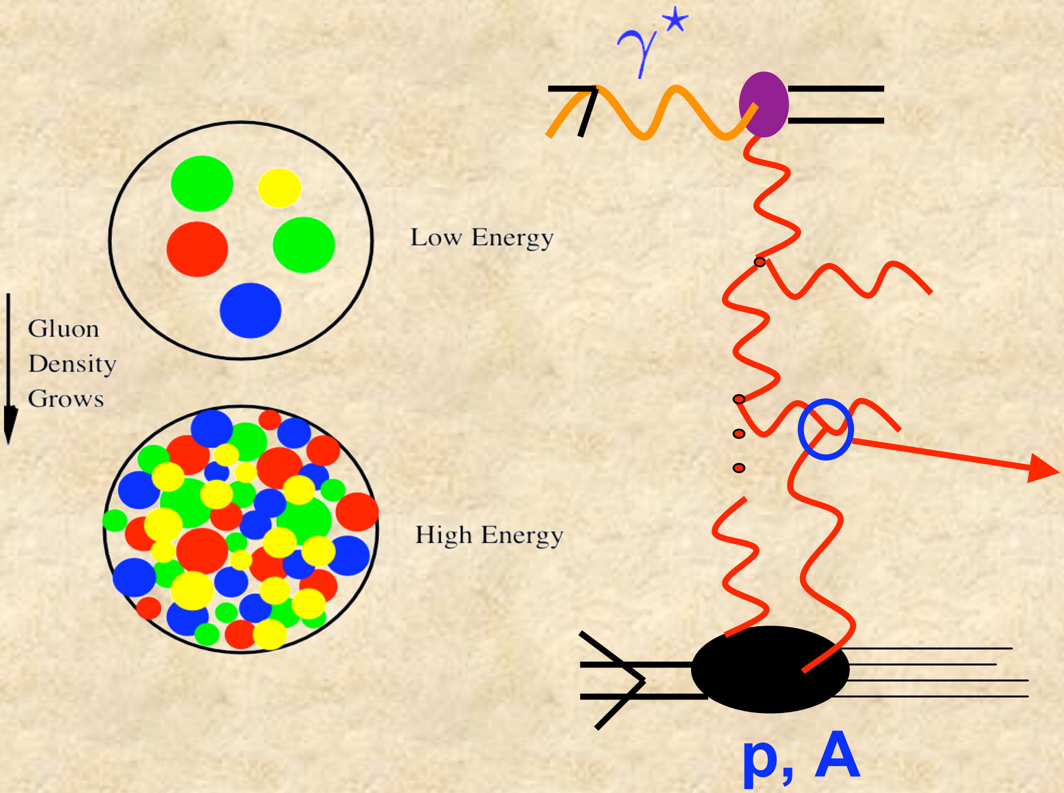
Valence modes- are static sources for wee modes

Born--Oppenheimer LC separation natural for EFT.

RG equations describe evolution of wavefunction with energy

EFT describes gluon saturation

Gribov,Levin,Ryskin
Mueller,Qiu



Large x - bremsstrahlung
linear evolution (DGLAP/BFKL)

Small x -gluon recombination
non-linear evolution
(BK/JIMWLK)

- Saturation when occupation # f ~ $1/\alpha_S$
- Saturation scale $Q_S(x)$ - dynamical scale below which non-linear ("higher twist") QCD dynamics is dominant -- arises naturally in EFT

The Color Glass Condensate

McLerran, RV
Iancu, Leonidov,McLerran

In the saturation regime:

Strongest fields in nature!

$$E^2 \sim B^2 \sim \frac{1}{\alpha_S}$$

CGC: *Classical* weak coupling effective theory of QCD describing dynamical gluon fields (A_μ)+ static color sources (ρ) in non-linear regime

- **A universal saturation scale Q_S arises naturally in the EFT**
- **Renormalization group equations (JIMWLK/BK) describe how the QCD dynamics changes with energy**

The CGC effective action

McLerran, RV

Scale separating sources & fields

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional for distribution of sources

$$S[A,\rho] = \frac{-1}{4}\int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c}\int d^2x_\perp dx^-\, \delta(x^-)\, \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

Dynamical wee fields

Coupling of wee fields to sources

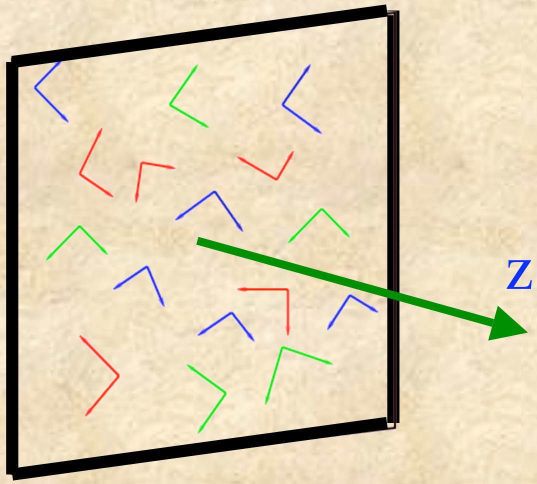
$$U_{-\infty,+\infty}[A^-] = \mathcal{P}\exp\left(ig\int dx^+ A^{-,a} T^a\right)$$

Other gauge invariant forms of the coupling of hard+soft modes...

Jalilian-Marian,Jeon,RV ; Fukushima

Classical field of a nucleus at high energies

$$(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu+}\,\delta(x^-)\,\rho^a(x_\perp)$$

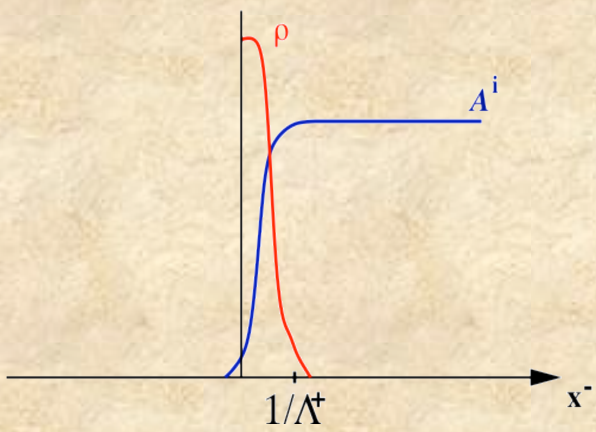


Analytical solution:

$$A^+ = A^- = 0$$

$$A^i = \frac{1}{ig}\,U(x_\perp,x^-)\nabla^i U^\dagger(x_\perp,x^-)$$

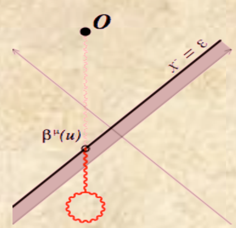
$$U(x_\perp,x^-) = \mathcal{P}\,\exp\left(ig\int_{-\infty}^{x^-} dx'^-\,\frac{1}{-\nabla_\perp^2}\tilde{\rho}(x_\perp,x'^-)\right)$$



non-Abelian Weizsäcker--Williams fields

RG evolution for a single nucleus: JIMWLK equation

$$\mathcal{O}_{\rm NLO} = \Bigg($$



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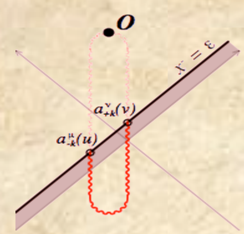


$\chi(x_\perp, y_\perp)$

$$\Bigg)\, \mathcal{O}_{\rm LO}$$

$$= \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}\, \mathcal{O}_{\rm LO}$$ **(keeping leading log divergences)**

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}] \left[\mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO}\right]$$

$$= \int [d\tilde{\rho}] \left\{ \left[1 + \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}\right] W_{\Lambda^+} \right\} \mathcal{O}_{\rm LO}$$

LHS independent of Λ^+ => $$\boxed{\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H}\, W[\tilde{\rho}]}$$

JIMWLK eqn.

Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner

Correlation Functions

$$< O[\alpha] >_Y = \int [d\alpha] O[\alpha] W_Y[\alpha]$$

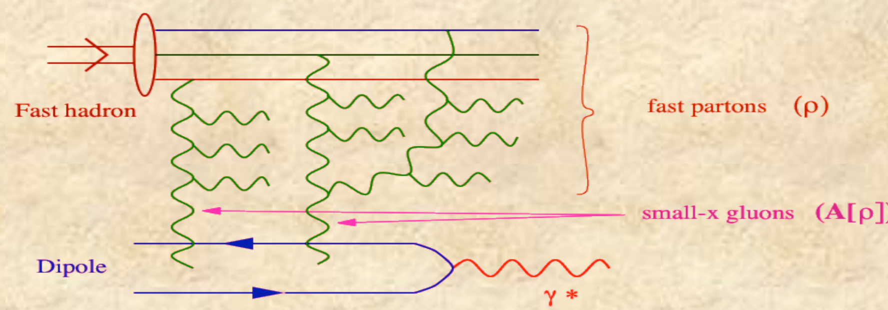
Brownian motion in functional space: Fokker-Planck equation!

"diffusion coefficient"

$$=> \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \underbrace{\frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta \alpha_Y^b(y)}}_{\mathcal{H}_{\text{JIMWLK}}} O[\alpha] >_Y$$

"time"

Mean field soln. of JIMWLK for 2 pt. Wilson line (dipole) correlators= BK (Balitsky-Kovchegov) eqn.

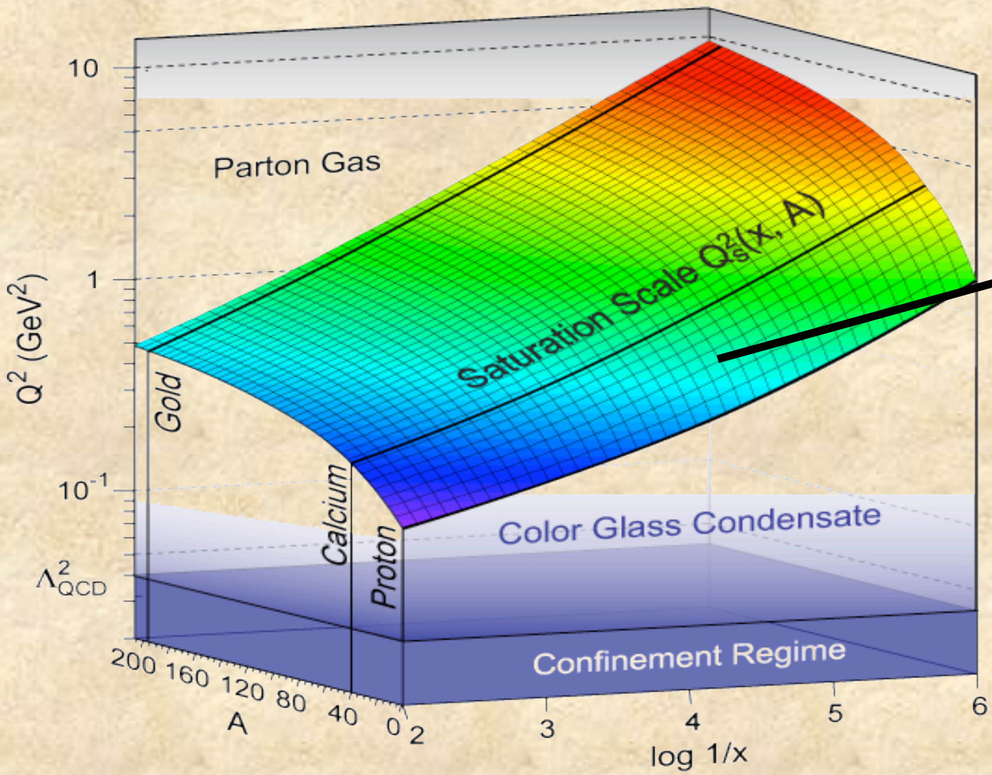


Dipole amplitude N satisfies

BFKL kernel

$$\frac{\partial \mathcal{N}}{\partial \ln(1/x)} = K * [\mathcal{N} - \mathcal{N}^2]$$

Q_S from dipole model fits to e+p, e+A, d+A data



$\times \frac{9}{4}$ **for glue**

$\alpha_S(Q_S^2) << 1$

Kowalski,Lappi,RV, PRL 100, 022303 (2008)
Kowalski,Lappi,Marquet,RV, arXiv:0805:4071

$\chi^2 \sim 1$ **Agreement with HERA diffractive and nuclear shadowing data**

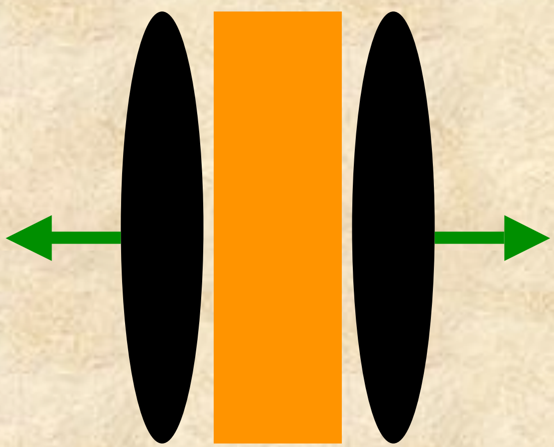
Note: Strong constraints from RHIC A+A: $N_{ch} \sim Q_S^2$ and $E_T \sim Q_S^3$ - "day 1" A+A at LHC will provide important confirmation

- **Careful analysis gives values consistent with above plot to ~15%** T. Lappi, arXiv:07113039 [hep-ph]

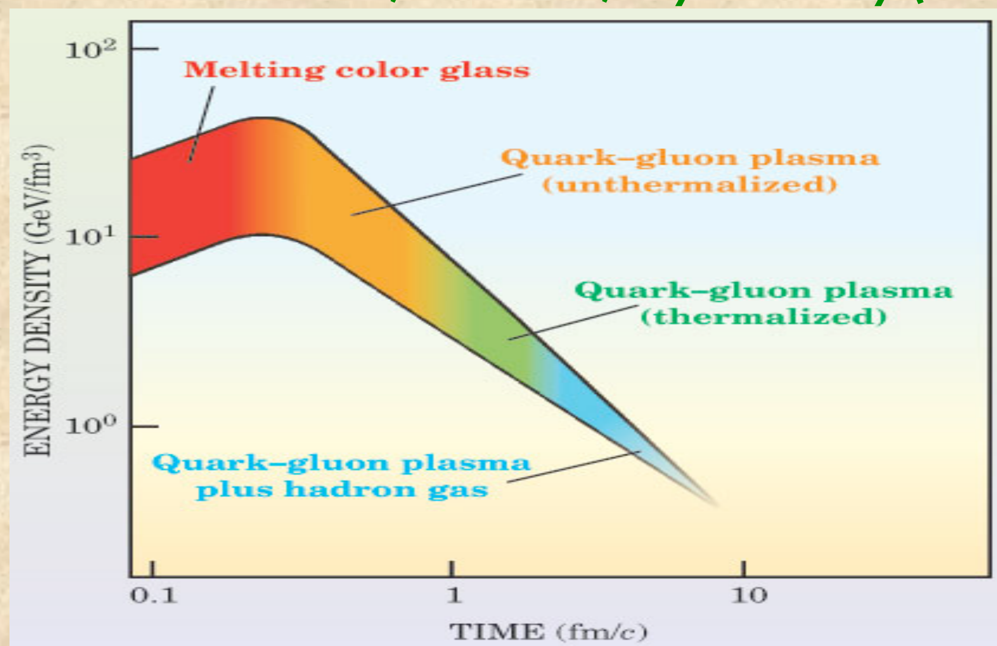
How Glasma is formed in a Little Bang

$\rho_1 \sim \frac{1}{g}$ $\rho_2 \sim \frac{1}{g}$

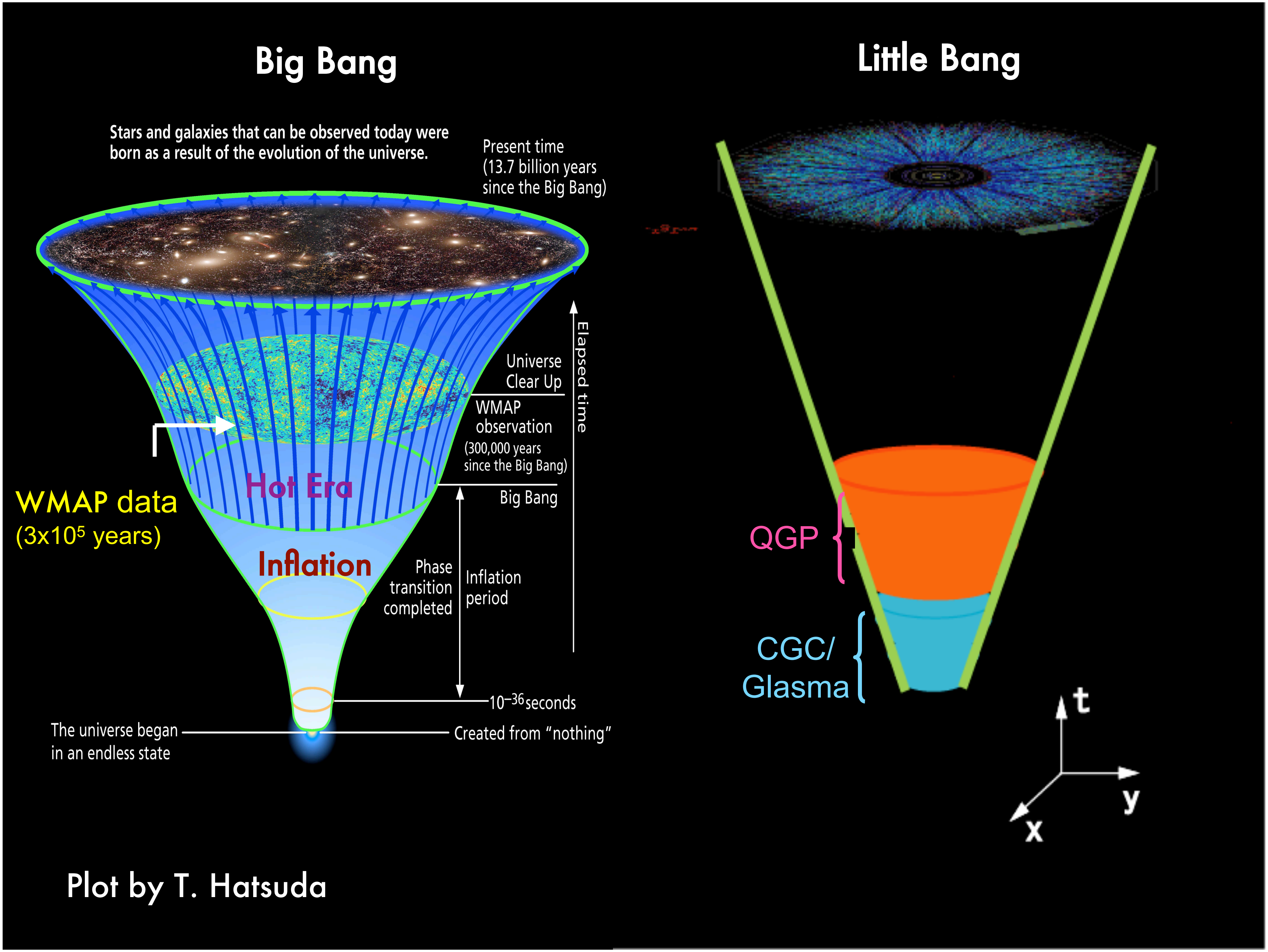
Glasma



Ludlam, McLerran, Physics Today (2003)



Glasma (\Glahs-maa\):
Noun: non-equilibrium matter
between Color Glass Condensate (CGC)
& Quark Gluon Plasma (QGP)



Big Bang
Stars and galaxies that can be observed today were born as a result of the evolution of the universe.
Present time (13.7 billion years since the Big Bang)
Elapsed time
Universe Clear Up
WMAP observation
(300,000 years since the Big Bang)
Big Bang
Hot Era
WMAP data
(3x10^5 years)
Inflation
Phase transition completed
Inflation period
10^-36 seconds
The universe began in an endless state
Created from "nothing"
Plot by T. Hatsuda
Little Bang
QGP
CGC/ Glasma
t
x
y

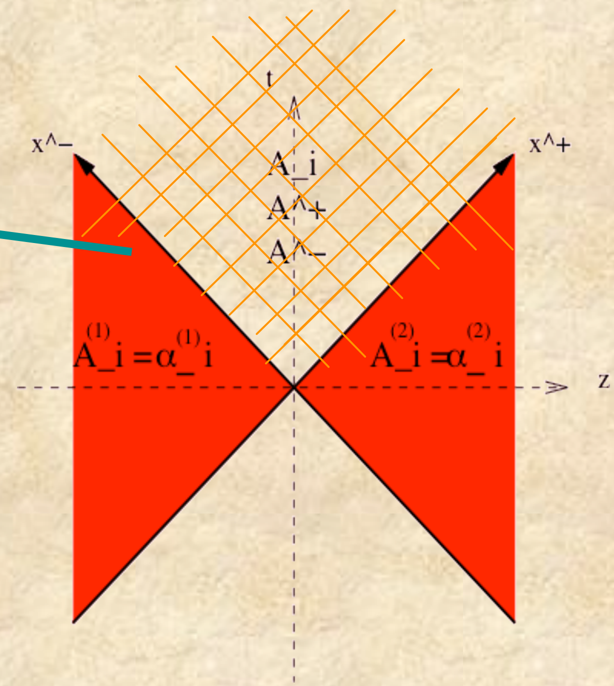
The Glasma at LO:Yang-Mills eqns. for two nuclei

(=$O(1/g^2)$ and all orders in $(g\rho)^n$)

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_1^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_2^a(x_\perp)\delta(x^+)$$

Glasma initial conditions from matching classical CGC wave-fns on light cone

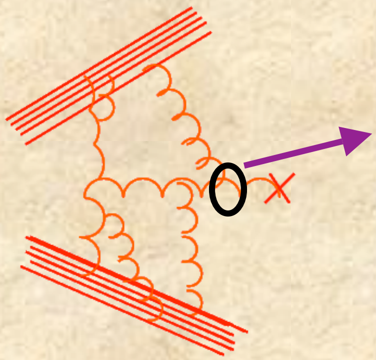
Kovner, McLerran, Weigert



Sources become *time dependent* after collision:
field theory formalism--
particle production in strong external fields
(e.g., Schwinger mechanism of e+e- production in strong QED fields).

Numerical Simulations of classical Glasma fields

Krasnitz, Nara, RV
Lappi



All such diagrams of order O(1/g)

$$E_p \frac{d\langle n\rangle_{LO}}{d^3p} = \frac{1}{16\pi^3} \lim_{x^0,y^0\to\infty} \int d^3x\, d^3y e^{ip\cdot(x-y)} (\partial_{x^0} - iE_p)(\partial_{y^0} + iE_p)$$
$$\times \sum_{\text{phys.}\Lambda} \varepsilon^{\lambda}_{\mu}(p) \varepsilon^{\star\lambda}_{\nu}(p) A^{\mu}_{a}(x) A^{\nu}_{c}(y)$$

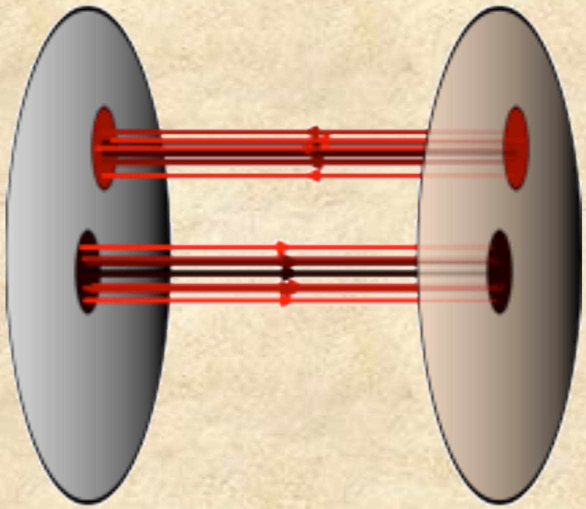
LO Glasma fields are boost invariant

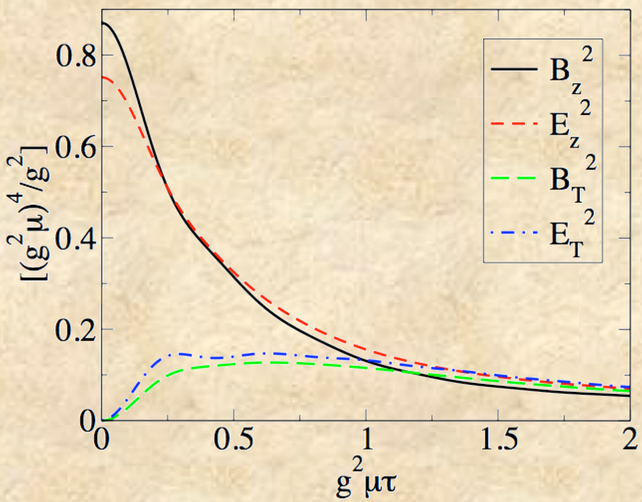
$$\varepsilon \approx 20 - 40\,\mathrm{GeV/fm^3} \text{ at } \tau \sim 0.3 \text{ fm}$$

for $Q_S^A \approx 1 - 1.2\,\mathrm{GeV}$

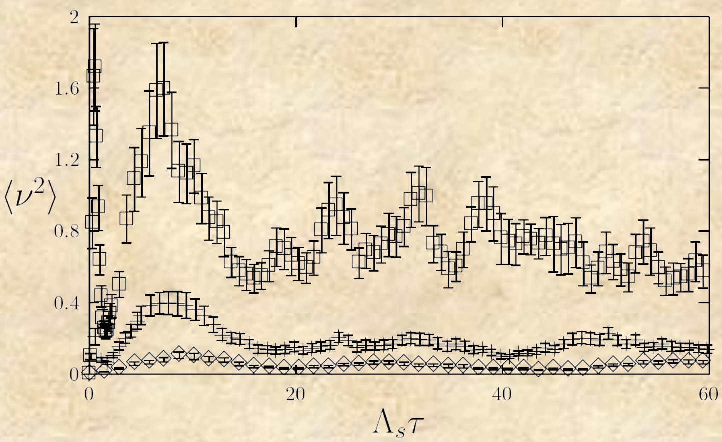
from extrapolating DIS data to RHIC energies

Numerical solns: Glasma flux tubes





Flux tubes of size $1/Q_S$ with parallel color E & B fields - generate Chern-Simons charge



$$\nabla \cdot E = \rho_{\text{electric}}$$

$$\nabla \cdot B = \rho_{\text{magnetic}}$$

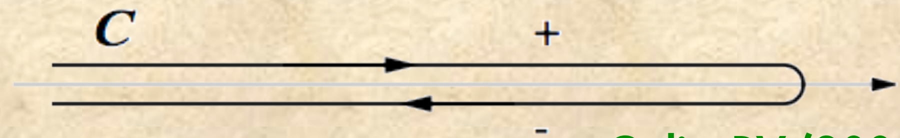
$$\rho_{\text{electric}} = ig[A^i, E^i]$$

$$\rho_{\text{magnetic}} = ig[A^i, B^i]$$

Kharzeev, Krasnitz, RV, Phys. Lett. B545 (2002)

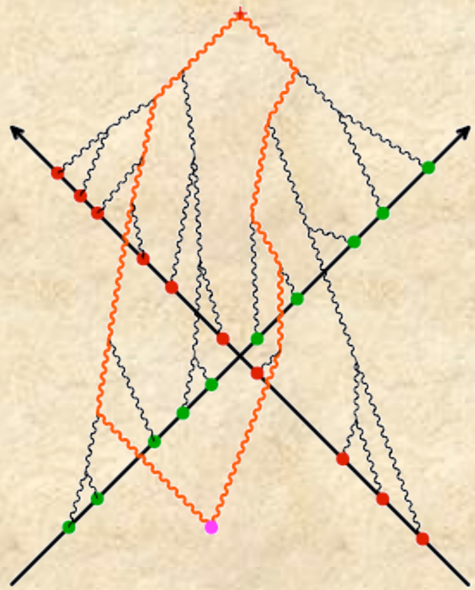
Multiplicity to NLO (=O(1) in g and all orders in $(g\rho)^n$)

Schwinger-Keldysh formalism



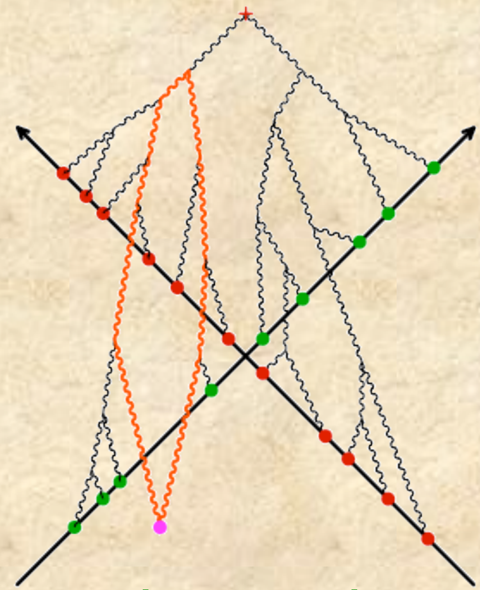
Gelis, RV (2006)

$$\langle n \rangle_{\text{NLO}} = $$



Gluon pair production

+



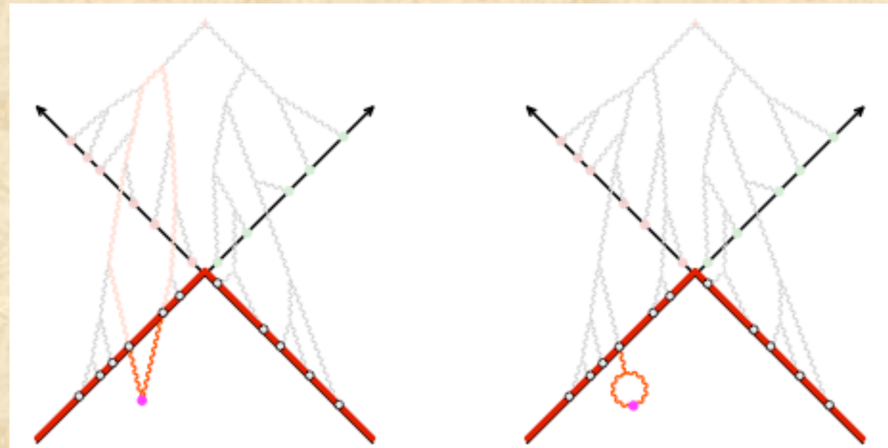
One loop contribution to classical field

Initial value problem **with retarded boundary conditions**
- can be solved on a lattice in real time

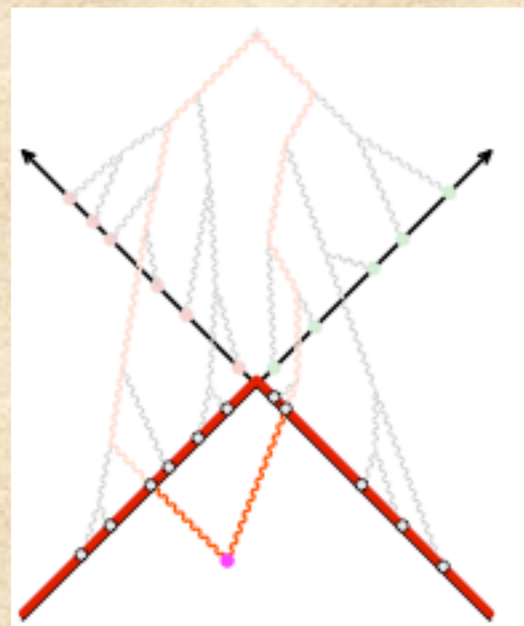
(*a la* Gelis,Kajantie,Lappi for Fermion pair production)

RG evolution for 2 nuclei

Log divergent contributions crossing nucleus 1 or 2:



Contributions across both nuclei are finite-no log divergences

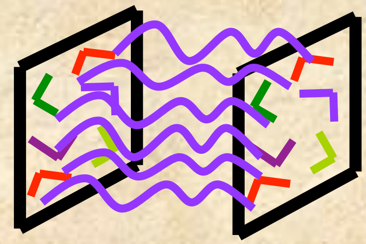


=> factorization

The unstable Glasma

- Small rapidity dependent quantum fluctuations of the LO Yang-Mills fields grow rapidly as

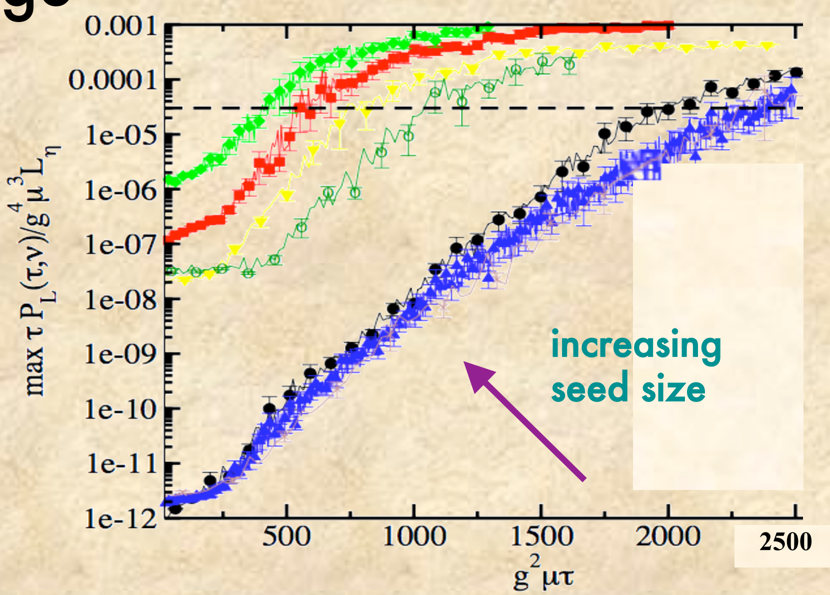
$$\sim e^{\sqrt{Q_s \tau}}$$



- $E_\perp$ and $B_\perp$ fields as large as E_L and B_L at time

$$\tau \sim \frac{1}{Q_S} \ln^2\left(\frac{1}{\alpha_S}\right)$$

Romatschke, RV:PRL,PRD(2006)

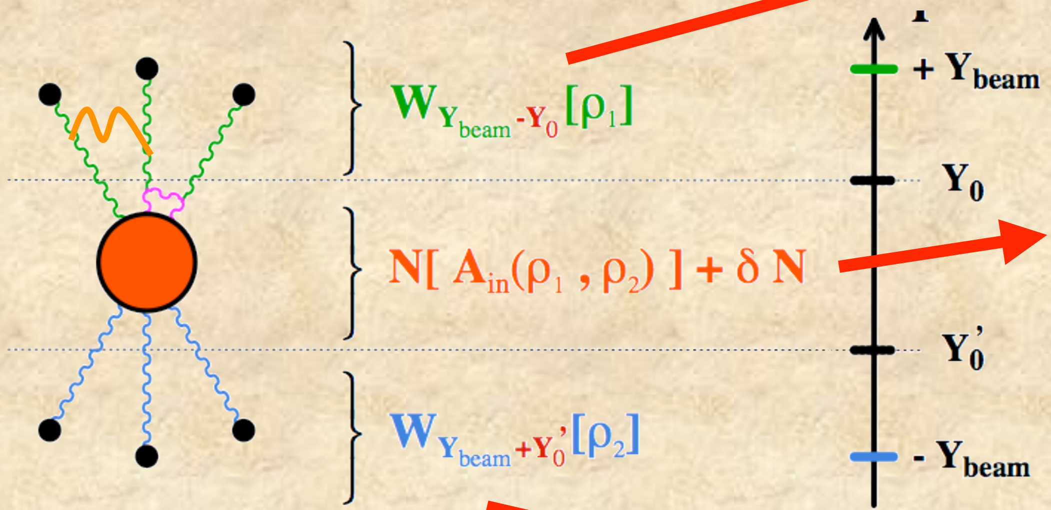


How instabilities arise
-talk by Itakura

NLO and QCD Factorization

What small fluctuations go into wave fn. and what go into particle production ?

Gelis,Lappi,RV
arXiv:0804.2630 [hep-ph]



Small x (JIMWLK) evolution of nucleus A – sum $(\alpha_S Y)^n$ & $(g_S\, \rho_1)^n$ terms

O(α_S) but may grow as $\alpha_S e^{\sqrt{Q_S \tau}}$

Small x (JIMWLK) evolution of nucleus B –sum $(\alpha_S Y)^n$ & $(g_S\, \rho_2)^n$ terms

From Glasma to Plasma

❖ NLO factorization formula:

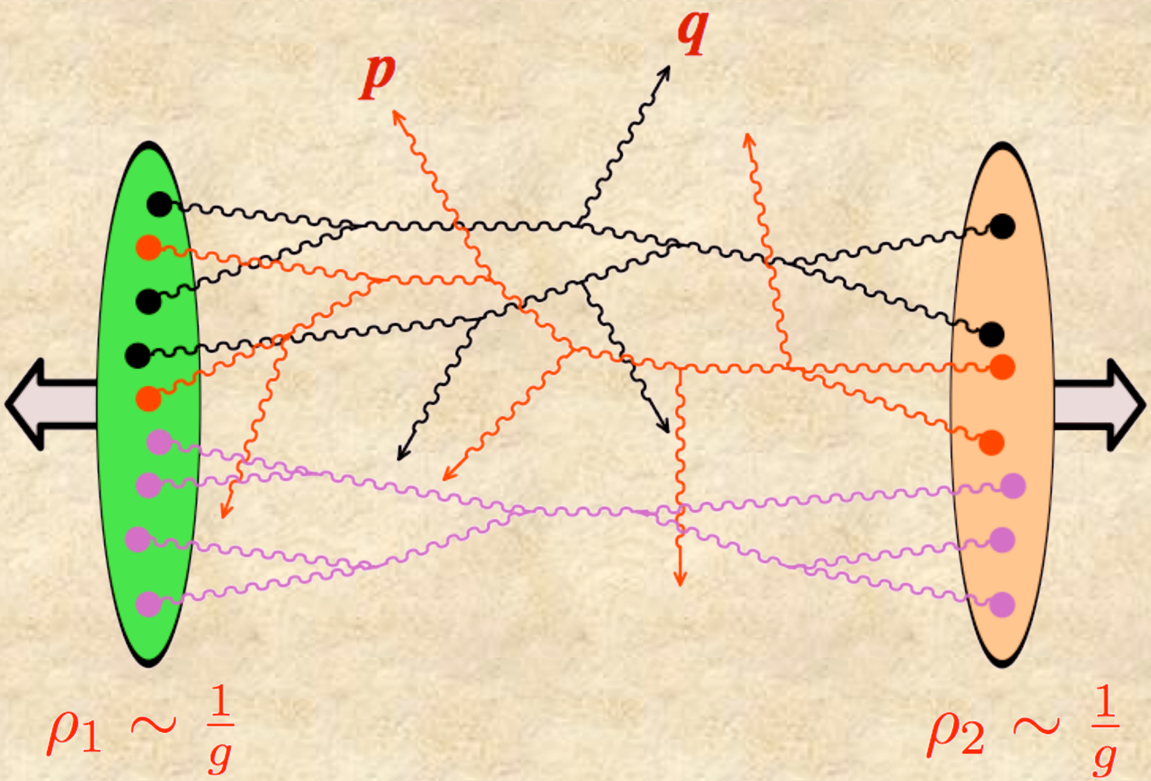
$$\frac{dN_{\rm LO+NLO}}{dY\, d^2p_\perp} = \int [D\rho_1]\,[D\rho_2] W_{Y_{\rm beam}-Y_0}[\rho_1] W_{Y_{\rm beam}+Y_0'}[\rho_2]$$

$$\times \int [Da(u)]\tilde{Z}[a]\frac{dN_{\rm LO}[\mathcal{A}(0,u)+a(u)]}{dY\, d^2p_\perp}|_{\rho_1,\rho_2}$$

"Holy Grail" spectrum of small fluctuations.

❖ With spectrum, can compute $T^{\mu\nu}$ - and match to hydro/kinetic theory–many subtle issues here...

Imaging the Glasma: two particle correlations and the near side Ridge

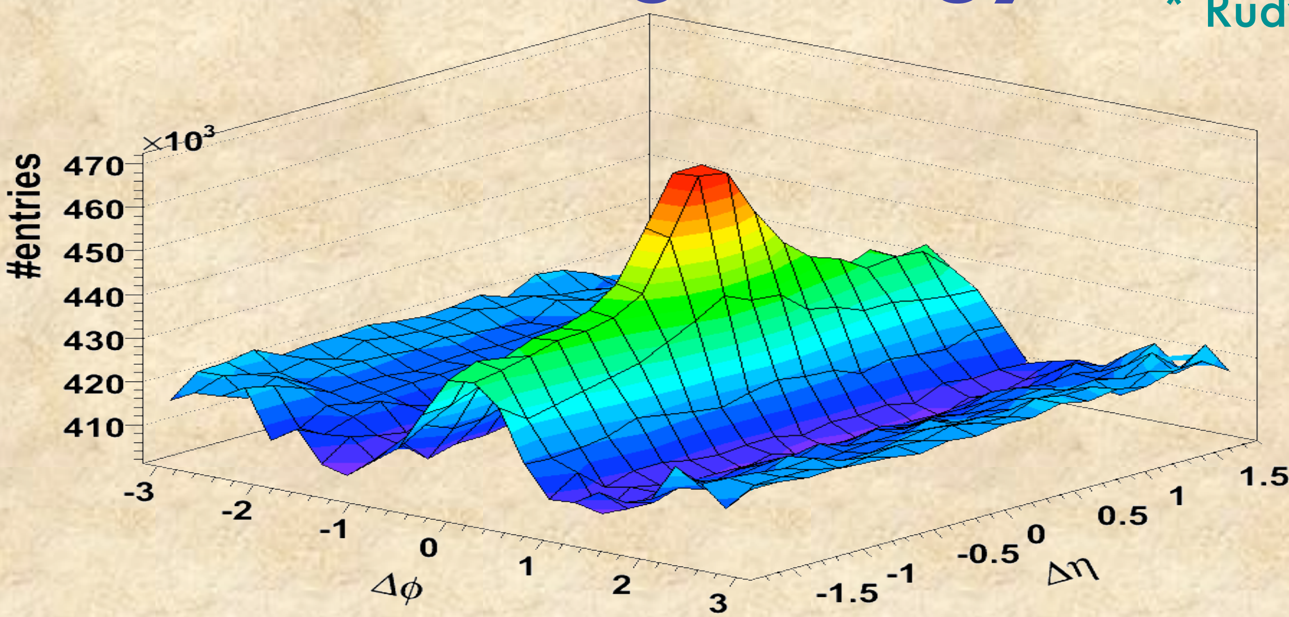


To all leading logs in x, JIMWLK factorization holds for inclusive multigluon production in a rapidity interval $\Delta Y \leq \alpha_s^{-1}$

Gelis, Lappi, RV, arXiv:0807.1306

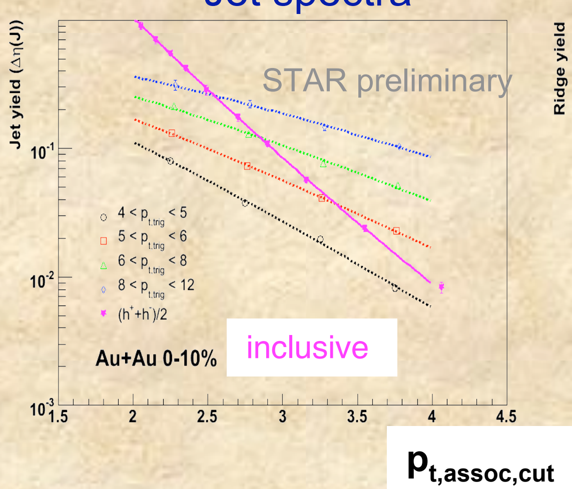
Ridgeology*

* Rudy Hwa

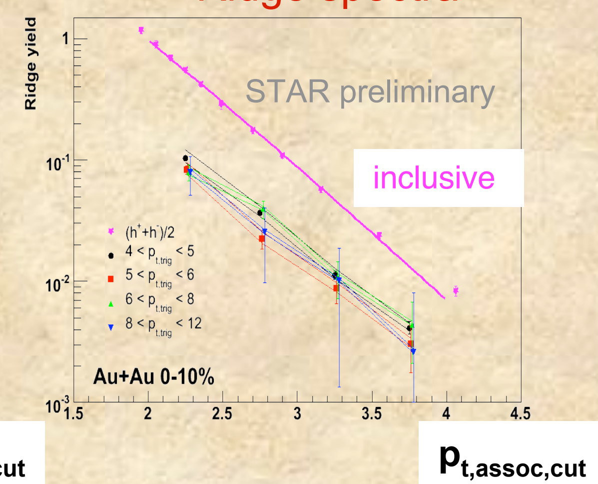


Near side peak+ ridge (from talk by J. Putschke,STAR collaboration)

Jet spectra

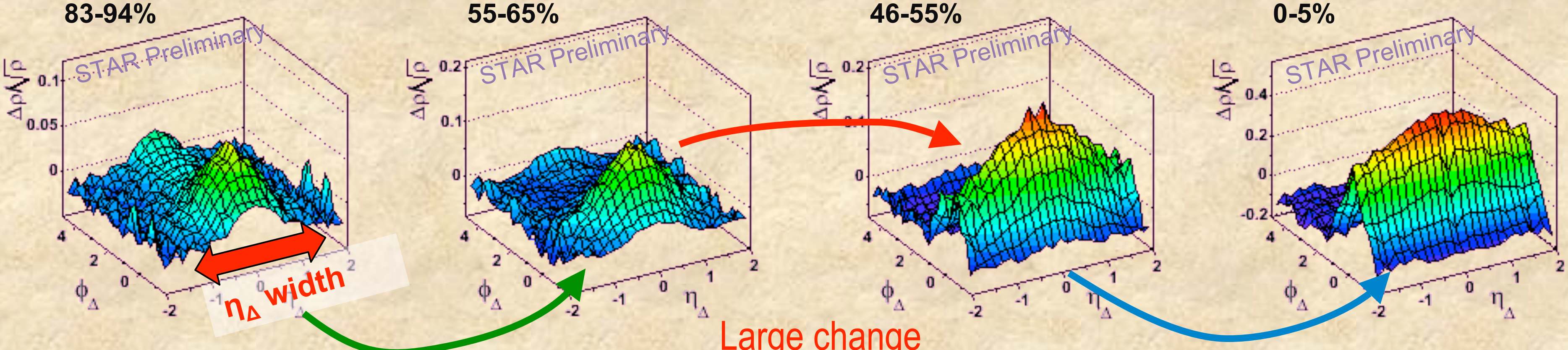


Ridge spectra



Evolution of mini-jet with centrality

Same-side peak



Little shape change from peripheral to 55% centrality

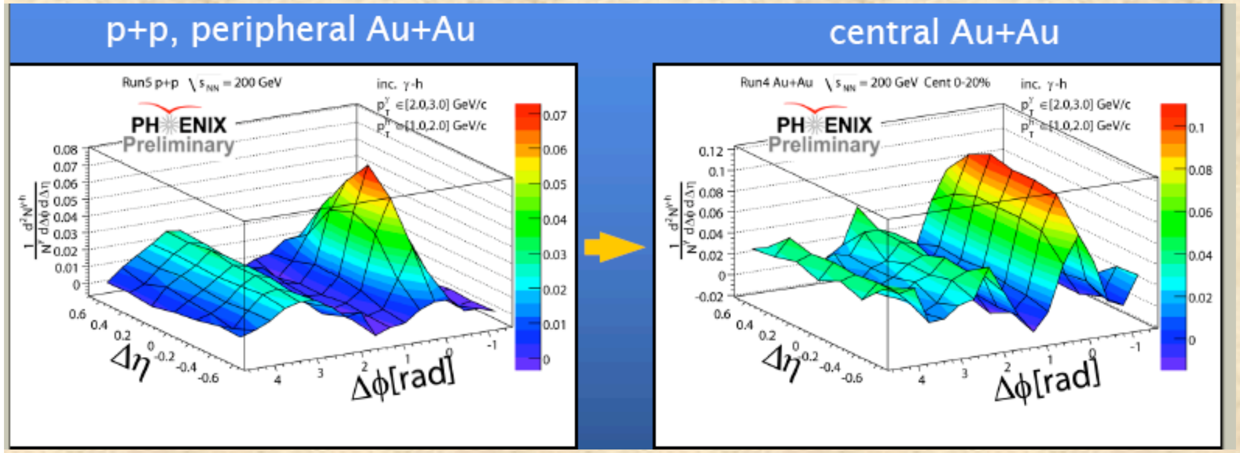
Large change within ~10% centrality

Smaller change from transition to most central

Binary scaling reference followed until sharp transition at ρ ~ 2.5
~30% of the hadrons in central Au+Au participate in the same-side correlation

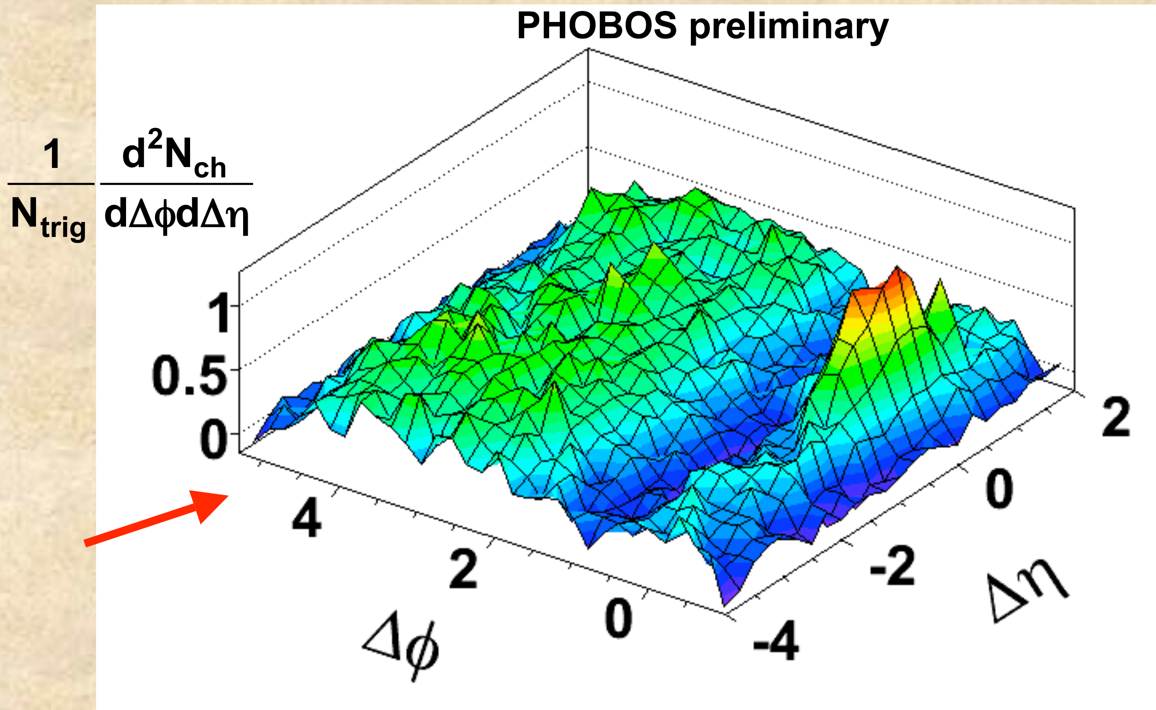
M. Daugherty Session IX, QM2008

Update: the ridge comes into its own

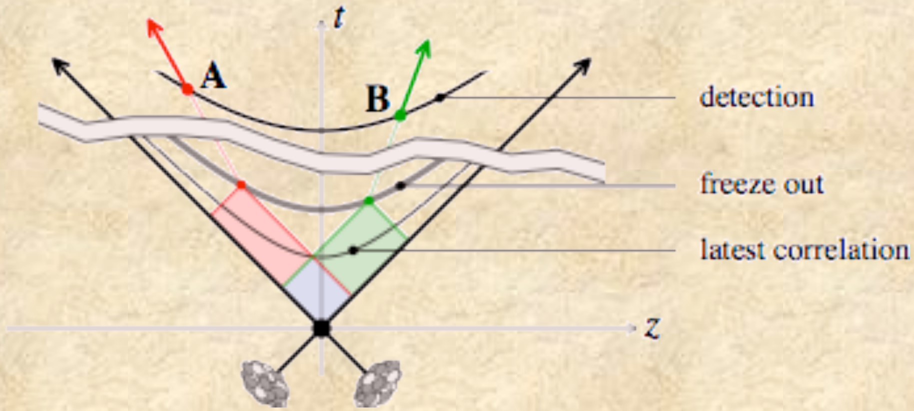


PHENIX: sees a ridge

Au+Au 200 GeV, 0 - 30%



PHOBOS: the ridge extends to very high rapidity



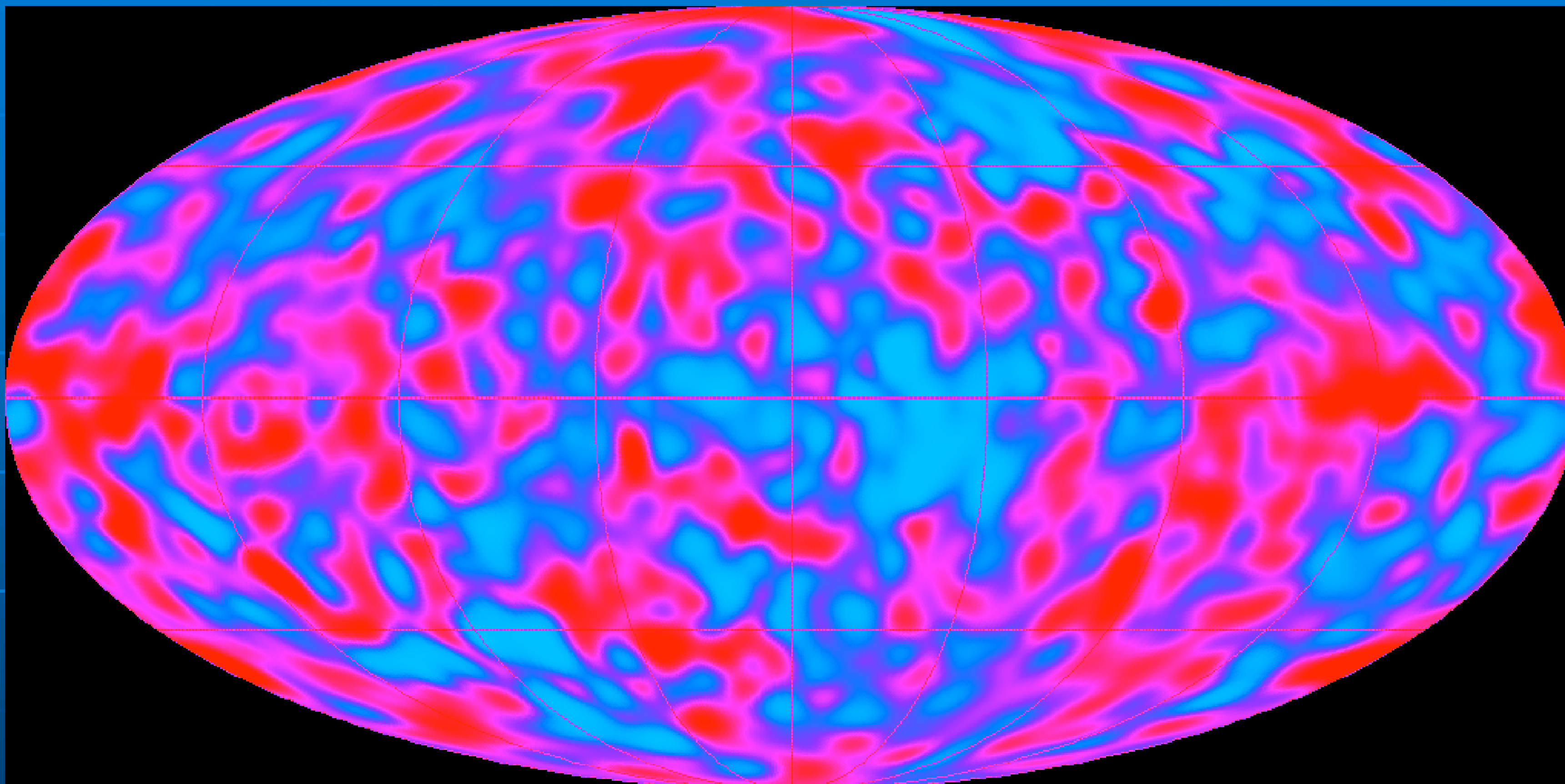
For particles to have been emitted from the same Event Horizon, causality dictates that

$$\tau \le \tau_{\mathrm{freeze-out}} \exp\left(-\frac{1}{2}|y_A - y_B|\right)$$

If ΔY is as large as (especially) suggested by PHOBOS, correlations were formed very early - in the Glasma…

An example of a small fluctuation spectrum...

COBE Fluctuations

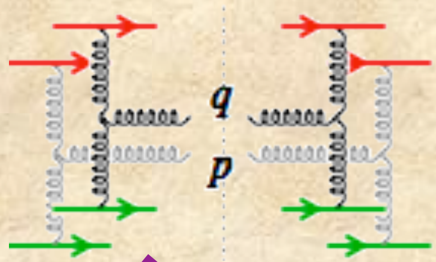


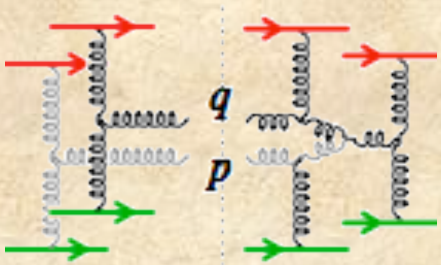
$\delta t/t < 10^{-5}$, i.e. much smoother than a baby's bottom!

2 particle correlations in the Glasma (I)

Dumitru, Gelis ,McLerran, RV, arXiv:0804.3858[hep-ph]

$$C(\mathbf{p},\mathbf{q}) = \left\langle \frac{dN_2}{dy_p\, d^2\mathbf{p}_\perp\, dy_q\, d^2\mathbf{q}_\perp} \right\rangle - \left\langle \frac{dN}{dy_p\, d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q\, d^2\mathbf{q}_\perp} \right\rangle$$

= 

+ 

Leading (classical) contribution

Note: *Interestingly, computing leading logs to all orders, both diagrams can be expressed as the first diagram with sources evolved a la JIMWLK Hamiltonian*

Gelis, Lappi, RV arXiv:0807.1306[hep-ph])

2 particle spectrum (II)

Simple "Geometrical" result:

$$\frac{C(\mathbf{p},\mathbf{q})}{\left\langle \frac{dN}{dy_p\, d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q\, d^2\mathbf{q}_\perp} \right\rangle} = \frac{\kappa}{S_\perp Q_S^2}$$

Ratio of transverse area of flux tube to nuclear area

$$\frac{\Delta\rho}{\sqrt{\rho_{\rm ref}}} = \left\langle \frac{dN}{dy} \right\rangle \frac{C(\mathbf{p},\mathbf{q})}{\left\langle \frac{dN}{dy_p\, d^2\mathbf{p}_\perp} \right\rangle \left\langle \frac{dN}{dy_q\, d^2\mathbf{q}_\perp} \right\rangle} = \frac{K_N}{\alpha_S(Q_S)}$$

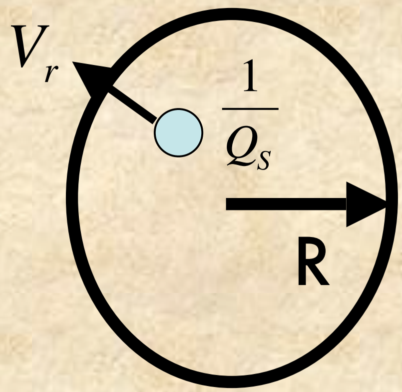
with K_N ≈ 0.3

2 particle spectrum (III)

Not the whole story… particle emission from the Glasma tubes is isotropic in the azimuth

Pairs correlated by transverse "Hubble flow" in final state - experience same boost

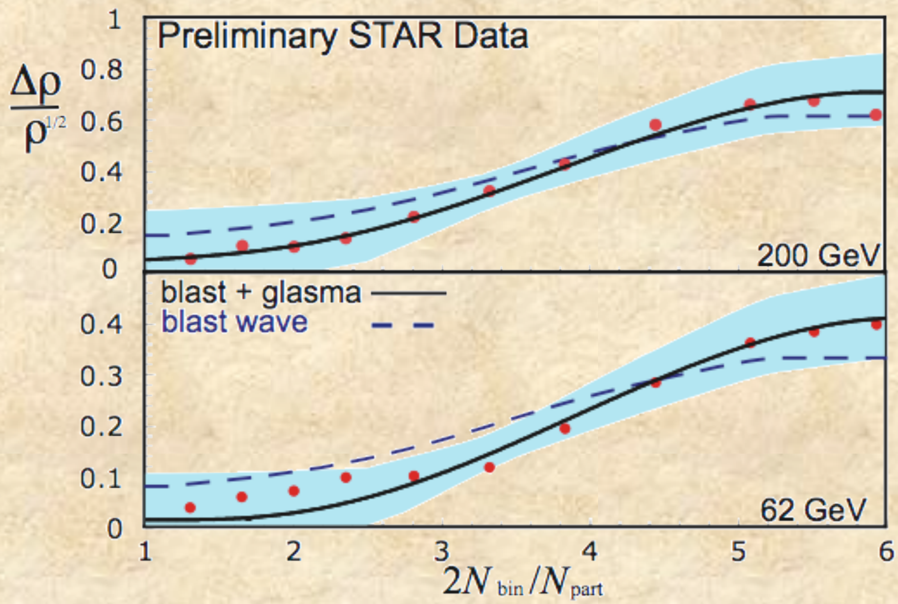
Voloshin, Shuryak
Gavin, Pruneau, Voloshin

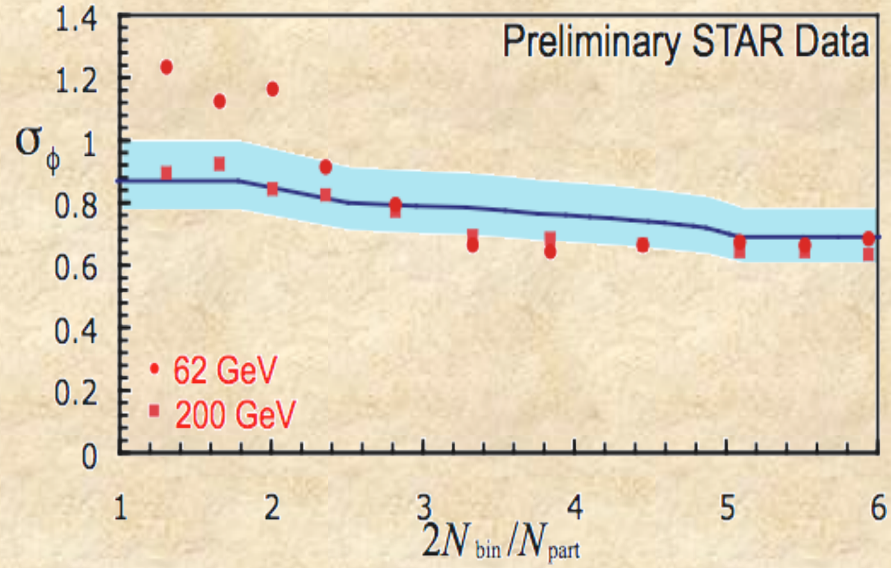


$$\gamma_B = \cosh \zeta_B$$

$$\int d\Phi \, \frac{\Delta\rho}{\sqrt{\rho_{\rm ref}}}(\Phi, \Delta\phi, y_p, y_q) = \frac{K_N}{\alpha_S(Q_S)} \, \frac{2\pi \cosh \zeta_B}{\cosh^2 \zeta_B - \sinh^2 \zeta_B \cos^2 \Delta\frac{\phi}{2}}$$

Ridge from flowing Glasma tubes





Gavin,McLerran,Moschelli, arXiv:0806.4718

Glasma flux tubes get additional qualitative features right:

i) Same flavor composition as bulk matter
ii) Ridge independent of trigger p_T-geometrical effect
iii) Signal for like and unlike sign pairs the same at large $\Delta\eta$

Summary

- Non-linear dynamics of QCD strongly enhanced in nuclei
- Factorization theorems linking these strong fields in the wavefunction to early time dynamics (Glasma) are becoming available
- These strong fields have important consequences for early thermalization

  -- may explain recent remarkable data from RHIC on long range rapidity correlations