Problem Set 3 (due Dec 9, 2017)

1. **Reading material** *Srednicki* Read Chapters 2, 33, 34, 35, and 36.

2. **Lie algebra for Lorentz transformations** *Srednicki* Problem 2.8 (a), (b), (c).

3. **Left handed spinor transformations** Define generators for left handed spinor transformations,

$$(S_L^{\mu\nu})_a{}^b = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)_a{}^b \ . \tag{1}$$

   (a) Prove that $S_L^{\mu\nu}$ have the correct commutation relations for the generators of the Lorentz Lie algebra

   (b) Write the transformation of a left handed spinor by a rotation by θ around the z axis. Compare with what was obtained in the class

   (c) Write the transformation of a left handed spinor by a boost by θ along the z axis. Compare with what was obtained in the class

4. **Right handed spinor transformations** Define generators for right handed spinor transformations,

$$(S_R^{\mu\nu})^{\dot{a}}{}_{\dot{b}} = -\frac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\nu)^{\dot{a}}{}_{\dot{b}} \ . \tag{2}$$

   (a) Prove that $S_R^{\mu\nu}$ have the correct commutation relations for the generators of the Lorentz Lie algebra

   (b) Write the transformation of a right handed spinor by a rotation by θ around the z axis. Compare with what was obtained in the class. (Hint: There might be a lowering of indices involved)

   (c) Write the transformation of a right handed spinor by a boost by θ along the z axis. Compare with what was obtained in the class

5. **Combining the Left and Right handed spinor transformations** Define generators for Dirac spinor transformations,

$$(S^{\mu\nu}) = \frac{i}{4}[\gamma^\mu, \gamma^\nu] \ . \tag{3}$$

   (a) Using the previous problems show that $S^{\mu\nu}$ have the correct commutation relations for the generators of the Lorentz Lie algebra

   (b) Show that the upper two components of the Dirac spinor transform as a left handed spinor and the lower two as a right handed spinor. Is it consistent with the definition of a four component Dirac field in Eq 36.19.