Problem Set 3 (due Sep 10, 2018)

1. **(Computational calculation of the path integral in imaginary time)**
   Consider the following lagrangian

$$\mathcal{L} = \frac{1}{2} m(\dot{x}^2 - \omega^2 x^2) \, . \tag{1}$$

   (a) Write the path integral for the lagrangian in imaginary time

   (b) Discretize time in steps $\delta\tau$ and rewrite the path integral in terms of dimensionless variables $\tilde{m} = m\delta\tau$, $\tilde{\omega} = \omega\delta\tau$, $\tilde{x}_i = x_i/\delta\tau$

   (c) For hints see *American Journal of Physics 86, 293 (2018); https://doi.org/10.1119/1.5024926*

   (d) Use the metropolis algorithm to evaluate the correlation function

$$G(\Delta\tau) = \langle x(\tau)x(\tau + \Delta\tau)\rangle \tag{2}$$

   for $\tilde{\omega} = \tilde{m} = 0.1$. Attach the plot.

   (e) Extract the correlation length ζ

$$\frac{1}{\zeta} = \frac{1}{2} \log\left(\frac{G(\Delta\tau - 1)}{G(\Delta\tau + 1)}\right) \tag{3}$$

   Attach the plot. Is the answer expected?

   (f) Calculate $\langle \tilde{x}^3 \rangle$. Attach the plot. Is the answer expected?

   (g) Calculate $\langle \tilde{x}^4 \rangle$. Attach the plot. Is the answer expected?